

Direct and Inverse Proportions

Understanding the Lesson

- Direct proportion.
- Indirect proportion.
- Compound variation.
- Time and work.
- Time and Distance.

Conceptual Facts

- The two quantities are said to be directly proportional if
 - (i) an increase in one leads to a corresponding increase in the other.
 - (ii) a decrease in one leads to a corresponding decrease in the other.
- If x varies directly as y then the ratio $\frac{x}{y} = k$ (constant) symbolically, $x \propto y$, where \propto is symbol of proportionality or $x = ky$ or $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = k$.
- The two quantities are said to be inversely proportional if
 - (i) an increase in one leads to a corresponding decrease in the other.
 - (ii) a decrease in one leads to a corresponding increase in the other.
- If x varies inversely with y , then $x \propto \frac{1}{y} \Rightarrow xy = k$ (constant) or $x_1y_1 = x_2y_2 = k$.
- $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ or $\text{Distance} = \text{Speed} \times \text{Time}$ or $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

TRY THESE (PAGE 204)

Q1. Observe the following table and find if x and y are directly proportional.

(i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

(ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

(iii)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

Sol. (i) $\frac{20}{40} = \frac{1}{2}$, $\frac{17}{34} = \frac{1}{2}$, $\frac{14}{28} = \frac{1}{2}$, $\frac{11}{22} = \frac{1}{2}$,

$\frac{8}{16} = \frac{1}{2}$, $\frac{5}{10} = \frac{1}{2}$ and $\frac{2}{4} = \frac{1}{2}$

Since the ratio of given quantities x and y is constant, i.e., $\frac{1}{2}$.

Hence x and y are directly proportional.

(ii) $\frac{6}{4} = \frac{3}{2}$, $\frac{10}{8} = \frac{5}{4}$, $\frac{14}{12} = \frac{7}{6}$, $\frac{18}{16} = \frac{9}{8}$,

$\frac{22}{20} = \frac{11}{10}$, $\frac{26}{24} = \frac{13}{12}$ and $\frac{30}{28} = \frac{15}{14}$

Since the ratio of x and y is different, i.e., it is not constant.

Hence x and y are not directly proportional.

$$(iii) \frac{5}{15} = \frac{1}{3}, \frac{8}{24} = \frac{1}{3}, \frac{12}{36} = \frac{1}{3}, \frac{15}{60} = \frac{1}{4},$$

$$\frac{18}{72} = \frac{1}{4} \text{ and } \frac{20}{100} = \frac{1}{5}$$

Since, the ratio of x and y is not constant.

Hence x and y are not directly proportional.

Q2. Principal = ₹ 1000, Rate = 8% per annum. Fill in the following table and find which type of interest (simple or compound) changes in direct proportion with time period.

Time period	1 year	2 years	3 years
Simple Interest (in ₹) $\frac{P \times r \times t}{100}$			
Compound Interest (in ₹) $P \left(1 + \frac{r}{100}\right)^t - P$			

Sol. We have $P = ₹ 1000$, $r = 8\%$ P.a.,

$$t = 1 \text{ year}$$

$$\therefore SI = \frac{P \times r \times t}{100}$$

$$= \frac{1000 \times 8 \times 1}{100} = ₹ 80$$

$$CI = P \left[\left(1 + \frac{r}{100}\right)^t - 1 \right]$$

$$= 1000 \left[\left(1 + \frac{8}{100}\right)^1 - 1 \right]$$

$$= 1000 \left[\frac{27}{25} - 1 \right]$$

$$= 1000 \left[\frac{27 - 25}{25} \right]$$

$$= 1000 \times \frac{2}{25} = ₹ 80$$

For $t = 2$ years

$$SI = \frac{P \times r \times t}{100} = \frac{1000 \times 8 \times 2}{100}$$

$$= ₹ 160$$

$$CI = P \left[\left(1 + \frac{r}{100}\right)^t - 1 \right]$$

$$= 1000 \left[\left(1 + \frac{8}{100}\right)^2 - 1 \right]$$

$$= 1000 \left[\left(\frac{27}{25}\right)^2 - 1 \right]$$

$$= 1000 \left[\frac{729}{625} - 1 \right]$$

$$= 1000 \left[\frac{729 - 625}{625} \right]$$

$$= \frac{1000 \times 104}{625} = \frac{832}{5}$$

$$= ₹ 166.40$$

For $t = 3$ years

$$SI = \frac{P \times r \times t}{100} = \frac{1000 \times 8 \times 3}{100}$$

$$= ₹ 240$$

$$CI = P \left[\left(1 + \frac{r}{100}\right)^t - 1 \right]$$

$$= 1000 \left[\left(1 + \frac{8}{100}\right)^3 - 1 \right]$$

$$= 1000 \left[\left(\frac{27}{25}\right)^3 - 1 \right]$$

$$= 1000 \left[\frac{19683}{15625} - 1 \right]$$

$$= 1000 \left[\frac{19683 - 15625}{15625} \right]$$

$$= \frac{1000 \times 4058}{15625}$$

$$= ₹ 259.71$$

Hence the completed table is

Time period	1 year	2 years	3 years
Simple Interest (in ₹)	80	160	240
Compound Interest (in ₹)	80	166.40	259.71

EXERCISE 13.1

Q1. Following are the car parking charges near a railway station upto.

