CHAPTER 3

Understanding Quadrilaterals

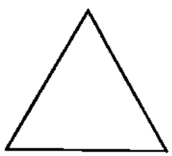
Understanding the Lesson

- Plane surfaces and plane curves.
- Polygons and their classification.
- Interior and exterior of a closed curve.
- Convex and Concave Polygons.
- Regular and irregular polygons.
- Angle sum property.
- Sum of the measure of exterior angles of a polygon.
- Kinds of quadrilateral: Trapezium, kite and parallelograms.
- Elements of a parallelogram.
- Angles and diagonals of a parallelogram.
- Special parallelograms: Rhombus, Rectangles, Squares.

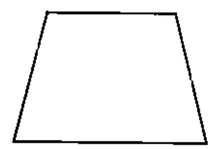
Conceptual Facts

- **Polygon:** A simple closed curve made up of only line segments is called a polygon.
- **Examples of Polygons:**

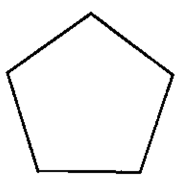
(*i*) Triangle



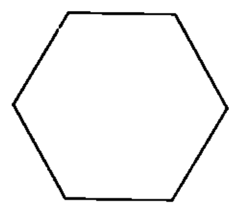
(*ii*) Quadrilateral



(*iii*) Pentagon

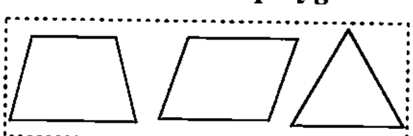


(*iv*) Hexagon



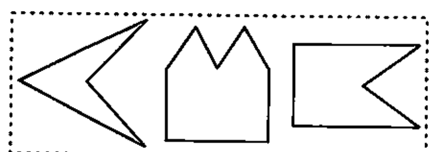
- **Convex and concave polygons**

(*i*)



Convex polygons

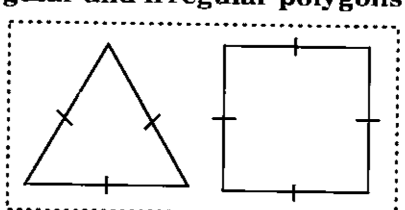
(*ii*)



Concave polygons

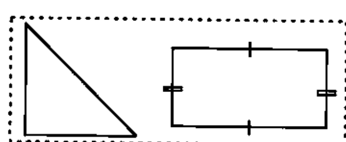
- **Regular and irregular polygons**

(*i*)



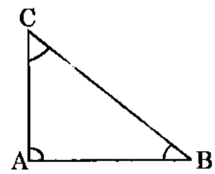
Regular polygons

(*ii*)

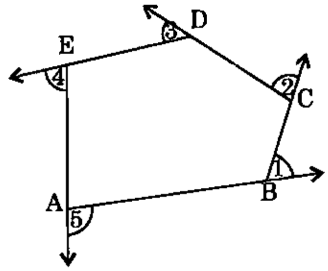


Irregular polygons

- **Angle sum property:** The sum of three angles of a triangle is 180°. In ΔABC, $\angle A + \angle B + \angle C = 180°$.

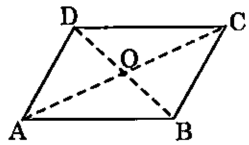


- Sum of all the exterior angles of a polygon is 360°. In the given polygon ABCDE, exterior angles $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360°$.



- **Kind of Quadrilaterals:**

(i) Parallelogram:



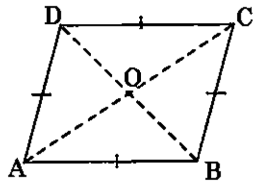
Properties

(a) Opposite angles are equal

(b) Opposite sides are equal

(c) Diagonals bisect each other

(ii) Rhombus:

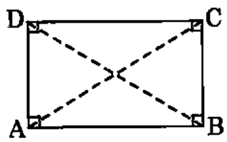


(a) All sides are equal

(b) Opposite angles are equal

(c) Diagonals bisect each other at 90°

(iii) Rectangle:

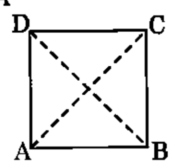


(a) It is a parallelogram having each angle of 90°

(b) Opposite sides are equal

(c) Diagonals are equal

(iv) Square:

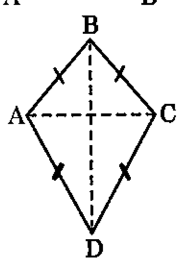


(a) All sides are equal

(b) Each angle is of 90°

(c) Diagonals are equal and bisect each other at 90°

(v) Kite:

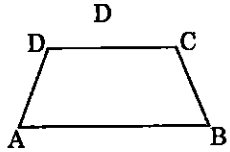


(a) Diagonals are perpendicular to each other

(b) One of the diagonals bisects the other

(c) $m\angle A = m\angle C$ but $m\angle B \neq m\angle D$

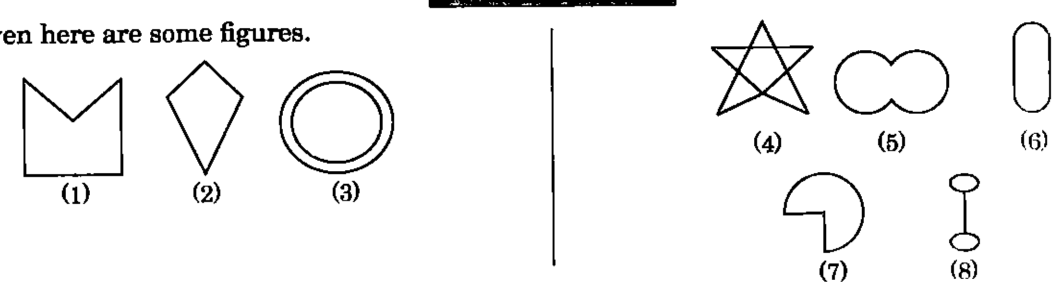
(vi) Trapezium:



A pair of opposite sides is parallel to each other.

EXERCISE 3.1

Q1. Given here are some figures.



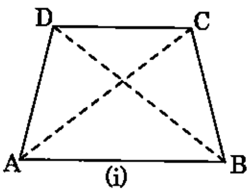
Classify each of above figure on the basis of the following:

(*a*) Simple curve (*b*) Simple closed curve
(*c*) Polygon (*d*) Convex polygon
(*e*) Concave polygon

Sol. (*a*) Simple curve: (1), (2), (5), (6) and (7)
(*b*) Simple closed curve: (1), (2), (5), (6) and (7)
(*c*) Polygon: (1) and (2)
(*d*) Convex polygon: (2)
(*e*) Concave polygon: (1)

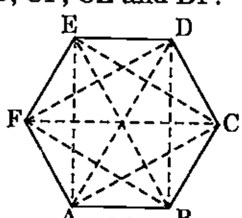
Q2. How many diagonals does each of the following have?
(*a*) A convex quadrilateral
(*b*) A regular hexagon
(*c*) A triangle

Sol. (*a*) In Fig. (*i*) ABCD is a convex quadrilateral which has two diagonals AC and BD.



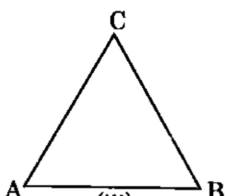
(i)

(*b*) In Fig. (*ii*) ABCDEF is a regular hexagon which has nine diagonals AE, AD, AC, BF, BE, BD, CF, CE and DF.



