

# 2

## Polynomials

### EXERCISE 2.1

Write the correct answer in each of the following:

1. Which one of the following is a polynomial?

(a)  $\frac{x^2}{2} - \frac{2}{x^2}$

(b)  $\sqrt{2x} - 1$

(c)  $x^2 + \frac{3x^2}{\sqrt{x}}$

(d)  $\frac{x-1}{x+1}$

Sol. (a)  $\frac{x^2}{2} - \frac{2}{x^2} = \frac{x^2}{2} - 2x^{-2}$

Second term is  $-2x^{-2}$ . Exponent of  $x^{-2}$  is  $-2$ , which is not a whole number. So, this algebraic expression is not a polynomial.

(b)  $\sqrt{2x} - 1 = \sqrt{2}x^{\frac{1}{2}} - 1$

First term is  $\sqrt{2}x^{\frac{1}{2}}$ . Here, the exponent of the second term, i.e.,  $x^{\frac{1}{2}}$  is  $\frac{1}{2}$ , which is not a whole number. So, this algebraic expression is not a polynomial.

(c)  $x^2 + \frac{3x^2}{\sqrt{x}} = x^2 + 3x$

In this expression, we have only whole numbers as the exponents of the variable in each term. Hence, the given algebraic expression is a polynomial.

2.  $\sqrt{2}$  is a polynomial of degree

(a) 2

(b) 0

(c) 1

(d)  $\frac{1}{2}$

Sol.  $\sqrt{2}$  is a constant polynomial. The only term here is  $\sqrt{2}$  which can be written as  $\sqrt{2} x^0$ . So, the exponent of  $x$  is zero. Therefore, the degree of the polynomial is 0.

Hence, (b) is the correct answer.

3. Degree of the polynomial of  $4x^4 + 0x^3 + 0x^5 + 5x + 7$  is

(a) 4

(b) 5

(c) 3

(d) 7

**Sol.** The highest power of the variable in a polynomial is called the degree of the polynomial. In this polynomial, the term with highest power of  $x$  is  $4x^4$ . Highest power of  $x$  is 4, so the degree of the given polynomial is 4.

4. Degree of the zero polynomial

(a) 0

(b) 1

(c) Any natural number

(d) Not defined.

**Sol.** Degree of the zero polynomial (0) is not defined.

Hence, (d) is the correct answer.

5. If  $p(x) = x^2 - 2\sqrt{2}x + 1$ , then  $p(2\sqrt{2})$  is equal to

(a) 0

(b) 1

(c)  $4\sqrt{2}$

(d)  $8\sqrt{2} + 1$

**Sol.** We have  $p(x) = x^2 - 2\sqrt{2}x + 1$

$$\begin{aligned}\therefore p(2\sqrt{2}) &= (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1 \\ &= 8 - 8 + 1 \\ &= 1\end{aligned}$$

Hence, (b) is the correct answer.

6. The value of the polynomial  $5x - 4x^2 + 3$ , when  $x = -1$  is

(a) -6

(b) 6

(c) 2

(d) -2

**Sol.** Let  $p(x) = 5x - 4x^2 + 3$

$$\text{Therefore, } p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$$

Hence, (a) is the correct answer.

7. If  $p(x) = x + 3$ , then  $p(x) + p(-x)$  is equal to

(a) 3

(b)  $2x$

(c) 0

(d) 6

**Sol.** We have  $p(x) = x + 3$ , then

$$p(-x) = -x + 3$$

$$\text{Therefore, } p(x) + p(-x) = x + 3 + (-x + 3) = x + 3 - x + 3 = 6$$

Hence, (d) is the correct answer.

8. Zero of the zero polynomial is

(a) 0

(b) 1

(c) Any real number

(d) Not defined

**Sol.** The zero (or degree) of the zero polynomial is undefined.

Hence, (d) is the correct answer.

9. Zero of the polynomial  $p(x) = 2x + 5$  is

(a)  $-\frac{2}{5}$

(b)  $-\frac{5}{2}$

(c)  $\frac{2}{5}$

(d)  $\frac{5}{2}$

**Sol.** Finding a zero of  $p(x)$  is the same as solving an equation  $p(x) = 0$ .

$$\text{Now, } p(x) = 0 \Rightarrow 2x + 5 = 0,$$

$$\text{which gives us } x = -\frac{5}{2}.$$

Therefore,  $-\frac{5}{2}$  is the zero of the polynomial.

Hence, (b) is the correct answer.

10. One of the zeroes of the polynomial  $2x^2 + 7x - 4$  is

- (a) 2            (b)  $\frac{1}{2}$             (c)  $-\frac{1}{2}$             (d) -2

**Sol.** We have  $p(x) = 2x^2 + 7x + 4$

$$\begin{aligned} \text{(a)} \quad p(2) &= 2(2)^2 + 7(2) - 4 \\ &= 8 + 14 - 4 \\ &= 18 \neq 0 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 4 \\ &= 2 \times \frac{1}{4} + \frac{7}{2} - 4 = \frac{1}{2} + \frac{7}{2} - 4 = 4 - 4 = 0 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad p\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right) - 4 \\ &= 2 \times \frac{1}{4} - \frac{7}{2} - 4 = \frac{1}{2} - \frac{7}{2} - 4 \\ &= -3 - 4 \\ &= -7 \neq 0 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad p(-2) &= 2(-2)^2 + 7(-2) - 4 \\ &= 8 - 14 - 4 = -10 \neq 0 \end{aligned}$$

As  $p\left(\frac{1}{2}\right) = 0$ , we say that  $\frac{1}{2}$  is a zero of the polynomial. Hence,  $\frac{1}{2}$

is one of the zeroes of the polynomial  $2x^2 + 7x - 4$ .

