CHAPTER 14

Factorisation

Understanding the Lesson

- Factors of Natural Number.
- Factors of Algebraic Expressions.
- Method of factorisation:
  (a) By common factors.
  (b) By regrouping the terms.
- Factorisation using identities
- Factors of the form $(x + a)(x + b)$
- Division of Algebraic Expressions
- Division of a monomial by another monomial.
- Division of a polynomial by a monomial.

Conceptual Facts

- Prime factors of 30 (say) = $2 \times 3 \times 5$
- Factors of algebraic expression $4xy = 4 \times x \times y$
- 1 is a factor of every expression.
- Method of common factors
$$4x + 24 = 4(x + 6)$$
- Method of factorisation by regrouping terms
$$6xy + 3y = 2 \times 3 \times x \times y + 3 \times y = 3 \times y\,(2x + 1) = 3y\,(2x + 1)$$
- Factorisation using identities
$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 - b^2 = (a + b)(a - b)$$

TRY THESE (*PAGE 219*)

Factorise:

(i) $12x + 36$ (ii) $22y - 33z$

(iii) $14pq + 35pqr$

Sol. (i) $12x + 36$
$= (2 \times 2 \times 3 \times x) + (2 \times 2 \times 3 \times 3)$
$= (2 \times 2 \times 3)(x + 3)$
$= 12(x + 3)$

(ii) $22y - 33z$
$= (2 \times 11 \times y) - (3 \times 11 \times z)$
$= 11(2 \times y - 3 \times z)$
$= 11(2y - 3z)$

(iii) $14pq + 35pqr$
$= (2 \times 7 \times p \times q) + (5 \times 7 \times p \times q \times r)$
$= (7 \times p \times q)(2 + 5r) = 7pq(2 + 5r)$

EXERCISE 14.1

Q1. Find the common factors of the given terms.

(i) $12x$, 36

(ii) $2y$, $22xy$

(iii) $14pq$, $28p^2q^2$

(iv) $2x$, $3x^2$, 4

(v) $6abc$, $24ab^2$, $12a^2b$

(vi) $16x^3$, $-4x^2$, $32x$

(vii) $10pq$, $20qr$, $30rp$

($viii$) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

Sol. (i) $12x$, 36
$\Rightarrow (2 \times 2 \times 3 \times x)$ and $(2 \times 2 \times 3 \times 3)$
$\therefore$ Common factors are $2 \times 2 \times 3 = 12$
Hence, the common factor = 12

(*ii*) $2y, 22xy$

$= (2 \times y)$ and $(2 \times 11 \times x \times y)$

$\therefore$ Common factors are $2 \times y = 2y$

Hence, the common factor $= 2y$

(*iii*) $14pq, 28p^2q^2$

$= (2 \times 7 \times p \times q)$ and $(2 \times 2 \times 7 \times p \times p \times q \times q)$

$\therefore$ Common factors are $2 \times 7 \times p \times q = 14pq$

Hence, the common factor $= 14pq$

(*iv*) $2x, 3x^2, 4$

$= (2 \times x)$, $(3 \times x \times x)$ and (2×2)

$\therefore$ Common factor is 1

Hence, the common factor = 1

[$\because$ 1 is a factor of every number]

(*v*) $6abc, 24ab^2, 12a^2b$

$= (2 \times 3 \times a \times b \times c)$, $(2 \times 2 \times 2 \times 3 \times a \times b \times b)$ and $(2 \times 2 \times 3 \times a \times a \times b)$

$\therefore$ Common factors are $2 \times 3 \times ab = 6ab$

Hence, the common factor $= 6ab$

(*vi*) $16x^3, -4x^2, 32x$

$= (2 \times 2 \times 2 \times 2 \times x \times x \times x)$, $-(2 \times 2 \times x \times x)$, $(2 \times 2 \times 2 \times 2 \times 2 \times x)$

$\therefore$ Common factors are $2 \times 2 \times x = 4x$

Hence, the common factor $= 4x$

(*vii*) $10pq, 20qr, 30rp$

$= (2 \times 5 \times p \times q)$, $(2 \times 2 \times 5 \times q \times r)$, $(2 \times 3 \times 5 \times r \times p)$

$\therefore$ Common factors are $2 \times 5 = 10$

Hence, the common factor = 10

(*viii*) $3x^2y^2, 10x^3y^2, 6x^2y^2z$

$= (3 \times x \times x \times y \times y)$, $(2 \times 5 \times x \times x \times x \times y \times y)$, $(2 \times 3 \times x \times x \times y \times y \times z)$

$\therefore$ Common factors are $x \times x \times y \times y = x^2y^2$

Hence, the common factor $= x^2y^2$.

Q2. Factorise the following expressions.

(*i*) $7x - 42$ (*ii*) $6p - 12q$

(*iii*) $7a^2 + 14a$ (*iv*) $-16z + 20z^3$

(*v*) $20l^2m + 30alm$ (*vi*) $5x^2y - 15xy^2$

(*vii*) $10a^2 - 15b^2 + 20c^2$ (*viii*) $-4a^2 + 4ab - 4ca$

(*ix*) $x^2yz + xy^2z + xyz^2$ (*x*) $ax^2y + bxy^2 + cxyz$

Sol. (*i*) $7x - 42 = 7(x - 6)$

(*ii*) $6p - 12q = 6(p - 2q)$

(*iii*) $7a^2 + 14a = 7a(a + 2)$

(*iv*) $-16z + 20z^3 = 4z(-4 + 5z^2)$

(*v*) $20l^2m + 30alm = 10lm(2l + 3a)$

(*vi*) $5x^2y - 15xy^2 = 5xy(x - 3y)$

(*vii*) $10a^2 - 15b^2 + 20c^2 = 5(2a^2 - 3b^2 + 4c^2)$

(*viii*) $-4a^2 + 4ab - 4ca = 4a(-a + b - c)$

(*ix*) $x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

(*x*) $ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$

Q3. Factorise:

(*i*) $x^2 + xy + 8x + 8y$

(*ii*) $15xy - 6x + 5y - 2$

(*iii*) $ax + bx - ay - by$

(*iv*) $15pq + 15 + 9q + 25p$

(*v*) $z - 7 + 7xy - xyz$

Sol. (*i*) $x^2 + xy + 8x + 8y$

Grouping the terms, we have

$x^2 + xy + 8x + 8y$

$= x(x + y) + 8(x + y)$

$= (x + y)(x + 8)$

Hence, the required factors $= (x + y)(x + 8)$

(*ii*) $15xy - 6x + 5y - 2$

Grouping the terms, we have

$(15xy - 6x) + (5y - 2)$

$= 3x(5y - 2) + (5y - 2)$

$= (5y - 2)(3x + 1)$

(*iii*) $ax + bx - ay - by$

Grouping the terms, we have

$= (ax - ay) + (bx - by)$

$= a(x - y) + b(x - y)$

$= (x - y)(a + b)$

Hence, the required factors $= (x - y)(a + b)$

(*iv*) $15pq + 15 + 9q + 25p$

Grouping the terms, we have

$(15pq + 25p) + (9q + 15)$

$= 5p(3q + 5) + 3(3q + 5)$

$= (3q + 5)(5p + 3)$

Hence, the required factors $= (3q + 5)(5p + 3)$

(*v*) $z - 7 + 7xy - xyz$

Grouping the terms, we have

$(-xyz + 7xy) + (z - 7)$

$= -xy(z - 7) + 1(z - 7)$

$= (-xy + 1)(z - 7)$

Hence the required factor $= -(1 - xy)(z - 7)$

EXERCISE 14.2

Q1. Factorise the following expressions.

