



Squares and Square Roots

Understanding the Lesson

- Squares and perfect square numbers.
- Properties of square numbers.
- Numbers between square numbers.
- Adding consecutive odd and even natural numbers.
- Product of two consecutive even and odd natural numbers.
- Square roots and method of finding the square roots.
- Finding square roots through repeated subtraction and prime factorisation.
- Finding square root by division method.
- Square roots of decimals.

Conceptual Facts

- If a number is multiplied by itself, the product that we get is called the square of the number.

For example:

$$\begin{array}{ll}
 4 \times 4 = 4^2 = 16 & (16 \text{ is square of } 4) \\
 20 \times 20 = 20^2 = 400 & (400 \text{ is square of } 20) \\
 \frac{3}{5} \times \frac{3}{5} = \left(\frac{3}{5}\right)^2 = \frac{9}{25} & \left(\frac{9}{25} \text{ is square of } \frac{3}{5}\right)
 \end{array}$$

- Square of a number is represented as the number raised to the power 2.
- A perfect square is a number that can be expressed as the product of two equal integers.

For example:

1, 4, 9, 16, 25, ..., are all perfect square numbers.

- **Properties of square numbers:**

- No square number ends with the digits 2, 3, 7 or 8 at its unit places.
- The square numbers must end with the digits 0, 1, 4, 5, 6, 9 but the number ending with 0, 1, 4, 5, 6, 9 may or may not be a perfect number.
For example: 36 is a perfect square but 56 is not.
256 is a perfect square but 346 is not.
- Square of even number is always an even and the square of odd number is odd.

For example: $4^2 = 16(\text{even})$

$$5^2 = 25(\text{odd})$$

- A perfect square can never be a negative number.

(v) For every natural number n , $(n + 1)^2 - n^2 = (n + 1) + n$.

$$\text{For example: } 14^2 - 13^2 = (13 + 1) + 13 = 14 + 13 = 27$$

$$26^2 - 25^2 = (25 + 1) + 25 = 26 + 25 = 51$$

- **Pythagorean Triplets:** A triplet (m, n, p) is called a Pythagorean triplet if $m^2 + n^2 = p^2$
For example: (3, 4, 5), (8, 15, 17) and (20, 21, 29)
- **Product of two consecutive even or odd natural numbers:**
Example: $11 \times 13 = 143 = 12^2 - 1$ (product of odds)
 $13 \times 15 = 195 = 14^2 - 1$ (product of odds)
 $44 \times 46 = 2024 = 45^2 - 1$ (product of evens)
- **Some Patterns in square numbers:**
Example: $1^2 = 1$
 $11^2 = 121$
 $111^2 = 12321$

.....
.....
.....

Similarly, $7^2 = 49$
 $67^2 = 4489$
 $667^2 = 444889$
 $6667^2 = 44448889$

.....
.....

TRY THESE (PAGE 90)

Q1. Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60.

Sol. Look at the following squares.

$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64$

(i) From the above data, we notice that $30 < 36 < 40$ or $30 < 6^2 < 40$
Hence, 36 is the perfect square number between 30 and 40.

(ii) From the above data, we notice that there is no perfect square number between 50 and 60.

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Q1. Can we say whether the following numbers are perfect squares? How do we know?

- (i) 1057 (ii) 23453 (iii) 7928
(iv) 222222 (v) 1069 (vi) 2061

Write five numbers which you can decide by looking at their one's digit that they are not square numbers.

Sol. (i) 1057 ends with 7 at unit place. So, it cannot be a perfect square.
(ii) 23453 ends with 3 at unit place. So it cannot be a perfect square.
(iii) 7928 ends with 8 at unit place. So it cannot be a perfect square.
(iv) 222222 ends with 2 at unit place. So it cannot be a perfect square.

(v) 1069 ends with 9 at unit place but it is not a perfect square.

(vi) 2061 ends with 1 at unit place but it is not a perfect square.

89, 106, 221, 147 and 216 are such examples which are not a square numbers.

Q2. Write five numbers which you cannot decide just by looking at their unit's digit (or one's place) whether they are square numbers or not.

Sol. The required numbers may be as follows:

109, 104, 306, 401, and 221

The one's place of the above numbers are 1, 4, 6 and 9 but they are not the perfect square numbers.

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Q1. Which of $123^2, 77^2, 82^2, 161^2, 109^2$ would end with digit 1?

Sol. 161^2 and 109^2 would end with digit 1.

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Q1. Which of the following numbers would have digit 6 at unit place.

- (i) 19^2 (ii) 24^2 (iii) 26^2
(iv) 36^2 (v) 34^2

Sol. (i) 19^2 does not have digit 6 at unit place.
(ii) 24^2 has digit 6 at unit place.
(iii) 26^2 has digit 6 at unit place.

(iv) 36^2 has digit 6 at unit place.

(v) 34^2 has digit 6 at unit place.

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Q1. What will be the “one’s digit” in the square of the following numbers?

(i) 1234 (ii) 26387 (iii) 52698

(iv) 99880 (v) 21222 (vi) 9106

Sol. (i) One’s digit of $1234^2 = 6$

(ii) One’s digit of $26387^2 = 9$

(iii) One’s digit of $52698^2 = 4$

(iv) One’s digit of $99880 = 0$

(v) One’s digit of $21222^2 = 4$

(vi) One’s digit of $9106^2 = 6$

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Q1. The square of which of the following numbers would be an odd number/an even number? Why?

(i) 727 (ii) 158 (iii) 269 (iv) 1980

Sol. (i) Since 727 is odd number. So 727^2 will also be odd number.

(ii) 158 is even number. So 158^2 will also be even.

(iii) 269 is odd number. So 269^2 will also be odd.

(iv) 1980 is even number. So 1980^2 will also be even.

Q2. What will be the number of zeros in the square of the following numbers?

(i) 60 (ii) 400

Sol. (i) 60 has one zero. So 60^2 will have 2 zeroes.

(ii) 400 has two zero. So 400^2 will have 4 zeroes.

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Q1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2 ?

Sol. (i) The natural numbers between 9^2 and $10^2 = (100 - 81) - 1 = 19 - 1 = 18$

(ii) The natural numbers between 11^2 and $12^2 = (144 - 121) - 1 = 23 - 1 = 22$

Q2. How many non square numbers lie between the following pairs of numbers

(i) 100^2 and 101^2

(ii) 90^2 and 91^2

(iii) 1000^2 and 1001^2

Sol. (i) Non-perfect square numbers between n^2 and $(n + 1)^2 = 2n$

$$\begin{aligned} & [(n + 1)^2 - n^2] - 1 \\ & [n^2 + 1 + 2n - n^2] - 1 \\ & = (2n + 1) - 1 = 2n + 1 - 1 = 2n \end{aligned}$$

\therefore Non-perfect square numbers between 100^2 and $101^2 = 2 \times 100 = 200$

(ii) Non-perfect square numbers between 90^2 and $91^2 = 2 \times 90 = 180$

(iii) Non-perfect square numbers between 1000^2 and $1001^2 = 2 \times 1000 = 2000$

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Q1. Find whether each of the following numbers is a perfect square or not?

(i) 121 (ii) 55 (iii) 81

(iv) 49 (v) 69

Sol. (i) Sum of consecutive odd numbers starting from 1 is

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 11^2 = 121$$

\therefore 121 is a square number.

(ii) Sum of consecutive odd numbers starting with 1 is

$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = 49$$

$$\text{and } 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2 = 64$$

Thus, 55 is not a perfect number.

(iii) Sum of consecutive odd numbers starting from 1 is

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$$

$$= 9^2 = 81$$

\therefore 81 is a perfect square number.

(iv) Sum of consecutive odd numbers starting from 1 is

$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = 49$$

\therefore 49 is a perfect square number.

(v) Sum of consecutive odd numbers starting from 1 is

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2 = 64$$

$$\text{and } 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$$

$$= 9^2 = 81$$

\therefore $64 < 69 < 81$

Thus 69 is not a perfect square number.

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Q1. Express the following as the sum of two consecutive integers.

