CHAPTER 7

Cubes and Cube Roots

Understanding the Lesson

- Cubes and perfect cubes.
- Prime factorisation of cubes.
- Cube roots of cube number
- Cube roots through prime factorisation.
- Hardy–Ramanujan number.
- Properties of cube numbers.
- Cube of a negative integers.
- Cube of a rational number.
- Properties of cube roots.

Conceptual Facts

- A natural number n is a perfect cube if there exists a natural number m such that $m \times m \times m = n$

  For example: 1, 8, 27 .... are all perfect cubes
- **Properties of Cubes of Numbers:**

  (*i*) Cubes of all odd numbers are odd. Thus $3^3 = 27$, $5^3 = 125$, etc.

  (*ii*) Cubes of all even numbers are even. Thus $2^3 = 8$, $4^3 = 64$, $6^3 = 216$, etc.

  (*iii*) Cubes of all negative numbers are always negative. Thus $(-1)^3 = -1$, $(-2)^3 = -8$, $(-3)^3 = -27$, etc.

  (*iv*) $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
- **Properties of Cube Roots**

  (*i*) $\sqrt[3]{-a^3} = -a$ (*ii*) $\sqrt[3]{ab} = \sqrt[3]{a} \circ \sqrt[3]{b}$ (*iii*) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}, b \neq 0$

  A pattern of cube

$$1^3 = 1$$
$$2^2 = 8 = 3 + 5$$
$$3^3 = 27 = 7 + 9 + 11$$
$$4^3 = 64 = 13 + 15 + 17 + 19$$
$$5^3 = 125 = 21 + 23 + 25 + 27 + 29$$

TRY THESE – PAGE 111

Q1. Find the one's digit of the cube of each of the following numbers.

(*i*) 3331 (*ii*) 8888
(*iii*) 149 (*iv*) 1005
(*v*) 1024 (*vi*) 77
(*vii*) 5022 (*viii*) 53

Sol. (*i*) Unit digit of 3331 is 1
and $1^3 = 1$
Thus, the one's digit of $(1331)^3 = 1$.

(*ii*) Unit digit of 8888 is 8
and $8^3 = 512$
Thus, the one's digit of $(8888)^3$ is 2.

(*iii*) Unit digit of 149 is 9 and $9^3 = 729$
Thus, the one's digit in $(149)^3 = 9$.

(*iv*) Unit digit of 1005 is 5 and $5^3 = 125$
Thus, the one's digit of $(1005)^3 = 5$.

(*v*) Unit digit of 1024 is 4 and $4^3 = 64$
Thus, one' digit of $(1024)^3 = 4$.

(*vi*) Unit digit of 77 is 7 and $7^3 = 343$
Thus, one's digit of $(77)^3 = 3$.

(*vii*) Unit digits of 5022 is 2 and $2^3 = 8$
Thus, one's digit of $(5022)^3 = 8$.

(*viii*) Unit place of 53 is 3 and $3^3 = 27$
Thus, the one's digit of $(53)^3 = 7$.

Try These – Page 111

Q1. (A) Express the following numbers as the sum of odd numbers using the above pattern given in conceptual fact.

(*a*) 6^3 (*b*) 8^3 (*c*) 7^3

(B) Consider the following pattern.

$2^3 - 1^3 = 1 + 2 \times 1 \times 3$
$3^3 - 2^3 = 1 + 3 \times 2 \times 3$
$4^3 - 3^3 = 1 + 4 \times 3 \times 3$

Using the above pattern, find the value of the following:

(*i*) $7^3 - 6^3$ (*ii*) $12^3 - 11^3$
(*iii*) $20^3 - 19^3$ (*iv*) $51^3 - 50^3$

Sol. (A) (*a*) $6^3 = 31 + 33 + 35 + 37 + 39 + 41$

(*b*) $8^3 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

(*c*) $7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

(B) (*i*) $7^3 - 6^3 = 1 + 7 \times 6 \times 3$

(*ii*) $12^3 - 11^3 = 1 + 12 \times 11 \times 3$

(*ii*) $51^3 - 50^3 = 1 + 51 \times 50 \times 3$

Try These – Page 112

Q. Which of the following are perfect cubes?

(*i*) 400 (*ii*) 3375 (*iii*) 8000 (*iv*) 15625
(*v*) 9000 (*vi*) 6859 (*vii*) 2025 (*viii*) 10648

Sol. (*i*) Prime factorisation of 400 is:

2	400
2	200
2	100
2	50
5	25
5	5
	1

$400 = \underline{2 \times 2 \times 2} \times 2 \times 5 \times 5$

In the above factorisation 2 and 5 do not form a group of three.

