Mathematics Standard X Sample Paper 05 Solved www.cbse.online

CLASS X (2020-21)
MATHEMATICS STANDARD (041)
SAMPLE PAPER-05

Time : 3 Hours
Maximum Marks : 80

General Instructions :
1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part–A :
1. It consists of two sections- I and II.
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part–B :
1. Question no. 21 to 26 are very short answer type questions of 2 mark each.
2. Question no. 27 to 33 are short answer type questions of 3 marks each.
3. Question no. 34 to 36 are long answer type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part - A
Section - I

1. Calculate the HCF of \(3^3 \times 5\) and \(3^2 \times 5^2\).
Ans :
We have \(3^3 \times 5 = 3^2 \times 5 \times 3\)
\(3^2 \times 5^2 = 3^2 \times 5 \times 5\)
HCF \((3^3 \times 5, 3^2 \times 5^2) = 3^2 \times 5 = 45\)

2. If the square of difference of the zeroes of the quadratic polynomial \(x^2 + px + 45\) is equal to 144, then what is the value of \(p\) ?
Ans :
We have \(f(x) = x^2 + px + 45\)
Then, \(\alpha + \beta = -\frac{p}{1} = -p\)
and \(\alpha \beta = \frac{45}{1} = 45\)
According to given condition, we have
\((\alpha + \beta)^2 = 144\)
\((\alpha + \beta)^2 - 4\alpha \beta = 144\)
\((-p)^2 - 4(45) = 144\)
\(p^2 = 144 + 180 = 324\)
\(\Rightarrow p = \pm 18\)

3. If \(\alpha\) and \(\beta\) are the roots of \(ax^2 - bx + c = 0\) \((a \neq 0)\), then calculate \(\alpha + \beta\).
Ans :
We know that
Sum of the roots \(= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}\)
Thus \(\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}\)

4. Value of the roots of the quadratic equation, \(x^2 - x - 6 = 0\) are ......... .
Ans :
\(x^2 - x - 6 = 0\)
\(x^2 - 3x + 2x - 6 = 0\)
\((x - 3) + 2(x - 3) = 0\)
\((x - 3)(x + 2) = 0 \Rightarrow x = 3\) and \(x = -2\)
or
If quadratic equation \(3x^2 - 4x + k = 0\) has equal roots, then the value of \(k\) is ...........
Ans :
Given, quadratic equation is \(3x^2 - 4x + k = 0\)
Comparing with \(ax^2 + bx + c = 0\), we get \(a = 3, b = -4\) and \(c = k\)
For equal roots, \(b^2 - 4ac = 0\)
\((-4)^2 - 4(3)(k) = 0\)
\(16 - 12k = 0\)
\(k = \frac{16}{12} = \frac{4}{3}\)

5. Which of the term of AP 5, 2, -1,...... is -49?
Ans :
Let the first term of an AP be \(a\) and common difference \(d\).
We have \(a = 5, d = -3\)
Now \(a_n = a + (n - 1)d\)
Substituting all values we have
\(-49 = 5 + (n - 1)(-3)\)
\(-49 = 5 - 3n + 3\)
\(3n = 49 + 5 + 3\)
\(n = \frac{57}{3} = 19^{th}\) term.
or
Find the first four terms of an AP whose first term is \(-2\) and common difference is \(-2\).

**Ans:** \[\text{Board Term-2 2012}\]

We have

\[a_1 = -2,\]
\[a_2 = a_1 + d = -2 + (-2) = -4,\]
\[a_3 = a_2 + d = -4 + (-2) = -6,\]
\[a_4 = a_3 + d = -6 + (-2) = -8,\]

Hence first four terms are \(-2, -4, -6, -8\)

6. If triangle \(ABC\) is similar to triangle \(DEF\) such that \(2AB = DE\) and \(BC = 8\ cm\) then find \(EF\).

**Ans:**

As per given condition we have drawn the figure below.

Here we have \(2AB = DE\) and \(BC = 8\ cm\)

Since \(\Delta ABC \sim \Delta DEF\), we have

\[
\frac{AB}{BC} = \frac{DE}{EF} \quad \text{and} \quad \frac{AB}{8} = \frac{2AB}{EF}.
\]

\[
EF = 2 \times 8 = 16 \ cm
\]

7. The co-ordinate of the point dividing the line segment joining the points \(A(1,3)\) and \(B(4,6)\) in the ratio 2 : 1 is .......... .

**Ans:** \[\text{Board 2020 OD Basic}\]

Let point \(P(x, y)\) divides the line segment joining the points \(A(1,3)\) and \(B(4,6)\) in the ratio 2 : 1.

