CHAPTER 12 Exponents and Powers

Understanding the Lesson

- Positive Integral Exponents.
- Negative Integral Exponents.
- Expanded form of a whole number.
- Laws of Exponents.
- Expressing small numbers in standard form.

Conceptual Facts

- A numeral $(a)^n$ is called an exponential expression where a is called base and n the exponent or power.
- $a^n = a \times a \times a \times a$ ..... $\times a$ (n times)
- A negative rational number raised to an even power is always positive, e.g., $\left(-\frac{1}{2}\right)^4 = \frac{1}{16}$
- A negative rational number raised to an odd power is always negative, i.e., $= \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$
- Laws of exponents:

$(i)\ \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$ $\quad (ii)\ \left[\left(\frac{a}{b}\right)^m\right]^n = \left(\frac{a}{b}\right)^{mn}$

$(iii)\ \left(\frac{a}{b}\right)^m \times \left(\frac{c}{d}\right)^m = \left(\frac{a}{b} \times \frac{c}{d}\right)^m$ $\quad (iv)\ \left(\frac{a}{b}\right)^m + \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$

$(v)\ \left(\frac{a}{b}\right)^{-m} = \frac{1}{\left(\frac{a}{b}\right)^m} = \left(\frac{b}{a}\right)^m$ $\quad (vi)\ \left(\frac{a}{b}\right)^0 = 1$ $\quad (vii)\ \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

TRY THESE (PAGE 194)

Q1. Find the multiplicative inverse of the following:
$(i)\ 2^{-4}$ $\quad (ii)\ 10^{-5}$ $\quad (iii)\ 7^{-2}$
$(iv)\ 5^{-3}$ $\quad (v)\ 10^{-100}$

Sol. (i) Multiplicative inverse of $2^{-4} = \frac{1}{2^{-4}} = 2^{-(-4)} = 2^4$

(ii) Multiplicative inverse of $10^{-5} = \frac{1}{10^{-5}} = 10^{-(-5)} = 10^5$

(iii) Multiplicative inverse of $7^{-2} = \frac{1}{7^{-2}} = 7^{-(-2)} = 7^2$

(iv) Multiplicative inverse of $5^{-3} = \frac{1}{5^{-3}} = 5^{-(-3)} = 5^3$

(v) Multiplicative inverse of $10^{-100} = \frac{1}{10^{-100}} = 10^{-(-100)} = 10^{100}$

TRY THESE (PAGE 194)

Q1. Expand the following numbers using exponents.
(i) 1025.63 $\quad (ii)$ 1256.249

Sol. (i) 1025.63

$= 1 \times 1000 + 2 \times 10 + 5 \times 1 + 6 \times \frac{1}{10} + 3 \times \frac{1}{100}$

$= 1 \times 10^3 + 2 \times 10 + 5 \times 1 + 6 \times 10^{-1} + 3 \times 10^{-2}$

(ii) 1256.249

$= 1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1 + 2 \times \frac{1}{10} + 4 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10 + 6 \times 1$
$+ 2 \times 10^{-1} + 4 \times 10^{-2} + 9 \times 10^{-3}$

TRY THESE (*PAGE 195*)

Q1. Simplify and write in exponential form.

(*i*) $(-2)^{-3} \times (-2)^{-4}$ (*ii*) $p^3 \times p^{-10}$

(*iii*) $3^2 \times 3^{-5} \times 3^6$

Sol. (*i*) $(-2)^{-3} \times (-2)^{-4} = (-2)^{(-3)+(-4)}$
$= (-2)^{-7}$

(*ii*) $p^3 \times p^{-10} = p^{3+(-10)} = p^{-7}$

(*iii*) $3^2 \times 3^{-5} \times 3^6$
$= (3)^{2-5+6} = (3)^3 = 3^3$

EXERCISE 12.1

Q1. Evaluate:

(*i*) 3^{-2} (*ii*) $(-4)^{-2}$ (*iii*) $\left(\frac{1}{2}\right)^{-5}$

Sol. (*i*) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

(*ii*) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

(*iii*) $\left(\frac{1}{2}\right)^{-5} = (2)^5 = 32$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$

Q2. Simplify and express the result in power notation with positive exponent.