(*i*) $a^2 + 8a + 16$

(*ii*) $p^2 - 10p + 25$

(*iii*) $25m^2 + 30m + 9$

(*iv*) $49y^2 + 84yz + 36z^2$

(*v*) $4x^2 - 8x + 4$

(*vi*) $121b^2 - 88bc + 16c^2$

(*vii*) $(l + m)^2 - 4lm$ (**Hint:** Expand $(l + m)^2$ first)

(*viii*) $a^4 + 2a^2b^2 + b^4$

Sol. (*i*) $a^2 + 8a + 16$

Here, $4 + 4 = 8$ and $4 \times 4 = 16$

$\therefore a^2 + 8a + 16$

$= a^2 + 4a + 4a + 4 \times 4$

$= (a^2 + 4a) + (4a + 16)$

$= a(a + 4) + 4(a + 4)$

$= (a + 4)(a + 4) = (a + 4)^2$

(*ii*) $p^2 - 10p + 25$

Here, $5 + 5 = 10$ and $5 \times 5 = 25$

$\therefore p^2 - 10p + 25$

$= p^2 - 5p - 5p + 5 \times 5$

$= (p^2 - 5p) + (-5p + 25)$

$= p(p - 5) - 5(p - 5)$

$= (p - 5)(p - 5) = (p - 5)^2$

(*iii*) $25m^2 + 30m + 9$

Here, $15 + 15 = 30$ and $15 \times 15 = 25 \times 9 = 225$

$\therefore 25m^2 + 30m + 9$

$= 25m^2 + 15m + 15m + 9$

$= (25m^2 + 15m) + (15m + 9)$

$= 5m(5m + 3) + 3(5m + 3)$

$= (5m + 3)(5m + 3)$

$= (5m + 3)^2$

(*iv*) $49y^2 + 84yz + 36z^2$

Here, $42 + 42 = 84$ and $42 \times 42 = 49 \times 36 = 1764$

$\therefore 49y^2 + 84yz + 36z^2$

$= 49y^2 + 42yz + 42yz + 36z^2$

$= 7y(7y + 6z) + 6z(7y + 6z)$

$= (7y + 6z)(7y + 6z)$

$= (7y + 6z)^2$

(*v*) $4x^2 - 8x + 4$

$= 4(x^2 - 2x + 1)$ [Taking 4 common]

$= 4(x^2 - x - x + 1)$

$= 4[x(x - 1) - 1(x - 1)]$

$= 4(x - 1)(x - 1)$

$= 4(x - 1)^2$

(*vi*) $121b^2 - 88bc + 16c^2$

Here, $44 + 44 = 88$ and $44 \times 44 = 121 \times 16 = 1936$

$\therefore 121b^2 - 88bc + 16c^2$

$= 121b^2 - 44bc - 44bc + 16c^2$

$= 11b(11b - 4c) - 4c(11b - 4c)$

$= (11b - 4c)(11b - 4c)$

$= (11b - 4c)^2$

(*vii*) $(l + m)^2 - 4lm$

Expanding $(l + m)^2$, we get

$l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2$

$= l^2 - lm - lm + m^2$

$= l(l - m) - m(l - m)$

$= (l - m)(l - m) = (l - m)^2$

(*viii*) $a^4 + 2a^2b^2 + b^4$

$= a^4 + a^2b^2 + a^2b^2 + b^4$

$= a^2(a^2 + b^2) + b^2(a^2 + b^2)$

$= (a^2 + b^2)(a^2 + b^2) = (a^2 + b^2)^2$

Q2. Factorise.

(*i*) $4p^2 - 9q^2$

(*ii*) $63a^2 - 112b^2$

(*iii*) $49x^2 - 36$

(*iv*) $16x^5 - 144x^3$

(*v*) $(l + m)^2 - (l - m)^2$

(*vi*) $9x^2y^2 - 16$

(*vii*) $(x^2 - 2xy + y^2) - z^2$

(*viii*) $25a^2 - 4b^2 + 28bc - 49c^2$

Sol. (*i*) $4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$= (2p - 3q)(2p + 3q)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

(*ii*) $63a^2 - 112b^2 = 7(9a^2 - 16b^2)$

$= 7[(3a)^2 - (4b)^2]$

$= 7(3a - 4b)(3a + 4b)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

(*iii*) $49x^2 - 36 = (7x)^2 - (6)^2$

$= (7x - 6)(7x + 6)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

(*iv*) $16x^5 - 144x^3 = 16x^3(x^2 - 9)$

$= 16x^3[(x)^2 - (3)^2]$

$= 16x^3(x - 3)(x + 3)$

$[\because a^2 - b^2 = (a + b)(a - b)]$

(v) $(l+m)^2-(l-m)^2$

$= [(l+m)-(l-m)]\,[(l+m)+(l-m)]$

$[\because\ a^2-b^2=(a+b)(a-b)$

$= (l+m-l+m)(l+m+l-m)$

$= (2m)(2l) = 4ml$

(vi) $9x^2y^2-16 = (3xy)^2-(4)^2$

$= (3xy-4)(3xy+4)$

$[\because\ a^2-b^2=(a+b)(a-b)]$

(vii) $(x^2-2xy+y^2)-z^2 = (x-y)^2-z^2$

$= (x-y-z)(x-y+z)$

$[\because\ a^2-b^2=(a+b)(a-b)]$

(viii) $25a^2-4b^2+28bc-49c^2$

$= 25a^2-(4b^2-28bc+49c^2)$

$= (5a)^2-(2b-7c)^2$

$= [5a-(2b-7c)][5a+(2b-7c)]$

$= (5a-2b+7c)(5a+2b-7c)$

Q3. Factorise the expressions.

(i) ax^2+bx

(ii) $7p^2+21q^2$

(iii) $2x^3+2xy^2+2xz^2$

(iv) $am^2+bm^2+bn^2+an^2$

(v) $(lm+l)+m+1$

(vi) $y(y+z)+9(y+z)$

(vii) $5y^2-20y-8z+2yz$

(viii) $10ab+4a+5b+2$

(ix) $6xy-4y+6-9x$

Sol. (i) $ax^2+bx = x(ax+b)$

(ii) $7p^2+21q^2 = 7(p^2+3q^2)$

(iii) $2x^3+2xy^2+2xz^2$

$= 2x(x^2+y^2+z^2)$

(iv) $am^2+bm^2+bn^2+an^2$

$= m^2(a+b)+n^2(a+b)$

$= (a+b)(m^2+n^2)$

(v) $(lm+l)+m+1$

$= l(m+1)+(m+1)$

$= (m+1)(l+1)$

(vi) $y(y+z)+9(y+z)$

$= (y+z)(y+9)$

(vii) $5y^2-20y-8z+2yz$

$= 5y^2-20y+2yz-8z$

$= 5y(y-4)+2z(y-4)$

$= (y-4)(5y+2z)$

(viii) $10ab+4a+5b+2$

$= 2a(5b+2)+1(5b+2)$

$= (5b+2)(2a+1)$

(ix) $6xy-4y+6-9x$

$= 6xy-4y-9x+6$

$= 2y(3x-2)-3(3x-2)$

$= (3x-2)(2y-3)$

Q4. Factorise.