Using section formula we have

\[
(x, y) = \left(\frac{m_2x_2 + m_1x_1}{m_1 + m_2}, \frac{m_2y_2 + m_1y_1}{m_1 + m_2}\right)
\]

\[
(x, y) = \left(\frac{2 \times 4 + 1 \times 1}{2 + 1}, \frac{2 \times 6 + 1 \times 3}{2 + 1}\right)
\]

\[
= \left(\frac{8 + 1}{3}, \frac{12 + 3}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3,5)
\]

8. Find the coordinates of a point \(A\), where \(AB\) is diameter of a circle whose centre is \((2, -3)\) and \(B\) is the point \((1, 4)\).

**Ans:** \[\text{Board 2019 Delhi}\]

As per question we have shown the figure below. Since, \(AB\) is the diameter, centre \(C\) must be the mid point of the diameter of \(AB\).

Let the co-ordinates of point \(A\) be \((x,y)\).

\[
x\text{-coordinate of } C = \frac{x+1}{2} = 2
\]

\[
x+1 = 4 \Rightarrow x = 3
\]

and y-coordinate of \(C\),

\[
y\text{-coordinate of } C = \frac{y+4}{2} = -3
\]

\[
y+4 = -6 \Rightarrow y = -10
\]

Hence, coordinates of point \(A\) are \((3, -10)\).

9. If \(\sin\theta + \sin^2\theta = 1\) then prove that \(\cos^2\theta + \cos^4\theta = 1\).

**Ans:** \[\text{Board 2020 OD Basic}\]

We have

\[
\sin\theta + \sin^2\theta = 1
\]

\[
\sin\theta + (1 - \cos^2\theta) = 1
\]

\[
\sin\theta - \cos^2\theta = 0
\]

\[
\sin\theta = \cos^2\theta
\]

Squaring both sides, we get

\[
\sin^2\theta = \cos^4\theta
\]

\[
1 - \cos^2\theta = \cos^2\theta
\]

\[
\cos^4\theta + \cos^2\theta = 1
\]

Hence Proved

10. If \(\tan(3x + 30^\circ) = 1\) then find the value of \(x\).

**Ans:** \[\text{Board Term-1 2015}\]

We have

\[
\tan(3x + 30^\circ) = 1 = \tan 45^\circ
\]

\[
3x + 30^\circ = 45^\circ
\]

\[
x = 5^\circ
\]

11. If \(k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)\), then find the value of \(k\).

**Ans:** \[\text{Board Term-1 2015}\]

We have

\[
k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)
\]

\[
= \sec^2\theta(1 - \sin^2\theta)
\]

\[
= \sec^2\theta \cdot \cos^2\theta
\]

\[
= \sec^2\theta \times \frac{1}{\sec^2\theta}
\]

\[
k + 1 = 1 \Rightarrow k = 1 - 1 = 0
\]

Thus \(k = 0\)

12. In the figure, \(QR\) is a common tangent to given circle which meet at \(T\). Tangent at \(T\) meets \(QR\) at \(P\). If \(QP = 3.8\ cm\), then find length of \(QR\).
Let us first consider large circle. Since length of tangents from external points are equal, we can write

\[ QP = PT \]

Thus

\[ QP = PT = 3.8 \text{ cm} \]  ....(1)

Now consider the small circle. For this circle we can also write using same logic,

\[ PR = PT \]

But we have \[ PT = 3.8 \text{ cm} \]

Thus

\[ PR = PT = 3.8 \text{ cm} \]

Now

\[ QR = QP + PR \]

\[ = 3.8 + 3.8 = 7.6 \text{ cm} \]

or

\[ PA \text{ and } PB \text{ are tangents from point } P \text{ to the circle with centre } O \text{ as shown in figure. At point } M, \text{ a tangent is drawn cutting } PA \text{ at } K \text{ and } PB \text{ at } N. \]

Prove that \( KN = AK + BN \)

\[ PA = PB, KA = KM, NB = NM, \]

\[ KA + NB = KM + NM \]

\[ AK + BN = KN. \] Hence Proved

13. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. \((\pi = 3.14)\)

Ans : [Board Term-2 2012]

Radius of circle \( r = 10 \text{ cm}, \) central angle \( = 90^\circ \)

Area of minor segment,

\[ = \frac{1}{2} \times 10^2 \times \left[ 3.14 \times 90 \times \frac{90^2}{180} \right] \]

\[ = \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2 \]
Today, she has planned a prime number game. She announce the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to second student. Second student also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number the last student got 173250. He told this number to Shalvi in class. Now she asked some questions to the students as given below.