(i) 21^2 (ii) 13^2 (iii) 11^2 (iv) 19^2

Sol. For n^2 , the two consecutive integers are:

$$\frac{n^2 - 1}{2} \text{ and } \frac{n^2 + 1}{2}$$

$$(i) \quad \frac{21^2 - 1}{2} = \frac{441 - 1}{2}$$

$$= \frac{440}{2} = 220$$

$$\text{and } \frac{21^2 + 1}{2} = \frac{441 + 1}{2}$$

$$= \frac{442}{2} = 221$$

$$\therefore 21^2 = 441 = \left(\frac{21^2 - 1}{2} + \frac{21^2 + 1}{2} \right)$$

$$= 220 + 221$$

$$(ii) 13^2 = 169 = \left(\frac{13^2 - 1}{2} + \frac{13^2 + 1}{2} \right)$$

$$= \left(\frac{169 - 1}{2} + \frac{169 + 1}{2} \right)$$

$$= \left(\frac{168}{2} + \frac{170}{2} \right) = 84 + 85$$

$$\therefore 13^2 = 169 = 84 + 85$$

$$(iii) 11^2 = 121 = \left(\frac{11^2 - 1}{2} + \frac{11^2 + 1}{2} \right)$$

$$= \left(\frac{121 - 1}{2} + \frac{121 + 1}{2} \right)$$

$$= \left(\frac{120}{2} + \frac{122}{2} \right) = (60 + 61)$$

$$\therefore 11^2 = 121 = 60 + 61$$

$$(iv) 19^2 = 361 = \left(\frac{19^2 - 1}{2} + \frac{19^2 + 1}{2} \right)$$

$$= \left(\frac{361 - 1}{2} + \frac{361 + 1}{2} \right)$$

$$= \left(\frac{360}{2} + \frac{362}{2} \right)$$

$$= (180 + 181)$$

$$\therefore 19^2 = 361 = 180 + 181$$

Q2. Do you think the reverse is true, i.e., is the sum of any two consecutive positive integers is perfect square of a number? Give example to support your answer.

Sol. No, it is not true.

Example (i) $11 + 12 = 23$ which is not a perfect square number.

Example (ii) $23 + 24 = 47$ which is not a perfect square number.

Example (iii) $50 + 51 = 101$ which is not a perfect square number.

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Q1. Write the square, making use of the pattern given in conceptual facts

(i) 111111^2 (ii) 1111111^2

Sol. The given pattern is:

$$1^2 = 1$$

$$11^2 = 121$$

$$111^2 = 12321$$

$$1111^2 = 1234321$$

$$11111^2 = 123454321$$

$$111111^2 = 12345654321$$

$$1111111^2 = 1234567654321$$

Thus (i) $111111^2 = 12345654321$

(ii) $1111111^2 = 1234567654321$

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Q1. Can you find the square of the following numbers using the pattern given in conceptual facts

(i) 6666667^2 (ii) 66666667^2

Sol. The given pattern is:

$$7^2 = 49$$

$$67^2 = 4489$$

$$667^2 = 444889$$

$$6667^2 = 44448889$$

$$66667^2 = 4444488889$$

$$666667^2 = 444444888889$$

$$6666667^2 = 44444448888889$$

$$66666667^2 = 4444444488888889$$

.....

.....

(i) yes, $6666667^2 = 44444448888889$

(ii) yes, $66666667^2 = 4444444488888889$

EXERCISE 6.1

Q1. What will be the unit digit of the squares of the following numbers?

(i) 81

(ii) 272

(iii) 799

(iv) 3853

(v) 1234

(vi) 26387

(vii) 52698

(viii) 99880

(ix) 12796

(x) 55555

Sol. (i) Unit digit of $81^2 = 1$

(ii) Unit digit of $272^2 = 4$

(iii) Unit digit of $799^2 = 1$

$$(ii) \text{ Numbers between } 25^2 \text{ and } 26^2 \\ = 2 \times 25 = 50 \quad (\because n = 25)$$

$$(iii) \text{ Number between } 99^2 \text{ and } 100^2 \\ = 2 \times 99 = 198 \quad (\because n = 29)$$

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Q1. Find the squares of the following numbers containing 5 in unit's place.

- (i) 15 (ii) 95 (iii) 105 (iv) 205

EXERCISE 6.2

Q1. Find the square of the following numbers.

(i) 32 (ii) 35 (iii) 86

(iv) 93 (v) 71 (vi) 46

Sol. (i) $32 = 30 + 2$

$$\begin{aligned} \therefore (32)^2 &= (30 + 2)^2 \\ &= 30(30 + 2) + 2(30 + 2) \\ &= 30^2 + 30 \times 2 + 2 \times 30 + 2^2 \\ &= 900 + 60 + 60 + 4 \\ &= 1024 \end{aligned}$$

Thus $(32)^2 = 1024$

(ii) $35 = (30 + 5)$

$$\begin{aligned} \therefore (35)^2 &= (30 + 5)^2 \\ &= 30(30 + 5) + 5(30 + 5) \\ &= (30)^2 + 30 \times 5 + 5 \times 30 + (5)^2 \\ &= 900 + 150 + 150 + 25 \\ &= 1225 \end{aligned}$$

Thus $(35)^2 = 1225$

(iii) $86 = (80 + 6)$

$$\begin{aligned} \therefore 86^2 &= (80 + 6)^2 \\ &= 80(80 + 6) + 6(80 + 6) \\ &= (80)^2 + 80 \times 6 + 6 \times 80 + (6)^2 \\ &= 6400 + 480 + 480 + 36 \\ &= 7396 \end{aligned}$$

Thus $(86)^2 = 7396$

(iv) $93 = (90 + 3)$

$$\begin{aligned} \therefore 93^2 &= (90 + 3)^2 \\ &= 90(90 + 3) + 3(90 + 3) \\ &= (90)^2 + 90 \times 3 + 3 \times 90 + (3)^2 \\ &= 8100 + 270 + 270 + 9 \\ &= 8649 \end{aligned}$$

Thus $(93)^2 = 8649$

(v) $71 = (70 + 1)$

$$\begin{aligned} \therefore 71^2 &= (70 + 1)^2 \\ &= 70(70 + 1) + 1(70 + 1) \\ &= (70)^2 + 70 \times 1 + 1 \times 70 + (1)^2 \\ &= 4900 + 70 + 70 + 1 \\ &= 5041 \end{aligned}$$

Thus $(71)^2 = 5041$

Sol.

(i) $15^2 = 225 = (2 \times 1) \text{ hundred} + 25$

(ii) $95^2 = 9025 = (10 \times 9) \text{ hundred} + 25$

(iii) $105^2 = 11025 = (11 \times 10) \text{ hundred} + 25$

(iv) $205^2 = 42025 = (21 \times 20) \text{ hundred} + 25$

(vi) $46 = (40 + 6)$

$$\begin{aligned} \therefore 46^2 &= (40 + 6)^2 \\ &= 40(40 + 6) + 6(40 + 6) \\ &= (40)^2 + 40 \times 6 + 6 \times 40 + (6)^2 \\ &= 1600 + 240 + 240 + 36 \\ &= 2116 \end{aligned}$$

Thus $(46)^2 = 2116$

Q2. Write a Pythagorean triplet whose one member is

(i) 6 (ii) 14 (iii) 16 (iv) 18

Sol. (i) Let $m^2 - 1 = 6$

[Triplets are in the form $2m, m^2 - 1, m^2 + 1$]

$\therefore m^2 = 6 + 1 = 7$

So, the value of m will not be an integer.

Now, let us try for $m^2 + 1 = 6$

$\Rightarrow m^2 = 6 - 1 = 5$

Also, the value of m will not be an integer.

Now we let $2m = 6$

$\therefore m = 3$ which is an integer.

\therefore Other members are:

$$m^2 - 1 = 3^2 - 1 = 8$$

and $m^2 + 1 = 3^2 + 1 = 10$

Hence, the required triplets are 6, 8 and 10

(ii) Let $m^2 - 1 = 14 \Rightarrow m^2 = 1 + 14 = 15$

\therefore The value of m will not be an integer.

Now take $2m = 14 \therefore m = 7$ which is an integer.

\therefore The member of triplets are

$$2m = 2 \times 7 = 14$$

$$m^2 - 1 = (7)^2 - 1 = 49 - 1 = 48$$

and $m^2 + 1 = (7)^2 + 1 = 49 + 1 = 50$

i.e., (14, 48, 50)

(iii) Let $2m = 16 \therefore m = 8$

\therefore The required triplets are

$$2m = 2 \times 8 = 16$$

$$m^2 - 1 = (8)^2 - 1 = 64 - 1 = 63$$

$$m^2 + 1 = (8)^2 + 1 = 64 + 1 = 65$$

i.e., (16, 63, 65)

(iv) Let $2m = 18 \therefore m = 9$

\therefore Required triplets are:

$$2m = 2 \times 9 = 18$$

$$m^2 - 1 = (9)^2 - 1 = 81 - 1 = 80$$

$$\text{and } m^2 + 1 = (9)^2 + 1 \\ = 81 + 1 = 82$$

i.e., (18, 80, 82)

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Q1. $11^2 = 121$. What is the square root of 121?