Thus, 400 is not a perfect cube.

(*ii*) Prime factorisation of 3375 is

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

In the above factorisation 3 and 5 have formed a group of three.

Thus, 3375 is a perfect cube.

(*iii*) Prime factorisation of 8000 is:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$8000 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

In the above factorisation 2 and 5 have formed a group of three.

Thus, 8000 is a perfect cube.

(*iv*) Prime factorisation of 15625 is:

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$15625 = \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

In the above prime factorisation, 5 has two groups of three.

Thus, 15625 is a perfect cube.

(*v*) Prime factorisation of 9000, is:

2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

$9000 = \underline{2 \times 2 \times 2} \times 3 \times 3 \times \underline{5 \times 5 \times 5}$

In the above factorisation 3 has not formed a group of three.

Thus, 9000 is not a perfect cube.

(*vi*) Prime factorsation of 6859, is

19	6859
19	361
19	19
	1

$6859 = \underline{19 \times 19 \times 19}$

In the above factorisation, 19 has formed a group of three.

Thus, 6859 is a perfect cube.

(*vii*) Prime factorisation of 2025, is:

3	2025
3	675
3	225
3	75
5	25
5	5
	1

$2025 = \underline{3 \times 3 \times 3} \times 3 \times 5 \times 5$

In the above factorisation 3 and 5 have not formed a group of three.

Thus, 2025 is not a perfect cube.

(*viii*) Prime factorisation of 10648 is

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$10648 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

In the above factorisation 2 and 11 have formed a group of three.

Thus, 10648 is a perfect cube.

EXERCISE 7.1

Q1. Which of the following numbers are not perfect cubes?

(*i*) 216 (*ii*) 128 (*iii*) 1000

(*iv*) 100 (*v*) 46656

Sol. (*i*) Prime factorisation of 216 is:

$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

In the above factorisation, 2 and 3 have formed a group of three.

Thus, 216 is a perfect cube.

2	216
2	108
2	54
3	27
3	9
3	3
	1

(*ii*) Prime factorisation of 128 is:

$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$

Here, 2 is left without making a group of three.

Thus 128 is not a perfect cube.

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(*iii*) Prime factorisation of 1000, is:

$1000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

Here, no number is left for making a group of three.

Thus, 1000 is a perfect cube.

2	1000
2	500
2	250
5	125
5	25
5	5
	1

(*iv*) Prime factorisation of 100, is:

$100 = \underline{2 \times 2} \times \underline{5 \times 5}$

Here 2 and 5 have not formed a group of three.

Thus, 100 is not a perfect cube.

2	100
2	50
5	25
5	5
	1

(*v*) Prime factorisation of 46656, is:

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$
$\times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

Here 2 and 3 have formed the groups of three.

Thus, 46656 is a perfect cube.

Q2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(*i*) 243 (*ii*) 256 (*iii*) 72

(*iv*) 675 (*v*) 100

Sol. (*i*) Prime factorisation of 243, is:

$243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

$= 3^3 \times 3 \times 3$

Here, number 3 is required to make 3×3 a group of three, i.e., $3 \times 3 \times 3$

Thus, the required smallest number to be multiplied is 3.

3	243
3	81
3	27
3	9
3	3
	1

(*ii*) Prime factorisation of 256, is:

$256 = \underline{2 \times 2 \times 2}$
$\times \underline{2 \times 2 \times 2} \times 2 \times 2$

$= 2^3 \times 2^3 \times 2 \times 2$

Here, a number 2 is needed to make 2×2 a group of three, i.e., $2 \times 2 \times 2$

Thus, the required smallest number to be multiplied is 2.