(i) How many students are in the class?
(a) 3 (b) 9 (c) 4 (d) 7

(ii) What is the highest prime number used by student?
(a) 11 (b) 7 (c) 5 (d) 3

(iii) What is the least prime number used by students?
(a) 2 (b) 7 (c) 5 (d) 3

(iv) Which prime number has been used maximum times?
(a) 2 (b) 7 (c) 5 (d) 3

(v) Which prime number has been used minimum times?
(a) 2 (b) 7 (c) 5 (d) 3

Ans:
(i) Prime factorization of 173250,
\[ 173250 = 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 11 \]
It includes 8 numbers. Number 2 has been used by Shalvi. Remaining 7 numbers have been by 7 students. Thus (d) is correct option.
(ii) Highest prime factor included in factorization of 173250 is 11.
Thus (a) is correct option.
(iii) Least prime factor included in factorization of 173250 is 2. But 2 is used by Shalvi, thus next least prime number used by students is 3.

18. Due to ongoing Corona virus outbreak, Wellness Medical store has started selling masks of decent quality. The store is selling two types of masks currently type A and type B.

The cost of type A mask is Rs. 15 and of type B mask is Rs. 20. In the month of April, 2020, the store sold 100 masks for total sales of Rs. 1650.

(i) How many masks of each type were sold in the month of April?
(a) 40 masks of type A, and 60 masks of type B
(b) 60 masks of type A, and 40 masks of type B
(c) 70 masks of type A, and 30 masks of type B
(d) 30 masks of type A, and 70 masks of type B

(ii) If the store had sold 50 masks of each type, what would be its sales in the month of April?
(a) Rs 550
(b) Rs 560
(c) Rs 1050
(d) Rs 1750

(iii) Due to great demand and short supply, the store has increased the price of each type by Rs. 5 from May 1, 2020. In the month of May, 2020, the store sold 310 masks for total sales of Rs. 6875. How many masks of each type were sold in the month of May?
(a) 175 masks of type A, and 135 masks of type B
(b) 200 masks of type A, and 110 masks of type B
(c) 110 masks of type A, and 200 masks of type B
(d) 135 masks of type A, and 175 masks of type B
(iv) What percent of masks of each type sale was increased in the month of May, compared with the sale of month April?
(a) 110 % in type A and 180 % in type B
(b) 180 % in type A and 110 % in type B
(c) 350 % in type A and 150 % in type B
(d) 150 % in type A and 350 % in type B

(v) What extra profit did store earn by increasing price in May month.
(a) Rs 1550  
(b) Rs 3100  
(c) Rs 1650  
(d) Rs 1825

Ans : 
(i) Let $x$ be the mask of type A sold and $y$ be the type of mask B sold in April.
Now $x + y = 100$ ...(1) and $15x + 20y = 1650$ ...(2)
Multiplying equation (1) by 15 and subtracting from (2) we obtain,
$5y = 150$  $\Rightarrow$  $y = 30$
$x = 100 - 30 = 70$
Hence 70 masks of type A, and 30 masks of type B were sold.
Thus (c) is correct option.

(ii) Total Sales $= 50 \times 15 + 50 \times 20 = 1750$
Thus (d) is correct option.

(iii) Let $x$ be the mask of type A sold and $y$ be the type of mask B sold in April.
Now, $x + y = 310$ ...(1) and $20x + 25y = 6875$ ...(ii)
Multiplying equation (1) by 20 and subtracting it from equation (2), we obtain
$5y = 675$  $\Rightarrow$  $y = 135$
$x = 310 - 135 = 175$
Thus (a) is correct option.

(iv) Increase in type A $= \frac{175 - 70}{70} \times 100 = 150 \%$
Increase in type B $= \frac{105 - 30}{30} \times 100 = 350 \%$
Thus (d) is correct option.

(v) Total sale value in May at old price $= 175 \times 15 + 135 \times 20 = 5325$
Total sale value in May at new price $= 6875$
Extra Profit $= 6875 - 5325 = 1550$

Alternative : 
Since extra profit is Rs 5 on per mask and total mask sold are 310, thus extra profit $= 310 \times 5 = 1550$.
Thus (a) is correct option.

19. A garden is in the shape of rectangle. Gardener grew sapling of Ashoka tree on the boundary of garden at the distance of 1 meter from each other. He want to decorate the garden with rose plants. He choose triangular region inside the park to grow rose plants. On the above situation, gardener took help from the students of class 10th. They made a chart for it which looks as the above figure.