Sol. Given that $11^2 = 121$

\therefore Square root of 121 = 11

[\because Square and square roots are under inversion operation]

Q2. $14^2 = 196$. What is the square root of 196?

Sol. Given that $14^2 = 196$

\therefore Square root of 196 = 14

[\because Square and square root are under the inversion operation]

TRY THESE (PAGE 100)

Q1. By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121 (ii) 55 (iii) 36

(iv) 49 (v) 90

Sol. (i) We have $\sqrt{121}$

$$\therefore 121 - 1 = 120, 120 - 3 = 117, 117 - 5 = 112, \\ 112 - 7 = 105, 105 - 9 = 96, 96 - 11 = 85, \\ 85 - 13 = 72,$$

$$72 - 15 = 57, 57 - 17 = 40, 40 - 19 = 21, \\ 21 - 21 = 0$$

Since the above process takes 11 steps to get 0.

$$\therefore \sqrt{121} = 11$$

(ii) We have $\sqrt{55}$

$$\therefore 55 - 1 = 54, 54 - 3 = 51, 51 - 5 = 46, 46 - 7 \\ = 39, 39 - 9 = 30, 30 - 11 = 19, 19 - 13 = 6, \\ 6 - 15 = -9 \neq 0$$

In the above process we did not get 0 in the last step.

\therefore 55 is not a perfect square number.

(iii) We have $\sqrt{36}$

$$\therefore 36 - 1 = 35, 35 - 3 = 32, 32 - 5 = 27, \\ 27 - 7 = 20, 20 - 9 = 11, 11 - 11 = 0$$

In the above process, we have taken 6 steps to get 0.

$$\therefore \sqrt{36} = 6 \text{ which is a perfect square number.}$$

(iv) We have $\sqrt{49}$

$$\therefore 49 - 1 = 48, 48 - 3 = 45, 45 - 5 = 40, 40 - 7 = 33, \\ 33 - 9 = 24, 24 - 11 = 13, 13 - 13 = 0$$

In the above process, we have 7 steps to get 0.

$$\therefore \sqrt{49} = 7 \text{ which is a perfect square number.}$$

(v) We have $\sqrt{90}$

$$\therefore 90 - 1 = 89, 89 - 3 = 86, 86 - 5 = 81, 81 - 7 \\ = 74, 74 - 9 = 65, 65 - 11 = 54, 54 - 13 = 41, \\ 41 - 15 = 26, 26 - 17 = 11 \neq 0$$

In the above process, we did not get 0 in the last step.

\therefore 90 is not a perfect square number.

EXERCISE 6.3

Q1. What could be the possible 'one's' digits of the square root of each of the following numbers?

(i) 9801 (ii) 99856

(iii) 998001 (iv) 657666025

Sol. (i) One's digit in the square root of 9801 may be 1 or 9.

(ii) One's digit in the square root of 99856 may be 4 or 6.

(iii) One's digit in the square root of 998001 may be 1 or 9.

(iv) One's digit in the square root of 657666025 can be 5.

Q2. Without doing any calculation, find the numbers which are surely not perfect squares.

(i) 153 (ii) 257

(iii) 408 (iv) 441

Sol. We know that the numbers ending with 2, 3, 7 or 8 are not perfect squares.

(i) 153 is not a perfect square number. (ending with 3)

(ii) 257 is not a perfect square number. (ending with 7)

(iii) 408 is not a perfect square number. (ending with 8)

(iv) 441 is a perfect square number.

Q3. Find the square roots of 100 and 169 by the method of repeated subtraction.

Sol. Using the method of repeated subtraction of consecutive odd numbers, we have

$$(i) \begin{aligned} 100 - 1 &= 99, 99 - 3 = 96, 96 - 5 = 91, 91 - 7 \\ &= 84, 84 - 9 = 75, 75 - 11 = 64, 64 - 13 = 51, \\ 51 - 15 &= 36, 36 - 17 = 19, 19 - 19 = 0 \end{aligned}$$

(Ten times repetition)

$$\text{Thus } \sqrt{100} = 10$$

$$(ii) \begin{aligned} 169 - 1 &= 168, 168 - 3 = 165, 165 - 5 = 160, \\ 160 - 7 &= 153, 153 - 9 = 144, 144 - 11 = 133, \\ 133 - 13 &= 120, 120 - 15 = 105, 105 - 17 = \\ 88, 88 - 19 &= 69, 69 - 21 = 48, 48 - 23 = 25, \\ 25 - 25 &= 0 \end{aligned}$$

(Thirteen times repetition)

$$\text{Thus } \sqrt{169} = 13$$

Q4. Find the square roots of the following numbers by the prime factorisation Method.

(i) 729 (ii) 400 (iii) 1764 (iv) 4096

(v) 7744 (vi) 9604 (vii) 5929 (viii) 9216

(ix) 529 (x) 8100

Sol. (i) We have 729

Prime factors of 729

$$\begin{aligned} 729 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^2 \times 3^2 \times 3^2 \end{aligned}$$

$$\therefore \sqrt{729} = 3 \times 3 \times 3 = 27$$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(ii) We have 400

Prime factors of 400

$$\begin{aligned} 400 &= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \\ &= 2^2 \times 2^2 \times 5^2 \end{aligned}$$

$$\therefore \sqrt{400} = 2 \times 2 \times 5 = 20$$

2	400
2	200
2	100
2	50
5	25
5	5
	1

(iii) 1764

$$\begin{aligned} 1764 &= 2 \times 2 \times 3 \times 3 \times 7 \times 7 \\ &= 2^2 \times 3^2 \times 7^2 \end{aligned}$$

$$\therefore \sqrt{1764} = 2 \times 3 \times 7 = 42$$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

(iv) 4096

$$\begin{aligned} 4096 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &\times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \end{aligned}$$

$$\therefore \sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(v) Prime factorisation of 7744 is

$$\begin{aligned} 7744 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11 \\ &= 2^2 \times 2^2 \times 2^2 \times 11^2 \end{aligned}$$

$$\therefore \sqrt{7744} = 2 \times 2 \times 2 \times 11 = 88$$

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
11	11
	1

(vi) Prime factorisation of 9604 is

$$\begin{aligned} 9604 &= 2 \times 2 \times 7 \times 7 \times 7 \times 7 \\ &= 2^2 \times 7^2 \times 7^2 \end{aligned}$$

$$\therefore \sqrt{9604} = 2 \times 7 \times 7 = 98$$

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

(vii) Prime factorisation of 5929 is

$$\begin{aligned} 5929 &= 7 \times 7 \times 11 \times 11 \\ &= 7^2 \times 11^2 \end{aligned}$$

$$\therefore \sqrt{5929} = 7 \times 11 = 77$$

7	5929
7	847
11	121
11	11
	1

(viii) Prime factorisation of 9216 is

$$\begin{aligned} 9216 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &\times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^2 \times 2^2 \times 2^2 \times 2^2 \\ &\times 2^2 \times 3^2 \end{aligned}$$

$$\therefore \sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(ix) Prime factorisation of 529 is	23	529
$529 = 23 \times 23 = 23^2$	23	23
$\therefore \sqrt{529} = 23$		1
(x) Prime factorisation of 8100 is	2	8100
$\therefore 8100 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$	2	4050
$\quad \times 5$	3	2025
$\quad = 2^2 \times 3^2 \times 3^2 \times 5^2$	3	675
$\therefore \sqrt{8100} = 2 \times 3 \times 3 \times 5$	3	225
$\quad = 90$	3	75
	5	25
	5	5
		1

Q5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square. Also find the square root of the square number so obtained.