(ii)

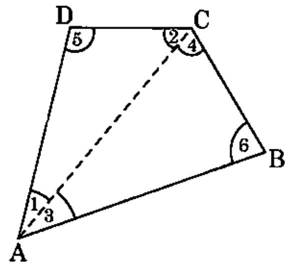
(*c*) In Fig. (*iii*) ABC is a triangle which has no diagonal.



(iii)

Q3. What is the sum of the measures of the angles of a convex quadrilateral ? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and verify!)

Sol. In the given figure, we have a quadrilateral ABCD. Join AC diagonal which divides the quadrilateral into two triangles ABC and ADC.



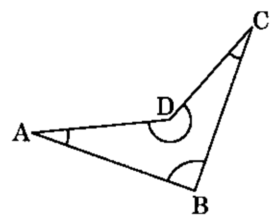
In ΔABC, $\angle 3 + \angle 4 + \angle 6 = 180°$ ...(*i*)
(angle sum property)

In ΔADC,
$\angle 1 + \angle 2 + \angle 5 = 180°$ ...(*ii*)
(angle sum property)

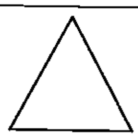
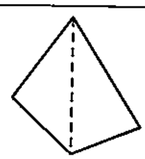
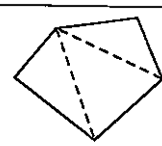
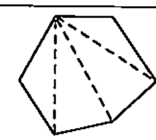
Adding, (*i*) and (*ii*)
$\angle 1 + \angle 3 + \angle 2 + \angle 4 + \angle 5 + \angle 6 = 180° + 180°$
$\Rightarrow \quad \angle A + \angle C + \angle D + \angle B = 360°$

Hence, the sum of all the angles of a convex quadrilateral = 360°.

Let us draw a non-convex quadrilateral. Yes, this property also holds true for a non-convex quadrilateral.



Q4. Examine the table. (Each figure is divided into triangles and the sum of the angles reduced from that).

Figure				
Side	3	4	5	6
Angle sum	180°	$2 \times 180° = (4-2) \times 180°$	$3 \times 180° = (5-2) \times 180°$	$4 \times 180° = (6-2) \times 180°$

What can you say about the angle sum of a convex polygon with number of sides?

(*a*) 7 (*b*) 8 (*c*) 10 (*d*) *n*

Sol. From the above table, we conclude that the sum of all the angles of a polygon of side '*n*'

$$= (n-2) \times 180°$$

(*a*) Number of sides = 7
Angles sum $= (7-2) \times 180° = 5 \times 180° = 900°$

(*b*) Number of sides = 8
Angle sum $= (8-2) \times 180°$
$= 6 \times 180° = 1080°$

(*c*) Number of sides = 10
Angle sum $= (10-2) \times 180°$
$= 8 \times 180° = 1440°$

(*d*) Number of sides = n

Angle sum = $(n-2) \times 180°$

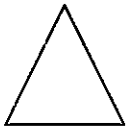
Q5. What is a regular polygon?

State the name of a regular polygon of

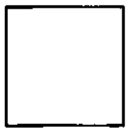
(*i*) 3 sides (*ii*) 4 sides (*iii*) 6 sides

Sol. A polygon with equal sides and equal angles is called a regular polygon.

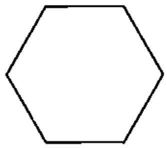
(*i*) Equilateral triangle



(*ii*) Square

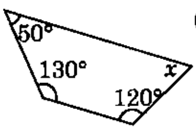


(*iii*) Regular hexagon

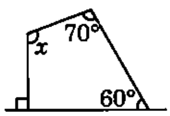


Q6. Find the angle measure x in the following figures:

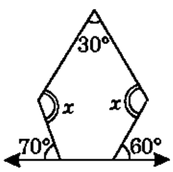
(*a*)



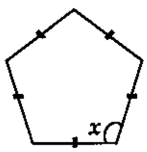
(*b*)



(*c*)



(*d*)



Sol. (*a*) Angle sum of a quadrilateral = 360°

$\Rightarrow \quad 50° + 130° + 120° + x = 360°$

$\Rightarrow \quad 300° + x = 360°$

$\therefore \quad x = 360° - 300° = 60°$

(*b*) Angle sum of a quadrilateral = 360°

$\Rightarrow \quad x + 70° + 60° + 90° = 360°$

[$\because$ 180° – 90° = 90°]

$\Rightarrow \quad x + 220° = 360°$

$\therefore \quad x = 360° - 220° = 140°$

(*c*) Angle sum of a pentagon = 540°

$\Rightarrow 30° + x + 110° + 120° + x = 540°$

[$\because$ 180° – 70° = 110°;
180° – 60° = 120°]

$\Rightarrow 2x + 260° = 540°$

$\Rightarrow \quad 2x = 540° - 260°$

$\Rightarrow \quad 2x = 280°$

$\therefore \quad x = \dfrac{280}{2} = 140°$

(*d*) Angle sum of a regular pentagon = 540°

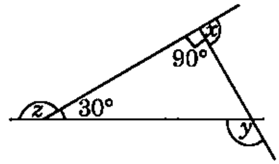
$\Rightarrow \quad x + x + x + x + x = 540°$

[All angles of a regular pentagon are equal]

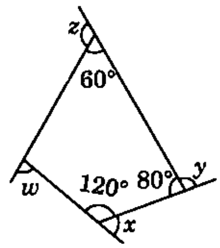
$\Rightarrow \quad 5x = 540°$

$\therefore \quad x = \dfrac{540}{5} = 108°$

Q7. (*a*) Find $x + y + z$

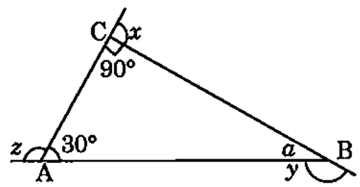


(*b*) Find $x + y + z + w$



Sol. (*a*) $\angle a$ + 30° + 90° = 180° [Angle sum property]

$\Rightarrow \angle a$ + 120° = 180°



$\Rightarrow \quad \angle a = 180° - 120° = 60°$

Now, $\quad y = 180° - a$ (Linear pair)

$= 180° - 60°$

$\therefore \quad y = 120°$

and, $z + 30° = 180°$ [Linear pair]

$\therefore \quad z = 180° - 30° = 150°$

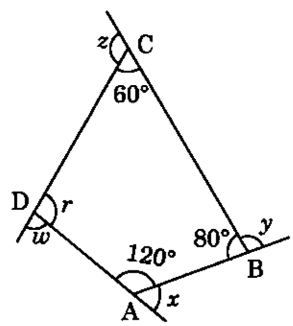
also, $x + 90° = 180°$ [Linear pair]

$\therefore \quad x = 180° - 90° = 90°$

Thus $x + y + z = 90° + 120° + 150° = 360°$

(*b*) $\angle r$ + 120° + 80° + 60° = 360°

[Angle sum property of a quadrilateral]



$\angle r + 260° = 360°$

$\therefore \quad \angle r = 360° - 260° = 100°$

Now $x + 120° = 180°$ (Linear pair)

$\therefore \quad x = 180° - 120° = 60°$

$y + 80° = 180°$ (Linear pair)

$\therefore \quad y = 180° - 80° = 100°$

$z + 60° = 180°$ (Linear pair)

$\therefore \quad z = 180° - 60° = 120°$

$w = 180° - \angle r$

$= 180° - 100° = 80°$

(Linear pair)

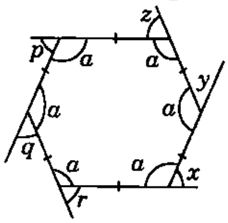
$\therefore \quad x + y + z + w$

$= 60° + 100° + 120° + 80°$

$= 360°$

TRY THESE (PAGE 43)