Hence, (b) is the correct answer.

11. If  $x^{51} + 51$  is divided by  $x + 1$ , the remainder is

- (a) 0            (b) 1            (c) 49            (d) 50

**Sol.** If  $p(x)$  is divided by  $x + a$ , then the remainder is  $p(-a)$ .

Here  $p(x) = x^{51} + 51$  is divided by  $x + 1$ , then

$$\text{Remainder} = p(-1) = (-1)^{51} + 51 = -1 + 51 = 50$$

Hence, (d) is the correct answer.

12. If  $x + 1$  is a factor of the polynomial  $2x^2 + kx$ , then the value of  $k$  is

- (a) -3            (b) 4            (c) 2            (d) -2

**Sol.** Let  $p(x) = 2x^2 + kx$

If  $x + 1$  is a factor of  $p(x)$ , then by factor theorem  $p(-1) = 0$

$$\text{Now,} \quad p(-1) = 0 \Rightarrow 2(-1)^2 + k(-1) = 0$$

$$\Rightarrow 2 - k = 0; \quad k = 2$$

Hence, (c) is the correct answer.

13.  $x + 1$  is a factor of the polynomial

(a)  $x^3 + x^2 - x + 1$  (b)  $x^3 + x^2 + x + 1$

(c)  $x^4 + x^3 + x^2 + 1$  (d)  $x^4 + 3x^3 + 3x^2 + x + 1$

**Sol.** If  $x + 1$  is a factor of  $p(x)$ , then  $p(-1) = 0$

(a) Let  $p(x) = x^3 + x^2 - x + 1$

$$\therefore p(-1) = (-1)^3 + (-1)^2 - (-1) + 1 \\ = -1 + 1 + 1 + 1 = 2 \neq 0$$

So,  $x + 1$  is not a factor of  $p(x)$ .

(b) Let  $p(x) = x^3 + x^2 + x + 1$

$$\therefore p(-1) = (-1)^3 + (-1)^2 + (-1) + 1 \\ = -1 + 1 - 1 + 1 = 0$$

(c) Let  $p(x) = x^4 + x^3 + x^2 + 1$

$$\therefore p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + 1 \\ = 1 - 1 + 1 + 1 = 2 \neq 0$$

(d) Let  $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$$\therefore p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 \\ = 1 - 3 + 3 - 1 + 1 = 1 \neq 0$$

Hence,  $x + 1$  is a factor of  $x^3 + x^2 + x + 1$ .

So, (b) is the correct answer.

14. One of the factors of  $(25x^2 - 1) + (1 + 5x)^2$  is

(a)  $5 + x$  (b)  $5 - x$  (c)  $5x - 1$  (d)  $10x$

**Sol.**  $(25x^2 - 1) + (1 + 5x)^2 = (5x)^2 - 1^2 + (5x + 1)^2$

$$= (5x - 1)(5x + 1) + (5x + 1)^2 = (5x + 1)(5x - 1 + 5x + 1)$$

$$= (5x + 1)(10x) = 10x(5x + 1)$$

Hence, one of the factors of  $(25x^2 - 1) + (1 + 5x)^2$  is  $10x$ . Therefore, (d) is the correct answer.

15. The value of  $249^2 - 248^2$  is

(a)  $1^2$  (b)  $477$  (c)  $487$  (d)  $497$

**Sol.**  $(249)^2 - (248)^2 = (249 + 248)(249 - 248)$   
 $= (497)(1) = 497$

Hence, (d) is the correct answer.

16. The factorisation of  $4x^2 + 8x + 3$  is

(a)  $(x + 1)(x + 3)$  (b)  $(2x + 1)(2x + 3)$

(c)  $(2x + 2)(2x + 5)$  (d)  $(2x - 1)(2x - 3)$

**Sol.**  $4x^2 + 8x + 3 = 4x^2 + 6x + 2x + 3$   
 $= 2x(2x + 3) + 1(2x + 3) = (2x + 1)(2x + 3)$

Hence, (b) is the correct answer.

17. Which of the following is a factor of  $(x + y)^3 - (x^3 + y^3)$ ?

(a)  $x^2 + y^2 + 2xy$  (b)  $x^2 + y^2 - xy$

(c)  $xy^2$  (d)  $3xy$

**Sol.**  $(x + y)^3 - (x^3 + y^3) = x^3 + y^3 + 3xy(x + y) - x^3 - y^3$   
 $= 3xy(x + y)$

So,  $3xy$  is a factor of  $(x+y)^3 - (x^3 + y^3)$ .

Hence, (d) is the correct answer.

18. The coefficient of  $x$  in the expansion of  $(x+3)^3$  is

- (a) 1                      (b) 9                      (c) 18                      (d) 27

Sol. Using  $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ , we get

$$\begin{aligned}(x+3)^3 &= x^3 + 3^3 + 3 \times x \times 3(x+3) \\ &= x^3 + 27 + 9x^2 + 27x\end{aligned}$$

Therefore, the coefficient of  $x$  is 27.

Hence, (d) is the correct answer.