4 hours	₹ 60
8 hours	₹ 100
12 hours	₹ 140
24 hours	₹ 180

Check if the parking charges are in direct proportions to the parking time.



Sol. We have the ratio of time period and the parking charge.

$$\frac{4}{60} = \frac{1}{15}, \quad \frac{8}{100} = \frac{2}{25}, \quad \frac{12}{140} = \frac{3}{35}, \quad \frac{24}{180} = \frac{2}{15}$$

$$\text{Since } \frac{1}{15} \neq \frac{2}{25} \neq \frac{3}{35} \neq \frac{2}{15}$$

Hence the given quantities are not directly proportional.

Q2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	-	-	-	-

Sol. Let the number to be filled in the blanks be a , b , c and d respectively.

$$\therefore \frac{1}{8} = \frac{4}{a} \Rightarrow a = 32$$

$$\frac{1}{8} = \frac{7}{b} \Rightarrow b = 56$$

$$\frac{1}{8} = \frac{12}{c} \Rightarrow c = 96$$

$$\frac{1}{8} = \frac{20}{d} \Rightarrow d = 160$$

Hence the required value of bases are

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

Q3. In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

Sol. Let the required red pigment be x part.

$$\frac{1}{x} = \frac{75 \text{ mL}}{1800 \text{ mL}}$$

$$\Rightarrow x \times 75 = 1 \times 1800$$

$$\Rightarrow x = \frac{1800}{75} = 24 \text{ Parts}$$

Hence, the required amount of red pigment = 24 parts.

Q4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Sol. Let the required number of bottles be x .

No. of bottles	Time in hours
840	6
x	5

Since the two quantities are directly proportional

$$\therefore \frac{840}{x} = \frac{6}{5}$$

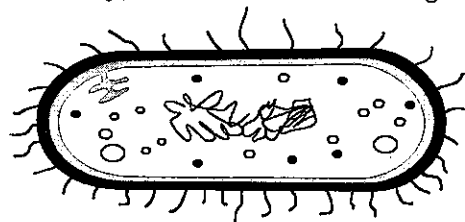
$$\Rightarrow 6 \times x = 5 \times 840$$

$$\therefore x = \frac{5 \times 840}{6}$$

$$= 5 \times 140 = 700$$

Hence the required number of bottles = 700.

Q5. A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?



Sol. Let the actual length be x cm.

Enlargement	Length
50,000 times	5 cm
20,000 times	x cm

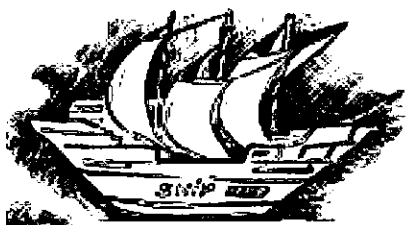
$$\text{Original length} = \frac{5}{50,000} \text{ cm}$$

$$= \frac{1}{10,000} = 10^{-4} \text{ cm.}$$

Since the two quantities are directly proportional.

$$\begin{aligned} \therefore \frac{50,000}{20,000} &= \frac{5}{x} \\ \Rightarrow 50,000 \times x &= 5 \times 20,000 \\ \therefore x &= \frac{5 \times 20,000}{50,000} \\ &= 2 \text{ cm.} \end{aligned}$$

- Q6. In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?



Sol. Let the required length of the model ship be x m.

Height of mast	Length of ship
9 cm	12 m
x cm	28

Since the two quantities are directly proportional.

$$\begin{aligned} \therefore \frac{9}{x} &= \frac{12}{28} \\ \Rightarrow 12x &= 9 \times 28 \\ \therefore x &= \frac{9 \times 28}{12} = 21 \text{ m.} \end{aligned}$$

- Q7. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?

Sol. Let x be the number of sugar crystals needed.

No. of sugar crystals	Weight of sugar
9×10^6	2 kg
x	5 kg

Since the two quantities are directly proportional.

$$\begin{aligned} \therefore \frac{9 \times 10^6}{x} &= \frac{2}{5} \\ \Rightarrow 2x &= 5 \times 9 \times 10^6 \\ \therefore x &= \frac{5 \times 9 \times 10^6}{2} \\ &= \frac{45}{2} \times 10^6 = 2.25 \times 10^7 \end{aligned}$$

Hence the required number of sugar crystals = 2.25×10^7 .

No. of sugar crystals	Weight of sugar
9×10^6	2 kg
y	1.2 kg

Since the two quantities are directly proportional.

$$\begin{aligned} \therefore \frac{9 \times 10^6}{y} &= \frac{2}{1.2} \\ \Rightarrow 2y &= 1.2 \times 9 \times 10^6 \\ \Rightarrow y &= \frac{1.2 \times 9 \times 10^6}{2} \\ &= 5.4 \times 10^6 \end{aligned}$$

Hence, the required number of sugar crystals = 5.4×10^6 .

- Q8. Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road of 72 km. What would be her distance covered in the map?

Sol. Let the required distance be x km.