- (i) 252 (ii) 180 (iii) 1008
 (iv) 2028 (v) 1458 (vi) 768

Sol. (i) Prime factorisation of 252 is	2	252
$252 = 2 \times 2 \times 3 \times 3 \times 7$	2	126
Here, the prime factorisation is not in pair. 7 has no pair.	3	63
Thus, 7 is the smallest whole number by which the given number is multiplied to get a perfect square number.	3	21
\therefore The new square number is $252 \times 7 = 1764$	7	7
Square root of 1764 is		1
$\sqrt{1764} = 2 \times 3 \times 7 = 42$		
(ii) Prime factorisation of 180 is	2	180
$180 = 2 \times 2 \times 3 \times 3 \times 5$	2	90
Here, 5 has no pair.	3	45
\therefore New square number	3	15
$\quad = 180 \times 5 = 900$	5	5
Square root of 900 is		1
$\therefore \sqrt{900} = 2 \times 3 \times 5 = 30$		
Thus, 5 is the smallest whole number by which the given number is multiplied to get a square number.	2	1008
(iii) Prime factorisation of 1008 is	2	504
$1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$	2	252
Here, 7 has no pair.	2	126
\therefore New square number	3	63
$\quad = 1008 \times 7 = 7056$	3	21
Thus, 7 is the required number.	7	7
Square root of 7056 is		1
$\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$	2	2028
(iv) Prime factorisation of 2028 is	2	1014
$2028 = 2 \times 2 \times 3 \times 13 \times 13$	3	507
Here, 3 is not in pair.	13	169
Thus, 3 is the required smallest whole number.	13	13
		1

\therefore New square number = $2028 \times 3 = 6084$

Square root of 6084 is

$$\sqrt{6084} = 2 \times 13 \times 3 = 78$$

(v) Prime factorisation of 1458 is	2	1458
$1458 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	3	729
Here, 2 is not in pair.	3	243
Thus, 2 is the required smallest whole number.	3	81
\therefore New square number = 1458×2	3	27
$\quad = 2916$	3	9
Square root of 1458 is		3
		1

$$\sqrt{2916} = 3 \times 3 \times 3 \times 2 = 54$$

(vi) Prime factorisation of 768 is	2	768
$768 = 2 \times 3$	2	384
Here, 3 is not in pair.	2	192
Thus, 3 is the required whole number.	2	96
\therefore New square number = 768×3	2	48
$\quad = 2304$	2	24
Square root of 2304 is	2	12
	2	6
	3	3
		1

$$\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3 = 48$$

Q6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

- (i) 252 (ii) 2925 (iii) 396
 (iv) 2645 (v) 2800 (vi) 1620

Sol. (i) Prime factorisation of 252 is	2	252
$252 = 2 \times 2 \times 3 \times 3 \times 7$	2	126
Here 7 has no pair.	3	63
\therefore 7 is the smallest whole number by which 252 is divided to get a square number.	3	21
\therefore New square number = $252 \div 7 = 36$	7	7
Thus, $\sqrt{36} = 6$		1
(ii) Prime factorisation of 2925 is	3	2925
$2925 = 3 \times 3 \times 5 \times 5 \times 13$	3	975
Here, 13 has no pair.	5	325
\therefore 13 is the smallest whole number by which 2925 is divided to get a square number.	5	65
\therefore New square number	13	13
$\quad = 2925 \div 13 = 225$		1
Thus $\sqrt{225} = 15$		

(iii) Prime factorisation of 396 is
 $396 = 2 \times 2 \times 3 \times 3 \times 11$
 Here 11 is not in pair.
 \therefore 11 is the required smallest whole number by which 396 is divided to get a square number.

2	396
2	198
3	99
3	33
11	11
	1

New square number = $396 \div 11 = 36$
 Thus $\sqrt{36} = 6$

(iv) Prime factorisation of 2645 is
 $2645 = 5 \times 23 \times 23$
 Here, 5 is not in pair.
 \therefore 5 is the required smallest whole number. By which 2645 is multiplied to get a square number

5	2645
23	529
23	23
	1

New square number = $2645 \div 5 = 529$
 Thus, $\sqrt{529} = 23$

(v) Prime factorisation of 2800 is
 $2800 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$
 Here, 7 is not in pair.
 \therefore 7 is the required smallest number. By which 2800 is multiplied to get a square number.

2	2800
2	1400
2	700
2	350
5	175
5	35
7	7
	1

New square number = $2800 \div 7 = 400$

Thus $\sqrt{400} = 20$

(vi) Prime factorisation of 1620 is
 $1620 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$
 Here, 5 is not in pair.
 \therefore 5 is the required smallest prime number. By which 1620 is multiplied to get a square number

2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

= $1620 \div 5 = 324$

Thus $\sqrt{324} = 18$

Q7. The students of class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Sol. Total amount of money donated = ₹ 2401

Total number of students in the class = $\sqrt{2401}$
 $= \sqrt{(7)^2 \times (7)^2}$
 $= 7 \times 7 \times 7 \times 7 = 7 \times 7 = 49$

7	2401
7	343
7	49
7	7
	1

Q8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Sol. Total number of rows = Total number of plants in each row

$= \sqrt{2025}$
 $= \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$
 $= \sqrt{3^2 \times 3^2 \times 5^2}$
 $= 3 \times 3 \times 5 = 45$
 Thus the number of rows and plants = 45

3	2025
3	675
3	225
3	75
5	25
5	5
	1

Q9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.

Sol. LCM of 4, 9, 10 = 180

\therefore The least number divisible by 4, 9 and 10 = 180

Now prime factorisation of 180 is
 $180 = 2 \times 2 \times 3 \times 3 \times 5$
 Here, 5 has no pair.

\therefore The required smallest square number = $180 \times 5 = 900$

2	4, 9, 10
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

Q10. Find the smallest number that is divisible by each of the numbers 8, 15 and 20.

Sol. The smallest number divisible by 8, 15 and 20 is equal to their LCM.

\therefore LCM = $2 \times 2 \times 2 \times 3 \times 5 = 120$

Here, 2, 3 and 5 have no pair.

\therefore The required smallest square number = $120 \times 2 \times 3 \times 5 = 120 \times 30 = 3600$

2	8, 15, 20
2	4, 15, 10
2	2, 15, 5
3	1, 15, 5
5	1, 5, 5
	1, 1, 1

TRY THESE (PAGE 105)

Q1. Without calculating square roots, find the number of digits in the square root of the following numbers.

(i) 25600 (ii) 100000000

(iii) 36864

Sol. (i) Here, number of digits in 25600 = 5 (odd)

\therefore Required number of digits in square root of 25600 = $\frac{5+1}{2} = 3$ digits

(ii) Here, number of digits in 100000000 = 9 (odd)

\therefore Required number of digits in square root of 100000000 = $\frac{9+1}{2} = 5$ digits

(vii)

$$\begin{array}{r} 76 \\ 7 \overline{) 5776} \\ \underline{49} \\ 146 \overline{) 876} \\ \underline{876} \\ 0 \end{array}$$

Thus, $\sqrt{5776} = 76$

(viii)

$$\begin{array}{r} 89 \\ 8 \overline{) 7921} \\ \underline{64} \\ 169 \overline{) 1521} \\ \underline{1521} \\ 0 \end{array}$$

Thus, $\sqrt{7921} = 89$

(ix)

$$\begin{array}{r} 24 \\ 2 \overline{) 576} \\ \underline{4} \\ 44 \overline{) 176} \\ \underline{176} \\ 0 \end{array}$$

Thus, $\sqrt{576} = 24$

(x)

$$\begin{array}{r} 32 \\ 3 \overline{) 1024} \\ \underline{9} \\ 62 \overline{) 124} \\ \underline{124} \\ 0 \end{array}$$

Thus, $\sqrt{1024} = 32$

(xi)

$$\begin{array}{r} 56 \\ 5 \overline{) 3136} \\ \underline{25} \\ 106 \overline{) 636} \\ \underline{636} \\ 0 \end{array}$$

Thus, $\sqrt{3136} = 56$

(xii)

$$\begin{array}{r} 30 \\ 3 \overline{) 900} \\ \underline{9} \\ 00 \end{array}$$

Thus, $\sqrt{900} = 30$

Q2. Find the number of digits in the square root of each of the following numbers (without any calculation)

- (i) 64 (ii) 144 (iii) 4489
(iv) 27225 (v) 390625

Sol. We know that if n is number of digits in a square number then

Number of digits in the square root = $\frac{n}{2}$ if n is even and $\frac{n+1}{2}$ if n is odd.

(i) 64

Here $n = 2$ (even)

\therefore Number of digits in $\sqrt{64} = \frac{2}{2} = 1$

(ii) 144

Here $n = 3$ (odd)

\therefore Number of digits in square root = $\frac{3+1}{2} = 2$

(iii) 4489

Here $n = 4$ (even)

\therefore Number of digits in square root = $\frac{4}{2} = 2$

(iv) 27225

Here $n = 5$ (odd)

\therefore Number of digits in square root = $\frac{5+1}{2} = 3$

(v) 390625

Here $n = 6$ (even)

\therefore Number of digits in square root = $\frac{6}{2} = 3$

Q3. Find the square root of the following decimal numbers.