19. If  $\frac{x}{y} + \frac{y}{x} = -1$ , the value of  $x^3 - y^3$  is

- (a) 1                      (b) -1                      (c) 0                      (d)  $\frac{1}{2}$

Sol.  $\frac{x}{y} + \frac{y}{x} = -1 \Rightarrow \frac{x^2 + y^2}{xy} = -1$

$$\Rightarrow x^2 + y^2 = -xy$$

$$\begin{aligned}\text{Now, } x^3 - y^3 &= (x-y)(x^2 + y^2 + xy) \\ &= (x-y)(-xy + xy) \quad [\because x^2 + y^2 = -xy] \\ &= (x-y)(0) \\ &= 0\end{aligned}$$

Hence, (c) is the correct answer.

20. If  $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$ , then the value of  $b$  is

- (a) 0                      (b)  $\frac{1}{\sqrt{2}}$                       (c)  $\frac{1}{4}$                       (d)  $\frac{1}{2}$

Sol.  $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$

$$\begin{aligned}\Rightarrow 49x^2 - b &= (7x)^2 - \left(\frac{1}{2}\right)^2 \\ &= 49x^2 - \frac{1}{4} \quad [\because (a+b)(a-b) = a^2 - b^2]\end{aligned}$$

So, we get  $b = \frac{1}{4}$ .

Hence, (c) is the correct answer.

21. If  $a + b + c = 0$ , then the value of  $a^3 + b^3 + c^3$  is equal to

- (a) 0                      (b)  $abc$                       (c)  $3abc$                       (d)  $2abc$

Sol. We know that

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{As } a+b+c=0, \text{ so, } a^3 + b^3 + c^3 - 3abc = (0)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$\text{Hence, } a^3 + b^3 + c^3 = 3abc.$$

Therefore, (c)  $3abc$  is the correct answer.

**EXERCISE 2.2**

1. Which of the following expressions are polynomials? Justify your answer.

(i) 8    (ii)  $\sqrt{3}x^2 - 2x$                           (iii)  $1 - \sqrt{5}x$

(iv)  $\frac{1}{5x^{-2}} + 5x + 7$                           (v)  $\frac{(x-2)(x-4)}{x}$                           (vi)  $\frac{1}{x+1}$

(vii)  $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$                           (viii)  $\frac{1}{2x}$

**Sol.** (i) 8 is a constant polynomial.

(ii)  $\sqrt{3}x^2 - 2x$

In each term of this expression, the exponent of the variable  $x$  is a whole number. Hence, it is a polynomial.

(iii)  $1 - \sqrt{5}x = 1 - \sqrt{5}x^{\frac{1}{2}}$

Here, the exponent of the second term, *i.e.*,  $x^{\frac{1}{2}}$  is  $\frac{1}{2}$ , which is not a whole number. Hence, the given algebraic expression is not a polynomial.

(iv)  $\frac{1}{5x^{-2}} + 5x + 7 = \frac{1}{5}x^2 + 5x + 7$

In each term of this expression, the exponent of the variable  $x$  is a whole number. Hence, it is a polynomial.

(v)  $\frac{(x-2)(x-4)}{x} = \frac{x^2 - 6x + 8}{x} = x - 6 + \frac{8}{x} = x - 6 + 8x^{-1}$

Here, the exponent of variable  $x$  in the third term, *i.e.*, in  $8x^{-1}$ , is  $-1$ , which is not a whole number. So, this algebraic expression is not a polynomial.

(vi)  $\frac{1}{x+1} = (x+1)^{-1}$  which cannot be reduced to an expression in which the exponent of the variable  $x$  have only whole numbers in each of its terms. So, this algebraic expression is not a polynomial.

(vii)  $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

In this expression, the exponent of  $a$  in each term is a whole number, so this expression is a polynomial.

(viii)  $\frac{1}{2x} = \frac{1}{2}x^{-1}$

Here, the exponent of the variable  $x$  is  $-1$ , which is not a whole number. So, this algebraic expression is not a polynomial.

2. Write whether the following statements are true or false. Justify your answer.

(i) A binomial can have atmost two terms.

(ii) Every polynomial is a binomial.

- (iii) A binomial may have degree 5.  
 (iv) Zero of a polynomial is always 0.  
 (v) A polynomial cannot have more than one zero.  
 (vi) The degree of the sum of two polynomials each of degree 5 is always 5.

**Sol.** (i) The given statement is false because binomial have exactly two terms.

(ii) A polynomial can be a monomial, binomial trinomial or can have finite number of terms. For example,  $x^4 + x^3 + x^2 + 1$  is a polynomial but not binomial.

Hence, the given statement is false.

(iii) The given statement is true because a binomial is a polynomial whose degree is a whole number  $\geq 1$ . For example,  $x^5 - 1$  is a binomial of degree 5.

(iv) The given statement is false, because zero of polynomial can be any real number.

(v) The given statement is false, because a polynomial can have any number of zeroes which depends on the degree of the polynomial.

(vi) The given statement is false. For example, consider the two polynomials  $-x^5 + 3x^2 + 4$  and  $x^5 + x^4 + 2x^3 + 3$ . The degree of each of these polynomials is 5. Their sum is  $x^4 + 2x^3 + 3x^2 + 7$ . The degree of this polynomial is not 5.

### EXERCISE 2.3

1. Classify the following polynomials as polynomials in one variable, two variables etc.

(i)  $x^2 + x + 1$

(ii)  $y^3 - 5y$

(iii)  $xy + yz + zx$

(iv)  $x^2 - 2xy + y^2 + 1$

**Sol.** (i)  $x^2 + x + 1$  is a polynomial in one variable.

(ii)  $y^3 - 5y$  is a polynomial in one variable.

(iii)  $xy + yz + zx$  is a polynomial in three variables.