(i) a^4-b^4 (ii) p^4-81

(iii) $x^4-(y+z)^4$ (iv) $x^4-(x-z)^4$

(v) $a^4-2a^2b^2+b^4$

Sol. (i) $a^4-b^4 = (a^2)^2-(b^2)^2$

$\boxed{\because\ a^2-b^2=(a-b)(a+b)]}$

$= (a^2-b^2)(a^2+b^2)$

$= (a-b)(a+b)(a^2+b^2)$

(ii) $p^4-81 = (p^2)^2-(9)^2$

$= (p^2-9)(p^2+9)$

$[\because\ a^2-b^2=(a+b)(a-b)]$

$= (p-3)(p+3)(p^2+9)$

(iii) $x^4-(y+z)^4 = (x^2)^2-[(y+z)^2]^2$

$[\because\ a^2-b^2=(a-b)(a+b)]$

$= [x^2-(y+z)^2]\,[x^2+(y+z)^2]$

$= [x-(y+z)][x+(y+z)]\,[x^2+(y+z)^2]$

$= (x-y-z)(x+y+z)\,[x^2+(y+z)^2]$

(iv) $x^4-(x-z)^4 = (x^2)^2-[(y-z)^2]^2$

$= [x^2-(y-z)^2]\,[x^2+(y-z)^2]$

$= (x-y+z)(x+y-z)(x^2+(y-z)^2]$

(v) $a^4-2a^2b^2+b^4$

$= a^4-a^2b^2-a^2b^2+b^4$

$= a^2(a^2-b^2)-b^2(a^2-b^2)$

$= (a^2-b^2)(a^2-b^2)$

$= (a^2-b^2)^2 = [(a-b)(a+b)]^2$

$= (a-b)^2(a+b)^2$

Q5. Factorise the following expressions.

(i) p^2+6p+8

(ii) $q^2-10q+21$

(iii) $p^2+6p-16$

Sol. (i) p^2+6p+8

Here, $2+4=6$ and $2\times 4=8$

$\therefore\ p^2+6p+8$

$= p^2+2p+4p+8$

$= p(p+2) + 4(p+2)$

$= (p+2)(p+4)$

(*ii*) $q^2 - 10q + 21$

Here, $3 + 7 = 10$ and $3 \times 7 = 21$

$\therefore\ q^2 - 10q + 21$

$= q^2 - 3q - 7q + 21$

$= q(q-3) - 7(q-3)$

$= (q-3)(q-7)$

(*iii*) $p^2 + 6p - 16$

Here, $8 - 2 = 6$ and $8 \times 2 = 16$

$\therefore\ p^2 + 6p - 16 = p^2 + 8p - 2p - 16$

$= p(p+8) - 2(p+8)$

$= (p+8)(p-2)$

TRY THESE (PAGE 225)

Q1. Divide.

(*i*) $24xy^2z^3$ by $6yz^2$

(*ii*) $63a^2b^4c^6$ by $7a^2b^2c^3$

Sol. (*i*) $24xy^2z^3 \div 6yz^2$

$$= \frac{\cancel{2} \times 2 \times 2 \times \cancel{3} \times x \times \cancel{y} \times \cancel{y} \times z \times \cancel{z} \times \cancel{z}}{\cancel{2} \times \cancel{3} \times \cancel{y} \times \cancel{z} \times \cancel{z}}$$

$= 4xyz$

(*ii*) $63a^2b^4c^6$ by $7a^2b^2c^3$

$63a^2b^4c^6 \div 7a^2b^2c^3$

$$= \frac{3 \times 3 \times \cancel{7} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times \cancel{c} \times \cancel{c} \times c \times c \times c}{\cancel{7} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{c} \times \cancel{c} \times \cancel{c}}$$

$= 3 \times 3 \times b \times b \times c \times c \times c = 9b^2c^3$

EXERCISE 14.3

Q1. Carry out the following divisions.

(*i*) $28x^4 \div 56x$

(*ii*) $-36y^3 \div 9y^2$

(*iii*) $66pq^2r^3 \div 11qr^2$

(*iv*) $34x^3y^3z^3 \div 51xy^2z^3$

(*v*) $12a^8b^8 \div (-6a^6b^4)$

Sol. (*i*) $28x^4 \div 56x = \dfrac{28x^4}{56x}$

$$= \frac{\cancel{2} \times \cancel{2} \times \cancel{7} \times \cancel{x} \times x \times x \times x}{2 \times \cancel{2} \times \cancel{2} \times \cancel{7} \times \cancel{x}} = \frac{x^3}{2}$$

(*ii*) $-36y^3 \div 9y^2 = \dfrac{-36y^3}{9y^2}$

$$= \frac{\cancel{(-)} \times 2 \times 2 \times \cancel{3} \times \cancel{3} \times y \times \cancel{y} \times \cancel{y}}{\cancel{(-)} \times (-) \times \cancel{3} \times \cancel{y} \times \cancel{3} \times \cancel{y}} = -4y$$

(*iii*) $66pq^2r^3 \div 11qr^2 = \dfrac{66pq^2r^3}{11qr^2}$

$$= \frac{2 \times 3 \times \cancel{11} \times p \times \cancel{q} \times q \times \cancel{r} \times \cancel{r} \times r}{\cancel{11} \times \cancel{q} \times \cancel{r} \times \cancel{r}}$$

$= 6pqr$

(*iv*) $34x^3y^3z^3 \div 51xy^2z^3 = \dfrac{34x^3y^3z^3}{51xy^2z^3}$

$$= \frac{2 \times \cancel{17} \times \cancel{x} \times x \times x \times \cancel{y} \times \cancel{y} \times y \times \cancel{z} \times \cancel{z} \times \cancel{z}}{3 \times \cancel{17} \times \cancel{x} \times \cancel{y} \times \cancel{y} \times \cancel{z} \times \cancel{z} \times \cancel{z}} = \frac{2}{3}x^2y$$

(*v*) $12a^8b^8 \div (-6a^6b^4) = \dfrac{12a^8b^8}{-6a^6b^4}$

$$= \frac{\cancel{(-)} \times (-) \times \cancel{2} \times 2 \times \cancel{3} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}{\cancel{(-)} \times \cancel{2} \times \cancel{3} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b}}$$

$= -2a^2b^4$

Q2. Divide the following polynomial by the given monomial.