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(*iii*) Prime factorisation of 72, is:

$72 = \underline{2 \times 2 \times 2} \times 3 \times 3$

$= 2^3 \times 3 \times 3$

Here, a number 3 is required to make 3×3 a group of three, i.e. $3 \times 3 \times 3$

Thus, the required smallest number to be multiplied is 3.

2	72
2	36
2	18
3	9
3	3
	1

(*iv*) Prime factorisation of 675, is:

$675 = \underline{3 \times 3 \times 3} \times 5 \times 5$

$= 3^3 \times 5 \times 5$

Here, a number 5 is required to make 5×5 a group of three to make it perfect cube, i.e. $5 \times 5 \times 5$

Thus, the required smallest number is 5.

3	675
3	225
3	75
5	25
5	5
	1

(*v*) Prime factorisation of 100, is:

$100 = 2 \times 2 \times 5 \times 5$

Here, number 2 and 5 are needed to multiplied $2 \times 2 \times 5 \times 5$ to make it a perfect cube, i.e.,

2	100
2	50
5	25
5	5
	1

$2 \times 2 \times 2 \times 5 \times 5 \times 5$

Thus, the required smallest number to be multiplied is $2 \times 5 = 10$.

Q3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(*i*) 81 (*ii*) 128 (*iii*) 135

(*iv*) 192 (*v*) 704

Sol. (*i*) Prime factorisation of 81, is:

$81 = \underline{3 \times 3 \times 3} \times 3 = 3^3 \times 3$

3	81
3	27
3	9
3	3
	1

Here, a number 3 is the number by which 81 is divided to make it a perfect cube, i.e., $81 \div 3 = 27$ which is a perfect cube.

Thus, the required smallest number to be divided is 3.

(*ii*) Prime factorisation of 128, is:

$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$

$= 2^3 \times 2^3 \times 2$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Here, a number 2 is the smallest number by which 128 is divided to make it a perfect cube, i.e., $128 \div 2 = 64$ which is a perfect cube.

Thus, 2 is the required smallest number.

(*iii*) Prime factorisation of 135 is:

$135 = \underline{3 \times 3 \times 3} \times 5$

$= 3^3 \times 5$

3	135
3	45
3	15
5	5
	1

Here, 5 is the smallest number by which 135 is divided to make a perfect cube, i.e., $135 \div 5 = 27$ which is a perfect cube.

Thus, 5 is the required smallest number.

(*iv*) Prime factorisation of 192 is:

$192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

$= 2^3 \times 2^3 \times 3$

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Here, 3 is the smallest number by which 192 is divided to make it a perfect cube, i.e., $192 \div 3 = 64$ which is a perfect cube.

Thus, 3 is the required smallest number.

(*v*) Prime factorisation of 704 is:

$704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

$= 2^3 \times 2^3 \times 11$

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

Here, 11 is the smallest number by which 704 is divided to make it a perfect cube, i.e., $704 \div 11 = 64$ which is a perfect cube.

Thus, 11 is the required smallest number.

Q4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will be needed to form a cube?

Sol. The sides of the cuboid are given as 5 cm, 2 cm and 5 cm.

$\therefore$ Volume of the cuboid

$= 5 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm} = 50 \text{ cm}^3$

For the prime factorisaiton of 50, we have

$50 = 2 \times 5 \times 5$

To make it a perfect cube, we must have

$2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= 20 \times (2 \times 5 \times 5)$

$= 20 \times$ volume of the given cuboid

Thus, the required number of cuboids = 20.

EXERCISE 7.2

Q1. Find the cube root of each of the following numbers by prime factorisation method.