Scale in cm	Distance in km
1	18
x	72

Since, the two quantities are directly proportional.

$$\begin{aligned} \therefore \frac{1}{x} &= \frac{18}{72} \\ \Rightarrow 18x &= 1 \times 72 \\ \therefore x &= \frac{1 \times 72}{18} = 4 \text{ cm.} \end{aligned}$$

Hence the distance covered in the map = 4 cm.

- Q9. A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high. (ii) the height of a pole which casts a shadow 5 m long.

Sol. (i) Let the required length of shadow be x m.

Height of Pole	Length of Shadow
5 m 60 cm	3 m 20 cm
10 m 50 cm	x m

Since the two quantities are directly proportional.

$$\begin{aligned} \therefore \frac{5 \text{ m } 60 \text{ cm}}{10 \text{ m } 50 \text{ cm}} &= \frac{3 \text{ m } 20 \text{ cm}}{x \text{ m}} \\ \frac{5.60}{10.50} &= \frac{3.20}{x} \end{aligned}$$

$$\Rightarrow 5.60 \times x = 3.20 \times 10.50$$

$$\Rightarrow x = \frac{3.20 \times 10.50}{5.60}$$

$$= \frac{320 \times 1050}{560} \times \frac{1}{100}$$

$$= 6 \text{ m.}$$

Hence the required length = 6 m.

(ii)

Height of Pole	Length of Shadow
5.60	3.20
y	5

Since the two quantities are directly proportional.

$$\therefore \frac{5.60}{y} = \frac{3.20}{5}$$

$$\Rightarrow 3.20 \times y = 5.60 \times 5$$

$$\Rightarrow y = \frac{5.60 \times 5}{3.20} = \frac{35}{4} \text{ m}$$

$$= 8 \text{ m } 75 \text{ cm}$$

Hence, the required length = 8 m 75 cm.

Q10. A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours.

Sol. Let the required distance be x km.

Distance	Time
14 km	25 minutes
x km	5 hours = 300 minutes

Since the two quantities are directly proportional.

$$\therefore \frac{14}{x} = \frac{25}{300}$$

$$\Rightarrow 25 \times x = 14 \times 300$$

$$\Rightarrow x = \frac{14 \times 300}{25}$$

$$= 14 \times 12 = 168 \text{ km.}$$

Hence the required distance = 168 km.

TRY THESE (PAGE 211)

Observe the following tables and find which pair of variables (here x and y) are in inverse proportion.

(i)

x	50	40	30	20
y	5	6	7	8

(ii)

x	100	200	300	400
y	60	30	20	15

(iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35

Sol. (i) We have

x	50	40	30	20
y	5	6	7	8

The conditions that the two quantities x and y is $xy = k$ (constant)

So, $xy = 50 \times 5 = 250$

$xy = 40 \times 6 = 240$

$xy = 30 \times 7 = 210$

and $xy = 20 \times 8 = 160$

Since no value of xy in pair is same.

Hence, no pair of x and y are in inverse proportion.

(ii) We have

x	100	200	300	400
y	60	30	20	15

Here $xy = 100 \times 60 = 6000$

$xy = 200 \times 30 = 6000$

$xy = 300 \times 20 = 6000$

and $xy = 400 \times 15 = 6000$

Since all the values of xy in each pair are constant, i.e., 6000.

Hence x is inversely proportional to y in each pair.

(iii) We have

x	90	60	45	30	20	5
y	10	15	20	25	30	35

Here, xy for first pair

$= 90 \times 10 = 900$

xy for second pair = $60 \times 15 = 900$

xy for third pair = $45 \times 20 = 900$

xy for fourth pair = $30 \times 25 = 750$

xy for fifth pair = $20 \times 30 = 600$

xy for sixth pair = $5 \times 35 = 175$

We observe that the value of xy for first, second and third pair of x and y is constant, i.e., 900.

Thus x varies inversely with y for three pairs only.

The values of xy for fourth, fifth and sixth pairs of x and y are different.

Thus x does not vary inversely with y for these pairs.

EXERCISE 13.2

Q1. Which of the following are in inverse proportion?

- (i) The number of workers on a job and the time to complete the job.
- (ii) The time taken for a journey and the distance travelled in a uniform speed.
- (iii) Area of cultivated land and the crop harvested.
- (iv) The time taken for a fixed journey and the speed of the vehicle.
- (v) The population of a country and the area of land per person.

Sol. (i) As the number of workers increase, the job will take less time to complete.

Hence, they are inversely proportional.

(ii) For more time, more distance to travel. Hence, they are not inversely proportional.

(iii) More area of land cultivated, more crop to harvest. Hence, they are not inversely proportional.

(iv) If speed is increased, it will take less time to complete the fixed journey.

Hence, they are inversely proportional.

(v) If the population of a country increases, then the area of land per person will be decreased.

Hence, they are inversely proportional.

Q2. In a Television game show, the prize money of ₹ 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	-	-	-	-	-

Sol. Let the blank spaces be denoted by a , b , c , d and e .