(iv)  $x^2 - 2xy + y^2 + 1$  is a polynomial in two variables.

2. Determine the degree of each of the following polynomials:

(i)  $2x - 1$

(ii)  $-10$

(iii)  $x^3 - 9x + 3x^5$

(iv)  $y^3(1 - y^4)$

**Sol.** (i) Since the highest power of  $x$  is 1, the degree of the polynomial  $2x - 1$  is 1.

(ii)  $-10$  is a non-zero constant. A non-zero constant term is always regarded as having degree 0.

(iii) Since the highest power of  $x$  is 5, the degree of the polynomial  $x^3 - 9x + 3x^5$  is 5.

(iv)  $y^3(1 - y^4) = y^3 - y^7$

Since the highest power of  $y$  is 7, the degree of the polynomial is 7.

3. For the polynomial  $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$ , write

- (i) the degree of the polynomial.
- (ii) the coefficient of  $x^3$ .
- (iii) the coefficient of  $x^6$ .
- (iv) the constant term.

**Sol.** (i) We know that highest power of variable in a polynomial is the degree of the polynomial.

In the given polynomial, the term with highest of  $x$  is  $-x^6$  and the exponent of  $x$  in this term is 6.

(ii) The coefficient of  $x^3$  is  $\frac{1}{5}$ .      (iii) The coefficient of  $x^6$  is  $-1$ .

(iv) The constant term is  $\frac{1}{5}$ .

4. Write the coefficient of  $x^2$  in each of the following:

(i)  $\frac{\pi}{6}x + x^2 - 1$

(ii)  $3x - 5$

(iii)  $(x-1)(3x-4)$

(iv)  $(2x-5)(2x^2-3x+1)$

**Sol.** (i) The coefficient of  $x^2$  in the given polynomial is 1.

(ii) The given polynomial can be written as  $0 \cdot x^2 + 3x - 5$ . So, the coefficient of  $x^2$  in the given polynomial is 0.

(iii) The given polynomial can be written as:

$$\begin{aligned}(x-1)(3x-4) &= 3x^2 - 4x - 3x + 4 \\ &= 3x^2 - 7x + 4\end{aligned}$$

So, the coefficient of  $x^2$  in the given polynomial is 3.

(iv) The given polynomial can be written as:

$$\begin{aligned}(2x-5)(2x^2-3x+1) &= 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5 \\ &= 4x^3 - 16x^2 + 17x - 5\end{aligned}$$

So, the coefficient of  $x^2$  in the given polynomial is  $-16$ .

5. Classify the following as a constant, linear, quadratic and cubic polynomials:

(i)  $2 - x^2 + x^3$

(ii)  $3x^3$

(iii)  $5t - \sqrt{7}$

(iv)  $4 - 5y^2$

(v) 3

(vi)  $2 + x$

(vii)  $y^3 - y$

(viii)  $1 + x + x^2$

(ix)  $t^2$

(x)  $\sqrt{2}x - 1$



**Sol.** We know that

(a) a polynomial in which exponent of the variable is zero, is called a constant term.

Here, (v) 3 is a constant polynomial because  $3 = 3x^0$ , exponent of the variable  $x$  is 0.

(b) A polynomial of degree 1 is called a linear polynomial.

$5t - \sqrt{7}$ ,  $2 + x$  and  $\sqrt{2}x + 1$  are linear polynomials.

(c) A polynomial of degree 2 is called a quadratic polynomial.

$4 - 5y^2$ ,  $1 + x + x^2$  and  $t^2$  are quadratic polynomials.

(d) A polynomial of degree 3 is called a cubic polynomial.

$2 - x + x^3$ ,  $3x^3$  and  $y^3 - y$  are cubic polynomials.

6. Give an example of a polynomial, which is:

(i) monomial of degree 1.

(ii) binomial of degree 20.

(iii) trinomial of degree 2.

**Sol.** We know that a polynomial having only one term is called a monomial, a polynomial having only two terms is called binomial, a polynomial having only three terms is called a trinomial.

(i)  $3x$  is a monomial of degree 1.

(ii)  $x^{20} - 7$  is a binomial of degree 20.

(iii)  $5x^2 + 3x - 1$  is a trinomial of degree 2.

7. Find the value of the polynomial  $3x^3 - 4x^2 + 7x - 5$ , when  $x = 3$  and also when  $x = -3$ .

**Sol.** Let  $p(x) = 3x^3 - 4x^2 + 7x - 5$

$$\begin{aligned} \therefore p(3) &= 3(3)^3 - 4(3)^2 + 7(3) - 5 \\ &= 3(27) - 4(9) + 21 - 5 \\ &= 81 - 36 + 21 - 5 \\ &= 61 \end{aligned}$$

$$\begin{aligned} \text{Now, } p(-3) &= 3(-3)^3 - 4(-3)^2 + 7(-3) - 5 \\ &= 3(-27) - 4(9) - 21 - 5 \\ &= -81 - 36 - 21 - 5 \\ &= -143 \end{aligned}$$

8. If  $p(x) = x^2 - 4x + 3$ , evaluate  $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

**Sol.** We have  $p(x) = x^2 - 4x + 3$

$$\therefore p(2) - p(-1) + p\left(\frac{1}{2}\right) = (2^2 - 4 \times 2 + 3)$$

$$- \{(-1)^2 - 4(-1) + 3\} + \left\{ \left(\frac{1}{2}\right)^2 - 4 \times \frac{1}{2} + 3 \right\}$$

$$\begin{aligned}
 &= (4-8+3)-(1+4+3)+\left(\frac{1}{4}-2+3\right) \\
 &= -1-8+\frac{5}{4} \\
 &= -9+\frac{5}{4}=\frac{-36+5}{4}=\frac{-31}{4}
 \end{aligned}$$