(*i*) 64 (*ii*) 512

(*iii*) 10648 (*iv*) 27000

(*v*) 15625 (*vi*) 13824

(*vii*) 110592 (*viii*) 46656

(*ix*) 175616 (*x*) 91125

Sol. (*i*) Prime factorisation of 64 is:

$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3$

$\therefore \sqrt[3]{64} = 2 \times 2 = 4$

2	64
2	32
2	16
2	8
2	4
2	2
	1

The cube root of 64 = 4

(*ii*) Prime factorisation of 512, is

$512 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3$

$\therefore \sqrt[3]{512} = 2 \times 2 \times 2$

$= 8$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Thus, the cube root of 512 = 8

(*iii*) Prime factorisation of 10648, is:

$10648 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

$= 2^3 \times 11^3$

$\therefore \sqrt[3]{10648} = 2 \times 11$

$= 22$

Thus, the cube root of 10648 = 22.

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

(*iv*) Prime factorisation of 27000, is:

$27000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 2^3 \times 3^3 \times 5^3$

$\therefore \sqrt[3]{27000} = 2 \times 3 \times 5$

$= 30$

Thus, the cube root of 27000 = 300.

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

(*v*) Prime factorisation of 15625 is:

$15625 = \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

$= 5^3 \times 5^3$

$\therefore \sqrt[3]{15625} = 5 \times 5$

$= 25$

Thus, the cube root of 15625 = 25.

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

(*vi*) Prime factorisation of 13824 is:

$13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 2^3 \times 3^3$

$\therefore \sqrt[3]{13824} = 2 \times 2 \times 2 \times 3$

$= 24$

Thus, the cube root of 13824 = 24.

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(*vii*) Prime factorisation of 110592 is:

$110592 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 2^3 \times 2^3 \times 3^3$

$\therefore \sqrt[3]{110592} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

Thus, the cube root of 110592 = 48.

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(*viii*) Prime factorisation of 46656, is:

$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 3^3 \times 3^3$

$\therefore \sqrt[3]{46656} = 2 \times 2 \times 3 \times 3$

$= 36$

Thus, the cube root of 46656 = 36.

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(*ix*) Prime factorisation of 175616 is:

$175616 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$= 2^3 \times 2^3 \times 2^3 \times 7^3$

$\therefore \sqrt[3]{175616} = 2 \times 2 \times 2 \times 7$

$= 56$

Thus, the cube root of 175616 = 56.

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

(*x*) Prime factorisation of 91125 is:

3	91125
3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$$91125 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$$
$$= 3^3 \times 3^3 \times 5^3$$

$\therefore \sqrt[3]{9125} = 3 \times 3 \times 5 = 45$

Thus, the cube root of 9125 = 45.

Q2. State True or False.

(*i*) Cube of any odd number is even.

(*ii*) A perfect cube does not end with two zeros.

(*iii*) If square of a number ends with 5, then its cube ends with 25.

(*iv*) There is no perfect cube which ends with 8.

(*v*) The cube of a two digit number may be a three digit number.

(*vi*) The cube of a two digit number may have seven or more digits.

(*vii*) The cube of a single digit number may be a single digit number.

Sol. (*i*) False – Cube of any odd number is always odd, e.g., $(7)^3 = 343$

(*ii*) True – A perfect cube does not end with two zeros.

(*iii*) True – If a square of a number ends with 5, then its cube ends with 25, .e.g., $(5)^2 = 25$ and $(5)^3 = 625$

(*iv*) False – $(12)^2 = 1728$ (ends with 8)

(*v*) False – $(10)^3 = 1000$ (4-digit number)

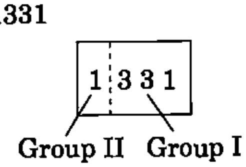
(*vi*) False – $(99)^3 = 970299$ (6-digit number)

(*vii*) True – $(2)^3 = 8$ (1-digit number)

Q3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768.

Sol. The given perfect cube = 1331

Forming groups of three from the right most digits of 1331



IInd group = 1

Ist group = 331

One's digit in first group = 1

$\therefore$ One's digit in the required cube root may be 1.

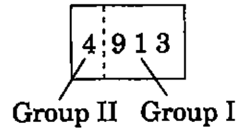
Second group has only 1.