So, we observe that $1 \times 100,000 = 2 \times 50,000$

$$\Rightarrow 1,00,000 = 1,00,000$$

Hence they are inversely proportional.

$$2 \times 50,000 = 4 \times a$$

$$\therefore a = \frac{2 \times 50,000}{4} = 25,000$$

$$2 \times 50,000 = 5 \times b$$

$$\therefore b = \frac{2 \times 50,000}{5} = 20,000$$

$$2 \times 50,000 = 8 \times c$$

$$\therefore c = \frac{2 \times 50,000}{8} = 12,500$$

$$2 \times 50,000 = 10 \times d$$

$$\therefore d = \frac{2 \times 50,000}{10} = 10,000$$

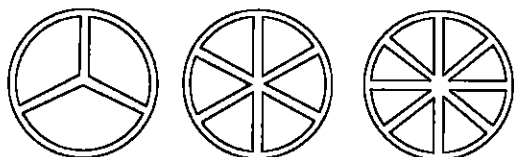
$$2 \times 50,000 = 20 \times e$$

$$\therefore e = \frac{2 \times 50,000}{20} = 5,000$$

Hence the required table is

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	25,000	20,000	12,500	10,000	5,000

Q3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.



Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°			

(i) Are the number of spokes and the angle formed between the pairs of consecutive spokes in inverse proportion.

- (ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.
- (iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40° ?

Sol. From the above table, we observe that

$$4 \times 90^\circ = 6 \times 60^\circ$$

$$360^\circ = 360^\circ$$

Thus the two quantities are inversely proportional. Let the blank spaces be denoted by a , b and c .

$$\therefore 4 \times 90^\circ = 8 \times a$$

$$\therefore a = \frac{4 \times 90^\circ}{8} = 45^\circ$$

$$4 \times 90^\circ = 10 \times b$$

$$\therefore b = \frac{4 \times 90^\circ}{10} = 36^\circ$$

$$4 \times 90^\circ = 12 \times c$$

$$\therefore c = \frac{4 \times 90^\circ}{12} = 30^\circ$$

Hence, the required table is

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

(i) Yes, they are in inverse proportion.

(ii) If number of spokes is 15, then

$$4 \times 90^\circ = 15 \times x$$

$$\therefore x = \frac{4 \times 90^\circ}{15} = 24^\circ$$

(iii) If angle between two consecutive spokes is 40° , then

$$4 \times 90^\circ = y \times 40^\circ$$

$$\therefore y = \frac{4 \times 90^\circ}{40^\circ} = 9 \text{ spokes.}$$

Thus the required number of spokes = 9.

Q4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is decreased by 4?

Number of children	Number of Sweets
24	5
(24 - 4) or 20	a

We observe that on increasing the number of children, number of sweets got by each will be less. So, they are in inverse proportion.

$$\therefore x_1 y_1 = x_2 y_2$$

where $x_1 = 24, y_1 = 5, x_2 = 20$

and $y_2 = a$ (let)

$$24 \times 5 = 20 \times a$$

$$\therefore a = \frac{24 \times 5}{20} = 6$$

Hence, the required number of sweets = 6.

Q5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Sol. If the number of animals increases, then it will take less days to last.

Then the two quantities are in inverse proportions.

Number of animals	Number of days
20	6
(20 + 10) or 30	p

Let the required number of days be p .

$$\therefore x_1 y_1 = x_2 y_2$$

where $x_1 = 20, y_1 = 6, x_2 = 30$

and $y_2 = p$ (let)

$$20 \times 6 = 30 \times p$$

$$\therefore p = \frac{20 \times 6}{30} = 4$$

Hence the required number of days = 4.

Q6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Sol. If the number of persons is increased, it will take less number of days to complete the job.

Thus, the two quantities are in inverse proportion.

Number of persons	Number of days
3	4
4	k

Let the required number of days be k .

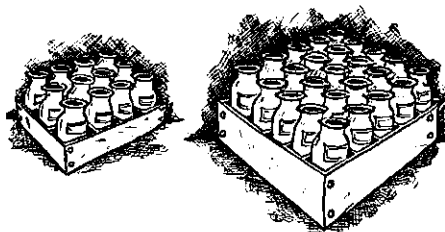
$$\therefore x_1 y_1 = x_2 y_2$$

$$3 \times 4 = 4 \times k$$

$$\therefore k = \frac{3 \times 4}{4} = 3 \text{ days.}$$

Hence, the required number of days = 3.

- Q7.** A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?



Sol. If number of bottles is increased then the required number of boxes will be decreased. Thus the two quantities are in inverse proportion.

Number of boxes	Number of bottles per box
25	12
x	20

Let the required number of boxes be x .

$$\begin{aligned} \therefore x_1 y_1 &= x_2 y_2 \\ 25 \times 12 &= x \times 20 \\ \Rightarrow x &= \frac{25 \times 12}{20} = 15 \end{aligned}$$

Hence, the required number of boxes = 15.

- Q8.** A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Sol. If number of machines is increased then less number of days would be required to produced same number of articles.

Thus, the two quantities are in inverse proportion.

Number of machines	Number of days
42	63
x	54

Let the required number of machines be x .

$$\begin{aligned} \therefore x_1 y_1 &= x_2 y_2 \\ 42 \times 63 &= x \times 54 \\ \therefore x &= \frac{42 \times 63}{54} = 49 \end{aligned}$$

Hence, the required number of machines is 49.