9. Find  $p(0)$ ,  $p(1)$ ,  $p(-2)$  for the following polynomials:

(i)  $p(x) = 10x - 4x^2 - 3$

(ii)  $p(y) = (y+2)(y-2)$

Sol. (i) We have

$$p(x) = 10x - 4x^2 - 3$$

$$\begin{aligned}
 \therefore p(0) &= 10(0) - 4(0)^2 - 3 \\
 &= 0 - 0 - 3 = -3
 \end{aligned}$$

And,

$$\begin{aligned}
 p(1) &= 10(1) - 4(1)^2 - 3 \\
 &= 10 - 4 - 3 = 10 - 7 = 3
 \end{aligned}$$

And,

$$\begin{aligned}
 p(-2) &= 10(-2) - 4(-2)^2 - 3 \\
 &= -20 - 4(4) - 3 = -20 - 16 - 3 = -39
 \end{aligned}$$

(ii) We have  $p(y) = (y+2)(y-2) = y^2 - 4$

$$\begin{aligned}
 \therefore p(0) &= (0)^2 - 4 \\
 &= 0 - 4 = -4
 \end{aligned}$$

And,

$$\begin{aligned}
 p(1) &= (1)^2 - 4 \\
 &= 1 - 4 = -3
 \end{aligned}$$

And,

$$\begin{aligned}
 p(-2) &= (-2)^2 - 4 \\
 &= 4 - 4 = 0
 \end{aligned}$$

10. Verify whether the following are True or False.

(i)  $-3$  is a zero of  $x - 3$ .

(ii)  $-\frac{1}{3}$  is a zero of  $3x + 1$ .

(iii)  $\frac{-4}{5}$  is a zero of  $4 - 5y$ .

(iv)  $0$  and  $2$  are the zeros of  $t^2 - 2t$ .

(v)  $-3$  is a zero of  $y^2 + y - 6$ .

Sol. A zero of a polynomial  $p(x)$  is a number  $c$  such that  $p(c) = 0$

(i) Let  $p(x) = x - 3$

$$\therefore p(-3) = -3 - 3 = -6 \neq 0$$

Hence,  $-3$  is not a zero of  $x - 3$ .

(ii) Let  $p(x) = 3x + 1$

$$\therefore p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

Hence,  $-\frac{1}{3}$  is a zero of  $p(x) = 3x + 1$ .

(iii) Let  $p(y) = 4 - 5y$

$$\therefore p\left(-\frac{4}{5}\right) = 4 - 5\left(-\frac{4}{5}\right) = 4 + 4 = 8 \neq 0$$

Hence,  $-\frac{4}{5}$  is not a zero of  $4 - 5y$ .

(iv) Let  $p(t) = t^2 - 2t$

$$\therefore p(0) = (0)^2 - 2(0) = 0$$

$$\text{and } p(2) = (2)^2 - 2(2) = 4 - 4 = 0$$

Hence, 0 and 2 are zeroes of the polynomial  $p(t) = t^2 - 2t$ .

(v) Let  $p(y) = y^2 + y - 6$

$$\therefore p(-3) = (-3)^2 + (-3) - 6 = 9 - 3 - 6 = 0$$

Hence,  $-3$  is a zero of the polynomial  $y^2 + y - 6$ .

11. Find the zeroes of the polynomial in each of the following:

(i)  $p(x) = x - 4$

(ii)  $g(x) = 3 - 6x$

(iii)  $q(x) = 2x - 7$

(iv)  $h(y) = 2y$

**Sol.** (i) Solving the equation  $p(x) = 0$ , we get

$$x - 4 = 0, \text{ which gives us } x = 4$$

So, 4 is a zero of the polynomial  $x - 4$ .

(ii) Solving the equation  $g(x) = 0$ , we get

$$3 - 6x = 0, \text{ which gives us } x = \frac{1}{2}$$

So,  $\frac{1}{2}$  is a zero of the polynomial  $3 - 6x$ .

(iii) Solving the equation  $q(x) = 0$ , we get

$$2x - 7 = 0, \text{ which gives us } x = \frac{7}{2}$$

So,  $\frac{7}{2}$  is a zero of the polynomial  $2x - 7$ .

(iv) Solving the equation  $h(y) = 0$ , we get

$$2y = 0, \text{ which gives us } y = 0$$

So, 0 is a zero of the polynomial  $2y$ .

12. Find the zeroes of the polynomial  $(x - 2)^2 - (x + 2)^2$ .

**Sol.** Let  $p(x) = (x - 2)^2 - (x + 2)^2$

As finding a zero of  $p(x)$ , is same as solving the equation  $p(x) = 0$

$$\text{So, } p(x) = 0 \Rightarrow (x - 2)^2 - (x + 2)^2 = 0$$

$$\Rightarrow (x - 2 + x + 2)(x - 2 - x - 2) = 0$$

$$\Rightarrow 2x(-4) = 0 \Rightarrow -8x = 0 \Rightarrow x = 0$$

Hence,  $x = 0$  is the only one zero of  $p(x)$ .

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial:  $x^4 + 1; x + 1$ .