(i) If A is taken as origin, What are the co-ordinates of triangle $PQR$?
(a) $P(4, 6)$, $Q(3, 2)$, $R(6, 5)$
(b) $P(6, 4)$, $Q(2, 3)$, $R(5, 6)$
(c) $P(5, 7)$, $Q(3, 3)$, $R(5, 5)$
(d) $P(6, 6)$, $Q(2, 3)$, $R(6, 6)$

(ii) If C is taken as origin, what is the co-ordinate of point $P$?
(a) $(-12, 2)$
(b) $(12, 2)$
(c) $(12, -2)$
(d) $(-12, -2)$

(iii) If B is taken as origin, what are the co-ordinates of $P$?
(a) $(4, 3)$
(b) $(4, -3)$
(c) $(-4, 3)$
(d) $(-4, -3)$

(iv) What is distance between $P$ and $Q$ if origin is taken $A$?
(a) $\sqrt{71}$
(b) $\sqrt{17}$
(c) $\sqrt{65}$
(d) $\sqrt{61}$

(v) What is distance between $P$ and $Q$ if origin is taken $B$?
(a) $\sqrt{50}$
(b) $\sqrt{71}$
(c) $\sqrt{17}$
(d) $\sqrt{61}$

Ans : 
(i) In following figure we have shown the co-ordinate taking $A$ as origin.
Thus (a) is correct option.
(ii) In following figure we have shown the co-ordinate taking \( C \) as origin.

Thus (d) is correct option.

Thus (c) is correct option.

(iv) \[ PQ = \sqrt{(4 - 3)^2 + (6 - 2)^2} = \sqrt{17} \]
Thus (b) is correct option.

(v) Distance dress not depend on origin. In this case this is \( \sqrt{17} \).
Thus (c) is correct option.

20. An air-to-surface missile (ASM) or air-to-ground missile (AGM or ATGM) is a missile designed to be launched from military aircraft and strike ground targets on land, at sea, or both. They are similar to guided glide bombs but to be deemed a missile,

A military fighter plane is flying at an altitude of 600 metres with the speed of 200 km/h. The pilot spots enemy tanks at point \( R \) on ground. After getting the permission from command centre to hit the target at \( R \), pilot fires a missile. Fighter plane was at point \( A \) at the time of fire of missile. Missile moves to target at enemy tanks stationed at \( R \) at an angle of 45° at a speed of 300 km/h.

(i) What is the horizontal distance between fighter plane at \( A \) and tank at \( R \) ?
(a) 300 metre
(b) \( 300\sqrt{3} \) metre
(c) 600 metre
(d) \( 600\sqrt{3} \) metre
(ii) How much time will missile take to hit the target $R$?
(a) $\frac{36\sqrt{2}}{5}$ sec  
(b) $\frac{5\sqrt{2}}{36}$ sec  
(c) $\frac{36\sqrt{3}}{5}$ sec  
(d) $\frac{34\sqrt{3}}{5}$ sec

(iii) Another enemy tank at point $S$ on ground moving with a speed of 90 km/h in straight line away from plane. Pilot fires another missile at an angle of $60^\circ$ from its flight path position $B$ at the instant when enemy’s tank was at $S$ and it hits this enemy tank at point $T$. How much time is taken by second missile to hit the enemy tank at point $T$?
(a) $24\sqrt{3}$ sec  
(b) $\frac{5\sqrt{2}}{24}$ sec  
(c) $24\sqrt{3}$ sec  
(d) $\frac{5\sqrt{3}}{24}$ sec

(iv) What is the horizontal distance between fighter plane at $B$ and tank at $T$?
(a) 200 metre  
(b) $100\sqrt{3}$ metre  
(c) 120 metre  
(d) $200\sqrt{3}$ metre

(v) What is the distance of point $T$ from $S$?
(a) $80\sqrt{3}$ metre  
(b) $120\sqrt{3}$ metre  
(c) $160\sqrt{3}$ metre  
(d) $240\sqrt{3}$ metre

Ans:
(i) We have shown this situation in following diagram

\[ PR = \tan 45^\circ = 1 \Rightarrow PR = PA = 600 \text{ m} \]
Thus (c) is correct option.

(ii) \[ \frac{AP}{AR} = \cos 45^\circ \]
\[ AR = \frac{AP}{\cos 45^\circ} = \frac{600}{\frac{\sqrt{2}}{2}} = 600\sqrt{2} \text{ m} \]
Speed of missile,
\[ s = 300 \text{ km/h} = \frac{300000}{3600} \text{ m/sec} = \frac{500}{6} \text{ m/s} \]
Time taken to hit missile.
\[ t = \frac{AR}{s} = \frac{600\sqrt{2}}{\frac{500}{6}} = \frac{36\sqrt{2}}{5} \text{ sec} \]
Thus (a) is correct option.
(iii) In this situation we have shown diagram below.