- Q9.** A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

Sol. On increasing the speed, it will take less time to travel a distance.

Thus the two quantities are in inverse proportions.

Speed in km/h	Time in hour
60	2
80	x

Let the required times be x hours.

$$\begin{aligned} \therefore x_1 y_1 &= x_2 y_2 \\ 60 \times 2 &= 80 \times x \\ \therefore x &= \frac{60 \times 2}{80} = \frac{3}{2} \text{ hours} \\ &= 1 \frac{1}{2} \text{ hrs.} \end{aligned}$$

Hence, the required time = $1 \frac{1}{2}$ hours.

- Q10.** Two persons could fit new windows in a house in 3 days.

- (i) One of the person fell ill before the work started. How long would the job take now?
 (ii) How many persons would be needed to fit the windows in one day?

Sol. On increasing the number of persons, less time will be required to complete a job.

Thus, the quantities are in inverse proportion.

Number of persons	Number of days
2	3
(i) $1(2-1)$	x
(ii) y	1

(i) Let the required number of days be x .

$$\begin{aligned} \therefore x_1 y_1 &= x_2 y_2 \\ 2 \times 3 &= 1 \times x \\ \therefore x &= 6 \end{aligned}$$

Hence, the required number of days = 6

(ii) Let the required number of persons be y .

$$\begin{aligned} \therefore x_1 y_1 &= x_2 y_2 \\ 2 \times 3 &= y \times 1 \\ \therefore y &= 6 \end{aligned}$$

Hence, the required number of persons = 6.

- Q11.** A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Sol. On increasing the duration of periods, the number of periods will be reduced.

Thus, the two quantities are in inverse proportion.

Number of periods	Duration of periods in minutes
8	45
9	x

Let the required duration of each period be x .

$$\therefore x_1 y_1 = x_2 y_2$$

$$8 \times 45 = 9 \times x$$

$$\therefore x = \frac{8 \times 45}{9} = 40 \text{ minutes}$$

Hence, the required duration of period = 40 minutes.

Learning More Q & A

I. VERY SHORT ANSWER (VSA) QUESTIONS

Q1. A train is moving at a uniform speed of 100 km/h. How far will it travel in 20 minutes?

Sol. Let the distance travelled by the train in 20 minutes be x km.

Distance travelled (in km)	100	x
Time taken (in minutes)	60	20

Since the speed is uniform, the distance travelled will be directly proportional to time.

$$\therefore \frac{100}{60} = \frac{x}{20}$$

$$\Rightarrow 60 \times x = 20 \times 100$$

$$\therefore x = \frac{20 \times 100}{60} = \frac{100}{3} \text{ km}$$

$$= 33\frac{1}{3} \text{ km.}$$

Hence, the required distance is $33\frac{1}{3}$ km.

Q2. Complete the table if x and y vary directly.

x	3.5	4	7.5	–
y	–	8	–	15

Sol. Let the blank spaces be filled with a , b and c .

x	3.5	4	7.5	c
y	a	8	b	15

Since x and y vary directly.

$$\therefore \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\Rightarrow \frac{3.5}{a} = \frac{4}{8}$$

$$\Rightarrow 4 \times a = 3.5 \times 8$$

$$\therefore a = \frac{3.5 \times 8}{4} = 7$$

$$\frac{4}{8} = \frac{7.5}{b}$$

$$\Rightarrow 4 \times b = 8 \times 7.5$$

$$\therefore b = \frac{8 \times 7.5}{4} = 15$$

$$\frac{4}{8} = \frac{c}{15}$$

$$\Rightarrow 8 \times c = 4 \times 15$$

$$\therefore c = \frac{4 \times 15}{8} = 7.5$$

Hence, the required values are $a = 7$, $b = 15$ and $c = 7.5$.

Q3. If the cost of 20 books is ₹ 180, how much will 15 books cost?

Sol. Let the required cost be ₹ x .

Here, the two quantities vary directly.

Number of books	20	15
Cost (in ₹)	180	x

$$\frac{20}{180} = \frac{15}{x}$$

$$\Rightarrow 20 \times x = 15 \times 180$$

$$\therefore x = \frac{15 \times 180}{20} = ₹ 135$$

Hence, the required cost = ₹ 135.

Q4. If $x_1 = 5$, $y_1 = 7.5$, $x_2 = 7.5$ then find y_2 if x and y vary directly.

Sol. Since x and y vary directly.

$$\therefore \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\Rightarrow \frac{5}{7.5} = \frac{7.5}{y_2}$$

$$\Rightarrow 5 \times y_2 = 7.5 \times 7.5$$

$$\therefore y_2 = \frac{7.5 \times 7.5}{5} = 11.25$$

Hence, the required value is 11.25.

Q5. If 3 kg of sugar contains 9×10^8 crystals. How many sugar crystals are there in 4 kg of sugar?

Sol. Let the required number of crystals be x .