Q. Take a regular hexagon given below:



1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?
2. Is $x = y = z = p = q = r$? Why?
3. What is the measure of each?
   (i) exterior angle (ii) interior angle
4. Repeat this activity for the cases of
   (i) a regular octagon
   (ii) a regular 20-gon

Sol. 1. We know that, sum of the measure of exterior angles of a polygon = 360°

$\therefore \quad m\angle x + m\angle y + m\angle z + m\angle p + m\angle q + m\angle r = 360°$

2. Since the given figure is a regular polygon.

$\therefore \quad m\angle x = m\angle y = m\angle z = m\angle p = m\angle q = m\angle r$

3. (i) Since the sum of all exterior angles = 360°

$\therefore$ Measure of each exterior angle

$= 360° \div 6 = 60°$

(ii) Sum of all interior angles of a regular polygon of sides $6 = (6 - 2) \times 180°$

$= 4 \times 180° = 720°$

$\therefore$ Measure of each interior angle

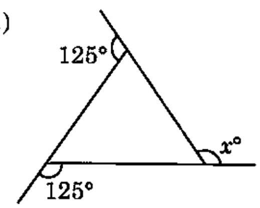
$= 720° \div 6 = 120°$

4. Since it is an activity. Try yourself.

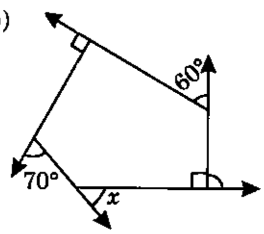
EXERCISE 3.2

Q1. Find x in the following figures.

(a)



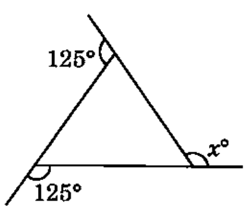
(b)



Sol. (a) We know that the sum of all the exterior angles of a polygon = 360°

$\therefore \quad 125° + 125° + x = 360°$

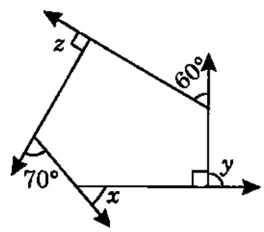
$250° + x = 360°$



$\therefore \quad x = 360° - 250° = 110$

Hence $\quad x = 110°$

(b) Here $\quad \angle y = 180° - 90° = 90°$



and $\quad \angle z = 90°$ (given)

$\therefore x + y + 60° + z + 70° = 360°$

[$\because$ Sum of all the exterior angles of a polygon = 360°]

$x + 90° + 60° + 90° + 70° = 360°$

$x + 310° = 360°$

$\therefore \quad x = 360° - 310° = 50°$

Hence $\quad x = 50°$

Q2. Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides

Sol. (i) We know the sum of all the exterior angles of polygon = 360°

$\therefore$ Measure of each angle of 9 sided regular polygon

$$= \frac{360^\circ}{9} = 40^\circ.$$

(*iii*) Sum of all the exterior angles of a polygon = 360°

$\therefore$ Measure of each angle of 15 sided regular polygon

$$= \frac{360^\circ}{15} = 24^\circ$$

Q3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?

Sol. Sum of all exterior angles of a regular polygon = 360°

$\therefore$ Number of sides

$$= \frac{360^\circ}{\text{measure of an angle}}$$

$$= \frac{360^\circ}{24} = 15$$

Hence, the number of sides = 15

Q4. How many sides does a regular polygon have if each of its interior angles is 165°?

Sol. Let n be the number of sides of a regular polygon.

$\therefore$ Sum of all interior angles

$$= (n-2) \times 180^\circ$$

and, measure of its each angle

$$= \frac{(n-2)\times 180^\circ}{n}$$

So, $\dfrac{(n-2)\times 180}{n} = 165$ (given)

$$\Rightarrow 180n - 2 \times 180 = 165n$$
$$\Rightarrow 180n - 360 = 165n$$
$$\Rightarrow 180n - 165n = 360$$
$$\Rightarrow 15n = 360$$
$$\therefore n = \frac{360}{15} = 24.$$

Hence, the number of sides = 24

Q5. (*a*) Is it possible to have a regular polygon with measure of each exterior angle a is 22°?

(*b*) Can it be an interior angle of a regular polygon? Why?

Sol. (*a*) Since, the sum of all the exterior angles of a regular polygon = 360° which is not divisible by 22°.

$\therefore$ It is not possible that a regular polygon must have its exterior angle 22°.

(*b*) Sum of all interior angles of a regular polygon of side n

$$= (n-2) \times 180^\circ$$

$\therefore$ Measure of each interior angle

$$= \frac{(n-2)\times 180^\circ}{n}$$

$$\frac{(n-2)\times 180^\circ}{n} = 22^\circ$$

$$\Rightarrow 180n - 2 \times 180 = 22n$$
$$\Rightarrow 180n - 22n = 360$$
$$\Rightarrow 158n = 360$$
$$\therefore n = \frac{360}{158} = \frac{180}{79}$$

not a whole number.

Since number of sides cannot be in fractions.

$\therefore$ It is not possible for a regular polygon to have its interior angle = 22°.

Q6. (*a*) What is the minimum interior angle possible for a regular polygon? Why?

(*b*) What is the maximum exterior angle possible for a regular polygon?

Sol. (*a*) Sum of all interior angles of a regular polygon of side n

$$= (n-2) \times 180^\circ$$

$\therefore$ Measure of each interior angle

$$= \frac{(n-2)\times 180^\circ}{n}$$

For minimum possible interior angle

$$\frac{(n-2)\times 180^\circ}{n} > 0$$

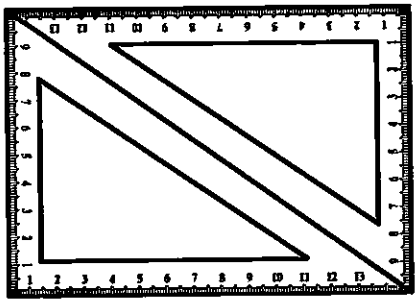
$$\Rightarrow n > 2$$

$\therefore$ The minimum measure the angle of an equilateral triangle ($n = 3$) = 60°.

(*b*) From part (*a*) we can conclude that the maximum exterior angle of a regular polygon = 180° – 60° = 120°.

Try These (Page 47)

Q. Take two identical set squares with angles 30° – 60° – 90° and place them adjacently to form a parallelogram as shown in figure. Does this help you to verify the above property?



Sol. It is an activity. You can try yourself.

EXERCISE 3.3

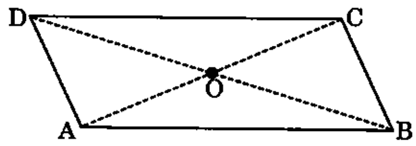
Q1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

(*i*) AD = ______

(*ii*) ∠DCB = ______

(*iii*) OC = ______

(*iv*) $m\angle DAB + m\angle CDA$ = ______



Sol. (*i*) AD = BC

[Opposite sides of a parallelogram are equal]

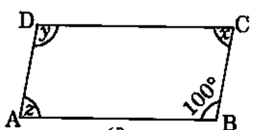
(*ii*) ∠DCB = ∠DAB [Opposite angles of a parallelogram are equal]

(*iii*) OC = OA

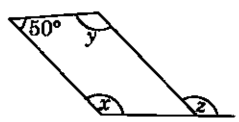
[Diagonals of a parallelogram bisect each other]

(*iv*) $m\angle DAB + m\angle CDA = 180°$ [Adjacent angles of a parallelogram are supplementary]

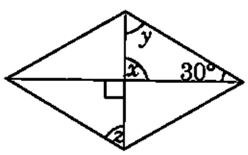
Q2. Consider the following parallelograms. Find the values of the unknowns *x*, *y*, *z*.