(*i*) $(5x^2 - 6x) \div 3x$

(*ii*) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(*iii*) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(*iv*) $(x^3 + 2x^2 + 3x) \div 2x$

(*v*) $(p^3q^6 - p^6q^3) \div p^3q^3$

Sol. (*i*) $(5x^2 - 6x) \div 3x = \dfrac{(5x^2 - 6x)}{3x}$

$$= \frac{\cancel{x}(5x-6)}{3\cancel{x}} = \frac{(5x-6)}{3}$$

(*ii*) $(3y^8 - 4y^6 + 5y^4) \div y^4$

$$= \frac{3y^8 - 4y^6 + 5y^4}{y^4}$$

$$= \frac{\cancel{y^4}(3y^4 - 4y^2 + 5)}{\cancel{y^4}}$$

$= 3y^4 - 4y^2 + 5$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

$$= \frac{8\left(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3\right)}{4x^2y^2z^2}$$

$$= \frac{\cancel{8}^{2}\,\cancel{x^2}\,\cancel{y^2}\,\cancel{z^2}\,(x+y+z)}{\cancel{4}\,\cancel{x^2}\,\cancel{y^2}\,\cancel{z^2}}$$

$$= 2(x + y + z)$$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

$$= \frac{x^3 + 2x^2 + 3x}{2x} = \frac{\cancel{x}\left(x^2 + 2x + 3\right)}{2\cancel{x}}$$

$$= \frac{x^2 + 2x + 3}{2}$$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3 = \dfrac{p^3q^6 - p^6q^3}{p^3q^3}$

$$= \frac{\cancel{p^3q^3}\left(q^3 - p^3\right)}{\cancel{p^3q^3}} = q^3 - p^3$$

Q3. Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Sol. (i) $(10x - 25) \div 5 = \dfrac{10x - 25}{5}$

$$= \frac{\cancel{5}\,(2x-5)}{\cancel{5}} = 2x - 5$$

(ii) $(10x - 25) \div (2x - 5) = \dfrac{(10x - 25)}{(2x - 5)}$

$$= \frac{5\cancel{(2x-5)}}{\cancel{(2x-5)}} = 5$$

(iii) $10y(6y + 21) \div 5(2y + 7)$

$$= \frac{10y(6y+21)}{5(2y+7)}$$

$$= \frac{\cancel{10y}^{2} \times 3\cancel{(2y+7)}}{\cancel{5}\,\cancel{(2y+7)}}$$

$$= 2y \times 3 = 6y$$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

$$= \frac{9x^2y^2(3z-24)}{27xy(z-8)}$$

$$= \frac{\cancel{9}\,\overset{x}{\cancel{x^2}}\,\overset{y}{\cancel{y^2}} \times \cancel{3}\,\cancel{(z-8)}}{\overset{9}{\cancel{27}}\,\cancel{x} \times \cancel{y}\,\cancel{(z-8)}} = xy$$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

$$= \frac{96abc(3a-12)(5b-30)}{144(a-4)(b-6)}$$

$$= \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times \cancel{3}abc \times \cancel{3}\,\cancel{(a-4)} \times 5\,\cancel{(b-6)}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{3}\;\cancel{(a-4)}\,\cancel{(b-6)}} = 10abc$$

Q4. Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

(ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$

(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

Sol. (i) $5(2x + 1)(3x + 5) \div (2x + 1)$

$$= \frac{5\cancel{(2x+1)}(3x+5)}{\cancel{(2x+1)}} = 5(3x + 5)$$

(ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

$$= \frac{26xy(x+5)\cancel{(y-4)}}{13x\cancel{(y-4)}}$$

$$= \frac{2 \times \cancel{13}\,\cancel{x}\,y(x+5)\,\cancel{(y-4)}}{\cancel{13}\,\cancel{x}\,\cancel{(y-4)}}$$

$$= 2y(x + 5)$$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

$$= \frac{52pqr(p+q)\;(q+r)\;(r+p)}{104pq(q+r)(r+p)}$$

$$= \frac{\cancel{2} \times \cancel{2} \times \cancel{13}\,\cancel{p}\,\cancel{q}\,r(p+q)\,\cancel{(q+r)}\,\cancel{(r+p)}}{\cancel{2} \times \cancel{2} \times 2 \times \cancel{13}\,\cancel{p}\,\cancel{q}\;\cancel{(q+r)}\,\cancel{(r+p)}}$$

$$= \frac{r(p+q)}{2}$$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$

$$= \frac{20(y+4)\left(y^2+5y+3\right)}{5(y+4)}$$

$$= \frac{2 \times 2 \times \cancel{5}\,\cancel{(y+4)}\,(y^2+5y+3)}{\cancel{5}\,\cancel{(y+4)}}$$

$$= 4(y^2+5y+3)$$

(v) $x(x+1)(x+2)(x+3) \div x(x+1)$

$$= \frac{\cancel{x}\,\cancel{(x+1)}\,(x+2)\,(x+3)}{\cancel{x}\,\cancel{(x+1)}}$$

$$= (x+2)(x+3)$$

Q5. Factorise the expressions and divide them as directed.

(i) $(y^2+7y+10) \div (y+5)$

(ii) $(m^2-14m-32) \div (m+2)$

(iii) $(5p^2-25p+20) \div (p-1)$

(iv) $4yz(z^2+6z-16) \div 2y(z+8)$

(v) $5pq(p^2-q^2) \div 2p(p+q)$

(vi) $12xy(9x^2-16y^2) \div 4xy(3x+4y)$

(vii) $39y^3(50y^2-98) \div 26y^2(5y+7)$

Sol. (i) $(y^2+7y+10) \div (y+5)$

$$= \frac{(y^2+7y+10)}{y+5}$$

$$= \frac{(y^2+5y+2y+10)}{(y+5)} \quad \left[\because 2+5=7,\ 2\times 5=10\right]$$

$$= \frac{y(y+5)+2(y+5)}{(y+5)}$$

$$= \frac{\cancel{(y+5)}\,(y+2)}{\cancel{(y+5)}}$$

$$= y+2$$

(ii) $(m^2-14m-32) \div (m+2)$

$$= \frac{(m^2-14m-32)}{(m+2)}$$

$$= \frac{(m^2-16m+2m-32)}{(m+2)} \quad \left[\begin{matrix}-16+2=-14\\ -16\times 2=-32\end{matrix}\right]$$