(*i*) $(-4)^5 \div (-4)^8$ (*ii*) $\left(\frac{1}{2^3}\right)^2$

(*iii*) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (*iv*) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(*v*) $2^{-3} \times (-7)^{-3}$

Sol. (*i*) $(-4)^5 \div (-4)^8$

$= (-4)^{5-8} = (-4)^{-3} = \frac{1}{(-4)^3}$

$= \left(-\frac{1}{4}\right)^3$ $[\because a^m \div a^n = a^{m-n}]$

(*ii*) $\left(\frac{1}{2^3}\right)^2 = \frac{(1)^2}{(2^3)^2} = \frac{1}{2^6} = \left(\frac{1}{2}\right)^6$

(*iii*) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{(5)^4}{(3)^4}$

$= \frac{(3)^4 \times (5)^4}{(3)^4} = 5^4$

(*iv*) $(3^{-7} \div 3^{-10}) \times 3^{-5}$
$= 3^{-7-(-10)} \times 3^{-5}$
$= 3^{-7+10} \times 3^{-5}$
$= 3^3 \times 3^{-5} = 3^{3-5}$
$= 3^{-2} = \frac{1}{3^2} = \left(\frac{1}{3}\right)^2$

(*v*) $2^{-3} \times (-7)^{-3}$
$= [2 \times (-7)]^{-3}$
$= (-14)^{-3} = -(14)^{-3}$
$= -\frac{1}{14^3} = \left(-\frac{1}{14}\right)^3$

Q3. Find the value of

(*i*) $(3^0 + 4^{-1}) \times 2^2$

(*ii*) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(*iii*) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(*iv*) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(*v*) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Sol. (*i*) $(3^0 + 4^{-1}) \times 2^2$

$= \left(1 + \frac{1}{4}\right) \times 4$

$= \left(\frac{4+1}{4}\right) \times 4$

$= \frac{5}{4} \times 4 = 5$

(*ii*) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

$= \left(\frac{1}{2} \times \frac{1}{4}\right) + \frac{1}{2^2}$

$= \frac{1}{8} \div \frac{1}{4}$

$= \frac{1}{8} \times \frac{4}{1} = \frac{1}{2}$

(*iii*) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$= 2^2 + 3^2 + 4^2$

$= 4 + 9 + 16 = 29$

(*iv*) $(3^{-1} + 4^{-1} + 5^{-1})^0$

$= \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^0$

$$= \left(\frac{20+15+12}{60}\right)^0$$

$$= \left(\frac{47}{60}\right)^0 = 1 \qquad \left[\because \left(\frac{a}{b}\right)^0 = 1\right]$$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

$$= \left(-\frac{2}{3}\right)^{-2\times 2} = \left(-\frac{2}{3}\right)^{-4}$$

$$= \left(-\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$$

Q4. Evaluate:

(i) $\frac{8^{-1}\times 5^3}{2^{-4}}$ (ii) $(5^{-1}\times 2^{-1})\times 6^{-1}$

Sol. (i) $\frac{8^{-1}\times 5^3}{2^{-4}}$

$$= \frac{\frac{1}{8}\times 125}{\frac{1}{2^4}} = \frac{125}{8}\times 2^4$$

$$= \frac{125}{\cancel{8}}\times \cancel{16}^2 = 250$$

(ii) $(5^{-1}\times 2^{-1})\times 6^{-1}$

$$= \left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6}$$

$$= \frac{1}{10}\times\frac{1}{6} = \frac{1}{60}$$

Q5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Sol. $5^m \div 5^{-3} = 5^5$

$\Rightarrow 5^{m-(-3)} = 5^5 \qquad [\because a^m \div a^n = a^{m-n}]$

$\Rightarrow 5^{m+3} = 5^5$

Comparing the powers of equal bases, we have $m + 3 = 5$

$\therefore m = 5 - 3 = 2$, i.e., $m = 2$

Q6. Evaluate:

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Sol. (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

$$= (3-4)^{-1}$$

$$= (-1)^{-1} = \frac{1}{-1} = -1$$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

$$= \left(\frac{8}{5}\right)^7\times\left(\frac{8}{5}\right)^{-4} = \left(\frac{8}{5}\right)^{7-4}$$

$$= \left(\frac{8}{5}\right)^3 = \frac{512}{125}$$

Q7. Simplify:

(i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}\ (t\neq 0)$

(ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

Sol. (i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}$

$$= \frac{5^2\times t^{-4}}{5^{-3}\times 10\times t^{-8}}$$

$$= \frac{5^{2+3}\times t^{-4+8}}{10} = \frac{5^5\times t^4}{10}$$

$$= \frac{^1\cancel{5}.5^4\times t^4}{\cancel{10}_2} = \frac{625}{2}t^4$$

(ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

$$= \frac{3^{-5}\times(2\times 5)^{-5}\times 125}{5^{-7}\times(2\times 3)^{-5}}$$

$$= \frac{3^{-5}\times 2^{-5}\times 5^{-5}\times 5^3}{5^{-7}\times 2^{-5}\times 3^{-5}}$$

$[\because (ab)^m = a^m\times b^m]$

$= (3)^{-5+5}\times(2)^{-5+5}\times(5)^{7-5+3}$

$= 3^0\times 2^0\times 5^5$

$= 1\times 1\times 5^5 = 5^5$

Try These (*Page 199*)

Q1. Write the following numbers in standard form.

(i) 0.000000564 (ii) 0.0000021

(iii) 21600000 (iv) 15240000

Sol. (i) 0.000000564

$$= \frac{564}{1000000000} = \frac{564}{10^9}$$

$$= \frac{5.64\times 10^2}{10^9}$$

$$= 5.64\times 10^{2-9}$$

$$= 5.64\times 10^{-7}$$

Hence, the required standard form $= 5.64\times 10^{-7}$

(*ii*) 0.0000021

$$= \frac{21}{10000000} = \frac{21}{10^7}$$

$$= \frac{2.1 \times 10}{10^7} = 2.1 \times 10^{-6}$$

Hence, the required standard form

$= 2.1 \times 10^{-6}$

(*iii*) 21600000

$= 2.16 \times 10000000$

$= 2.16 \times 10^7$

Hence, the required standard form

$= 2.16 \times 10^7$

(*iv*) 15240000

$= 1.524 \times 10000000$

$= 1.524 \times 10^7$

Hence, the required standard form

$= 1.524 \times 10^7$

Q2. Write all the facts given is the standard form.

Sol. Facts;

(*i*) Exponent is positive when we express large numbers in standard form.

(*ii*) Exponent is negative when we express smaller numbers (<1) in standard form.

EXERCISE 12.2

Q1. Express the following numbers in standard form:

(*i*) 0.0000000000085

(*ii*) 0.00000000000942

(*iii*) 6020000000000000

(*iv*) 0.00000000837

(*v*) 31860000000

Sol. (*i*) 0.0000000000085

$$= \frac{85}{10000000000000} = \frac{8.5 \times 10}{10^{13}}$$

$= 8.5 \times 10^{1-13} = 8.5 \times 10^{-12}$

Hence, the required standard form

$= 8.5 \times 10^{-12}$

(*ii*) 0.00000000000942

$$= \frac{942}{100000000000000}$$

$$= \frac{9.42 \times 100}{100000000000000} = \frac{9.42 \times 10^2}{10^{14}}$$

$= 9.42 \times 10^{2-14} = 9.42 \times 10^{-12}$

Hence, the required standard form

$= 9.42 \times 10^{-12}$

(*iii*) 6020000000000000

$= 6.02 \times 1000000000000000$

$= 6.02 \times 10^{15}$

Hence, the required standard form

$= 6.02 \times 10^{15}$

(*iv*) 0.00000000837

$$= \frac{837}{100000000000}$$

$$= \frac{8.37 \times 100}{100000000000} = \frac{8.37 \times 10^2}{10^{11}}$$

$= 8.37 \times 10^{2-11} = 8.37 \times 10^{-9}$

Hence, the required standard from

$= 8.37 \times 10^{-9}$

(*v*) 31860000000

$= 3.186 \times 10000000000$

$= 3.186 \times 10^{10}$

Hence, the required standard from

$= 3.186 \times 10^{10}$

Q2. Express the following numbers in usual form.