(i) 2.56 (ii) 7.29 (iii) 51.84

(iv) 42.25 (v) 31.36

Sol. (i)

$$\begin{array}{r} 1.6 \\ 1 \overline{) 2.56} \\ \underline{1} \\ 26 \overline{) 156} \\ \underline{156} \\ 0 \end{array}$$

Thus, $\sqrt{2.56} = 1.6$

(ii)

$$\begin{array}{r} 2.7 \\ 2 \overline{) 7.29} \\ \underline{4} \\ 47 \overline{) 329} \\ \underline{329} \\ 0 \end{array}$$

Thus, $\sqrt{7.29} = 2.7$

$$(iii) \begin{array}{r} 7.2 \\ 7 \overline{) 51.84} \\ \underline{49} \\ 142 \overline{) 284} \\ \underline{284} \\ 0 \end{array}$$

Thus, $\sqrt{51.84} = 7.2$

$$(iv) \begin{array}{r} 6.5 \\ 6 \overline{) 42.25} \\ \underline{36} \\ 125 \overline{) 625} \\ \underline{625} \\ 0 \end{array}$$

Thus, $\sqrt{42.25} = 6.5$

$$(v) \begin{array}{r} 5.6 \\ 5 \overline{) 31.36} \\ \underline{25} \\ 106 \overline{) 636} \\ \underline{636} \\ 0 \end{array}$$

Thus, $\sqrt{31.36} = 5.6$

Q4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 402 (ii) 1989 (iii) 3250

(iv) 825 (v) 4000

Sol. (i)

$$\begin{array}{r} 20 \\ 2 \overline{) 402} \\ \underline{4} \\ 4 \overline{) 02} \end{array}$$

Here remainder is 2

\therefore 2 is the least required number to be subtracted from 402 to get a perfect square

\therefore New number = $402 - 2 = 400$

Thus, $\sqrt{400} = 20$

$$(ii) \begin{array}{r} 44 \\ 4 \overline{) 1989} \\ \underline{16} \\ 84 \overline{) 389} \\ \underline{336} \\ 53 \end{array}$$

Here remainder is 53

\therefore 53 is the least required number to be subtracted from 1989.

\therefore New number = $1989 - 53 = 1936$

Thus, $\sqrt{1936} = 44$

$$(iii) \begin{array}{r} 57 \\ 5 \overline{) 3250} \\ \underline{25} \\ 107 \overline{) 750} \\ \underline{749} \\ 1 \end{array}$$

Here remainder is 1

\therefore 1 is the least required number to be subtracted from 3250 to get a perfect square.

\therefore New number = $3250 - 1 = 3249$

Thus, $\sqrt{3249} = 57$

$$(iv) \begin{array}{r} 28 \\ 2 \overline{) 825} \\ \underline{4} \\ 48 \overline{) 425} \\ \underline{384} \\ 41 \end{array}$$

Here, the remainder is 41

\therefore 41 is the least required number which can be subtracted from 825 to get a perfect square.

New number = $825 - 41 = 784$

Thus, $\sqrt{784} = 28$

$$(v) \begin{array}{r} 63 \\ 6 \overline{) 4000} \\ \underline{36} \\ 123 \overline{) 400} \\ \underline{369} \\ 31 \end{array}$$

Here, the remainder is 31

\therefore 31 is the least required number which should be subtract from 4000 to get a perfect square.

New number = $4000 - 31 = 3969$

Thus, $\sqrt{3969} = 63$

Q5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 525 (ii) 1750 (iii) 252

(iv) 1825 (v) 6412

Sol. (i)

$$\begin{array}{r} 22 \\ 2 \overline{) 525} \\ \underline{4} \\ 42 \overline{) 125} \\ \underline{84} \\ 41 \end{array}$$

Here remainder is 41

∴ It represents that square of 22 is less than 525. Next number is 23 and $23^2 = 529$

Hence, the number to be added
 $= 529 - 525 = 4$

New number = 529

Thus, $\sqrt{529} = 23$

(ii)

$$\begin{array}{r} 41 \\ 4 \overline{) 1750} \\ \underline{16} \\ 81 \overline{) 150} \\ \underline{81} \\ 69 \end{array}$$

Here the remainder is 69

∴ It represents that square of 41 is less than 1750. The next number is 42 and $42^2 = 1764$

Hence, number to be added to 1750 = $1764 - 1750 = 14$

∴ Required perfect square = 1764

$$\sqrt{1764} = 42$$

(iii)

$$\begin{array}{r} 15 \\ 1 \overline{) 252} \\ \underline{16} \\ 25 \overline{) 152} \\ \underline{125} \\ 27 \end{array}$$

Here the remainder is 27.

∴ It represents that square of 15 is less than 252. The next number is 16 and $16^2 = 256$

Hence, number to be added to 252 = $256 - 252 = 4$

New number = $252 + 4 = 256$

∴ Required perfect square = 256

and $\sqrt{256} = 16$

(iv)

$$\begin{array}{r} 42 \\ 4 \overline{) 1825} \\ \underline{16} \\ 82 \overline{) 225} \\ \underline{164} \\ 61 \end{array}$$

The remainder is 61. It represents that square of 42 is less than 1825.

Next number is 43 and $43^2 = 1849$

Hence, number to be added to 1825 = $1849 - 1825 = 24$

The required perfect square is 1849 and $\sqrt{1849} = 43$

(v)

$$\begin{array}{r} 80 \\ 8 \overline{) 6412} \\ \underline{64} \\ 160 \overline{) 12} \\ \underline{0} \\ 12 \end{array}$$

Here, the remainder is 12.

∴ It represents that square of 80 is less than 6412.

The next number is 81 and $81^2 = 6561$

Hence the number to be added
 $= 6561 - 6412 = 149$

The required perfect square is 6561 and $\sqrt{6561} = 81$

Q6. Find the length of the side of a square whose area = 441 m^2

Sol. Let the length of the side of the square be $x \text{ m}$.

∴ Area of the square = $(\text{side})^2 = x^2 \text{ m}^2$

$$\therefore x^2 = 441 \Rightarrow x = \sqrt{441}$$

$$\begin{array}{r} 21 \\ 2 \overline{) 441} \\ \underline{4} \\ 41 \overline{) 41} \\ \underline{41} \\ 0 \end{array}$$

Thus, $x = 21 \text{ m}$

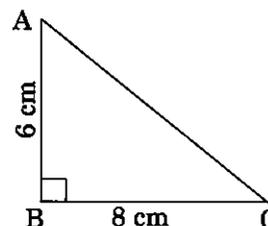
Hence the length of the side of square = 21 m.

Q7. In a right triangle ABC, $\angle B = 90^\circ$.

(a) If AB = 6 cm, BC = 8 cm, find AC

(b) If AC = 13 cm, BC = 5 cm, find AB

Sol. (a) In right triangle ABC



$$AC^2 = AB^2 + BC^2$$

[By Pythagoras Theorem]

$$\Rightarrow AC^2 = (6)^2 + (8)^2$$

$$= 36 + 64 = 100$$

$$\therefore AC = \sqrt{100} = 10$$

Thus, $AC = 10$ cm.

(b) In right triangle ABC

$$AC^2 = AB^2 + BC^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow (13)^2 = AB^2 + (5)^2$$

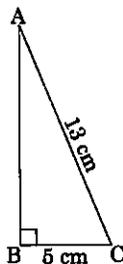
$$\Rightarrow 169 = AB^2 + 25$$

$$\Rightarrow 169 - 25 = AB^2$$

$$\Rightarrow 144 = AB^2$$

$$\therefore AB = \sqrt{144} = 12 \text{ cm}$$

Thus, $AB = 12$ cm.



Q8. A gardener has 1000 plants. He wants to plant these in such a way that number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

Sol. Let number of rows be x .

And number of columns also be x .

$$\therefore \text{Total number of plants} = x \times x = x^2$$

$$x^2 = 1000$$

$$\therefore x = \sqrt{1000}$$

$$\begin{array}{r} 31 \\ 3 \overline{) 1000} \\ \underline{9} \\ 61 \\ \underline{61} \\ 39 \end{array}$$

Here the remainder is 39

So square of 31 is less than 1000.

Next number is 32 and $32^2 = 1024$

Hence the number to be added

$$= 1024 - 1000 = 24$$

Thus the minimum number of plants required by him = 24.

Alternative method:

$$\begin{array}{r} 32 \\ 3 \overline{) 1000} \\ \underline{-9} \\ 100 \\ \underline{-124} \\ -24 \end{array}$$

\therefore The minimum number of plants required by him = 24.