In triangle $\triangle BQT$, \[ \angle QBT = 90^\circ - 60^\circ = 30^\circ \]
\[ \frac{BQ}{BT} = \cos 30^\circ \]
\[ BT = \frac{BQ}{\cos 30^\circ} = \frac{600}{\frac{\sqrt{3}}{2}} = \frac{1200}{\sqrt{3}} = 400\sqrt{3} \text{ m} \]
Time taken to hit tank at $T$,
\[ t_1 = \frac{BT}{s} = \frac{400\sqrt{3}}{\frac{500}{6}} = \frac{4\sqrt{3} \times 6}{5} = \frac{24\sqrt{3}}{5} \text{ sec} \]
Thus (a) is correct option.

(iv) In figure this distance is given by $QT$
\[ \frac{QT}{QB} = \tan 30^\circ \]
\[ QT = QB \tan 30^\circ = 600 \times \frac{1}{\sqrt{3}} = 200\sqrt{3} \text{ m} \]
Thus (d) is correct option.

(v) Speed of tank, $s_t = 90 \text{ km/hour}$
\[ = \frac{90000}{3600} \text{ m/sec} = 25 \text{ m/sec} \]
Distance $ST$ travelled by tank in $\frac{24\sqrt{3}}{5}$ sec
\[ = 25 \times \frac{24\sqrt{3}}{5} = 120\sqrt{3} \text{ m} \]
Thus (b) is correct option.

Part - B

All questions are compulsory. In case of internal choices, attempt anyone.

21. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

Ans:

We have $p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$
\[ = \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3} \]
\[ = \sqrt{3}(x-2\sqrt{3}) - 2(x-2\sqrt{3}) \]
\[ = (\sqrt{3}x - 2)(x - 2\sqrt{3}) \]
Substituting $p(x) = 0$, we have
\[ (\sqrt{3}x - 2)(x - 2\sqrt{3}) = 0 \]
Solving we get $x = \frac{2}{\sqrt{3}}, 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.
Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

\[ \text{Ans: } \quad \text{[Board Term-1 2016]} \]

 Sum of zeroes, \( \alpha + \beta = 6 \)

 Product of zeroes \( \alpha \beta = 9 \)

 Now \( p(x) = x^2 - (\alpha + \beta)x + \alpha \beta \)

 Thus \( = x^2 - 6x + 9 \)

 Thus quadratic polynomial is \( x^2 - 6x + 9 \).

 Now \( p(x) = x^2 - 6x + 9 \)

 \( = (x-3)(x-3) \)

 Substituting \( p(x) = 0 \), we get \( x = 3,3 \)

 Hence zeroes are 3, 3

 For what value of \( k \), the system of equations \( kx + 3y = 1 \), \( 12x + ky = 2 \) has no solution.

\[ \text{Ans: } \quad \text{[Board Term-1 2011, NCERT]} \]

 The given equations can be written as \( kx + 3y = 1 = 0 \) and \( 12x + ky = 2 = 0 \)

 Here, \( a_1 = k, b_1 = 3, c_1 = -1 \)

 and \( a_2 = 12, b_2 = k, c_2 = -2 \)

 The equation for no solution if

\[ \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \]

 or,

\[ k \frac{12}{6} = \frac{3}{k} \neq \frac{-1}{2} \]

 From \( k \frac{12}{6} = \frac{3}{k} \), we have \( k^2 = 36 \Rightarrow k = \pm 6 \)

 From \( \frac{3}{k} \neq \frac{-1}{2} \), we have \( k \neq 6 \)

 Thus \( k = -6 \)

 Solve the following pair of linear equations by cross multiplication method:

\[ x + 2y = 2 \]
\[ x - 3y = 7 \]

\[ \text{Ans: } \quad \text{[Board Term-1 2016]} \]

 We have \( x + 2y - 2 = 0 \)
\[ x - 3y - 7 = 0 \]

 Using the formula

\[ \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \]

 we have

\[ \frac{x}{-14 - 6} = \frac{y}{2 + 7} = \frac{1}{3 - 2} \]

\[ \Rightarrow x = -2 + 7 = -3 \]
\[ \Rightarrow x = 4 \]
\[ \Rightarrow y = -1 \]

 The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110. Find the true mean and median.

\[ \text{Ans: } \quad \text{[Board Term-1 2016]} \]

 We have

\[ 5 \cos^2 \theta = 7 \]
\[ \cos \theta = \frac{7}{5} \]
\[ \sin \theta = \frac{5}{7} \]

\[ \cos \theta = \frac{1}{\sin \theta} \]

\[ \sin \theta + \cos^2 \theta - 1 = \sin \theta - (1 - \cos^2 \theta) \]
\[ = \sin \theta - \sin^2 \theta \]
\[ = \frac{5}{7} - \frac{25}{49} = \frac{10}{49} \]