(*i*) 3.02×10^{-6} (*ii*) 4.5×10^4

(*iii*) 3×10^{-8} (*iv*) 1.0001×10^9

(*v*) 5.8×10^{12} (*vi*) 3.61492×10^6

Sol. (*i*) 3.02×10^{-8}

$$= 3.02 \times \frac{1}{10^6} = \frac{302}{100 \times 10^6}$$

$$= \frac{302}{10^2 \times 10^6} = \frac{302}{10^{2+6}} = \frac{302}{10^8}$$

$= 302 \times 10^{-8}$

$= 0.00000302$

Hence, $3.02 \times 10^{-6} = 0.00000302$

(*ii*) 4.5×10^4

$$= \frac{45}{10} \times 10^4 = 45 \times 10^{4-1}$$

$= 45 \times 10^3 = 45000$

Hence, $4.5 \times 10^4 = 45000$

(*iii*) 3×10^{-8}

$$= 3 \times \frac{1}{10^8} = \frac{3}{100000000}$$

$= 0.00000003$

Hence, $3 \times 10^{-8} = 0.00000003$

(*iv*) 1.0001×10^9

$$= \frac{10001}{10000} \times 10^9 = \frac{10001}{10^4} \times 10^9$$

$= 10001 \times 10^{9-4} = 10001 \times 10^5$

Hence, $1.0001 \times 10^9 = 1000100000$

(v) 5.8×10^{12}

$= \frac{58}{10} \times 10^{12} = 58 \times 10^{12-1}$

$= 58 \times 10^{11}$

$= 5800000000000$

Hence, $5.8 \times 10^{12} = 5800000000000$

(vi) 3.61492×10^6

$= \frac{361492}{100000} \times 10^6$

$= \frac{361492 \times 10^6}{10^5}$

$= 361492 \times 10^{6-5}$

$= 361492 \times 10$

$= 3614920$

Hence, $3.61492 \times 10^6 = 3614920$

Q3. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm.

Sol. (i) 1 micron $= \frac{1}{1000000}$

$= \frac{1}{10^6}$ m

$= 10^{-6}$ m

(ii) Charge of an election

$= 0.000,000,000,000,000,000,16$

$= \frac{16}{1,000,000,000,000,000,000,00}$

$= \frac{1.6 \times 10}{1,000,000,000,000,000,000,00}$

$= \frac{1.6 \times 10}{10^{20}} = 1.6 \times 10^{1-20} = 1.6 \times 10^{-19}$

(iii) Size of a bacteria = 0.0000005 m

$= \frac{5}{10000000}$ m

$= \frac{0.5 \times 10}{10^7}$ m $= 0.5 \times 10^{1-7}$ m

$= 0.5 \times 10^{-6}$ m $= 5 \times 10^{-7}$ m

(iv) Size of a plant cell = 0.00001275 m

$= \frac{1275}{100000000}$ m

$= \frac{1.275 \times 10^3}{10^8}$ m

$= 1.275 \times 10^{3-8}$

$= 1.275 \times 10^{-5}$ m

(v) Thickness of a thick paper = 0.07 mm

$= \frac{7}{100}$ mm

$= \frac{0.7 \times 10}{10^2} = 0.7 \times 10^{1-2}$

$= 0.7 \times 10^{-1}$ mm $= 7 \times 10^{-2}$ mm

Q4. In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Sol. Thickness of books $= 5 \times 20 = 100$ mm

Thickness of 5 paper sheets

$= 5 \times 0.016$ mm

$= 0.080$ mm.

$\therefore$ Total thickness of the stack

$= 100$ mm $+ 0.080$ mm

$= 100.080$ mm

$= 1.0008 \times 10^2$ mm

Learning More Q & A

I. Very Short Answer (VSA) Questions

Q1. Find the multiplicative inverse of:

(i) 3^{-3} (ii) 10^{-10}

Sol. (i) Multiplicative inverse of $3^{-3} = \frac{1}{3^{-3}} = 3^3$

(ii) Multiplicative inverse of $10^{-10} = \frac{1}{10^{-10}} = 10^{10}$

Q2. Expand the following using exponents.

(i) 0.0523 (ii) 32.005

Sol. (i) $0.0523 = \frac{5}{100} + \frac{2}{1000} + \frac{3}{10000}$

$= 5 \times \frac{1}{100} + 2 \times \frac{1}{1000} + 3 \times \frac{1}{10000}$

$= 5 \times \frac{1}{10^2} + 2 \times \frac{1}{10^3} + 3 \times \frac{1}{10^4}$

$= 5 \times 10^{-2} + 2 \times 10^{-3} + 3 \times 10^{-4}$

$$(ii)\ 32.005 = 3 \times 10 + 2 \times 1 + \frac{5}{1000}$$
$$= 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{1000}$$
$$= 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{10^3}$$
$$= 3 \times 10 + 2 \times 1 + \times 5 \times 10^{-3}$$

Q3. Simplify and write in exponential form.