Q9. There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement?

Sol. Let number of children in a row be x . And also that of in a column be x .

$$\therefore \text{Total number of students} = x \times x = x^2$$

$$\therefore x^2 = 500 \Rightarrow x = \sqrt{500}$$

$$\begin{array}{r} 22 \\ 2 \overline{) 500} \\ \underline{4} \\ 42 \\ \underline{42} \\ 16 \end{array}$$

Here the remainder is 16

$$\text{New Number} = 500 - 16 = 484$$

$$\text{and, } \sqrt{484} = 22$$

Thus, 16 students will be left out in this arrangement.

Learning More Q & A

I. VERY SHORT ANSWER (VSA) QUESTIONS

Q1. Find the perfect square numbers between 40 and 50.

Sol. Perfect square numbers between 40 and 50 = 49.

Q2. Which of the following 24^2 , 49^2 , 77^2 , 131^2 or 189^2 end with digit 1?

Sol. Only 49^2 , 131^2 and 189^2 end with digit 1.

Q3. Find the value of each of the following without calculating squares.

(i) $27^2 - 26^2$ (ii) $118^2 - 117^2$

Sol. (i) $27^2 - 26^2 = 27 + 26 = 53$

(ii) $118^2 - 117^2 = 118 + 117 = 235$

Q4. Write each of the following numbers as difference of the square of two consecutive natural numbers.

- (i) 49 (ii) 75 (iii) 125

Sol. (i) $49 = 2 \times 24 + 1$

$\therefore 49 = 25^2 - 24^2$

(ii) $75 = 2 \times 37 + 1$

$\therefore 75 = 38^2 - 37^2$

(iii) $125 = 2 \times 62 + 1$

$\therefore 125 = 63^2 - 62^2$

Q5. Write down the following as sum of odd numbers.

- (i) 7^2 (ii) 9^2

Sol. (i) $7^2 = \text{Sum of first 7 odd numbers}$

$= 1 + 3 + 5 + 7 + 9 + 11 + 13$

(ii) $9^2 = \text{Sum of first 9 odd numbers}$

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

Q6. Express the following as the sum of two consecutive integers.

- (i) 15^2 (ii) 19^2

Sol. (i) $15^2 = 225 = 112 + 113$

$$\left[\because 112 = \frac{15^2 - 1}{2} \text{ and } 113 = \frac{15^2 + 1}{2} \right]$$

(ii) $19^2 = 361 = 180 + 181$

$$\left[\because 180 = \frac{19^2 - 1}{2} \text{ and } 181 = \frac{19^2 + 1}{2} \right]$$

Q7. Find the product of the following:

- (i) 23×25 (ii) 41×43

Sol. (i) $23 \times 25 = (24 - 1)(24 + 1)$

$= 24^2 - 1$

$= 576 - 1 = 575$

(ii) $41 \times 43 = (42 - 1)(42 + 1)$

$= 42^2 - 1 = 1764 - 1 = 1763$

Q8. Find the squares of:

- (i) $-\frac{3}{7}$ (ii) $-\frac{9}{17}$

Sol. (i) $\left(-\frac{3}{7}\right)^2 = \left(-\frac{3}{7}\right)\left(-\frac{3}{7}\right) = \frac{9}{49}$

(ii) $\left(-\frac{9}{17}\right)^2 = \left(-\frac{9}{17}\right)\left(-\frac{9}{17}\right) = \frac{81}{289}$

Q9. Check whether (6, 8, 10) is a Pythagorean triplet.

Sol. $2m, m^2 - 1$ and $m^2 + 1$ represent the Pythagorean triplet.

Let $2m = 6 \Rightarrow m = 3$

$m^2 - 1 = (3)^2 - 1 = 9 - 1 = 8$

and $m^2 + 1 = (3)^2 + 1 = 9 + 1 = 10$

Hence (6, 8, 10) is a Pythagorean triplet.

Alternative Method

$(6)^2 + (8)^2 = 36 + 64 = 100 = (10)^2$

$\Rightarrow (6, 8, 10)$ is a Pythagorean triplet.

Q10. Using property, find the value of the following:

- (i) $19^2 - 18^2$ (ii) $23^2 - 22^2$

Sol. (i) $19^2 - 18^2 = 19 + 18 = 37$

(ii) $23^2 - 22^2 = 23 + 22 = 45$

II. SHORT ANSWER (SA) QUESTIONS

Q11. Using the prime factorisation method, find which of the following numbers are not perfect squares.

- (i) 768 (ii) 1296

Sol. (i)

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$768 = 2 \times 3$

Here, 3 is not in pair.

$\therefore 768$ is not a perfect square.

(ii)

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Here, there is no number left to make pair.

$\therefore 1296$ is a perfect square.

Q12. Which of the following triplets are Pythagorean?

- (i) (14, 48, 50) (ii) (18, 79, 82)

Sol. We know that $2m, m^2 - 1$ and $m^2 + 1$ make Pythagorean triplets.

- (i) For (14, 48, 50),

Put $2m = 14 \Rightarrow m = 7$

$\therefore m^2 - 1 = (7)^2 - 1$

$= 49 - 1 = 48$

$m^2 + 1 = (7)^2 + 1$

$= 49 + 1 = 50$

Hence (14, 48, 50) is a Pythagorean triplet.

(ii) For (18, 79, 82)

$$\text{Put } 2m = 18 \Rightarrow m = 9$$

$$m^2 - 1 = (9)^2 - 1$$

$$= 81 - 1 = 80$$

$$m^2 + 1 = (9)^2 + 1$$

$$= 81 + 1 = 82$$

Hence (18, 79, 82) is not a Pythagorean triplet.

Q13. Find the square root of the following using successive subtraction of odd numbers starting from 1.

(i) 169 (ii) 81 (iii) 225

Sol. (i) $169 - 1 = 168, 168 - 3 = 165,$

$$165 - 5 = 160, 160 - 7 = 153,$$

$$153 - 9 = 144, 144 - 11 = 133,$$

$$133 - 13 = 120, 120 - 15 = 105,$$

$$105 - 17 = 88, 88 - 19 = 69,$$

$$69 - 21 = 48, 48 - 23 = 25,$$

$$25 - 25 = 0$$

We have subtracted odd numbers 13 times to get 0.

$$\therefore \sqrt{169} = 13$$

(ii) $81 - 1 = 80, 80 - 3 = 77,$

$$77 - 5 = 72, 72 - 7 = 65,$$

$$65 - 9 = 56, 56 - 11 = 45,$$

$$45 - 13 = 32, 32 - 15 = 17,$$

$$17 - 17 = 0$$

We have subtracted 9 times to get 0.

$$\therefore \sqrt{81} = 9$$

(iii) $225 - 1 = 224, 224 - 3 = 221,$

$$221 - 5 = 216, 216 - 7 = 209,$$

$$209 - 9 = 200, 200 - 11 = 189,$$

$$189 - 13 = 176, 176 - 15 = 161,$$

$$161 - 17 = 144, 144 - 19 = 125,$$

$$125 - 21 = 104, 104 - 23 = 81,$$

$$81 - 25 = 56, 56 - 27 = 29,$$

$$29 - 29 = 0$$

We have subtracted 15 times to get 0.

$$\therefore \sqrt{225} = 15$$

Q14. Find the square root of the following using prime factorisation

(i) 441

(ii) 2025

(iii) 7056

(iv) 4096

Sol. (i) $441 = 3 \times 3 \times 7 \times 7$

$$\therefore \sqrt{441} = 3 \times 7$$

$$= 21$$

3	441
3	147
7	49
7	7
	1

(ii) $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$

$$\therefore \sqrt{2025} = 3 \times 3 \times 5 = 45$$

3	2025
3	675
3	225
3	75
5	25
5	5
	1

(iii) $7056 = 2 \times 2 \times 2 \times 2$

$$\times 3 \times 3 \times 7 \times 7$$

$$\therefore \sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$$

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

(iv) $4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$\times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$\therefore \sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Q15. Find the least square number which is divisible by each of the number 4, 8 and 12.

Sol. LCM of 4, 8, 12 is the least number divisible by each of them.

$$\therefore \text{LCM of } 4, 8 \text{ and } 12 = 24$$

$$\therefore 24 = 2 \times 2 \times 2 \times 3$$

To make it perfect square multiply 24 by the product of unpaired numbers, i.e., $2 \times 3 = 6$

$$\therefore \text{Required number} = 24 \times 6 = 144$$

2	4, 8, 12
2	2, 4, 6
2	1, 2, 3
3	1, 1, 3
	1, 1, 1

Q16. Find the square roots of the following decimal numbers

(i) 1056.25

(ii) 10020.01

Sol.