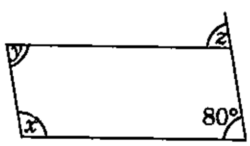
(*i*)



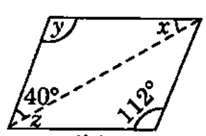
(*ii*)



(*iii*)

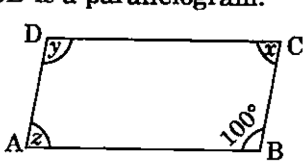


(*iv*)



(*iv*)

Sol. (*i*) ABCD is a parallelogram.



∴ ∠B = ∠D [Opposite angles of a parallelogram are equal]

∴ ∠D = 100° ⇒ $y = 100°$

∠A + ∠B = 180° [Adjacent angles of a parallelogram are supplementary]

⇒ $z + 100° = 180°$

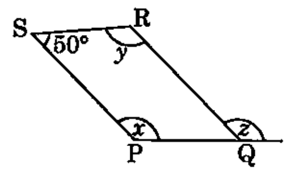
∴ $z = 180° - 100° = 80°$

∠A = ∠C [Opposite angles of a ||gm]

∴ $x = 80°$

Hence $x = 80°$, $y = 100°$ and $z = 80°$

(*ii*) PQRS is a parallelogram.



∴ ∠P + ∠S = 180°

[Adjacent angles of parallelogram]

⇒ $x + 50° = 180°$

∴ $x = 180° - 50° = 130°$

Now, ∠P = ∠R [Opposite angles are equal]

⇒ $x = y$ ⇒ $y = 130°$

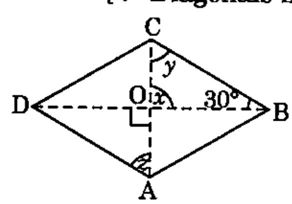
Also, $y = z$ [Alternate angles]

∴ $z = 130°$

Hence, $x = 130°$, $y = 130°$ and $z = 130°$

(*iii*) ABCD is a rhombus.

[∵ Diagonals intersect at 90°]



∴ $x = 90°$

Now in ΔOCB,

$x + y + 30° = 180°$

(Angle sum property)

⇒ $90° + y + 30° = 180°$

⇒ $y + 120° = 180°$

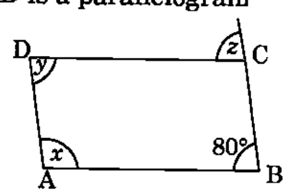
∴ $y = 180° - 120° = 60°$

$y = z$ (Alternate angles)

∴ $z = 60°$

Hence, $x = 90°$, $y = 60°$ and $z = 60°$.

(*iv*) ABCD is a parallelogram



∴ ∠A + ∠B = 180°

(Adjacent angles of a parallelogram are supplementary)

$\Rightarrow \quad x + 80° = 180°$

$\therefore \quad x = 180° - 80° = 100°$

Now, $\angle D = \angle B$ [Opposite angles of a ||gm]

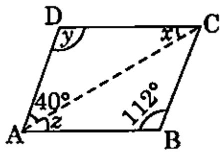
$\Rightarrow \quad y = 80°$

Also, $z = \angle B = 80°$

(Alternate angles)

Hence $x = 100°, y = 80°$ and $z = 80°$

(*v*) ABCD is a parallelogram.



$\therefore \quad \angle D = \angle B$

[Opposite angles of a ||gm]

$\therefore \quad y = 112°$

$x + y + 40° = 180°$

[Angle sum property]

$x + 112° + 40° = 180°$

$x + 152° = 180°$

$x = 180° - 152 = 28°$

$z = x = 28°$

(Alternate angles)

Hence $x = 28°, y = 112°, z = 28°$.

Q3. Can a quadrilateral ABCD be a parallelogram if

(*i*) $\angle D + \angle B = 180°$?

(*ii*) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

(*iii*) $\angle A = 70°$ and $\angle C = 65°$?

Sol. (*i*) For $\angle D + \angle B = 180$, quadrilateral ABCD may be a parallelogram if following conditions are also fulfilled.

(*a*) The sum of measures of adjacent angles should be 180°.

(*b*) Opposite angles should also be of same measures

So, ABCD can be but need not be a parallelogram.

(*ii*) Given: AB = DC = 8 cm, AD = 4 cm, BC = 4.4 cm

In a parallelogram, opposite sides are equal.

Here $AD \neq BC$

Thus, ABCD cannot be a parallelogram.

(*iii*) $\angle A = 70°$ and $\angle C = 65°$

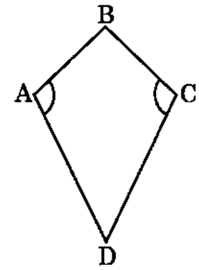
Since $\angle A \neq \angle C$

Opposite angles of quadrilateral are not equal.

Hence, ABCD is not a parallelogram.

Q4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

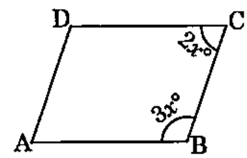
Sol. ABCD is a rough figure of a quadrilateral in which $m\angle A = m\angle C$ but it is not a parallelogram. It is a kite.



Q5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

Sol. Let ABCD is parallelogram such that

$m\angle B : m\angle C = 3 : 2$



Let $m\angle B = 3x°$ and $m\angle C = 2x°$

$m\angle B + m\angle C = 180°$

(Sum of adjacent angles = 180°)

$\therefore \quad 3x + 2x = 180°$

$5x = 180$

$\therefore \quad x = \frac{180}{5} = 36°$

Thus, $\angle B = 3 \times 36 = 108°$,

$\angle C = 2 \times 36° = 72°$

$\angle B = \angle D = 108°$

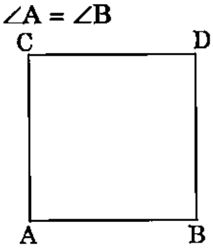
and $\angle A = \angle C = 72°$

Hence, the measures of the angles of the parallelogram are 108°, 72°, 108° and 72°.

Q6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Sol. Let ABCD be a parallelogram in which

$\angle A = \angle B$



We know $\angle A + \angle B = 180°$

[Sum of adjacent angles = 180°]

$\therefore \quad \angle A + \angle A = 180°$

$$2\angle A = 180°$$

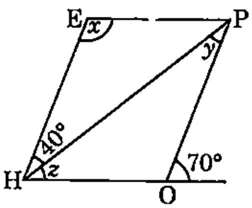
$$\therefore \quad \angle A = \frac{180°}{2} = 90°$$

Thus, $\angle A = \angle C = 90°$

and $\angle B = \angle D = 90°$

[Opposite angles of a parallelogram are equal]

Q7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



Sol. $\angle y = 40°$ (Alternate angles)

$\angle z + 40° = 70°$ (Exterior angle property)

$\therefore \quad \angle z = 70° - 40° = 30°$

$z = \angle EPH$ (Alternate angle)

In ΔEPH

$\angle x + 40° + \angle z = 180°$ (Adjacent angles)

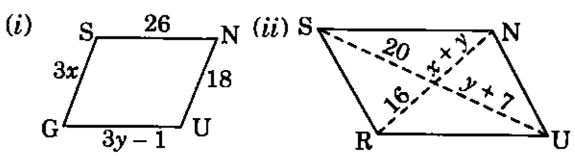
$\angle x + 40° + 30° = 180°$

$\angle x + 70° = 180°$

$\therefore \quad \angle x = 180° - 70° = 110°$

Hence $x = 110°$, $y = 40°$ and $z = 30°$.