 The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110. Find the true mean and median.

\[ \text{Ans: } \quad \text{[Board Term-1 2016]} \]

 Mean, \( M = \frac{\sum fx}{\sum f} \)

 We have

\[ 15x^2 - 10\sqrt{6}x + 10 = 0 \]

\[ 3x^2 - 2\sqrt{6}x + 2 = 0 \]
\[ 3x^2 - 2\sqrt{6}x + 2 = 0 \]
\[ \sqrt{3}(x - \sqrt{2}) - \sqrt{2} = 0 \]
\[ \sqrt{3}(x - \sqrt{2}) = 0 \]
\[ \sqrt{3}(x - \sqrt{2}) = 0 \]
\[ \sqrt{3}(x - \sqrt{2}) = 0 \]

 or

 24. In the given figure, in a triangle \( PQR \), \( ST \parallel QR \) and \( \frac{PS}{SQ} = \frac{1}{2} \) and \( PR = 28 \) cm, find \( PT \).

\[ \text{Ans: } \quad \text{[Board Term-1 2011]} \]

 We have

\[ \frac{PS}{SQ} = \frac{3}{5} \]
\[ \frac{PS}{PS + SQ} = \frac{3}{3 + 5} \]
\[ \frac{PS}{PQ} = \frac{3}{8} \]

 We also have, \( ST \parallel QR \), thus by BPT we get

\[ \frac{PS}{PQ} = \frac{PT}{PR} \]
\[ PT = \frac{PS}{PQ} \times PR \]
\[ = \frac{3 \times 28}{5} = 10.5 \text{ cm} \]

 25. If \( \theta \) be an acute angle and \( 5 \cos \theta = 7 \), then evaluate \( \sin \theta + \cos^2 \theta - 1 \).

\[ \text{Ans: } \quad \text{[Board Term-1 2012]} \]

 We have

\[ 5 \cos \theta = 7 \]
\[ \cos \theta = \frac{7}{5} \]
\[ \sin \theta = \frac{5}{7} \]

\[ \cos \theta = \frac{1}{\sin \theta} \]

\[ \sin \theta + \cos^2 \theta - 1 = \sin \theta - (1 - \cos^2 \theta) \]
\[ = \sin \theta - \sin^2 \theta \]
\[ = \frac{5}{7} - \frac{25}{49} = \frac{10}{49} \]

26. The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110. Find the true mean and median.

\[ \text{Ans: } \quad \text{[Board Term-1 2016]} \]

 Mean, \( M = \frac{\sum fx}{\sum f} \)
27. Solve the following equation: \( \frac{1}{x} - \frac{1}{x-2} = 3, \ x \neq 0, 2 \)

Ans:

We have \( \frac{1}{x} - \frac{1}{x-2} = 3 \) \( (x \neq 0, 2) \)

\[ \frac{x-2-x}{x(x-2)} = 3 \]

\[ \frac{-2}{x(x-2)} = 3 \]

\[ 3x(x-2) = -2 \]

\[ 3x^2 - 6x + 2 = 0 \]

Comparing it by \( ax^2 + bx + c \), we get \( a = 3 \), \( b = -6 \) and \( c = 2 \).

Now, \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

\[ x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} \]

\[ = \frac{6 \pm \sqrt{36 - 24}}{6} \]

\[ = \frac{6 \pm 2\sqrt{3}}{6} \]

\[ = \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3} \]

28. The sum of four consecutive number in AP is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

Ans:

Let the four consecutive terms of AP be \( (a - d) \), \( (a + d) \), \( (a + d) \) and \( (a + 3d) \).

As per question statement we have

\[ a - 3d + a - d + a + d + a + 3d = 32 \]

\[ 4a = 32 \Rightarrow a = 8 \]

and

\( \frac{(a - d)(a + 3d)}{(a - d)(a + d)} = \frac{7}{15} \)

\[ \frac{a^2 - 4ad + 3d^2}{a^2 - d^2} = \frac{7}{15} \]

\[ \frac{64 - 9d^2}{64 - d^2} = \frac{7}{15} \]

\[ 960 - 135d^2 = 448 - 7d^2 \]

\[ 7d^2 - 135d^2 = 448 - 960 \]

\[ -128d^2 = -512 \]

\[ d^2 = 4 \Rightarrow d = \pm 2 \]

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

29. In the given figure, \( CB \parallel QR \) and \( CA \parallel PR \). If \( AQ = 12 \) cm, \( AR = 20 \) cm, \( PB = CQ = 15 \) cm, calculate \( PC \) and \( BR \).