	32.5
3	$\overline{1056.25}$
	9
62	156
	124
645	3225
	3225
	0

$$\text{Hence } \sqrt{1056.25} = 32.5$$

$$(ii) \begin{array}{r} 100.1 \\ 1 \overline{) 10020.01} \\ \underline{100} \\ 2001 \\ \underline{2001} \\ 0 \end{array}$$

Hence $\sqrt{10020.01} = 100.1$

Q17. What is the least number that must be subtracted from 3793 so as to get a perfect square? Also, find the square root of the number so obtained.

Sol. First, we find the square root of 3793 by division method.

$$\begin{array}{r} 61 \\ 6 \overline{) 3793} \\ \underline{36} \\ 121 \\ \underline{121} \\ 72 \end{array}$$

Here, we get a remainder 72

$$\therefore 61^2 < 3793$$

$$\therefore \text{Required perfect square number} = 3793 - 72 = 3721 \text{ and } \sqrt{3721} = 61$$

Q18. Fill in the blanks:

(a) Perfect square number between 60 and 70 is _____.

(b) The square root of 361 ends with digit _____.

(c) The sum of first n odd numbers is _____.

(d) The number of digits in the square root of 4096 is _____.

(e) If $(-3)^2 = 9$, then the square root of 9 is _____.

(f) Number of digits in the square root of 1002001 is _____.

(g) Square root of $\frac{36}{625}$ is _____.

(h) The value of $\sqrt{63 \times 28} =$ _____.

Sol. (a) 64 (b) 9 (c) n^2 (d) 2

(e) ± 3 (f) 4 (g) $\frac{6}{25}$ (h) 42

Q19. Simplify: $\sqrt{900} + \sqrt{0.09} + \sqrt{0.000009}$

Sol. We know that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$$\begin{aligned} \therefore \sqrt{900} &= \sqrt{9 \times 100} \\ &= \sqrt{9} \times \sqrt{100} = 3 \times 10 = 30 \end{aligned}$$

$$\sqrt{0.09} = \sqrt{0.3 \times 0.3} = 0.3$$

$$\sqrt{0.000009} = \sqrt{0.003 \times 0.003} = 0.003$$

$$\begin{aligned} \therefore \sqrt{900} + \sqrt{0.09} + \sqrt{0.000009} \\ &= 30 + 0.3 + 0.003 \\ &= 30.303 \end{aligned}$$

III. HIGHER ORDER THINKING SKILLS (HOTS) QUESTIONS

Q20. Find the value of x if

$$\sqrt{1369} + \sqrt{0.0615 + x} = 37.25$$

Sol. We have $\sqrt{1369} + \sqrt{0.0615 + x} = 37.25$

$$\therefore \sqrt{1369} = 37$$

$$\begin{array}{r} 37 \\ 3 \overline{) 1369} \\ \underline{9} \\ 67 \\ \underline{67} \\ 0 \end{array}$$

$$\therefore 37 + \sqrt{0.0615 + x} = 37.25$$

$$\Rightarrow \sqrt{0.0615 + x} = 37.25 - 37$$

$$\Rightarrow \sqrt{0.0615 + x} = 0.25$$

Squaring both sides, we have

$$0.0615 + x = 0.0625$$

$$\Rightarrow x = 0.0625 - 0.0615$$

$$\therefore x = 0.0010$$

Hence $x = 0.001$

Q21. Simplify: $\sqrt{\frac{(0.105 + 0.024 - 0.008) \times 0.85}{1.7 \times 0.022 \times 0.25}}$

$$\text{Sol. } \sqrt{\frac{(0.105 + 0.024 - 0.008) \times 0.85}{1.7 \times 0.022 \times 0.25}}$$

$$= \sqrt{\frac{(0.129 - 0.008) \times 0.85}{1.7 \times 0.022 \times 0.25}}$$

$$= \sqrt{\frac{0.121 \times 0.85}{1.7 \times 0.22 \times 0.25}}$$

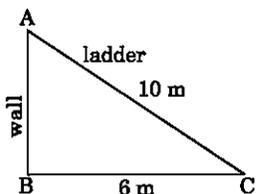
$$= \sqrt{\frac{121^{11} \times 85^5}{17 \times 22_2 \times 25_5}}$$

[Removing the decimals]

$$= \sqrt{\frac{11}{10}} = \sqrt{1.1}$$

Hence, the required result = $\sqrt{1.1}$.

- Q22.** A ladder 10 m long rests against a vertical wall. If the foot of the ladder is 6 m away from the wall and the ladder just reaches the top of the wall, how high is the wall? (NCERT Exemplar)



Sol. Let AC be the ladder.
Therefore, $AC = 10$ m
Let BC be the distance between the foot of the ladder and the wall.
Therefore, $BC = 6$ m
 $\triangle ABC$ forms a right angled triangle, right angled at B.

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = AB^2 + 6^2$$

or $AB^2 = 10^2 - 6^2 = 100 - 36 = 64$

or $AB = \sqrt{64} = 8$ m

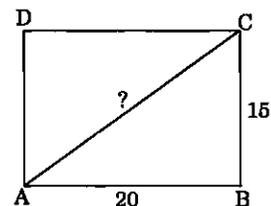
Hence, the wall is 8 m high.

- Q23.** Find the length of a diagonal of a rectangle with dimensions 20 m by 15 m. (NCERT Exemplar)

Sol. Using Pythagoras theorem, we have
Length of diagonal of the rectangle

$$= \sqrt{(l^2 + b^2)} \text{ units}$$

$$\begin{aligned} &= \sqrt{(20^2 + 15^2)} \text{ m} \\ &= \sqrt{400 + 225} \text{ m} \\ &= \sqrt{625} \text{ m} \\ &= 25 \text{ m} \end{aligned}$$



Hence, the length of diagonal is 25 m.

- Q24.** The area of a rectangular field whose length is twice its breadth is 2450 m^2 . Find the perimeter of the field.

Sol. Let the breadth of the field be x metres. Then length of the field is $2x$ metres.

Therefore, area of the rectangular field

$$= \text{length} \times \text{breadth}$$

$$= (2x)(x) = (2x^2) \text{ m}^2$$

Given that area is 2450 m^2 .

Therefore, $2x^2 = 2450$

$$x^2 = \frac{2450}{2}$$

$$x = \sqrt{1225} \text{ or } x = 35 \text{ m}$$

Hence, breadth = 35 m

and length = $35 \times 2 = 70$ m

Perimeter of the field = $2(l + b)$

$$= 2(70 + 35) \text{ m}$$

$$= 2 \times 105 \text{ m} = 210 \text{ m}$$

Test Yourself

I. VERY SHORT ANSWER (VSA) QUESTIONS

- Q1.** What is the one's digit in the square root of the following?

(i) 361

(ii) 484

(iii) 1600

(iv) 961

- Q2.** What will be the number of zeros in the square of the following numbers?

(i) 50

(ii) 500

(iii) 1040

(iv) 1001

- Q3.** Find the perfect square numbers between 70 and 90.

- Q4.** Write the following square as the sum of consecutive natural numbers.

(i) 3^2

(ii) 5^2

(iii) 7^2

(iv) 11^2

- Q5.** Find the sum of first 10 odd numbers.

- Q6.** Express 121 as the sum of 11 consecutive odd numbers.

- Q7.** Write the Pythagorean triplets whose one member is :

(i) 6

(ii) 16

(iii) 8

(iv) 20

- Q8.** Find the estimated values of the following:

(i) $\sqrt{80}$

(ii) $\sqrt{250}$

(iii) $\sqrt{1023}$

(iv) $\sqrt{360}$

- Q9.** Find the number of digits in the square roots of the following numbers (without calculations)

(i) 576

(ii) 3136

(iii) 390625

(iv) 27225

- Q10.** Find the square root of the following:

(i) 6.25

(ii) 51.84

(iii) 7.29

(iv) 31.36

II. SHORT ANSWER (SA) QUESTIONS

Q11. Which of the following numbers is not a perfect square?

(i) 64 (ii) 728

(iii) 291 (iv) 256

Q12. Find the square root of the following:

(i) 7744 (ii) 0.0625

(iii) 0.000009 (iv) 7.84

Q13. Find the square root of $56\frac{569}{1225}$.

Q14. Find the square root of 3881.29.

Q15. What should be subtracted from 6249 to get a perfect square number? Also find its square root.