$$= \frac{m(m-16)+2(m-16)}{m+2}$$

$$= \frac{(m-16)\,\cancel{(m+2)}}{\cancel{(m+2)}}$$

$$= m-16$$

(iii) $(5p^2-25p+20) \div (p-1)$

$$= \frac{(5p^2-25p+20)}{p-1}$$

$$= \frac{5p^2-20p-5p+20}{p-1}$$

$$= \frac{5p(p-4)-5(p-4)}{p-1}$$

$$= \frac{(p-4)(5p-5)}{p-1}$$

$$= \frac{(p-4)5\cancel{(p-1)}}{\cancel{(p-1)}}$$

$$= 5(p-4)$$

(iv) $4yz(z^2+6z-16) \div 2y(z+8)$

$$= \frac{4yz(z^2+6z-16)}{2y(z+8)}$$

$$= \frac{4yz(z^2+8z-2z-16)}{2y(z+8)}$$

$$= \frac{\overset{2}{\cancel{4y}}\,z[z(z+8)-2(z+8)]}{\cancel{2y}\,(z+8)}$$

$$= \frac{2z\,\cancel{(z+8)}\,(z-2)}{\cancel{(z+8)}}$$

$$= 2z(z-2)$$

(v) $5pq(p^2-q^2) \div 2p(p+q)$

$$= \frac{5pq(p^2-q^2)}{2p(p+q)}$$

$$= \frac{5\cancel{p}q\,\cancel{(p+q)}\,(p-q)}{2\cancel{p}\,\cancel{(p+q)}}$$

$$= \frac{5q(p-q)}{2}$$

(vi) $12xy(9x^2-16y^2) \div 4xy(3x+4y)$

$$= \frac{12xy(9x^2-16y^2)}{4xy(3x+4y)}$$

$$= \frac{12xy[(3x)^2-(4y)^2]}{4xy(3x+4y)}$$

$$= \frac{\overset{3}{\cancel{12}}\ \cancel{xy}\ \cancel{(3x+4y)}\ (3x-4y)}{\cancel{4}\ \cancel{xy}\ \cancel{(3x+4y)}}$$

$[\because\ a^2 - b^2 = (a+b)(a-b)]$

$= 3(3x - 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

$$= \frac{39y^3\left(50y^2 - 98\right)}{26y^2\left(5y+7\right)}$$

$$= \frac{3 \times 13y^3 \times 2\left(25y^2 - 49\right)}{2 \times 13y^2\left(5y+7\right)}$$

$$= \frac{3 \times 13y^3 \times 2\left[(5y)^2 - (7)^2\right]}{2 \times 13 \times y^2\left(5y+7\right)}$$

$$= \frac{\cancel{2} \times 3 \times \cancel{13}\ \overset{y}{\cancel{y^2}}\ \cancel{(5y+7)}\ (5y-7)}{\cancel{2} \times \cancel{13}\ \cancel{y^2}\ \cancel{(5y+7)}}$$

$= 3y(5y - 7)$

EXERCISE 14.4

Find and correct the errors in the following mathematical statements.

Mathematical Statement	Correction
1. $4(x - 5) = 4x - 5$	$4(x - 5) = 4x - 20$
2. $x(3x + 2) = 3x^2 + 2$	$x(3x + 2) = 3x^2 + 2x$
3. $2x + 3y = 5xy$	$2x + 3y = 2x + 3y$
4. $x + 2x + 3x = 5x$	$x + 2x + 3x = 6x$
5. $5y + 2y + y - 7y = 0$	$5y + 2y + y - 7y = y$
6. $3x + 2x = 5x^2$	$3x + 2x = 5x$
7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$	$(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$
8. $(2x)^2 + 5x = 4x + 5x = 9x$	$(2x)^2 + 5x = 4x^2 + 5x$
9. $(3x + 2)^2 = 3x^2 + 6x + 4$	$(3x + 2)^2 = 9x^2 + 12x + 4$
10. Substituting $x = -3$ in	
(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$	$x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2$
(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$	$x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$
(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$	$x^2 + 5x$ gives $(-3)^2 + 5(-3) = 9 - 15 = -6$
11. $(y - 3)^2 = y^2 - 9$	$(y - 3)^2 = y^2 - 6y + 9$
12. $(z + 5)^2 = z^2 + 25$	$(z + 5)^2 = z^2 + 10z + 25$
13. $(2a + 3b)(a - b) = 2a^2 - 3b^2$	$(2a + 3b)(a - b)$ $= a(2a + 3b) - b(2a + 3b)$ $= 2a^2 + 3ab - 2ab - 3b^2$ $= 2a^2 + ab - 3b^2$
14. $(a + 4)(a + 2) = a^2 + 8$	$(a + 4)(a + 2) = a^2 + (4 + 2)a + 8$ $= a^2 + 6a + 8$
15. $(a - 4)(a - 2) = a^2 - 8$	$(a - 4)(a - 2) = a^2 + (-4 - 2)a + 8$ $= a^2 - 6a + 8$

16. $\frac{3x^2}{3x^2} = 0$	$\frac{3x^2}{3x^2} = 1$
17. $\frac{3x^2+1}{3x^2} = 1+1=2$	$\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2}$
18. $\frac{3x}{3x+2} = \frac{1}{2}$	$\frac{3x}{3x+2} = \frac{3x}{3x+2}$ (No error)
19. $\frac{3}{4x+3} = \frac{1}{4x}$	$\frac{3}{4x+3} = \frac{3}{4x+3}$
20. $\frac{4x+5}{4x} = 5$	$\frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x}$
21. $\frac{7x+5}{5} = 7x$	$\frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1$

Learning More Q & A

I. Very Short Answer (VSA) Questions

Q1. Find the common factors of the following terms.

(a) $25x^2y$, $30xy^2$ (b) $63m^3n$, $54mn^4$

Sol. (a) $25x^2y$, $30xy^2$

$25x^2y = 5 \times 5 \times x \times x \times y$

$30xy^2 = 2 \times 3 \times 5 \times x \times y \times y$

Common factors are $5 \times x \times y = 5xy$

(b) $63m^3n$, $54mn^4$

$63m^3n = 3 \times 3 \times 7 \times m \times m \times m \times n$

$54mn^4 = 2 \times 3 \times 3 \times 3 \times m \times n \times n \times n \times n$

Common factors are $3 \times 3 \times m \times n = 9mn$

Q2. Factorise the following expressions.

(a) $54m^3n + 81m^4n^2$

(b) $15x^2y^3z + 25x^3y^2z + 35x^2y^2z^2$

Sol. (a) $54m^3n + 81m^4n^2$

$= 2 \times 3 \times 3 \times 3 \times m \times m \times m \times n + 3 \times 3 \times 3 \times 3 \times m \times m \times m \times m \times n \times n$

$= 3 \times 3 \times 3 \times m \times m \times m \times n \times (2 + 3mn)$

$= 27m^3n\,(2 + 3mn)$

(b) $15x^2y^3z + 25x^3y^2z + 35x^2y^2z^2$

$= 5x^2y^2z\,(3y + 5x + 7)$

Q3. Factorise the following polynomials.

(a) $6p(p-3) + 1(p-3)$

(b) $14(3y-5z)^3 + 7(3y-5z)^2$

Sol. (a) $6p(p-3) + 1(p-3)$

$= (p-3)(6p+1)$

(b) $14(3y-5z)^3 + 7(3y-5z)^2$

$= 7(3y-5z)^2\,[2(3y-5z)+1]$

$= 7(3y-5z)^2\,(6y-10z+1)$

Q4. Factorise the following:

(a) $p^2q - pr^2 - pq + r^2$

(b) $x^2 + yz + xy + xz$

Sol. (a) $p^2q - pr^2 - pq + r^2$

$= (p^2q - pq) + (-pr^2 + r^2)$

$= pq(p-1) - r^2(p-1)$

$= (p-1)(pq - r^2)$

(b) $x^2 + yz + xy + xz$

$= x^2 + xy + xz + yz$

$= x(x+y) + z(x+y)$

$= (x+y)(x+z)$

Q5. Factorise the following polynomials.