Ans:

In \( \triangle PQR \), \( CA \parallel PR \)

By BPT similarity we have

\[ \frac{PC}{CQ} = \frac{RA}{AQ} \]

\[ \frac{PC}{15} = \frac{20}{12} \]

\[ PC = \frac{15 \times 20}{12} = 25 \text{ cm} \]

In \( \triangle PQR \), \( CB \parallel QR \)

Thus

\[ \frac{PC}{CQ} = \frac{PR}{BR} \]

\[ \frac{25}{15} = \frac{15}{BR} \]

\[ BR = \frac{15 \times 15}{25} = 9 \text{ cm} \]

30. The rod of TV disc antenna is fixed at right angles to wall \( AB \) and a rod \( CD \) is supporting the disc as shown in Figure. If \( AC = 1.5 \) m long and \( CD = 3 \) m, find (i) \( \tan \theta \) (ii) \( \sec \theta + \csc \theta \).

Ans:

From the given information we draw the figure as below
In right angle triangle \( \triangle CAD \), applying Pythagoras theorem,
\[
AD^2 + AC^2 = DC^2
\]
\[
AD^2 + (1.5)^2 = (3)^2
\]
\[
AD^2 = 9 - 2.25 = 6.75
\]
\[
AD = \sqrt{6.75} = 2.6 \text{ m (Approx)}
\]
(i) \(
\tan \theta = \frac{AC}{AD} = \frac{2}{1.5} = \frac{4}{3}
\)
(ii) \(
\sec \theta + \csc \theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}
\)
Prove that : \[
\cot \theta + \cosec \theta - \frac{1}{\sin \theta} = 1 + \cot \theta
\]
Ans : [Board 2020 Delhi Standard]

LHS = \[
\frac{\cot \theta + \cosec \theta - 1}{\cot \theta - \cosec \theta + 1}
\]
\[
= \frac{\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} - 1}{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} + 1}
\]
\[
= \frac{\sin \theta (\cos \theta + 1 - \sin \theta)}{\sin \theta (\cos \theta - 1 + \sin \theta)}
\]
\[
= \frac{\sin \theta \cos \theta + \sin \theta - (1 - \cos^2 \theta)}{\sin \theta (\cos \theta + \sin \theta - 1)}
\]
\[
= \frac{\sin \theta (\cos \theta + 1) - (1 - \cos^2 \theta)}{\sin \theta (\cos \theta + \sin \theta - 1)}
\]
\[
= \frac{(1 + \cos \theta)(\sin \theta - 1 + \cos \theta)}{\sin \theta (\cos \theta + \sin \theta - 1)}
\]
\[
= \frac{1 + \cos \theta}{\sin \theta} = \text{RHS}
\]

31. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.
Ans : [Board Term-2 Foreign 2015, OD 2016]

Steps of Construction :
1. Draw a circle with centre \( O \) and radius 6 cm.
2. Draw two radii \( OA \) and \( OB \) inclined to each other at an angle of 120°.
3. Draw \( AP \perp OA \) at \( A \) and \( BP \perp OB \) at \( B \), which meet at \( P \).
4. \( PA \) and \( PB \) are the required tangents inclined to each other an angle of 60°.

or

Draw a circle of radius 3 cm. From a point \( P \), 7 cm away from centre draw two tangents to the circle.
Measure the length of each tangent.

32. In the given figure, \( AOB \) is a sector of angle 60° of a circle with centre \( O \) and radius 17 cm. If \( AP \perp OB \) and \( AP = 15 \) cm, find the area of the shaded region.

Ans : [Board Term-2 2016]

Here \( OA = 17 \) cm \( AP = 15 \) cm and \( \triangle OPA \) is right triangle

Using Pythagoras theorem, we have
\[
OP = \sqrt{17^2 - 15^2} = 8 \text{ cm}
\]

Area of the shaded region
\[
= \text{Area of the sector } \triangle OAB - \text{Area of } \triangle OPA
\]
\[
= \frac{60^\circ}{360^\circ} \times \pi r^2 - \frac{1}{2} \times b \times h
\]
\[
= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15
\]
\[
= 151.38 - 60 = 91.38 \text{ cm}^2
\]

33. If the median of the following data is 240, then find the value of \( f \):

<table>
<thead>
<tr>
<th>Classes</th>
<th>0-100</th>
<th>100-200</th>
<th>200-300</th>
<th>300-400</th>
<th>400-500</th>
<th>500-600</th>
<th>600-700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>15</td>
<td>17</td>
<td>f</td>
<td>12</td>
<td>9</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Ans : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.
34. In Figure, $PQ$ is a chord of length 8 cm of a circle of radius 5 cm and centre $O$. The tangents at $P$ and $Q$ intersect at point $T$. Find the length of $TP$.