$\therefore$ Estimated cube root of 1331 = 11

Thus $\sqrt[3]{1331} = 11$

(*i*) Given perfect cube = 4913

Forming groups of three from the right most digit of 4913



IInd group = 4

Ist group = 913

One's place digit in 913 is 3.

One's place digit in the cube root of the given number may be 7.

Now in IInd goup digit is 4

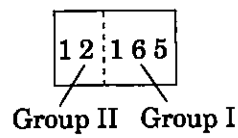
$$1^3 < 4 < 2^3$$

$\therefore$ Ten's place must be the smallest number 1.

Thus, the estimated cube root of 4913 = 17.

(*ii*) Given perfect cube = 12167

Forming group of three from the right most digits of 12167



We have IInd group = 12

Ist group = 167

The one's place digit in 167 is 7.

$\therefore$ One's place digit in the cube root of the given number may be 3.

Now in IInd group, we have 12

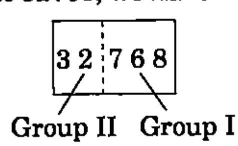
$$2^3 < 12 < 3^3$$

$\therefore$ Ten's place of the required cube root of the given number = 2.

Thus, the estimated cube root of 12167 = 23.

(*iii*) Given perfect cube = 32768

Forming groups of three from the right most digits of 32768, we have



IInd group = 32

Ist group = 768

One's place digit in 768 is 8.

∴ One's place digit in the cube root of the given number may be 2.

Now in IInd group, we have 32

$$3^3 < 32 < 4^3$$

∴ Ten's place of the cube root of the given number = 3.

Thus, the estimated cube root of 32768 = 32.

Learning More Q & A

I. Very Short Answer (VSA) Questions

Q1. Find the cubes of the following:

(*a*) 12 (*b*) – 6 (*c*) $\frac{2}{3}$ (*d*) $-\frac{5}{6}$

Sol. (*a*) $12^3 = 12 \times 12 \times 12 = 1728$

(*b*) $(-6)^3 = (-6) \times (-6) \times (-6) = -216$

(*c*) $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

(*d*) $\left(\frac{-5}{6}\right)^3 = \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) = \frac{-125}{216}$

Q2. Find the cubes of the following:

(*a*) 0.3 (*b*) 0.8

(*c*) .001 (*d*) 2 – 0.3

Sol. (*a*) $(0.3)^3 = 0.3 \times 0.3 \times 0.3 = 0.027$

(*b*) $(0.8)^3 = 0.8 \times 0.8 \times 0.8 = 0.512$

(*c*) $(0.001)^3 = (0.001) \times (0.001) \times (0.001)$

$= 0.000000001$

(*d*) $(2 - 0.3)^3 = (1.7)^3 = 1.7 \times 1.7 \times 1.7$

$= 4.913$

Q3. Is 135 a perfect cube?

Sol. Prime factorisation of 135, is:

$135 = \underline{3 \times 3 \times 3} \times 5$

3	135
3	45
3	15
5	5
	1

We find that on making triplet, the number 5 does not make group of triplet.

Hence, 135 is not a perfect cube.

Q4. Find the cube roots of the following:

(*a*) 1728

(*b*) 3375

Sol. (*a*) Prime factorisation of 1728 is:

$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 3^3$

∴ $\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(*b*) We find the prime factorisation of 3375 as follows:

$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 3^3 \times 5^3$

∴ $\sqrt[3]{3375}$

$= 3 \times 5$

$= 15$

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

Q5. Examine if (*i*) 200 (*ii*) 864 are perfect cubes.

Sol. (*i*) $200 = \underline{2 \times 2 \times 2} \times 5 \times 5$

If we form triplet of equal factors, the number 2 forms a group of three where-as 5 does not do it.

Therefore, 200 is not a perfect cube.

2	200
2	100
2	50
5	25
5	5
	1

(*ii*) We have $864 = \underline{2 \times 2 \times 2} \times \boxed{2 \times 2} \times \underline{3 \times 3 \times 3}$

If we form triplet of equal factors, the number 2 and 3 form a group of three whereas another group of 2's does not do so.

Therefore, 864 is not a perfect cube.