$(i)\ (-5)^2 \times (-5)^{-3}$ $(ii)\ \left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2}$

Sol. $(i)\ (-5)^2 \times (-5)^{-3}$
$= (-5)^{2+(-3)} = (-5)^{2-3}$
$= (-5)^{-1} = -\frac{1}{5}$

$(ii)\ \left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^{(-3)+(-2)}$
$= \left(\frac{1}{2}\right)^{-3-2} = \left(\frac{1}{2}\right)^{-5}$
$= \frac{1}{2^{-5}} = 2^5$

Q4. Simplify the following and write in exponential form.

$(i)\ 2^3 \times 3^3$ $(ii)\ \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{6}\right)^5$

Sol. $(i)\ 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$

$(ii)\ \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{6}\right)^5 = \left(\frac{4}{5} \times \frac{5}{6}\right)^5$
$= \left(\frac{4}{6}\right)^5 = \left(\frac{2}{3}\right)^5$

Q5. Express 8^{-4} as a power with the base 2.

Sol. We have $8 = 2 \times 2 \times 2 = 2^3$
$\therefore\ 8^{-4} = (2^3)^{-4} = 2^{3\times(-4)} = 2^{-12}$

Q6. Simplify the following and write in exponential form.

$(i)\ (3^6 \div 3^8)^4 \times 3^{-4}$ $(ii)\ \frac{1}{27} \times 3^{-3}$

Sol. $(i)\ (3^6 \div 3^8)^4 \times 3^{-4}$
$= (3^{6-8})^4 \times 3^{-4} = 3^{-2\times4} \times 3^{-4}$
$= 3^{-8} \times 3^{-4} = 3^{-8-4} = 3^{-12}$

$(ii)\ \frac{1}{27} \times 3^{-3}$
$= \frac{1}{3^3} \times 3^{-3}$
$= 3^{-3} \times 3^{-3} = 3^{-3-3} = 3^{-6}$

Q7. Find the value of k if
$(-2)^{k+1} \times (-2)^3 = (-2)^7$

Sol. $(-2)^{k+1} \times (-2)^3 = (-2)^7$
$\Rightarrow\ (-2)^{k+1+3} = (-2)^7$
$\Rightarrow\ (-2)^{k+4} = (-2)^7$
$\Rightarrow\ k + 4 = 7$
$\Rightarrow\ k = 7 - 4 = 3$
Hence, $k = 3$

Q8. Simplify the following:

$(i)\ \left\{\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

$(ii)\ \left(\frac{2}{3}\right)^{-6} \times \left(\frac{3}{2}\right)^{-4}$

Sol. $(i)\ \left\{\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$
$= \left\{\frac{1^{-3}}{4^{-3}} - \frac{1^{-3}}{3^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$
$\Rightarrow\ \left(\frac{4^3}{1^3} - \frac{3^3}{1^3}\right) \div \frac{4^2}{1^2}$
$\Rightarrow\ (64 - 27) \div 16$
$\Rightarrow\ 37 \div 16 = \frac{37}{16}$

$(ii)\ \left(\frac{2}{3}\right)^{-6} \times \left(\frac{3}{2}\right)^{-4}$
$= \frac{2^{-6}}{3^{-6}} \times \frac{3^{-4}}{2^{-4}} = \frac{3^6}{2^6} \times \frac{2^4}{3^4}$
$= \frac{3^{6-4}}{2^{6-4}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$

Q9. Find the value of $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

Sol. $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2 = \left(-\frac{3}{4}\right)^{-4}$
$= (-1)^4 \times \left(\frac{3}{4}\right)^{-4}$
$= 1 \times \frac{3^{-4}}{4^{-4}} = \frac{4^4}{3^4} = \frac{256}{81}$

Q10. Write the following in standard form
$(i)\ 0.0035$ $(ii)\ 365.05$

Sol. $(i)\ 0.0035 = \frac{35}{10000}$
$= \frac{3.5 \times 10}{10^4} = 3.5 \times 10^{1-4} = 3.5 \times 10^{-3}$

$(ii)\ 365.05 = \frac{36505}{100} = \frac{3.6505 \times 10^4}{10^2}$
$= 3.6505 \times 10^2$