Q16. What least number must be added to 594 to make the sum a perfect square?

Q17. If $\frac{x}{\sqrt{2.25}} = 550$, find the value of x .

Q18. Find the value of:

$$\sqrt{15625} + \sqrt{156.25} + \sqrt{1.5625}$$

Q19. If $\sqrt{625} = \sqrt{5^x}$, find the value of x .

Q20. Find the least number that must be added to 9598 to make it a perfect square.

ANSWERS

1. (i) 9 (ii) 2 (iii) 0 (iv) 1

2. (i) 2 (ii) 4 (iii) 2 (iv) No zeroes

3. 81

4. (i) 4 + 5 (ii) 12 + 13

(iii) 24 + 25 (iv) 60 + 61

5. 100

6. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21

7. (i) (6, 8, 10) (ii) (16, 63, 65)

(iii) (8, 15, 17) (iv) (20, 99, 101)

8. (i) 9 (ii) 16 (iii) 32 (iv) 19

9. (i) 2 (ii) 2 (iii) 3 (iv) 3

10. (i) 2.5 (ii) 7.2 (iii) 2.7 (iv) 5.6

11. (ii) and (iii)

12. (i) 88 (ii) 0.25

(iii) 0.003 (iv) 2.8

13. $7\frac{18}{35}$ 14. 62.3 15. 8, 79

16. 31, 25 17. 825 18. 138.75

19. 4 20. 6

Internal Assessment

Q1. Fill in the blanks:

(a) $1 + 3 + 5 + 7 + 9 + 11 + 13 = \underline{\hspace{2cm}}$.

(b) The unit digit in the square root of 66564 will be either $\underline{\hspace{1cm}}$ or $\underline{\hspace{1cm}}$.

(c) The number of zeros at the end of a perfect square is always $\underline{\hspace{1cm}}$.

(d) Square of $-0.07 = \underline{\hspace{1cm}}$.

(e) $99^2 - 98^2 = \underline{\hspace{1cm}}$.

(f) $\sqrt{0.0081} = \underline{\hspace{1cm}}$.

Q2. Answer True (T) or False (F).

(a) The number of digits in the square root of 66564 is 4.

(b) All the numbers ending with even number of zeros are always perfect squares.

(c) There is one perfect square number between 50 and 60.

(d) The value of $\frac{3}{\sqrt{0.09}}$ is 10.

(e) $\sqrt{400 \times 4} = 40$

Q3. The number 12 and 21 are mirror image of each other such that their squares 144 and 441 are also mirror images of each other. Find one such other pair.

Q4. Find the least number of four digits that is a perfect square. (NCERT Exemplar)

Q5. Find the greatest number of three digits that is a perfect square. (NCERT Exemplar)

Q6. Find the least square number which is exactly divisible by 3, 4, 5, 6 and 8. (NCERT Exemplar)

Q7. Find the square root of 2.9929.

Q8. Observe the pattern and fill in the blanks.

$$1^2 = 1$$

$$11^2 = 121$$

$$111^2 = 12321$$

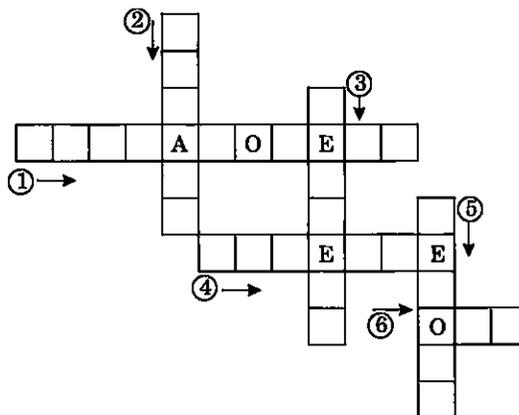
$$1111^2 = \underline{\hspace{1cm}}$$

$$11111^2 = \underline{\hspace{1cm}}$$

Q9. Complete the following puzzle.

Directions

1. (6, 8, 10) are _____ triplet.
2. The _____ of 21 is 441.
3. 441 is a _____ square number.
4. Square root is the _____ operation of square.
5. The number ending with odd number of _____ cannot be a perfect square.
6. Square of every odd natural number is always _____ natural number.



ANSWERS

- | | | | | | |
|--------------------|------------|-------------|-------------|------------|-----------------------|
| 1. (a) 49 | (b) 2, 8 | (c) even | (d) 0.0049 | (e) 197 | (f) 0.09 |
| 2. (a) F | (b) F | (c) F | (d) T | (e) T | |
| 3. 13 and 31 | 4. 1024 | 5. 61 | 6. 3600 | 7. 1.73 | 8. 1234321, 123454321 |
| 9. (1) PYTHAGOREAN | (2) SQUARE | (3) PERFECT | (4) INVERSE | (5) ZEROES | (6) ODD |

◆ Periodic Assessment

SET-1

Time: 1 hour

M.M.: 20

General Instructions

- All questions are compulsory. However there is an internal choice.
- Section-A consists of 4 questions carrying 1 mark each.
- Section-B consists of 8 questions carrying 2 mark each.
- Calculator is not permitted.

SECTION-A

1. Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.
2. Find one rational number between $\frac{1}{4}$ and $\frac{1}{2}$.
3. Simplify: $\frac{x+1}{2x+3} = \frac{3}{8}$
4. Construct a parallelogram ABCD if AB = 5 cm, BC = 6 cm and $\angle D = 85^\circ$.

SECTION-B

5. In the given figure (i) find the value of x .

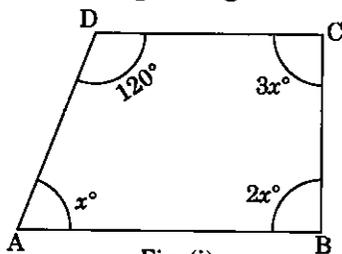


Fig. (i)

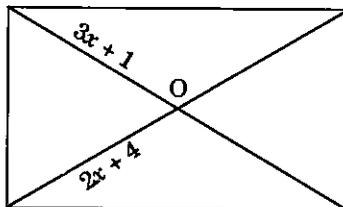


Fig. (ii)

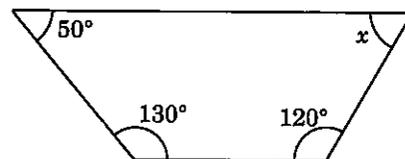


Fig. (iii)

6. In the given fig. (ii) ABCD is a rectangle. If $OA = 2x + 4$, $OD = 3x + 1$, then find x .
7. Find the value of x in the given fig. (iii).
8. Find the value of y if $\frac{7y+4}{y+2} = \frac{-4}{3}$.
9. A man's age is three times his son's age. Ten years ago he was 5 times his son's age. Find their present ages.
10. If you subtract $\frac{1}{2}$ from a number and multiply the results by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?

OR

The difference between two numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers?

11. Find the rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.
12. Simplify: $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$.

◇ Periodic Assessment

SET-2

Time: 1 hour

M.M.: 20

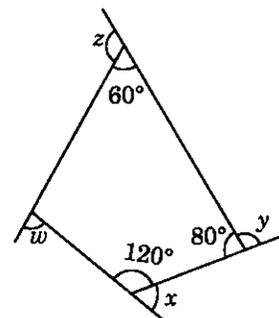
General Instructions: Same as paper-1

SECTION-A

- Using property, simplify: $\left\{ \frac{7}{5} \times \left(\frac{-3}{12} \right) \right\} + \left\{ \frac{7}{5} \times \frac{5}{12} \right\}$.
- Write five rational numbers less than 2.
- Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.
- Solve for x : $\frac{15}{4} - 7x = 9$.

SECTION-B

- A grandfather is ten times older than his grand daughter. He is also 54 years older than her. Find their present ages.
- Solve: $0.16(5x - 2) = 0.4x + 7$.
- In the adjoining figure, find $x + y + z + w$.



- How many sides does a regular polygon has if each of its interior angles is 165° ?
- The three angles of a quadrilateral are 76° , 54° and 108° . Find the measure of the fourth angle.
- Construct a rhombus whose diagonals are 10 cm and 8 cm.

OR

Construct a quadrilateral ABCD in which $BC = 4.5$ cm, $AB = 4$ cm, $AD = 4$ cm, $\angle B = 75^\circ$, $\angle A = 90^\circ$ and $\angle C = 120^\circ$.

- Find: $\frac{-4}{5} \times \frac{3}{7} \times \frac{15}{16} \times \left(\frac{-14}{9} \right)$.
- The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

OR

The sum of three consecutive multiples of 11 is 363. Find these multiples.