2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Q6. Find the smallest number by which 1323 may be multiplied so that the product is a perfect cube.

Sol. $1323 = \underline{3 \times 3 \times 3} \times \boxed{7 \times 7}$

Since, we required one more 7 to make a triplet of 7.

Therefore 7 is the smallest number by which 1323 may be multiplied to make it a perfect cube.

3	1323
3	441
3	147
7	49
7	7
	1

Q7. What is the smallest number by which 2916 should be divided so that the quotient is a perfect cube?

Sol. Prime factorisation of

$2916 = \boxed{2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Since, we required one more 2 to make a triplet

Therefore, the required smallest number by which 2916 should be divided to make it a perfect cube is $2 \times 2 = 4$, i.e., $2916 \div 4 = 729$ which is a perfect cube.

Q8. Check whether 1728 is a perfect cube by using prime factorisation. *(NCERT Exemplar)*

Sol. Prime factorisation of 1728 is

$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Since all prime factors can be grouped in triplets.

Therefore, 1728 is a perfect cube.

Q9. Using prime factorisation, find the cube root of 5832. *(NCERT Exemplar)*

Sol. The prime factorisation of 5832 is

$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Therefore,

$$\sqrt[3]{5832} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$$

$$= 2 \times 3 \times 3 = 18$$

Q10. Show that $\sqrt[3]{27} \times \sqrt[3]{125} = \sqrt[3]{27 \times 125}$

Sol. LHS $= \sqrt[3]{27} \times \sqrt[3]{125}$

$$= \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{5 \times 5 \times 5}$$

$$= \sqrt[3]{3^3} \times \sqrt[3]{5^3} = 3 \times 5 = 15$$

RHS $= \sqrt[3]{27 \times 125}$

$$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}$$

$$= \sqrt[3]{3^3 \times 5^3} = 3 \times 5 = 15$$

Hence, LHS = RHS

Short Answers (SA) Questions

Q11. Simplify: $\sqrt[3]{5 - \frac{10}{27}}$

Sol. $\sqrt[3]{5 - \frac{10}{27}} = \sqrt[3]{\frac{5 \times 27 - 10}{27}}$

$$= \sqrt[3]{\frac{135 - 10}{27}} = \sqrt[3]{\frac{125}{27}}$$

$$= \sqrt[3]{\frac{5 \times 5 \times 5}{3 \times 3 \times 3}} = \sqrt[3]{\frac{5^3}{3^3}} = \frac{5}{3}$$

Q12. Find the cube roots of

(*i*) $4\frac{12}{125}$ (*ii*) -0.729

Sol. (*i*) $\sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{4 \times 125 + 12}{125}} = \sqrt[3]{\frac{500 + 12}{125}}$

$$= \sqrt[3]{\frac{512}{125}}$$

$$= \sqrt[3]{\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{5 \times 5 \times 5}}$$

$$= \sqrt[3]{\frac{2^3 \times 2^3 \times 2^3}{5^3}}$$

$$= \frac{2 \times 2 \times 2}{5} = \frac{8}{5}$$

(*ii*) $\sqrt[3]{-0.729} = \sqrt[3]{\frac{-729}{1000}}$

$$= \sqrt[3]{-\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{10 \times 10 \times 10}}$$

$$= \sqrt[3]{-\frac{3^3 \times 3^3}{10^3}}$$

$$= -\frac{3 \times 3}{10} = -\frac{9}{10} = -0.9$$

Q13. Express the following numbers as the sum of odd numbers using the given pattern

$$5^3 - 4^3 = 1 + \frac{5 \times 4}{2} \times 6 = 61$$

$$7^3 - 6^3 = 1 + \frac{7 \times 6}{2} \times 6 = 127$$

(*i*) $9^3 - 8^3$ = ________ = ________

(*ii*) $12^3 - 11^3$ = ________ = ________

(*iii*) $51^3 - 50^3$ = ________ = ________

Sol. (*i*) $9^3 - 8^3 = 1 + \frac{9 \times 8}{2} \times 6 = 217$

(*ii*) $12^3 - 11^3 = 1 + \frac{12 \times 11}{2} \times 6 = 397$

(*iii*) $51^3 - 50^3 = 1 + \frac{51 \times 50}{2} \times 6 = 7651$

Q14. Observe the following pattern and complete the blank spaces.