II. Short Answer (SA) Questions

Q11. Find the value of P if

$$\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2P-1}$$

Sol. $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2P-1}$

$\Rightarrow \quad \left(\frac{2}{5}\right)^{3-6} = \left(\frac{2}{5}\right)^{2P-1}$

$\Rightarrow \quad \left(\frac{2}{5}\right)^{-3} = \left(\frac{2}{5}\right)^{2P-1}$

Equating the powers of the same base

$2P - 1 = -3$

$2P = -3 + 1$

$2P = -2$

$\therefore \quad P = -1$

Q12. If $\left(\frac{x}{y}\right) = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{3}{7}\right)^0$, find the value of $\left(\frac{x}{y}\right)^{-3}$.

Sol. $\left(\frac{x}{y}\right) = \left(\frac{3^{-2}}{2^{-2}}\right) + 1 \qquad \left[\because \left(\frac{a}{b}\right)^0 = 1\right]$

$= \frac{2^2}{3^2} = \frac{4}{9}$

$\therefore \quad \left(\frac{x}{y}\right)^{-3} = \left(\frac{4}{9}\right)^{-3}$

$= \frac{4^{-3}}{9^{-3}} = \frac{9^3}{4^3}$

$= \frac{729}{64}$

Q13. Find the value of x if

$$\left(\frac{125}{27}\right) \times \left(\frac{125}{27}\right)^x = \left(\frac{5}{3}\right)^{18}$$

Sol. $\left(\frac{125}{27}\right) \times \left(\frac{125}{27}\right)^x = \left(\frac{5}{3}\right)^{18}$

$\Rightarrow \quad \left(\frac{5^3}{3^3}\right) \times \left(\frac{5^3}{3^3}\right)^x = \left(\frac{5}{3}\right)^{18}$

$\Rightarrow \quad \left(\frac{5}{3}\right)^3 \times \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^{18}$

$\Rightarrow \quad \left(\frac{5}{3}\right)^{3+x} = \left(\frac{5}{3}\right)^{18}$

$\Rightarrow \quad 3 + x = 18$

[Equating the powers of same base]

$\therefore \quad x = 18 - 3 = 15$

Q14. Solve the following: $81^{-4} \div 729^{2-x} = 9^{4x}$

Sol. $(81)^{-4} \div (729)^{2-x} = (9)^{4x}$

$\Rightarrow \quad (9^2)^{-4} \div (9^3)^{2-x} = (9)^{4x}$

$\Rightarrow \quad 9^{-8} \div 9^{6-3x} = 9^{4x}$

$\Rightarrow \quad 9^{-8-(6-3x)} = 9^{4x}$

$\Rightarrow \quad 9^{-8-6+3x} = 9^{4x}$

$\Rightarrow \quad 9^{-14+3x} = 9^{4x}$

Equating the power of same base, we have

$-14 + 3x = 4x$

$\Rightarrow \quad 4x - 3x = -14$

$\therefore \quad x = -14$

Q15. Simplify: $\frac{(x^{m+n})^2 \times (x^{n+p})^2 \times (x^{p+m})^2}{(x^m \cdot x^n \cdot x^p)^3}$

Sol. $\frac{(x^{m+n})^2 \times (x^{n+p})^2 \times (x^{p+m})^2}{(x^m \cdot x^n \cdot x^p)^3}$

$= \frac{x^{2m+2n} \times x^{2n+2p} \times x^{2p+2m}}{x^{3m} \cdot x^{3n} \cdot x^{3p}}$

$[\because (x^a)^b = (x^{ab})]$

$= \frac{x^{2m+2n+2n+2p+2p+2m}}{x^{3m+3n+3p}}$

$[\because x^a \times x^b = x^{a+b}]$

$= \frac{x^{4m+4n+4p}}{x^{3m+3n+3p}}$

$= x^{(4m+4n+4p)-(3m+3n+3p)}$

$= x^{4m+4n+4p-3m-3n-3p}$

$= x^{m+n+p} \qquad \left[\because \frac{x^a}{x^b} = x^{a-b}\right]$

Q16. Simplify: $\frac{(-2)^3 \times (-2)^7}{3 \times 4^6}$ *(NCERT Exemplar)*

Sol. $\frac{(-2)^3 \times (-2)^7}{3 \times 4^6} = \frac{(-2)^{3+7}}{3 \times (2^2)^6}$ $\{a^m \times a^n = a^{m+n}\}$