Q8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)



Sol. (i) GU = SN (Opposite sides of a parallelogram)

$$3y - 1 = 26$$

$$\Rightarrow \quad 3y = 26 + 1$$

$$\Rightarrow \quad 3y = 27$$

$$\therefore \quad y = \frac{\cancel{27}^{9}}{\cancel{3}} = 9$$

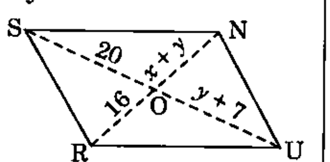
Similarly, GS = UN

$$3x = 18$$

$$\therefore \quad x = \frac{\cancel{18}^{6}}{\cancel{3}} = 6$$

Hence, $x = 6$ cm and $y = 9$ cm

(ii) Since, the diagonals of a parallelogram bisect each other



$\therefore \quad$ OU = OS

$$\Rightarrow \quad y + 7 = 20$$

$$\Rightarrow \quad y = 20 - 7 = 13$$

Also, ON = OR

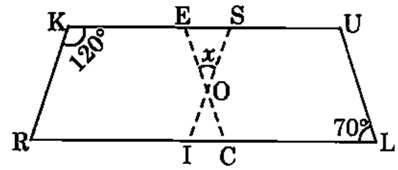
$$\Rightarrow \quad x + y = 16$$

$$\Rightarrow \quad x + 13 = 16$$

$$\therefore \quad x = 16 - 13 = 3$$

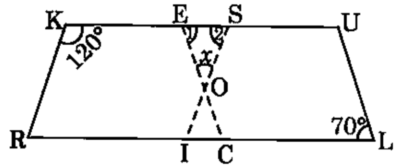
Hence, $x = 3$ cm and $y = 13$ cm.

Q9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

Sol. Here RISK and CLUE are two parallelograms.



$\angle 1 = \angle L = 70°$ (Opposite angles of a parallelogram)

$\angle K + \angle 2 = 180°$

Sum of adjacent angles is 180°

$120° + \angle 2 = 180°$

$\therefore \quad \angle 2 = 180° - 120° = 60°$

In ΔOES,

$\angle x + \angle 1 + \angle 2 = 180°$ (Angle sum property)

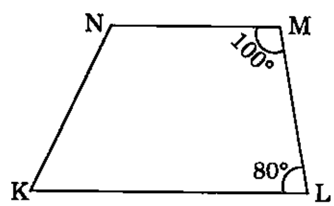
$\Rightarrow \quad \angle x + 70° + 60° = 180°$

$\Rightarrow \quad \angle x + 130° = 180°$

$\therefore \quad \angle x = 180° - 130° = 50°$

Hence $\quad x = 50°$

Q10. Explain how this figure is a trapezium. Which of its two sides are parallel?



Sol. $\angle M + \angle L = 100° + 80° = 180°$

$\because \angle M$ and $\angle L$ are the adjacent angles.

and sum of adjacent interior angles is 180°

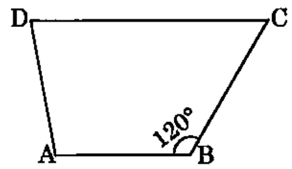
$\therefore$ KL is parallel to NM

Hence KLMN is a trapezium.

Q11. Find $m\angle C$ in below figure if $\overline{AB} \parallel \overline{DC}$

Sol. Given that $\overline{AB} \parallel \overline{DC}$

$\therefore m\angle B + m\angle C = 180°$ (Sum of adjacent angles of a parallelogram is 180°)

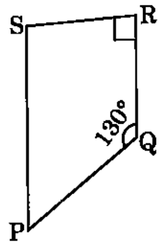


$120° + m\angle C = 180°$

$\therefore \quad m\angle C = 180° - 120° = 60°$

Hence $\quad m\angle C = 60°$

Q12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in figure, is there any other method to find $m\angle P$?)



Sol. Given that $\angle Q = 130°$ and $\angle R = 90°$

$\overline{SP} \parallel \overline{RQ}$ (given)

$\therefore \quad \angle P + \angle Q = 180°$ (Adjacent angles)

$\Rightarrow \quad \angle P + 130° = 180°$

$\Rightarrow \quad \angle P = 180° - 130° = 50°$

and, $\angle S + \angle R = 180°$ (Adjacent angles)

$\Rightarrow \quad \angle S + 90° = 180°$

$\therefore \quad \angle S = 180° - 90° = 90°$

Alternate Method:

$\angle Q = 130°, \angle R = 90°$

and $\quad \angle S = 90°$

We know that

$\angle P + \angle Q + \angle R + \angle Q = 360°$

(Angle sum property of quadrilateral)

$\angle P + 130° + 90° + 90° = 360°$

$\angle P + 310° = 360°$

$\therefore \quad \angle P = 360° - 310° = 50°$

Hence $\quad m\angle P = 50°$

EXERCISE 3.4

Q1. State whether True or False.

(*a*) All rectangles are squares.

(*b*) All rhombuses are parallelograms.

(*c*) All squares are rhombuses and also rectangles.

(*d*) All squares are not parallelograms.

(*e*) All kites are rhombuses.

(*f*) All rhombuses are kites.

(*g*) All parallelograms are trapeziums.

(*h*) All squares are trapeziums.

Sol. (*a*) False (*b*) True (*c*) True

(*d*) False (*e*) False (*f*) True

(*g*) True (*h*) True

Q2. Identify all the quadrilaterals that have

(*a*) four sides of equal length

(*b*) four right angles

Sol. (*a*) Squares and rhombuses.

(*b*) Rectangles and squares.

Q3. Explain how a square is

(*i*) a quadrilateral (*ii*) a parallelogram

(*iii*) a rhombus (*iv*) a rectangle

Sol. (*i*) Square is a quadrilateral because it is closed with four line segments.

(*ii*) Square is a parallelogram due to the following properties:

(*a*) Opposite sides are equal and parallel.

(*b*) Opposite angles are equal.

(*iii*) Square is a rhombus because its all sides are equal and opposite sides are parallel.

(*iv*) Square is a rectangle because its opposite sides are equal and has equal diagonal.

Q4. Name the quadrilaterals whose diagonals

(*i*) bisect each other

(*ii*) are perpendicular bisectors of each other

(*iii*) are equal

Sol. (*i*) Parallelogram, rectangle, square and rhombus

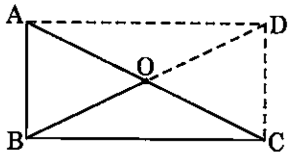
(*ii*) Square and rhombus

(*iii*) Rectangle and square

Q5. Explain why a rectangle is a convex quadrilateral.

Sol. In a rectangle, both of its diagonal lie in its interior. Hence, it is a convex quadrilateral.

Q6. ABC is a right-angled triangle and O is the midpoint of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Sol. Since, the right-angled triangle ABC makes a rectangle ABCD by the dotted lines.

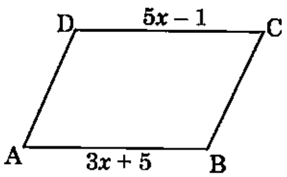
Therefore OA = OB = OC = OD[Diagonals of a rectangle are equal and bisect each other]

Hence, O is equidistant from A, B and C.

Learning More Q & A

I. Very Short Answer (VSA) Questions

Q1. In the given figure, ABCD is a parallelogram. Find x.