$1^3 = 1$

$2^3 - 1^3 = 1 + \frac{2\times1}{2}\times6 = 7$

$\therefore \quad 2^3 = 1 + 7 = 8$

$3^3 - 2^3 = 1 + \frac{3\times2}{2}\times6 = 19$

$\therefore \quad 3^3 = 2^3 + 19$

$\Rightarrow \quad 3^3 = 1 + 7 + 19$

(*i*) 4^4 = ________ (*ii*) 6^3 = ________

(*iii*) 7^3 = ________ (*iv*) 9^3 = ________

(*v*) 11^3 = ________

Sol. Using the formula $n^3 - (n-1)^3$

$$= 1 + \frac{n\times(n-1)}{2}\times6 \text{, we get}$$

(*i*) $4^3 - 3^3 = 1 + \frac{4\times3}{2}\times6 = 37$

$\therefore \quad 4^3 = 3^3 + 37 = 1 + 7 + 19 + 37$

(*ii*) $6^3 - 5^3 = 1 + \frac{6\times5}{2}\times6 = 91$

$\therefore \quad 6^3 = 5^3 + 91$

$= 1 + 7 + 19 + 37 + 61 + 91$

(*iii*) $9^3 = 9^3 - 8^3 = 1 + \frac{9\times8}{2}\times6 = 217$

$\therefore \quad 9^3 = 8^3 + 217$

$= 1 + 7 + 19 + 37 + 61 + 91 + 127 + 169 + 217$

(*iv*) $11^3 - 10^3 = 1 + \frac{11\times10}{2}\times6 = 331$

$\therefore \quad 11^3 = 10^3 + 331$

$= 1 + 7 + 19 + 37 + 61 + 91 + 127 + 169 + 217 + 271 + 331$

Test Yourself

I. Very Short Answer (VSA) Questions

Q1. Find the cubes of the following:

(*i*) $-\frac{1}{2}$ (*ii*) $\frac{6}{7}$

(*iii*) –1 (*iv*) 0

Q2. Which of the following are perfect cube?

(*i*) 64 (*ii*) 72

(*iii*) 144 (*iv*) 125

Q3. Find the cube root of –1728.

Q4. Find the cubes of the following:

(*i*) 0.01 (*ii*) –0.2

Q5. Find the cube root of 64 × 27 × 125.

Q6. Without actual calculations, estimate the cube root of the following:

(*i*) 4913 (*ii*) –4096

Q7. Find the cube root of the following by successive subtraction of the numbers 1, 7, 19, 37, 61, 91, 127, 169 ......

(*i*) 216 (*ii*) 512

(*iii*) 1728 (*iv*) 343

Hint: For $\sqrt[3]{64}$ we have 64 – 1 = 63, 63 – 7 = 56, 56 –19 = 37, 37 – 37 = 0, we find that by successive subtraction of given numbers four times, we get 0

$\therefore \sqrt[3]{64} = 4$

Q8. Find the values of the following:

(*i*) $\sqrt[3]{0.008}$ (*ii*) $\sqrt[3]{0.000216}$

Short Answer (SA) Questions

Q9. What is the smallest number by which 2916 should be divided so that the quotient is a perfect cube?

Q10. Evaluate: $\sqrt{\sqrt[3]{0.000729}} + \sqrt[3]{0.008}$

Q11. If $\frac{\sqrt[3]{0.216}}{x} = \sqrt[3]{1000000}$ then find the value of x.

Q12. Find the value of $\sqrt[3]{\frac{64\times125\times1728}{512\times729}}$

Q13. If the cube root of 175616 is 56 then find the value of

$$\sqrt[3]{175.616} + \sqrt[3]{0.175616} + \sqrt[3]{0.000175616}$$

Q14. Find the cube roots of the following through estimates

(*i*) 17576 (*ii*) 857375

Q15. State True or False

(*a*) The cube root of –64 = 4.