![Diagram](image)

**Ans :**

We redraw the given figure as shown below. Here $OT$ is perpendicular bisector of $PQ$.

![Diagram](image)

Since, $OT$ is perpendicular bisector of $PQ$,

$PR = QR = 4 \text{ cm}$

In right angle triangle $\triangle OPT$ and $\triangle PTR$, we have

$TP^2 = TR^2 + PR^2$ 

...(1)

Also,

$OT^2 = TP^2 + OP^2$

Substituting $TP^2$ from equation (1) we have

$OT^2 = (TR^2 + PR^2) + OP^2$

Now

$TR^2 = OP^2 - PR^2$

$= 5^2 - 4^2 = 9$

Thus

$OR = 3 \text{ cm}$

Thus substituting $OR = 3 \text{ cm}$ we have

$(TR + 3)^2 = TR^2 + 4^2 + 5^2$

$TR^2 + 9 + 6TR = TR^2 + 16 + 25$

$6TR = 32$

$TR = \frac{16}{3}$

Now, from (1),

$TP^2 = TR^2 + PR^2$

$= \left(\frac{16}{3}\right)^2 + 4^2$

$= \frac{256}{9} + 16 = \frac{400}{9}$

$TP = \frac{20}{3} \text{ cm}$

or

If the angle between two tangents drawn from an external point $P$ to a circle of radius $r$ and centre $O$, is $60^\circ$, then find the length of $OP$.

**Ans :** [Board 2020 SQP STD]

As per the given question we draw the figure as below.

![Diagram](image)

Tangents are always equally inclined to line joining the external point $P$ to centre $O$.

$\angle APO = \angle BPO = \frac{60^\circ}{2} = 30^\circ$

Also radius is also perpendicular to tangent at point of contact.

In right $\triangle OAP$ we have,

$\angle APO = 30^\circ$

Now,

$\sin 30^\circ = \frac{OA}{OP}$

Here $OA$ is radius whose length is $a$, thus

$\frac{1}{2} = \frac{a}{OP}$

or

$OP = 2a$

35. Hence, radius of the ice-cream cone is 3 cm A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 m high embankment. Find the width of the embankment.

**Ans :** [Board Term-2 2012]
Depth of well, \( d = 14 \) m.
Radius, \( r = 12 \) m.

Volume of earth taken out,
\[
\pi r^2 h = \frac{22}{7} \times (2)^2 \times 14
\]
\[
= \frac{22}{7} \times 2 \times 2 \times 14
\]
\[
= 176 \text{ m}^3
\]

Let \( r \) be the width of embankment. The radius of outer circle of embankment
\[
= 2 + r
\]

Area of upper surface of embankment
\[
= \pi \left[(2 + r)^2 - (2)^2\right]
\]

Volume of embankment = Volume of earth taken out
\[
\pi \left[(2 + r)^2 - (2)^2\right] \times 0.4 = 176
\]
\[
\pi[4 + r^2 + 4r - 4] \times 0.4 = 176
\]
\[
0.4 \times \frac{22}{7} (r^2 + 4r) = 176
\]
\[
r^2 + 4r = \frac{176}{0.4} \times \frac{7}{22} = 140
\]
\[
r^2 + 4r - 140 = 0
\]
\[
(r + 14)(r - 10) = 0 \Rightarrow r = 10
\]

Hence width of embankment is 10 m.

36. A box contains 125 shirts of which 110 are good 12 have minor defects and 3 have major defects. Ram Lal will buy only those shirts which are good while Naveen will reject only those which have major defects. A shirt is taken out at random from the box. Find the probability that:
(i) Ram Lal will buy it
(ii) Naveen will buy it

Ans : [Board Term-2 OD 2017]

For both case total shirt,
\[
n(S) = 125
\]

(i) Ram Lal will buy it

Ramlal will buy only a good shirt.
No. of all possible outcomes,
\[
n(E_1) = 110
\]

\[
P(\text{Ramlal will buy a shirt}) = \frac{n(E_1)}{n(S)} = \frac{110}{125} = \frac{22}{25}
\]

(ii) Naveen will buy it

Naveen will reject the shirt which have major defects and will buy all other shirts.
No. of favourable outcomes,
\[
n(E_2) = 125 - 3 = 122
\]

\[
P(\text{Naveen will buy the shirt}) = \frac{n(E_2)}{n(S)} = \frac{122}{125}
\]

Download unsolved version of this paper from www.cbse.online