$= \frac{(-2)^{10}}{3 \times 2^{12}}$ $\{(a^m)^n = a^{m \times n}\}$

$= \frac{(-2)^{10}}{3 \times 2^{12}} = \frac{2^{10-12}}{3}$ $\{a^m \div a^n = a^{m-n}, (-2)^{10} = 2^{10}\}$

$= \frac{2^{-2}}{3} = \frac{1}{3 \times 2^2} = \frac{1}{12}$

Q17. Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Sol. $(-5)^{x+1} \times (-5)^5 = (-5)^7$ *(NCERT Exemplar)*

$(-5)^{x+1+5} = (-5)^7$ $\{a^m \times a^n = a^{m+n}\}$

$(-5)^{x+6} = (-5)^7$

On both sides, powers have the same base, so their exponents must be equal.

Therefore, $x + 6 = 7$

$x = 7 - 6 = 1$

$x = 1$.

Test Yourself

I. Very Short Answer (VSA) Questions

Q1. Express each of the following as a rational numbers.

(a) $\left(-\frac{2}{3}\right)^4$ (b) $\left(\frac{1}{3}\right)^3$

Q2. Express the following in power notation.

(a) $-\frac{27}{125}$ (b) $\frac{16}{81}$

Q3. Find the value of the following:

(a) $(-2)^5 \div \left(-\frac{1}{3}\right)^4$ (b) $(3^2 - 2^2) \div \left(\frac{1}{5}\right)^2$

Q4. Find the reciprocal of the following:

(a) $\left(-\frac{3}{8}\right)^3$ (b) $(-3)^3$

Q5. Find the multiplicative inverse of the following.

(a) $(-3)^4$ (b) $(6)^3$

Q6. Express the following in exponential form.

(a) $-\frac{5}{7} \times -\frac{5}{7} \times -\frac{5}{7} \times -\frac{5}{7}$

(b) $\frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8}$

Q7. Find the value of

(a) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$ (b) $\left(-\frac{1}{5}\right)^4 \times \left(-\frac{3}{2}\right)^4$

Q8. Find the value of

(a) $\left[\left(\frac{2}{5}\right)^{-3}\right]^4$ (b) $\left[\left(-\frac{6}{13}\right)^{-5}\right]^{-3}$

Q9. Find the value of x if $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-11} = \left(\frac{2}{3}\right)^{-7x}$.

Q10. Simplify:

(a) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}$

(b) $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$

II. Short Answer (SA) Questions

Q11. If $\left(\frac{6}{11}\right)^{-3} \div \left(\frac{6}{11}\right)^{-8} = \left(\frac{6}{11}\right)^{2m-13}$, find m.

Q12. By what number should $\left(\frac{5}{4}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{15}{16}\right)^{-2}$?

Q13. Compare: $4.67 \times 10^{-3} \square 4.67 \times 10^{-4}$.

Q14. Write the following numbers in scientific notation.

(a) 0.00412 (b) 0.00000082

Q15. Write the following in standard form:

(a) 6.74×10^{-5} (b) 2.302×10^{-8}

Q16. Solve for x: $(125^{-2} \div 625^{1-x}) = 5^{2x}$.

Q17. Mass of the Earth is 5.97×10^{24} kg and the mass of Moon is 7.35×10^{22} kg. Find the total mass.

Q18. Simplify: $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$.

Q19. Find the value of k if $(-3)^{k+1} \times (-3)^5 = (-3)^7$.

Q20. The distance between Sun and Earth is 1.496×10^{11} m and the distance between Earth and Moon is 3.84×10^8 m. Find the distance between Sun and Moon.

Q21. The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Compare the diameter of the Earth with the diameter of the Sun.