Sugar (in kg)	3	4
Number of crystals	9×10^8	x

Since the two quantities are directly proportional to each other.

$$\therefore \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$\Rightarrow \frac{3}{9 \times 10^8} = \frac{4}{x}$$

$$\Rightarrow 3 \times x = 4 \times 9 \times 10^8$$

$$\therefore x = \frac{4 \times 9 \times 10^8}{3}$$

$$= 12 \times 10^8 = 1.2 \times 10^9$$

Hence, the required number of crystals
 $= 1.2 \times 10^9$.

Q6. If 15 men can do a work in 12 days, how many men will do the same work in 6 days?

Sol. Let the required number of men be x .

Less days \rightarrow more men.

Thus the two quantities are inversely proportional to each other.

Men	15	x
days	12	6

$$x_1 y_1 = x_2 y_2$$

$$\Rightarrow 15 \times 12 = x \times 6$$

$$\therefore x = \frac{15 \times 12}{6} = 30$$

Hence, the required number of days = 30.

Q7. A train travels 112 km in 1 hour 30 minutes with a certain speed. How many kilometres it will travel in 4 hours 45 minutes with the same speed?

Sol. Let the required distance be x km.

More distance \rightarrow more time

Thus, the two quantities are directly proportional.

Distance (in km)	112	x
Time (in hours)	$\frac{3}{2}$ h	$\frac{19}{4}$ h

$$\left[\begin{array}{l} \therefore 1 \text{ hr } 30 \text{ min} = \frac{3}{2} \text{ h} \\ 4 \text{ hrs } 45 \text{ min} = \frac{19}{4} \text{ h} \end{array} \right]$$

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

$$y_1 = \frac{3}{2}$$

$$\Rightarrow \frac{112}{x} = \frac{2}{19}$$

$$\Rightarrow \frac{3}{2} x = 112 \times \frac{19}{4}$$

$$\therefore x = \frac{56}{1} \times \frac{19}{4} = \frac{1064}{4} = 354.6 \text{ km.}$$

Hence, the required distance = 354.6 km.

Q8. The scale of a map is given as 1 : 50,000. Two villages are 5 cm apart on the map. Find the actual distance between them.

Sol. Let the map distance be x cm and actual distance be y .

$$\therefore 1 : 50,000 = x : y$$

$$\Rightarrow \frac{1}{50,000} = \frac{x}{y}$$

Since $x = 5$, So $\frac{1}{50,000} = \frac{5}{y}$

$$\therefore y = 5 \times 50,000$$

$$= 2,50,000 \text{ cm}$$

$$= 250 \text{ km}$$

Hence, the required distance = 250 km.

Q9. 8 pipes are required to fill a tank in 1 hr 20 min. How long will it take if only 6 pipes of the same type are used?

Sol. Let the required time be t hours.

Number of pipe	8	6
Time (in hours)	$\frac{4}{3}$	t

$$[\therefore 1 \text{ hr } 20 \text{ min} = \frac{4}{3} \text{ h}]$$

Less number of pipes \rightarrow more time

$$\therefore x_1 y_1 = x_2 y_2$$

$$\Rightarrow 8 \times \frac{4}{3} = 6 \times t$$

$$\Rightarrow t = \frac{8 \times 4}{6 \times 3} = \frac{16}{9} \text{ h}$$

$$= 1 \frac{7}{9} \text{ h.}$$

Hence, the required time = $1 \frac{7}{9}$ h.

Q10. 15 men can build a wall in 42 hours, how many workers will be required to the same work in 30 hours?

Sol. Let the required number of workers be x .
More number of workers, faster will they do the work.

So, the two quantities are inversely proportional.

Number of hours	42	30
Number of men	15	x

$$\therefore x_1 y_1 = x_2 y_2$$

$$\Rightarrow 42 \times 15 = 30 \times x$$

$$\Rightarrow x = \frac{42 \times 15}{30} = 21$$

Hence, the required number of men = 21.

II. SHORT ANSWER (SA) QUESTIONS

Q11. The volume of a gas V varies inversely as the pressure P for a given mass of the gas. Fill in the blank spaces in the following table:

	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Volume (in cm^3)	–	48	60	–	100	–	200
Pressure (in atmosphere)	2	–	$\frac{3}{2}$	1	–	$\frac{1}{2}$	–

Sol. Since volume and pressure are inversely proportional.

$$\therefore PV = K$$

$$\text{From (C)} \quad 60 \times \frac{3}{2} = K \therefore K = 90$$

$$\text{From (A)} \quad P = 2, K = 90$$

$$\therefore V = \frac{K}{P} = \frac{90}{2} = 45 \text{ cm}^3$$

$$\text{From (B)} \quad V = 48, K = 90$$

$$\therefore P = \frac{K}{V} = \frac{90}{48}$$

$$= \frac{15}{8} \text{ atm}$$

$$\text{From (D)} \quad P = 1, K = 90$$

$$\therefore V = \frac{K}{P} = \frac{90}{1} = 90 \text{ cm}^3$$

$$\text{From (E)} \quad V = 100, K = 90$$

$$\therefore P = \frac{K}{V} = \frac{90}{100} = \frac{9}{10} \text{ atm}$$

$$\text{From (F)} \quad P = \frac{1}{2}, K = 90$$

$$\therefore V = \frac{K}{P} = \frac{90}{\frac{1}{2}} = 180 \text{ cm}^3$$

$$\text{From (G)} \quad V = 200, K = 90$$

$$\therefore P = \frac{K}{V} = \frac{90}{200} = \frac{9}{20} \text{ atm}$$

Hence, the completed table is

	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Volume (in cm^3)	45	48	60	90	100	180	200
Pressure (in atmosphere)	2	$\frac{15}{8}$	$\frac{3}{2}$	1	$\frac{9}{10}$	$\frac{1}{2}$	$\frac{9}{20}$

Q12. The cost of 5 metres of cloth is ₹ 210. Tabulate the cost of 2, 4, 10 and 13 metres of cloth of the same type.