(a) $xy(z^2+1) + z(x^2+y^2)$

(b) $2axy^2 + 10x + 3ay^2 + 15$

Sol. (a) $xy(z^2+1) + z(x^2+y^2)$

$= xyz^2 + xy + zx^2 + zy^2$

$= (xyz^2 + zx^2) + (xy + zy^2)$

$= zx(yz + x) + y(x + yz)$

$= zx(x + yz) + y(x + yz)$

$= (x + yz)(zx + y)$

(b) $2axy^2 + 10x + 3ay^2 + 15$

$= (2axy^2 + 3ay^2) + (10x + 15)$

$= ay^2(2x+3) + 5(2x+3)$

$= (2x+3)(ay^2+5)$

Q6. Factorise the following expressions.

(a) $x^2 + 4x + 8y + 4xy + 4y^2$

(b) $4p^2 + 2q^2 + p^2q^2 + 8$

Sol. (a) $x^2 + 4x + 8y + 4xy + 4y^2$

$= (x^2 + 4xy + 4y^2) + (4x + 8y)$

$= (x + 2y)^2 + 4(x + 2y)$

$= (x + 2y)(x + 2y + 4)$

(b) $4p^2 + 2q^2 + p^2q^2 + 8$

$= (4p^2 + 8) + (p^2q^2 + 2q^2)$

$= 4(p^2 + 2) + q^2(p^2 + 2)$

$= (p^2 + 2)(4 + q^2)$

Q7. Factorise:

(a) $a^2 + 14a + 48$ (b) $m^2 - 10m - 56$

Sol. (a) $a^2 + 14a + 48$

$= a^2 + 6a + 8a + 48$ $\quad \begin{bmatrix} 6+8=14 \\ 6\times 8 = 48 \end{bmatrix}$

$= a(a + 6) + 8(a + 6)$

$= (a + 6)(a + 8)$

(b) $m^2 - 10m - 56$

$= m^2 - 14m + 4m - 56$ $\quad \begin{bmatrix} 14-4=10 \\ 14\times 4 = 56 \end{bmatrix}$

$= m(m - 14) + 6(m - 14)$

$= (m - 14)(m + 6)$

Q8. Factorise:

(a) $x^4 - (x - y)^4$

(b) $4x^2 + 9 - 12x - a^2 - b^2 + 2ab$

Sol. (a) $x^4 - (x - y)^4$

$= (x^2)^2 - [(x - y)^2]^2$

$= [x^2 - (x - y)^2][x^2 + (x - y)^2]$

$= [x + (x - y)][x - (x - y)][x^2 + x^2 - 2xy + y^2]$

$= (x + x - y)(x - x + y)[2x^2 - 2xy + y^2]$

$= (2x - y)\, y(2x^2 - 2xy + y^2)$

$= y(2x - y)(2x^2 - 2xy + y^2)$

(b) $4x^2 + 9 - 12x - a^2 - b^2 + 2ab$

$= (4x^2 - 12x + 9) - (a^2 + b^2 - 2ab)$

$= (2x - 3)^2 - (a - b)^2$

$= [(2x - 3) + (a - b)][(2x - 3) - (a - b)]$

$= (2x - 3 + a - b)(2x - 3 - a + b)$

II. Short Answer (SA) Questions

Q9. Factorise the following polynomials.

(a) $16x^4 - 81$ (b) $(a - b)^2 + 4ab$

Sol. (a) $16x^4 - 81 = (4x^2)^2 - (9)^2$

$= (4x^2 + 9)(4x^2 - 9)$

$= (4x^2 + 9)[(2x)^2 - (3)^2]$

$= (4x^2 + 9)(2x + 3)(2x - 3)$

(b) $(a - b)^2 + 4ab$

$= a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2 = (a + b)^2$

Q10. Factorise:

(a) $14m^5n^4p^2 - 42m^7n^3p^7 - 70m^6n^4p^3$

(b) $2a^2(b^2 - c^2) + b^2(2c^2 - 2a^2) + 2c^2(a^2 - b^2)$

Sol. (a) $14m^5n^4p^2 - 42m^7n^3p^7 - 70m^6n^4p^3$

$= 14m^5n^3p^2(n - 3m^2p^5 - 5mnp)$

(b) $2a^2(b^2 - c^2) + b^2(2c^2 - 2a^2) + 2c^2(a^2 - b^2)$

$= 2a^2(b^2 - c^2) + 2b^2(c^2 - a^2) + 2c^2(a^2 - b^2)$

$= 2[a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)]$

$= 2\Big[\cancel{a^2b^2} - \cancel{a^2c^2} + \cancel{b^2c^2} - \cancel{a^2b^2} + \cancel{a^2c^2} - \cancel{b^2c^2}\Big]$

$= 2 \times 0 = 0$

Q11. Factorise:

(a) $(x + y)^2 - 4xy - 9z^2$

(b) $25x^2 - 4y^2 + 28yz - 49z^2$

Sol. (a) $(x + y)^2 - 4xy - 9z^2$

$= x^2 + 2xy + y^2 - 4xy - 9z^2$

$= (x^2 - 2xy + y^2) - 9z^2$

$= (x - y)^2 - (3z)^2$

$= (x - y + 3z)(x - y - 3z)$

(b) $25x^2 - 4y^2 + 28yz - 49z^2$

$= 25x^2 - (4y^2 - 28yz + 49z^2)$

$= (5x)^2 - (2y - 7)^2$

$= (5x + 2y - 7)[5x - (2y - 7)]$

$= (5x + 2y - 7)(5x - 2y + 7)$

Q12. Evaluate the following divisions:

(a) $(3b - 6a) \div (30a - 15b)$

(b) $(4x^2 - 100) \div 6(x + 5)$

Sol. (a) $(3b - 6a) \div (30a - 15b)$

$= \dfrac{3b - 6a}{30a - 15b} = \dfrac{-\cancel{3}\,\cancel{(2a - b)}}{\cancel{15}\,\cancel{(2a - b)}}$ ($15 \to 5$)

$= \dfrac{-1}{5}$

(b) $(4x^2 - 100) \div 6(x + 5)$

$= \dfrac{4x^2 - 100}{6(x + 5)} = \dfrac{4(x^2 - 25)}{6(x + 5)}$

$= \dfrac{\overset{2}{\cancel{4}}\,(x - 5)\,\cancel{(x + 5)}}{\underset{3}{\cancel{6}}\,\cancel{(x + 5)}}$

$= \dfrac{2}{3}(x - 5)$

Q13. Simplify the following expressions:

(a) $\dfrac{(x-1)(x-2)\left(x^2-9x+14\right)}{(x-7)\left(x^2-3x+2\right)}$

(b) $\dfrac{\left(x^2-8x+12\right)\left(x^2-16\right)}{\left(x^2-36\right)\left(x^2-4\right)}$