(*b*) There is no cube which ends with 4.

(*c*) Cube of any o dd is also odd.

(*d*) A perfect cube does not end with one zero.

(*e*) The cube of an even is always even.

(*f*) 0.01 is a perfect cube.

ANSWERS

1. (i) $-\frac{1}{8}$ (ii) $\frac{216}{343}$
 (iii) –1 (iv) 0
2. (i), (iv)
3. –12
4. (i) 0.000001 (ii) –0.008
5. 60
6. (i) 17 (ii) –16
7. (i) 6 (ii) 8 (iii) 12 (iv) 7
8. (i) 0.2 (ii) 0.06
9. 4
10. 0.5
11. 0.006
12. $\frac{10}{3}$
13. 6.216
14. (i) 26 (ii) 95
15. (a) F (b) F (c) T
 (d) T (e) T (f) F

Internal Assessment

Q1. Multiple Choice Questions (MCQs)

(i) The cube root of $-\frac{216}{125}$ is

(a) $\frac{6}{5}$ (b) $-\frac{6}{5}$ (c) $\frac{5}{6}$ (d) $-\frac{5}{6}$

(ii) The cube root of 0.008 is

(a) 0.2 (b) 0.002

(c) 0.0002 (d) $\frac{2}{100}$

(iii) The cube root of 0.001331 is

(a) 1.1 (b) 0.011

(c) 0.11 (d) 0.101

(iv) The volume of a cubical box is 32.768 cm^3, its side is

(a) 3.2 (b) 4.2 (c) 3.6 (d) 4.8

(v) The value of $(0.3)^3 + (0.2)^3$ is

(a) 0.35 (b) 0.035

(c) 3.5 (d) 0.0035

Q2. Fill in the blanks:

(i) Cube of a positive number is always ________.

(ii) Cube root of a negative number is always ________.

(iii) Write the next two numbers

$\sqrt[3]{1}, \sqrt[3]{8}, \sqrt[3]{27}, ..., ...$

(iv) The cube root of 125 × 512 = ________.

(v) The cube root of ________ = –32

Q3. Evaluate the following:

(a) $(10)^3 - (4)^3$ (b) $\sqrt[3]{0.001} \times 10$

Q4. If $6^{x-2} = 216$, then find the value of x.

Q5. Find the value of $\sqrt[3]{\frac{-192}{81}}$.

Q6. Being a student of mathematics, you must have heard about two great mathematicians Prof. G.H. Hardy and Dr. S. Ramanujan. Once Prof. G. H. Hardy came to visit Dr. Ramanujan by a Taxi with a number 1729. Prof. Hardy described 1729, a dull number but Dr. Ramanujan described this number as an interesting number as it can be expressed as a sum of two cubes in two different ways

$1729 = 1728 + 1 = 12^3 + 1^3$

$1729 = 1000 + 729 = 10^3 + 9^3$

Now, express the following numbers in the above pattern

(i) 4104 (ii) 13832

Q7. If one side of a cube is 15 m in length, find its volume. *(NCERT Exemplar)*

Q8. Find the length of each side of a cube if its volume is 512 cm^3. *(NCERT Exemplar)*

Q9. Three numbers are in the ratio 1:2:3 and the sum of their cubes is 4500. Find the numbers. *(NCERT Exemplar)*

Q10. Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, find the cube root of the larger number. *(NCERT Exemplar)*

ANSWERS

1. (i) (b) (ii) (a) (iii) (c)
 (iv) (a) (v) (b)
2. (i) positive (ii) negative (iii) $\sqrt[3]{64}, \sqrt[3]{125}$
 (iv) 40 (v) –32768
3. (a) 936 (b) 1
4. 5
5. $-\frac{4}{3}$
6. (i) $16^3 + 2^3$ and $15^3 + 9^3$
 (ii) $18^3 + 20^3$ and $2^3 + 24^3$
7. 3,375
8. 8 cm
9. 5, 10 and 15
10. 6