Sol. By actual division, we have

$$\begin{array}{r}
 x^3 + x^2 + x + 1 \\
 x - 1 \overline{) x^4 + 1} \\
 \underline{- x^4 + x^3} \phantom{+ 1} \\
 x^3 + 1 \\
 \underline{- x^3 + x^2} \phantom{+ 1} \\
 x^2 + 1 \\
 \underline{- x^2 + x} \phantom{+ 1} \\
 x + 1 \\
 \underline{- x + 1} \\
 2
 \end{array}$$

14. By Remainder Theorem find the remainder, when  $p(x)$  is divided by  $g(x)$ , where

(i)  $p(x) = x^3 - 2x^2 - 4x - 1, g(x) = x + 1$

(ii)  $p(x) = x^3 - 3x^2 + 4x + 50, g(x) = x - 3$

(iii)  $p(x) = 4x^3 - 12x^2 + 14x - 3, g(x) = 2x - 1$

(iv)  $p(x) = x^3 - 6x^2 + 2x - 4, g(x) = 1 - \frac{3}{2}x$

Sol. (i) We have  $g(x) = x + 1$

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

$$\text{Remainder} = p(-1)$$

$$= (-1)^3 - 2(-1)^2 - 4(-1) - 1 = -1 - 2 + 4 - 1 = 0$$

(ii) We have  $g(x) = x - 3$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

$$\text{Remainder} = p(3)$$

$$= (3)^3 - 3(3)^2 + 4(3) + 50 = 27 - 27 + 12 + 50 = 62$$

(iii) We have  $g(x) = 2x - 1$

$$\Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\begin{aligned}
 \text{Remainder} &= p\left(\frac{1}{2}\right) \\
 &= 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3 \\
 &= 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 7 - 3 \\
 &= \frac{1}{2} - 3 + 7 - 6 = \frac{1}{2} - 2 = \frac{-3}{2}
 \end{aligned}$$

$$(iv) \ g(x) = 0 \quad \Rightarrow \ 1 - \frac{3}{2}x = 0; \ x = \frac{2}{3}$$

$$\begin{aligned}
 \text{Remainder} &= p\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{24}{9} + \frac{4}{3} - 4 \\
 &= \frac{8 - 72 + 36 - 108}{27} = \frac{-136}{27}
 \end{aligned}$$

15. Check whether  $p(x)$  is a multiple of  $g(x)$  or not:

$$(i) \ p(x) = x^3 - 5x^2 + 4x - 3, \ g(x) = x - 2$$

$$(ii) \ p(x) = 2x^3 - 11x^2 - 4x + 5, \ g(x) = 2x + 1$$

**Sol.** (i)  $p(x)$  will be a multiple of  $g(x)$  if  $g(x)$  divides  $p(x)$ .

Now,  $g(x) = x - 2$  gives  $x = 2$

$$\begin{aligned}
 \text{Remainder} &= p(2) = (2)^3 - 5(2)^2 + 4(2) - 3 \\
 &= 8 - 5(4) + 8 - 3 = 8 - 20 + 8 - 3 \\
 &= -7
 \end{aligned}$$

Since remainder  $\neq 0$ , so  $p(x)$  is not a multiple of  $g(x)$ .

(ii)  $p(x)$  will be a multiple of  $g(x)$  if  $g(x)$  divides  $p(x)$ .

Now,  $g(x) = 2x + 1$  give  $x = -\frac{1}{2}$

$$\begin{aligned}
 \text{Remainder} &= p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 11\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 5 \\
 &= 2\left(-\frac{1}{8}\right) - 11\left(\frac{1}{4}\right) + 2 + 5 = \frac{-1}{4} - \frac{11}{4} + 7 \\
 &= \frac{-1 - 11 + 28}{4} = \frac{16}{4} = 4
 \end{aligned}$$

Since remainder  $\neq 0$ , so  $p(x)$  is not a multiple of  $g(x)$ .

16. Show that:

(i)  $x + 3$  is a factor of  $69 + 11x - x^2 + x^3$ .

(ii)  $2x - 3$  is a factor of  $x + 2x^3 - 9x^2 + 12$ .

**Sol.** (i) Let  $p(x) = 69 + 11x - x^2 + x^3$ ,  $g(x) = x + 3$ .

$$g(x) = x + 3 = 0 \text{ gives } x = -3$$

$g(x)$  will be a factor of  $p(x)$  if  $p(-3) = 0$  (Factor theorem)

$$\begin{aligned} \text{Now, } p(-3) &= 69 + 11(-3) - (-3)^2 + (-3)^3 \\ &= 69 - 33 - 9 - 27 \\ &= 0 \end{aligned}$$

Since,  $p(-3) = 0$ , so,  $g(x)$  is a factor of  $p(x)$ .

(ii) Let  $p(x) = x + 2x^3 - 9x^2 + 12$  and  $g(x) = 2x - 3$

$$g(x) = 2x - 3 = 0 \text{ gives } x = \frac{3}{2}$$

$g(x)$  will be a factor of  $p(x)$  if  $p\left(\frac{3}{2}\right) = 0$  (Factor theorem)

$$\begin{aligned} \text{Now, } p\left(\frac{3}{2}\right) &= \frac{3}{2} + 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 12 = \frac{3}{2} + 2\left(\frac{27}{8}\right) - 9\left(\frac{9}{4}\right) + 12 \\ &= \frac{3}{2} + \frac{27}{4} - \frac{81}{4} + 12 = \frac{6 + 27 - 81 + 48}{4} = \frac{0}{4} = 0 \end{aligned}$$

Since,  $p\left(\frac{3}{2}\right) = 0$ , so,  $g(x)$  is a factor of  $p(x)$ .