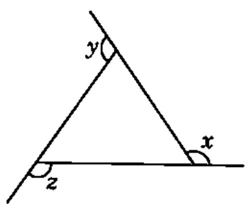
Sol. AB = DC [Opposite sides of a parallelogram]

$\therefore \quad 3x + 5 = 5x - 1$

$\Rightarrow \quad 3x - 5x = -1 - 5$

$\Rightarrow \quad -2x = -6 \quad \therefore \quad x = 3$

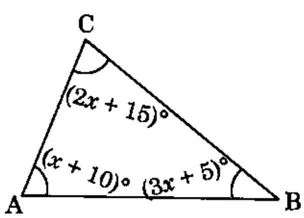
Q2. In the given figure find $x + y + z$.



Sol. We know that the sum of all the exterior angles of a polygon = 360°

$\therefore \quad x + y + z = 360°$

Q3. In the given figure, find x.



Sol. $\angle A + \angle B + \angle C = 180°$ [Angle sum property]

$\therefore \quad (x + 10)° + (3x + 5)° + (2x + 15)° = 180°$

$\Rightarrow \quad x + 10 + 3x + 5 + 2x + 15 = 180$

$\Rightarrow \quad 6x + 30 = 180$

$\Rightarrow \quad 6x = 180 - 30$

$\Rightarrow \quad 6x = 180 - 30$

$\Rightarrow \quad 6x = 150$

$\therefore \quad x = \dfrac{\cancel{150}^{25}}{\cancel{6}} = 25$

Q4. The angles of a quadrilateral are in the ratio of 2 : 3 : 5 : 8. Find the measure of each angle.

Sol. Sum of all interior angles of a quadrilateral = 360°

Let the angles of the quadrilateral be $2x°$, $3x°$, $5x°$ and $8x°$.

$\therefore \quad 2x + 3x + 5x + 8x = 360°$

$\Rightarrow \quad 18x = 360°$

$\therefore \quad x = \dfrac{\cancel{360}^{20}}{\cancel{18}} \Rightarrow x = 20°$

Hence the angles are 2 × 20 = 40°, 3 × 20 = 60°, 5 × 20 = 100° and 8 × 20 = 160°.

Q5. Find the measure of an interior angle of a regular polygon of 9 sides.

Sol. Measure of an interior angle of a regular polygon of n sides $= \dfrac{(n-2)\times 180°}{n}$

For $n = 9$, we have

$$\frac{(9-2)\times 180°}{9} = \frac{7\times 180°}{9}$$

$$= 7 \times 20° = 140°$$

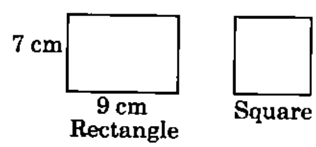
Hence, the angle is 140°.

Q6. Length and breadth of a rectangular wire are 9 cm and 7 cm respectively. If the wire is bent into a square, find the length of its side.

Sol. Perimeter of the rectangle

= 2 [length + breadth]

= 2[9 + 7] = 2 × 16 = 32 cm.



Now perimeter of the square

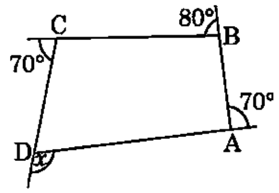
= Perimeter of rectangle

= 32 cm.

$\therefore$ Side of the square $= \dfrac{\cancel{32}^{8}}{\cancel{4}} = 8$ cm.

Hence, the length of side of square = 8 cm.

Q7. In the given figure ABCD, find the value of x.



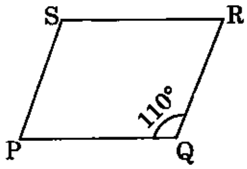
Sol. Sum of all the exterior angles of a polygon = 360°

$\therefore \quad x + 70° + 80° + 70° = 360°$

$\Rightarrow \quad x + 220° = 360°$

$\therefore \quad x = 360° - 220° = 140°$

Q8. In the parallelogram given alongside if $m\angle Q = 110°$, find all the other angles.



Sol. Given $\quad m\angle Q = 110°$

Then $\quad m\angle S = 110°$

(Opposite angles are equal)

Since $\angle P$ and $\angle Q$ are supplementary.

Then $m\angle P + m\angle Q = 180°$

$\Rightarrow m\angle P + 110° = 180°$

$\therefore \quad m\angle P = 180° - 110° = 70°$

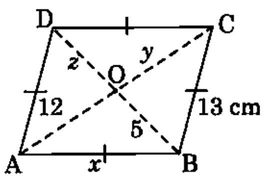
$m\angle P = m\angle R = 70°$

(Opposite angles)

Hence $\quad m\angle P = 70, m\angle R = 70°$

and $\quad m\angle S = 110°$

Q9. In the given figure, ABCD is a rhombus. Find the values of x, y and z.



Sol. AB = BC (Sides of a rhombus)

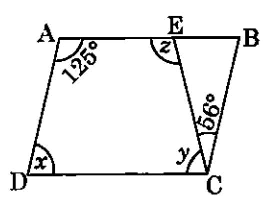
$\therefore \quad x = 13$ cm.

Since the diagonals of a rhombus bisect each other

$\therefore \quad z = 5$ and $y = 12$

Hence, $x = 13$ cm, $y = 12$ cm and $z = 5$ cm.

Q10. In the given figure, ABCD is a parallelogram. Find x, y and z.



Sol. $\angle A + \angle D = 180°$ (Adjacent angles)

$\Rightarrow \quad 125° + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 125°$

$\therefore \quad x = 55°$

$\angle A = \angle C$ [Opposite angles of a parallelogram]

$125° = y + 56°$

$\Rightarrow \quad y = 125° - 56°$

$\Rightarrow \quad y = 69°$

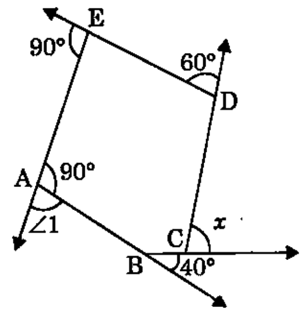
$\angle z + \angle y = 180°$ (Adjacent angles)

$\therefore \quad \angle z + 69° = 180°$

$\Rightarrow \quad \angle z = 180° - 69° = 111°$

Hence the angles $x = 55°, y = 69°$ and $z = 111°$

Q11. Find x in the following figure. (*NCERT Exemplar*)



Sol. In the given figure $\angle 1 + 90° = 180°$ (linear pair)

$\angle 1 = 90°$

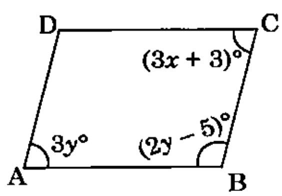
Now, sum of exterior angles of a polygon is 360°, therefore, $x + 60° + 90° + 90° + 40° = 360°$

$x + 280° = 360°$

$x = 80°$

II. Short Answer (SA) Questions

Q12. In the given parallelogram ABCD, find the value of x and y.



Sol. $\angle A + \angle B = 180°$ (Adjacent angles)

$3y + 2y - 5 = 180°$

$\Rightarrow \quad 5y - 5 = 180°$

$\Rightarrow \quad 5y = 180 + 5°$

$\Rightarrow \quad 5y = 185°$

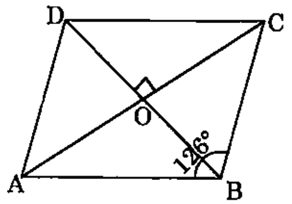
$\therefore \quad y = \dfrac{\cancel{185}^{37}}{\cancel{5}} \Rightarrow y = 37°$

Now $\quad \angle A = \angle C$ [Opposite angles of a parallelogram]

$$3y = 3x + 3$$
$$\Rightarrow \quad 3 \times 37 = 3x + 3$$
$$\Rightarrow \quad 111 = 3x + 3$$
$$\Rightarrow \quad 111 - 3 = 3x$$
$$\Rightarrow \quad 108 = 3x$$
$$\therefore \quad x = \frac{\cancel{108}^{36}}{\cancel{3}} \Rightarrow x = 36°$$

Hence, $x = 36°$ and $y = 37°$.