ANSWERS

1. (a) $\frac{16}{81}$ (b) $\frac{1}{27}$
2. (a) $\left(-\frac{3}{5}\right)^3$ (b) $\left(\frac{2}{3}\right)^4$
3. (a) -2592 (b) 125
4. (a) $\left(-\frac{8}{3}\right)^3$ (b) $\left(-\frac{1}{3}\right)^3$
5. (a) $\left(-\frac{1}{3}\right)^4$ (b) $\left(\frac{1}{6}\right)^3$
6. (a) $\left(-\frac{5}{7}\right)^4$ (b) $\left(\frac{3}{8}\right)^5$
7. (a) $\frac{32}{243}$ (b) $\frac{81}{10000}$

8. (a) $\left(\frac{5}{2}\right)^{12}$ (b) $\left(-\frac{6}{13}\right)^{15}$
9. $x = 2$
10. (a) 90 (b) 30
11. $m = 9$ 12. $\frac{9}{20}$
13. $4.67 \times 10^{-3} > 4.67 \times 10^{-4}$
14. (a) 4.12×10^{-3} (b) 8.2×10^{-7}
15. (a) 0.0000674 (b) 0.00000002302
16. $x = 5$ 17. 604.35×10^{22} kg
18. 5^5 19. 1
20. 1492.16×10^8 m 21. 100 times (approx.)

Internal Assessment

Q1. Fill in the blanks:

(a) 2.67×10^{-3} ..... 2.67×10^{-4}

(b) 1.16×10^{-3} ..... 1.16×10^{-2}

(c) 5.2×10^{-5} ..... 0.52×10^{-6}

(d) $[3^{-1} + 5^{-1} - 6^{-1}]^0$ = ...

(e) $\left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)$

= ..... .

II. Multiple Choice Questions (MCQ)

Q2. The value of $(6 \times 10^{-2} + 0.49 \times 10^{-4})$ is

(a) 0.006049 (b) 6.049×10^{-1}

(c) 0.06049 (d) 6.0049×10^{-2}

Q3. If $\left(-\frac{2}{3}\right)^4 \times \left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)^{3m}$, then m is equal to

(a) 2 (b) 0 (c) 3 (d) 1

Q4. If $\left(\frac{125}{27}\right) \times \left(\frac{125}{17}\right)^x = \left(\frac{5}{3}\right)^{18}$, then x is equal to

(a) 6 (b) 5 (c) 4 (d) 1

Q5. $\left[\left\{\left(-\frac{2}{3}\right)^2\right\}^{-1}\right]^{-3}$ is equal to

(a) $\frac{64}{729}$ (b) $\frac{729}{64}$ (c) $-\frac{64}{729}$ (d) $-\frac{729}{64}$

Q6. Value of $(5^{-1} \div 4^{-1})^3$

(a) $\frac{125}{64}$ (b) $\frac{64}{125}$ (c) $-\frac{125}{64}$ (d) $-\frac{64}{125}$

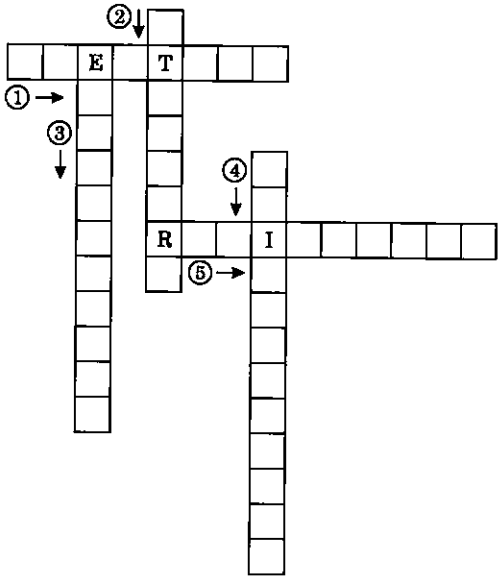
Q7. The value of $(4^2 + 4^0) + 3^0(5^1 + 5^0) + 5^0 - (11^1 - 11)^0$ is

(a) 33 (b) 23 (c) 32 (d) 0

Q8. Complete the cross-puzzle.

Hint:

1. The charge of an ..... is 1.6×10^{-19} coulombs.
2. The ..... form of 0.00037 is 3.7×10^{-4}.



3. The ..... form of $\frac{125}{729}$ is $\left(\frac{5}{9}\right)^3$.
4. The ..... notation of 0.00002 is 2.0×10^{-5}.
5. The ..... of $\left(\frac{2}{3}\right)^5$ is $\left(\frac{3}{2}\right)^5$.

ANSWERS

1. (a) > (b) < (c) = (d) 1 (e) $\left(-\frac{1}{2}\right)^4$
2. (d) 3. (a)
4. (b) 5. (a)
6. (b) 7. (b)
8. (1) ELECTRON
   (2) STANDARD
   (3) EXPONENTIAL
   (4) SCIENTIFICAL
   (5) RECIPROCAL