17. Determine which of the following polynomials has  $x - 2$  a factor:

(i)  $3x^2 + 6x - 24$

(ii)  $4x^2 + x - 2$

**Sol.** We know that if  $(x - a)$  is a factor of  $p(x)$ , then  $p(a) = 0$ .

(i) Let  $p(x) = 3x^2 + 6x - 24$

If  $x - 2$  is a factor of  $p(x) = 3x^2 + 6x - 24$ , then  $p(2)$  should be equal to 0.

$$\begin{aligned} \text{Now, } p(2) &= 3(2)^2 + 6(2) - 24 \\ &= 3(4) + 6(2) - 24 \\ &= 12 + 12 - 24 \\ &= 0 \end{aligned}$$

$\therefore$  By factor theorem,  $(x - 2)$  is a factor of  $3x^2 + 6x - 24$ .

(ii) Let  $p(x) = 4x^2 + x - 2$ .

If  $x - 2$  is a factor of  $p(x) = 4x^2 + x - 2$ , then,  $p(2)$  should be equal to 0.

$$\text{Now, } p(2) = 4(2)^2 + 2 - 2$$

$$\begin{aligned}
 &= 4(4) + 2 - 2 \\
 &= 16 + 2 - 2 \\
 &= 16 \neq 0
 \end{aligned}$$

$\therefore x - 2$  is not a factor of  $4x^2 + x - 2$ .

**18.** Show that  $p - 1$  is a factor of  $p^{10} - 1$  and also of  $p^{11} - 1$ .

**Sol.** If  $p - 1$  is a factor of  $p^{10} - 1$ , then  $(1)^{10} - 1$  should be equal to zero.

Now,  $(1)^{10} - 1 = 1 - 1 = 0$

Therefore,  $p - 1$  is a factor of  $p^{10} - 1$ .

Again, if  $p - 1$  is a factor of  $p^{11} - 1$ , then  $(1)^{11} - 1$  should be equal to zero.

Now,  $(1)^{11} - 1 = 1 - 1 = 0$

Therefore,  $p - 1$  is a factor of  $p^{11} - 1$ .

Hence,  $p - 1$  is a factor of  $p^{10} - 1$  and also of  $p^{11} - 1$ .

**19.** For what value of  $m$  is  $x^3 - 2mx^2 + 16$  divisible by  $x + 2$ ?

**Sol.** If  $x^3 - 2mx^2 + 16$  is divisible by  $x + 2$ , then  $x + 2$  is a factor of  $x^3 - 2mx^2 + 16$ .

Now, let  $p(x) = x^3 - 2mx^2 + 16$ .

As  $x + 2 = x - (-2)$  is a factor of  $x^3 - 2mx^2 + 16$

so  $p(-2) = 0$

Now, 
$$\begin{aligned}
 p(-2) &= (-2)^3 - 2m(-2)^2 + 16 \\
 &= -8 - 8m + 16 = 8 - 8m
 \end{aligned}$$

Now,  $p(-2) = 0$

$\Rightarrow 8 - 8m = 0$

$\Rightarrow m = 8 \div 8$

$\Rightarrow m = 1$

Hence, for  $m = 1$ ,  $x + 2$  is a factor of  $x^3 - 2mx^2 + 16$ , so  $x^3 - 2mx^2 + 16$  is completely divisible by  $x + 2$ .

**20.** If  $x + 2a$  is a factor of  $x^5 - 4a^2x^3 + 2x + 2a + 3$ , find  $a$ .

**Sol.** Let  $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$

If  $x - (-2a)$  is a factor of  $p(x)$ , then  $p(-2a) = 0$

$$\begin{aligned}
 \therefore p(-2a) &= (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 \\
 &= -32a^5 + 32a^5 - 4a + 2a + 3 \\
 &= -2a + 3
 \end{aligned}$$

Now,  $p(-2a) = 0$

$\Rightarrow -2a + 3 = 0$

$\Rightarrow a = \frac{3}{2}$

**21.** Find the value of  $m$  so that  $2x - 1$  be a factor of  $8x^4 + 4x^3 - 16x^2 + 10x + m$ .

**Sol.** Let  $p(x) = 8x^4 + 4x^3 - 16x^2 + 10x + m$ .

As  $(2x - 1)$  is a factor of  $p(x)$

$$\therefore p\left(\frac{1}{2}\right) = 0 \quad \text{[By factor theorem]}$$

$$\Rightarrow 8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m = 0$$

$$\Rightarrow 8\left(\frac{1}{16}\right) + 4\left(\frac{1}{8}\right) - 16\left(\frac{1}{4}\right) + 5 + m = 0$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0$$

$$\Rightarrow 2 + m = 0 \Rightarrow m = -2$$

**22.** If  $x + 1$  is a factor of  $ax^3 + x^2 - 2x + 4a - 9$ , find the value of  $a$ .

**Sol.** Let  $p(x) = ax^3 + x^2 - 2x + 4a - 9$ .

As  $(x + 1)$  is a factor of  $p(x)$

$$\therefore p(-1) = 0 \quad \text{[By factor theorem]}$$

$$\Rightarrow a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0$$

$$\Rightarrow a(-1) + 1 + 2 + 4a - 9 = 0$$

$$\Rightarrow -a + 4a - 6 = 0$$

$$\Rightarrow 3a - 6 = 0 \Rightarrow 3a = 6 \Rightarrow a = 2$$

**23.** Factorise:

(i)  $x^2 + 9x + 18$

(ii)  $6x^2 + 7x - 3$

(iii)  $2x^2 - 7x - 15$

(iv)  $84 - 2r - 2r^2$

**Sol.** (i) In order to factorise  $x^2 + 9x + 18$ , we have to find two numbers  $p$  and  $q$  such that  $p + q = 9$  and  $pq = 18$ .