Q13. ABCD is a rhombus with $\angle ABC = 126°$, find the measure of $\angle ACD$.



Sol. $\angle ABC = \angle ADC$

(Opposite angles of a rhombus)

$$\therefore \quad \angle ADC = 126°$$
$$\angle ODC = \frac{1}{2} \times \angle ADC$$

(Diagonal of rhombus bisects the respective angles)

$$= \frac{1}{2} \times 126° = 63°$$
$$\therefore \quad \angle DOC = 90°$$

($\because$ Diagonals of a rhombus bisect each other at 90°)

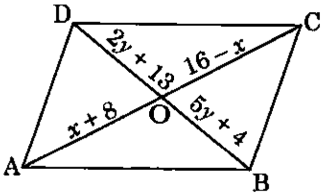
$\therefore$ In ΔOCD,
$$\angle OCD + \angle ODC + \angle DOC = 180°$$

(Angle sum property)

$$\therefore \quad \angle OCD + 63° + 90° = 180°$$
$$\angle OCD + 153° = 180°$$
$$\angle OCD = 180° - 153° = 27°$$

Hence $\angle OCD$ or $\angle ACD = 27°$

Q14. Find the values of x and y in the following parallelogram.



Sol. Since, the diagonals of a parallelogram bisect each other.

$$\therefore \quad OA = OC$$
$$x + 8 = 16 - x$$
$$x + x = 16 - 8$$
$$2x = 8$$
$$\therefore \quad x = \frac{8}{2} = 4$$

Similarly $\quad OB = OD$
$$5y + 4 = 2y + 13$$
$$\Rightarrow \quad 5y - 2y = 13 - 4$$
$$\Rightarrow \quad 3y = 9$$
$$\therefore \quad y = \frac{9}{3} = 3$$

Hence, $x = 4$ and $y = 3$

Q15. Write true and false against each of the given statements.

(*a*) Diagonals of a rhombus are equal. ______

(*b*) Diagonals of a rectangles are equal. ______

(*c*) Kite is a parallelogram. ______

(*d*) Sum of the interior angles of a triangle is 180°. ______

(*e*) Trapezium is a parallelogram. ______

(*f*) Sum of all the exterior angles of a polygon is 360°. ______

(*g*) Diagonals of a rectangle are perpendicular to each other. ______

(*h*) Triangle is possible with angles 60°, 80° and 100°. ______

(*i*) In a parallelogram, the opposite sides are equal. ______

Sol. (*a*) False (*b*) True (*c*) False
(*d*) True (*e*) False (*f*) True
(*g*) False (*h*) False (*i*) True

Q16. The sides AB and CD of a quadrilateral ABCD are extended to points P and Q respectively. Is $\angle ADQ + \angle CBP = \angle A + \angle C$? Give reason.

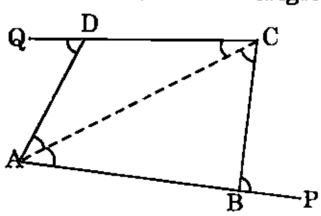
(*NCERT Exemplar*)

Sol. Join AC, then

$$\angle CBP = \angle BCA + \angle BAC$$

and $\quad \angle ADQ = \angle ACD + \angle DAC$

(Exterior angles of triangles)



Therefore,
$$\angle CBP + \angle ADQ = \angle BCA + \angle BAC + \angle ACD + \angle DAC$$
$$= (\angle BCA + \angle ACD) + (\angle BAC + \angle DAC)$$
$$= \angle C + \angle A$$

Higher Order Thinking Skills (HOTS) Questions

Q17. The diagonal of a rectangle is thrice its smaller side. Find the ratio of its sides.

Sol. Let AD = x cm

$\therefore$ diagonal BD = $3x$ cm

In right-angled triangle DAB,

$$AD^2 + AB^2 = BD^2$$

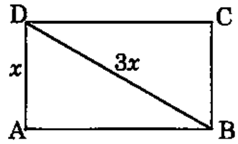
(Using Pythagoras Theorem)

$$x^2 + AB^2 = (3x)^2$$

$$\Rightarrow \quad x^2 + AB^2 = 9x^2$$

$$\Rightarrow \quad AB^2 = 9x^2 - x^2$$

$$\Rightarrow \quad AB^2 = 8x^2$$



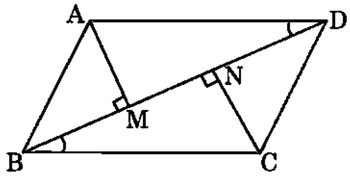
$$\therefore \quad AB = \sqrt{8}x = 2\sqrt{2}x$$

$\therefore$ Required ratio of AB : AD

$$= 2\sqrt{2}x : x = 2\sqrt{2} : 1$$

Q18. If AM and CN are perpendiculars on the diagonal BD of a parallelogram ABCD, Is ΔAMD $\cong$ ΔCNB? Give reason. (*NCERT Exemplar*)

Sol.



In triangles AMD and CNB,

AD = BC (opposite sides of parallelogram)

$\angle$AMB = $\angle$CNB = 90°

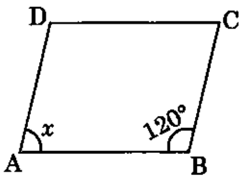
$\angle$ADM = $\angle$NBC (AD || BC and BD is transversal.)

So, ΔAMD $\cong$ ΔCNB (AAS)

Test Yourself

Multiple Choice Questions (MCQs)

Q1. ABCD is a parallelogram. The value of x is



(*a*) 70° (*b*) 80°

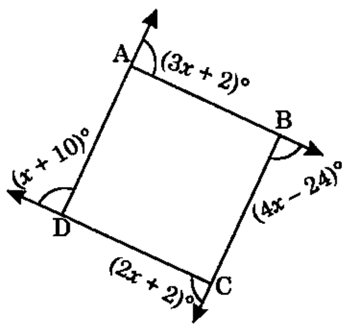
(*c*) 60° (*d*) 100°

Q2. The measure of three angles of a quadrilateral are 39°, 141° and 67°. The fourth angle is

(*a*) 113° (*b*) 103°

(*c*) 131° (*d*) 133°

Q3. The measure of $\angle$ADC is



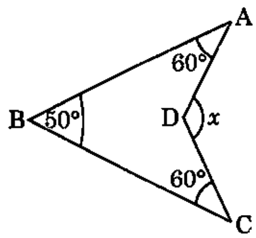
(*a*) 37° (*b*) 47° (*c*) 133° (*d*) 43°

Q4. The angles of a quadrilateral are in the ratio 2 : 3 : 5 : 8. The smallest angle is:

(*a*) 60° (*b*) 40° (*c*) 50° (*d*) 20°

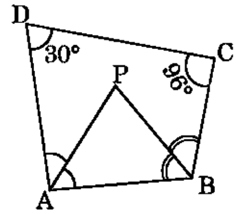
Q5. In the given figure, the value of x is

(*a*) 170° (*b*) 190° (*c*) 90° (*d*) 100°



Q6. If the angles of a quadrilateral are $x°$, $(2x + 13)°$, $(3x + 10)°$ and $(x - 6)°$, find x.