Sol. (a) $\dfrac{(x-1)(x-2)\left(x^2-9x+14\right)}{(x-7)\left(x^2-3x+2\right)}$

$= \dfrac{(x-1)(x-2)\left(x^2-7x-2x+14\right)}{(x-7)\left(x^2-2x-x+2\right)}$

$= \dfrac{(x-1)(x-2)\left[x(x-7)-2(x-7)\right]}{(x-7)\left[x(x-2)-1(x-2)\right]}$

$= \dfrac{\cancel{(x-1)}\,(x-2)\,\cancel{(x-7)}\,\cancel{(x-2)}}{\cancel{(x-7)}\,\cancel{(x-2)}\,\cancel{(x-1)}} = (x-2)$

(b) $\dfrac{\left(x^2-8x+12\right)\left(x^2-16\right)}{\left(x^2-36\right)\left(x^2-4\right)}$

$= \dfrac{\left(x^2-6x-2x+12\right)(x-4)(x+4)}{(x-6)(x+6)(x-2)(x+2)}$

$= \dfrac{\left[x(x-6)-2(x-6)\right](x-4)(x+4)}{(x-6)(x+6)(x-2)(x+2)}$

$= \dfrac{\cancel{(x-2)}\,\cancel{(x-6)}\,(x-4)(x+4)}{\cancel{(x-6)}\,(x+6)\,\cancel{(x-2)}\,(x+2)}$

$= \dfrac{(x-4)(x+4)}{(x+6)(x+2)}$

Q14. Factorise the given expressions and divide that as indicated.

(a) $39n^3(50n^2-98) \div 26n^2(5n-7)$

(b) $44(p^4-5p^3-24p^2) \div 11p(p-8)$

Sol. (a) $39n^3(50n^2-98) \div 26n^2(5n+7)$

$= \dfrac{39n^3\left(50n^2-98\right)}{26n^2(5n-7)}$

$= \dfrac{39n^3 \times 2\left(25n^2-49\right)}{26n^2(5n-7)}$

$= \dfrac{3\times 13n^3 \times 2\left[(5n)^2-(7)^2\right]}{2\times 13n^2(5n-7)}$

$= \dfrac{3\times \cancel{13}\,n^{\cancel{3}^{n}} \times \cancel{2}\,(5n+7)\,\cancel{(5n-7)}}{\cancel{2}\times \cancel{13}\,n^{\cancel{2}}\,\cancel{(5n-7)}}$

$= 3n(5n+7)$

(b) $44(p^4-5p^3-24p^2) \div 11p(p-8)$

$= \dfrac{44\left(p^4-5p^3-24p^2\right)}{11p(p-8)}$

$= \dfrac{44\times p^2\left(p^2-5p-24\right)}{11p(p-8)}$

$= \dfrac{44p^2\left(p^2-8p+3p-24\right)}{11p(p-8)}$

$= \dfrac{44p^2\left[p(p-8)+3(p-8)\right]}{11p(p-8)}$

$= \dfrac{\overset{4}{\cancel{44}}\,\overset{p}{\cancel{p^2}}\,\cancel{(p-8)}\,(p+3)}{\cancel{11}\,\cancel{p}\,\cancel{(p-8)}}$

$= 4p(p+3)$

Q15. If one of the factors of $(5x^2+70x-160)$ is $(x-2)$. Find the other factor.

Sol. Let the other factor be m.

$\therefore\ (x-2)\times m = 5x^2+70x-160$

$\Rightarrow\quad m = \dfrac{5x^2+70x-160}{x-2}$

$= \dfrac{5\left(x^2+14x-32\right)}{x-2}$

$= \dfrac{5\left(x^2+16x-2x-32\right)}{x-2}$

$= \dfrac{5\left[x(x+16)-2(x+16)\right]}{x-2}$

$= \dfrac{5(x+16)\,\cancel{(x-2)}}{\cancel{(x-2)}}$

$= 5(x+16)$

Test Yourself

Q1. Factorise the following polynomials.

(*a*) $75 - 25x$ (*b*) $6xy + 3y^2$

(*c*) $8x^3y - 32xy^3$

Q2. Factorise by regrouping the terms.

(*a*) $x^2 + yz + xy + zx$

(*b*) $m^2n - ms^2 - mn + s^2$

Q3. Factorise the following by splitting the middle terms.

(*a*) $x^2 - x - 6$ (*b*) $2x^2 - x - 6$

(*c*) $2x^2 - x - 1$

Q4. Factorise the following trinomials.

(*a*) $1 - 8ax + 16a^2x^2$

(*b*) $x^2 + x + \frac{1}{4}$

(*c*) $16p^2 - 40pq + 25q^2$

Q5. Factorise the following polynomials.

(*a*) $\frac{x^2}{4y^2} + \frac{2}{3} + \frac{4y^2}{9x^2}$

(*b*) $\frac{a^2}{b^2} + 2 + \frac{b^2}{a^2}$

Q6. Factorise the following expressions.

(*a*) $x(x - 2z) + y(x - 2z) + (2z - x)$

(*b*) $16(3x - 5y)^3 + 8(3x - 5y)^2$

Q7. Factorise the following expressions.

(*a*) $2b^2 + 8ab + 4ac + bc$

(*b*) $6pm + 9mq + 8pn + 12qn$

Q8. Factorise:

(*a*) $4x^2 - 144$

(*b*) $x^4 - 81$

(*c*) $3x^3y - 12xy^3$

Q9. Factorise the following:

(*a*) $15p^4 + 3p^2 - 18$ (*b*) $-5x^2 - x + 4$

Q10. Factorise the following polynomials.

(*a*) $ab(x^2 + y^2) - xy(a^2 + b^2)$

(*b*) $100 - x^2 - y^2 - 2xy$

Q11. Divide $25x^3y - 16xy^3$ by $5x - 4y$.

Q12. Divide $x^3 + 6x^2 + 11x + 6$ by $x + 2$.

Q13. Area of a rectangle is $(15x^2 + 23x + 4)$ units. If its length is $(5x + 1)$ units, find its breadth.

Q14. Factorise the following:

(*a*) $a^4 - (a - b)^4$

(*b*) $16x^2 + 40xy - 81 + 25y^2$

Q15. Simplify the following fractions.