Sol. Let the length of the cloth be x m and its cost be ₹ y . We have the following table.

x	2	4	5	10	13
y	y_1	y_2	210	y_3	y_4

Since x and y are directly proportional.

$$\therefore \frac{2}{y_1} = \frac{5}{210}$$

$$\Rightarrow 5 \times y_1 = 2 \times 210$$

$$\Rightarrow y_1 = \frac{2 \times 210}{5}$$

$$\Rightarrow y_1 = ₹ 84$$

Similarly, $\frac{4}{y_2} = \frac{5}{210}$

$\Rightarrow 5 \times y_2 = 4 \times 210$

$\Rightarrow y_2 = \frac{4 \times 210}{5}$

$\Rightarrow y_2 = ₹ 168$

$\frac{10}{y_3} = \frac{5}{210}$

$\Rightarrow 5 \times y_3 = 10 \times 210$

$\Rightarrow y_3 = \frac{10 \times 210}{5} = ₹ 420$

$\frac{13}{y_4} = \frac{5}{210}$

$\Rightarrow 5 \times y_4 = 13 \times 210$

$\Rightarrow y_4 = \frac{13 \times 210}{5} = ₹ 546$

Q13. Six pumps working together empty a tank in 28 minutes. How long will it take to empty the tank if 4 such pumps are working together?

Sol. Let the required time be t minutes.

Number of pumps	6	4
Time (in minutes)	28	x

Less pump \rightarrow More time

Since there is an inverse variation.

$\therefore x_1 y_1 = x_2 y_2$

$\Rightarrow 6 \times 28 = 4 \times x$

$\Rightarrow x = \frac{6 \times 28}{4} = 42$

Hence, the required time = 42 minutes.

Q14. Mohit deposited a sum of ₹ 12000 in a Bank at a certain rate of interest for 2 years and earns an interest of ₹ 900. How much interest would be earned for a deposits of ₹ 15000 for the same period and at the same rate of interest?

Sol. Let the required amount of interest be ₹ x .

Deposit (in ₹)	12000	15000
Interest (in ₹)	900	x

Since there is a direct variations.

$\therefore \frac{x_1}{y_1} = \frac{x_2}{y_2}$

$\Rightarrow \frac{12000}{900} = \frac{15000}{x}$

$\Rightarrow 12000 \times x = 900 \times 15000$

$\Rightarrow x = \frac{900 \times 15000}{12000}$

$\Rightarrow x = 1125$

Hence, the required amount of interest = ₹ 1125.

Q15. A garrison of 120 men has provisions for 30 days. At the end of 5 days, 5 more men joined them. How many days can they sustain on the remaining provision?

Sol. Let the number of days be x .

Number of men	120	125
Number of days	25	x

[\therefore Remaining days = 30 - 5 = 25]

[Total men = 120 + 5 = 125]

Since there is an inverse variation.

$\therefore x_1 y_1 = x_2 y_2$

$\Rightarrow 120 \times 25 = 125 \times x$

$\Rightarrow x = \frac{120 \times 25}{125} = 24$

Hence, the required number of days be 24.

Q16. In a scout camp, there is food provision for 300 cadets for 42 days. If 50 more persons join the camp, for how many days will the provision last?

(NCERT Exemplar)

Sol. More the persons, the sooner would be the provision exhausted. So, this is a case of inverse proportion.

Let the required number of days be x .

Hence, $300 \times 42 = (300 + 50) \times x$

$\Rightarrow 300 \times 42 = 350 \times x$

$\Rightarrow \frac{300 \times 42}{350} = x$

$\therefore x = 36$

Q17. If two cardboard boxes occupy 500 cubic centimetres space, then how much space is required to keep 200 such boxes?

(NCERT Exemplar)

Sol. As the number of boxes increases, the space required to keep them also increases.

So, this is a case of direct proportion.

Number of boxes	2	200
Space occupied (in cubic centimetres)	500	x

So $\therefore \frac{2}{500} = \frac{200}{x}$

$\Rightarrow 2x = 500 \times 200$

$\Rightarrow x = \frac{500 \times 200}{2}$

$\therefore x = 50,000$

Thus, the required space is 50,000 cubic centimetres.

- Q18.** Under the condition that the temperature remains constant, the volume of gas is inversely proportional to its pressure. If the volume of gas is 630 cubic centimetres at a pressure of 360 mm of mercury, then what will be the pressure of the gas if its volume is 720 cubic centimetres at the same temperature? (NCERT Exemplar)

Sol. Given that, at constant temperature pressure and volume of a gas are inversely proportional. Let the required pressure be x .