Q7. In the given figure, the bisectors of $\angle$A and $\angle$B meet at a point P. If $\angle$C = 96° and $\angle$D = 30°, find $\angle$APB.



Q8. One side of a parallelogram is greater than the other by 25 cm. If the perimeter of the parallelogram is 150 cm, find the length of all sides of the parallelogram.

Q9. Which of the following groups of angles forms a quadrilateral? Give reason.

(*a*) 45°, 75°, 110°, 90°

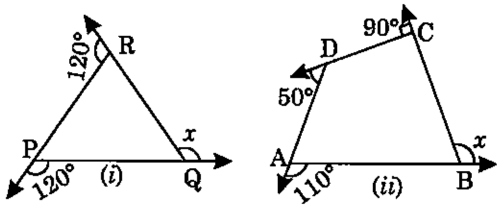
(*b*) 125°, 180°, 25°, 40°

(*c*) 95°, 115°, 60°, 90°

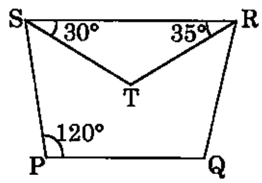
Q10. In a quadrilateral ABCD, $\angle$A = 55° and $\angle$B = 85°. If $\angle$C · $\angle$D = 5 : 6, find angles C and D.

Q11. What is the sum of the angles of a convex polygon whose number of sides are (*a*) 10 (*b*) 8

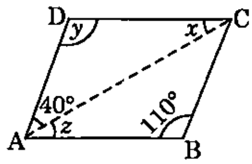
Q12. Find the value of x in each of the following cases:



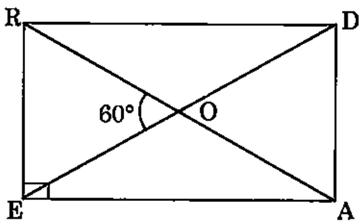
Q13. In the given trapezium PQRS, ST and RT are the bisectors of ∠PSR and ∠SRQ respectively. Find ∠PQR.



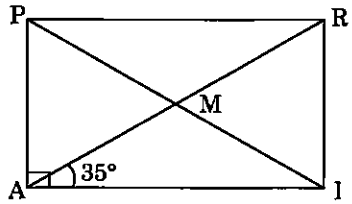
Q14. Find the value of x, y and z in the following figure.



Q15. In rectangle READ, find ∠EAR, ∠RAD and ∠ROD (*NCERT Exemplar*)



Q16. In rectangle PAIR, find ∠ARI, ∠RMI and ∠PMA. (*NCERT Exemplar*)



Q17. Fill in the blanks:

(*i*) The sum of the adjacent angles of a parallelogram is ______.

(*ii*) The diagonals of a rectangle ______ each other.

(*iii*) The diagonals of a rhombus bisect each other at ______.

(*iv*) A pair of opposite sides of a trapezium is ______.

(*v*) The sum of all the angles of a polygon of 8 sides is ______.

(*vi*) The sum of all the exterior angles of a polygon is ______.

(*vii*) In a square ABCD if AC = $(7x - 2)$ cm and BD = $(11x - 10)$ cm, then x = ______.

(*viii*) The two diagonals of a kite are not ______.

(*ix*) The sum of all interior angles of a pentagon is equal to ______.

(*x*) The diagonals of an isosceles trapezium are ______.

ANSWERS

1. (*c*) 2. (*a*) 3. (*c*)
4. (*b*) 5. (*a*)
6. 49 7. 63
8. 25 cm, 50 cm, 25 cm, 50 cm
9. (*c*) 10. 100° and 120°
11. (*a*) 1440° (*b*) 1080°
12. (*i*) 120° (*ii*) 110° 13. 110°
14. $x = 30°, y = 110°, z = 30°$
15. 30°, 60°, 120° 16. 55°, 70°, 70°
17. (*i*) 180° (*ii*) bisect (*iii*) 90° (*iv*) parallel (*v*) 1080° (*vi*) 360° (*vii*) 2 (*viii*) equal (*ix*) 540° (*x*) equal

Internal Assessment

Q1. Which of the following is a true statement?

(*a*) The diagonals of a rhombus are equal.

(*b*) The diagonals of a square are equal.

(*c*) The diagonals of a parallelogram bisect each other at 90°.

(*d*) The diagonals of a rectangle are perpendicular to each other.

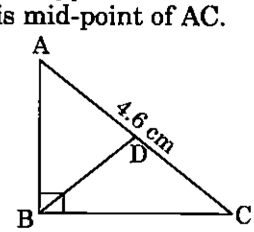
Q2. In a right angled ΔABC, the hypotenuse AC = 4.6 cm, find BD, where D is mid-point of AC.

(*a*) 3.2 cm

(*b*) 2.3 cm

(*c*) 4.6 cm

(*d*) 3.5 cm



Q3. One angle of a parallelogram is 3 times its adjacent angle. Find all the angles of the parallelogram.

Q4. In a quadrilateral, sum of the three angles is equal to twice the fourth angle. Find the fourth angle.

Q5. The measure of an angle of a regular polygon is 108°. Find the number of sides of the polygon.

Q6. Fill in the blanks:

(*i*) The greatest angle of triangle is less than ______.

(*ii*) One of the acute angle of a right angled triangle is 30°, then the measure of other acute angle is ______.

(*iii*) In an isosceles trapezium, a pair of opposite sides is parallel and the pair of other two sides is ______.

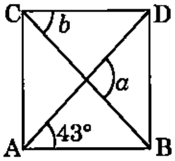
(*iv*) The sum of the measures of exterior angles of a polygon is ______.

(*v*) If all angles of a rhombus are right angles then it is a ______.

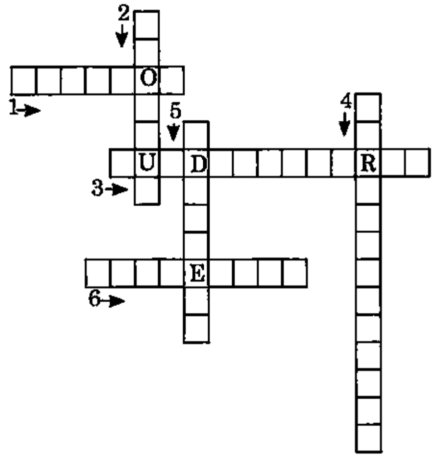
Q7. One side of a rectangle is $2\frac{1}{2}$ times the other. If its perimeter is 98 cm, find its sides.

Q8. In a rectangle ABCD, diagonals AC and BD meet at O. If OA = $2x + 7$ and OD = $3x - 5$, find x.

Q9. In the given figure ABCD is a rhombus, find the value of $(a - b)$.



Q10. Complete the following puzzle.



Hints:

1. A figure enclosed by a number of line segments is called ______.
2. ______ has all its sides equal.
3. The sum of all interior angles of a ______ is 360°.
4. Opposite sides of a ______ are equal.
5. The sum of ______ angles of a parallelogram is 180°.
6. A pair of opposite sides of a ______ are parallel to each other.

ANSWERS

1. (*b*) 2. (*b*)
3. 45°, 135°, 45°, 135°
4. 120° 5. 5
6. (*i*) 180° (*ii*) 60° (*iii*) equal
   (*iv*) 360° (*v*) square
7. 14 cm, 35 cm 8. 12 9. 43°
10. (1) POLYGON
    (2) RHOMBUS
    (3) QUADRILATERAL
    (4) PARALLELOGRAM
    (5) ADJACENT
    (6) TRAPEZIUM