(*i*) $\frac{6.25x^2 - 1.21y^2}{2.5x - 1.1y}$ (*ii*) $\frac{16x^4 - 625y^4}{4x^2 + 25y^2}$

ANSWERS

1. (*a*) $25(3 - x)$ (*b*) $3y(2x + y)$
   (*c*) $8xy(x + 2y)(x - 2y)$
2. (*a*) $(x + y)(x + z)$ (*b*) $(m - 1)(mn - s^2)$
3. (*a*) $(x - 3)(x + 2)$ (*b*) $(x - 2)(2x + 3)$
   (*c*) $(x - 1)(2x + 1)$
4. (*a*) $(1 - 4ax)^2$ (*b*) $\left(x + \frac{1}{2}\right)^2$
   (*c*) $(4p - 5q)^2$
5. (*a*) $\left(\frac{x}{2y} - \frac{2y}{3x}\right)^2$ (*b*) $\left(\frac{a}{b} + \frac{b}{a}\right)^2$
6. (*a*) $(x + y - 1)(x - 2z)$
   (*b*) $8(3x - 5y)^2(6x - 10y + 1)$
7. (*a*) $(4a + b)(2b + c)$
   (*b*) $(2p + 3q)(3m + 4n)$
8. (*a*) $4(x - 6)(x + 6)$
   (*b*) $(x^2 + 9)(x + 3)(x - 3)$
   (*c*) $3xy(x + 2y)(x - 2y)$
9. (*a*) $3(5p^2 + 6)(p^2 - 1)$
   (*b*) $-(x + 1)(5x - 4)$
10. (*a*) $(ax - by)(bx - ay)$
    (*b*) $(10 - x - y)(10 + x + y)$
11. $xy(5x + 4y)$
12. $(x^2 + 4x + 3)$
13. $(3x + 4)$ units
14. (*a*) $b(2a - b)(2a^2 + b^2 - 2ab)$
    (*b*) $(4x + 5y + 9)(4x + 5y - 9)$
15. (*i*) $(2.5x + 1.1y)$
    (*ii*) $(2x + 5y)(2x - 5y)$

Internal Assessment

Q1. Fill in the blanks:

(a) $x^2 - 2x$ = ...

(b) $4x^3yz^2 \div 2xyz$ = ...

(c) $25x^2 - 16$ = ...

(d) $x^2 + 16x + 64$ = ...

(e) $-x^3 + x^2$ = ...

Q2. Write True (T) or False (F).

(a) $(x - 2)$ is one of the factors of $x^2 + 4$.

(b) $x^2 - 9$ is completely divisible by $x + 3$.

(c) The factors of $x^2 - 1$ are $(x - 1)(x + 1)$.

(d) The factors of $x^2 + x + 1 = (x + 1)(x + 1)$.

(e) Factors of $6x^2 - 216 = 6(x - 6)(x + 6)$.

Multiple Choice Questions (MCQs)

Q3. If $x - 2$ is a factor of the expression $x^2 - 10x + 16$, then the other factor is

(a) $x + 2$ (b) $x^2 - 2$

(c) $x^2 + 2$ (d) $x - 8$

Q4. The factors of $x^4 - 1$ are

(a) $(x + 1)(x - 1)(x^2 + 1)$

(b) $(x - 1)(x^2 + 1)(x - 2)$

(c) $(x + 1)^2(x^2 + 1)$

(d) $(x - 1)^2(x^2 + 1)$

Q5. The factors of $a^2 + b^2 - c^2 - 2ab$ are

(a) $(a - b - c)(a + b + c)$

(b) $(a - b + c)(a + b - c)$

(c) $(a - b + c)(a - b - c)$

(d) $(a - b - c)(a + b - c)$

Q6. The factors of $\frac{x^2}{a^2} - 2 + \frac{a^2}{x^2}$ are

(a) $\left(\frac{x}{a} + \frac{a}{x}\right)^2$ (b) $\left(\frac{x}{a} - \frac{a}{x}\right)^2$

(c) $\left(\frac{a}{x} - \frac{x}{a}\right)\left(\frac{a}{x} + \frac{x}{a}\right)$ (d) $\left(\frac{x}{a} - \frac{a}{x}\right)\left(\frac{x}{a} + \frac{a}{x}\right)$

Q7. The value of $\frac{x^2 - a^2}{x + a}$ is

(a) $(x + a)$ (b) $(x - a)$

(c) $(x^2 + a^2)$ (d) $(x^2 + a^2)$

Q8. The value of $24x^3yz^5 \div 6xyz$ is

(a) $4x^2z^4$ (b) $4x^3yz$

(c) $4xy^3z^2$ (d) $4yz^5$

ANSWERS

1. (a) $x(x - 2)$ (b) $2x^2z$
   (c) $(5x - 4)(5x + 4)$ (d) $(x + 8)^2$
   (e) $-x^2(x - 1)$
2. (a) F (b) T
   (c) T (d) F
   (e) T
3. (d) 4. (a)
5. (c) 6. (b)
7. (b) 8. (a)

◆ Periodic Assessment

SET-3

Time: 1 hour *M.M.: 20*

General Instructions

- *All questions are compulsory. However there is an internal choice.*
- *Section-A consists of 4 questions carrying 1 mark each.*
- *Section-B consists of 8 questions carrying 2 mark each.*
- *Calculator is not permitted.*

SECTION–A

1. Perimeter of a rhombus is 20 cm. If the length of one diagonal is 8 cm, find the length of the other diagonal.
2. Using the formula $a^2 - b^2 = (a + b)(a - b)$, find $(1.02)^2 - (0.98)^2$
3. Multiply $(x - 4)$ and $(2x + 3)$.
4. Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$.

SECTION–B

5. Show that: $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
6. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find 103×104.
7. Using Euler's formula, find the value of F.

Faces	Vertices	Edges
F	6	12

8. Find the perimeter of a semi-circular paper of diameter 2.8 cm.
9. The area of a trapezium shaped field is 480 m^2, the distance between two parallel sides is 15 m and one of its parallel sides is 20 m. Find the other parallel side.
10. Find the height of a right circular cylinder whose radius is 7 cm and the total surface area is 968 cm^2.

OR

Two equal cubes of edge 6 cm each are joined with a common face. Find the volume of the resulting cuboid.

11. A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.
12. Find the value of m for which $\left(-\frac{2}{3}\right)^{2m+1} \div \left(-\frac{2}{3}\right)^{-3} = \left(-\frac{2}{3}\right)^2$.

◇ Periodic Assessment

SET-4

Time: 1 hour *M.M.: 20*

General Instructions: *Same as paper–1*

SECTION–A

1. Add: $7x^2 - 4x + 5$ and $9x - 10$.
2. Find the product: $\left(\frac{-10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$
3. Show that $(3x + 7)^2 - 84x = (3x - 7)^2$
4. Using Euler's formula, find the number of Edges if it has 8 vertices and 6 faces.

SECTION–B

5. An octahedron is having 20 triangular faces and 12 vertices. Find the number of its faces.
6. Fill in the blanks:
   (a) A regular octahedron has _____ vertices _____ faces.
   (b) An hexagonal pyramid has _____ faces.
   (c) A tetrahedron has _____ vertices.
   (d) A cylinder has _____ faces.
7. Find the area of a rhombus whose diagonals are of lengths 10 cm and 8.2 cm.
8. Find the height of a cylinder whose radius is 7 cm and the total surface area is 968 cm^2.
9. Find the height of the cylinder whose volume is 1.54 m^2 and diameter of the base is 140 cm.
10. Simplify: $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

OR

Simplify: $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

11. Find the side of a cube whose total surface area is 600 cm^2.
12. Find the area of the quadrilateral PQRS shown in given figure.