Volume of gas (in cubic centimetres)	630	720
Pressure of gas (in mm)	360	x

$$\text{Then, } 630 \times 360 = 720 \times x$$

$$\frac{630 \times 360}{720} = x$$

$$\therefore x = 315$$

Therefore, the required pressure is 315 mm of mercury.

Test Yourself

II. SHORT ANSWER (SA) QUESTIONS

- Q1.** Find the values of a and b in the following table if x and y vary directly.

x	3	a	18
y	5	35	b

- Q2.** Fill in the blank spaces in the given table if x and y vary directly.

x	6	8	10	—
y	15	20	—	40

- Q3.** If a varies inversely as $b + 2$ and if $a = 8$, when $b = 1.5$, find a when $b = 5$.
- Q4.** If c is inversely proportional to d , and if $c = 18$ when $d = \frac{2}{3}$, find d when $c = \frac{6}{7}$.
- Q5.** If y varies inversely as x , and if $y = 4$ when $x = 72$, find the following:
(i) The constant of variation
(ii) The value of y when $x = 9$
- Q6.** 25 horses eat 5 bags of corn in 12 days, how much corn will 10 horses eat in 18 days?
- Q7.** A car travels 370 km in 20 litres of petrol. How far will it travel in 5 litres, 10 litres, 15 litres and 25 litres of petrol?
- Q8.** If 10 teachers can evaluate 1200 answer sheets in a week, then how many teachers would be required to evaluate 4200 sheets in the same time?

- Q9.** A machine in a soft drink factory fill 840 bottles in 6 hours. How many bottles will it fill in 5 hours?

- Q10.** Oranges cost ₹ 60 for 4 dozens in the super market. What is the cost of 12 oranges?

HIGHER ORDER THINKING SKILLS (HOTS) QUESTIONS

- Q11.** Ramesh goes on foot from a village A to village B at a speed of 4 km/h. If he walks at a speed of 5 km/h, he reaches village B seven minutes earlier. Find the distance between them.
- Q12.** A uniform pipe can fill a tank in 4 hours and pipe B can empty it in 5 hours. If both pipes are opened at the same time, when will the tank be full?
- Q13.** Mahesh can do a piece of work in 5 days and Rakesh can do the same piece of work independently in 6 days. How long will they take, if both work together?
- Q14.** Water tap x can fill a tank in 4 hours and tap y can empty the tank in 6 hours. How long would it take to fill the empty tank if both of them are opened together?
- Q15.** An army camp spends ₹ 1920 on wheat for 12 soldiers for 30 days. If they spends ₹ 1440 for 18 soldiers, for how many days will the wheat last?

ANSWERS

1. $a = 21, b = 30$ 2. 25, 16
3. 4 4. 14
5. (i) 288, (ii) 32 6. 3 bags
7. 92.5 km, 185 km, 277.5 km, 462.5 km
8. 35 9. 700 bottles

10. ₹ 15 11. 28 km
12. 20 hours 13. $2\frac{8}{11}$ hours
14. 12 hours 15. 15 days

Internal Assessment

Q1. Fill in the blanks.

(a) If x and y vary directly then $\frac{x}{y} = \dots\dots\dots$

(b) If x and y vary inversely then $\dots\dots\dots = k$.

(c) If $x_1 = 4, y_1 = 10, x_2 = 2$ and x and y vary directly, the value of y_2 is $\dots\dots\dots$

(d) If $\frac{20}{15} = \frac{8}{x}$ then $x = \dots\dots\dots$

(e) Speed = $\frac{\text{Distance}}{\dots\dots\dots}$

Q2. Multiple Choice Questions (MCQs)

(i) 10 men can dig a trench in 6 hours. Number of men who can dig the same trench in 12 hours is

- (a) 20 (b) 5
(c) 7 (d) 15

(ii) 36 men can complete a work in 20 days. The number of days in which the work will be done by 12 men is

- (a) $\frac{20}{3}$ day (b) 40 days
(c) 60 days (d) 8 days

(iii) If 5 labours can do a work in 28 days, then the number of labours to do the same work in 7 days is

- (a) 15 (b) 30
(c) 20 (d) 35

(iv) A pipe can fill a tank in 12 hours, the portion of tank filled in 9 hours is

- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{1}{12}$ (d) $\frac{1}{9}$

Q3. Write True (T) or False (F):

(a) The two quantities x and y are said to vary inversely if their quotient $\frac{x}{y}$ remains constant.

(b) If the cost of x metres of wire is ₹ p , then the cost of y metres of wire is ₹ $\frac{yp}{x}$.

(c) If 20 bars of soap weigh 5 kg, the weight of 4 bars will be $\frac{20 \times 5}{4}$ kg.

(d) The number of workers working to complete a piece of work and the time taken are inversely proportional.

(e) Area of a circle varies directly as the square of its radius.

Q4. There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Q5. Six pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

ANSWERS

1. (a) Constant (b) xy (c) 5
(d) 6 (e) Time
2. (i) b (ii) c
(iii) c (iv) a

3. (a) F (b) T (c) F
(d) T (e) T
4. 16 days 5. 1 hr 36 min.