Clearly,  $6 + 3 = 9$  and  $6 \times 3 = 18$ .

So, we write the middle term  $9x$  as  $6x + 3x$ .

$$\begin{aligned} \therefore x^2 + 9x + 18 &= x^2 + 6x + 3x + 18 \\ &= x(x + 6) + 3(x + 6) \\ &= (x + 6)(x + 3) \end{aligned}$$

(ii) In order to factorise  $6x^2 + 7x - 3$ , we have to find two numbers  $p$  and  $q$  such that  $p + q = 7$  and  $pq = -18$ .

Clearly,  $9 + (-2) = 7$  and  $9 \times (-2) = -18$ .



So, we write the middle term  $7x$  as  $9x + (-2x)$ , i.e.,  $9x - 2x$ .

$$\begin{aligned}\therefore 6x^2 + 7x - 3 &= 6x^2 + 9x - 2x - 3 \\ &= 3x(2x + 3) - 1(2x + 3) \\ &= (2x + 3)(3x - 1)\end{aligned}$$

(iii) In order to factorise  $2x^2 - 7x - 15$ , we have to find two numbers  $p$  and  $q$  such that  $p + q = -7$  and  $pq = -30$ .

Clearly,  $(-10) + 3 = -7$  and  $(-10) \times 3 = -30$ .

So, we write the middle term  $-7x$  as  $(-10x) + 3x$ .

$$\begin{aligned}\therefore 2x^2 - 7x - 15 &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x - 5) + 3(x - 5) \\ &= (x - 5)(2x + 3)\end{aligned}$$

(iv) In order to factorise  $84 - 2r - 2r^2$ , we have to find two numbers  $p$  and  $q$  such that  $p + q = -2$  and  $pq = -168$ .

Clearly,  $(-14) + 12 = -2$  and  $(-14) \times 12 = -168$ .

So, we write the middle term  $-2r$  as  $(-14r) + 12r$ .

$$\begin{aligned}\therefore 84 - 2r - 2r^2 &= -2r^2 - 2r + 84 \\ &= -2r^2 - 14r + 12r + 84 \\ &= -2r(r + 7) + 12(r + 7) \\ &= (r + 7)(-2r + 12) \\ &= -2(r + 7)(r - 6) = -2(r - 6)(r + 7)\end{aligned}$$

#### 24. Factorise:

$$(i) 2x^3 - 3x^2 - 17x + 30 \qquad (ii) x^3 - 6x^2 + 11x - 6$$

$$(iii) x^3 + x^2 - 4x - 4 \qquad (iv) 3x^3 - x^2 - 3x + 1$$

**Sol.** (i) Let  $f(x) = 2x^3 - 3x^2 - 17x + 30$  be the given polynomial. The factors of the constant term  $+30$  are  $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$ . The factor of coefficient of  $x^3$  is 2. Hence, possible rational roots of  $f(x)$  are:

$$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}.$$

$$\begin{aligned}\text{We have } f(2) &= 2(2)^3 - 3(2)^2 - 17(2) + 30 \\ &= 2(8) - 3(4) - 17(2) + 30 \\ &= 16 - 12 - 34 + 30 = 0\end{aligned}$$

$$\begin{aligned}\text{and } f(-3) &= 2(-3)^3 - 3(-3)^2 - 17(-3) + 30 \\ &= 2(-27) - 3(9) - 17(-3) + 30 \\ &= -54 - 27 + 51 + 30 = 0\end{aligned}$$

So,  $(x - 2)$  and  $(x + 3)$  are factors of  $f(x)$ .

$\Rightarrow x^2 + x - 6$  is a factor of  $f(x)$ .

Let us now divide  $f(x) = 2x^3 - 3x^2 - 17x + 30$  by  $x^2 + x - 6$  to get the other factors of  $f(x)$ .

By long division, we have

$$\begin{array}{r} x^2 + x - 6 \overline{) 2x^3 - 3x^2 - 17x + 30} \quad 2x - 5 \\ \underline{2x^3 + 2x^2 - 12x} \phantom{+ 30} \\ -5x^2 - 5x + 30 \\ \underline{+ 5x^2 + 5x - 30} \\ 0 \end{array}$$

$$\therefore 2x^3 - 3x^2 - 17x + 30 = (x^2 + x - 6)(2x - 5)$$

$$\Rightarrow 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$$

$$\text{Hence, } 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$$

(ii) Let  $f(x) = x^3 - 6x^2 + 11x - 6$  be the given polynomial. The factors of the constant term  $-6$  are  $\pm 1, \pm 2, \pm 3$  and  $\pm 6$ .

$$\text{We have, } f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$$

$$\text{and, } f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$$

So,  $(x - 1)$  and  $(x - 2)$  are factors of  $f(x)$ .

$\Rightarrow (x - 1)(x - 2)$  is also a factor of  $f(x)$ .

$\Rightarrow x^2 - 3x + 2$  is a factor of  $f(x)$ .

Let us now divide  $f(x) = x^3 - 6x^2 + 11x - 6$  by  $x^2 - 3x + 2$  to get the other factors of  $f(x)$ .

By long division, we have

$$\begin{array}{r} x^2 - 3x + 2 \overline{) x^3 - 6x^2 + 11x - 6} \quad x - 3 \\ \underline{x^3 - 3x^2 + 2x} \phantom{- 6} \\ -3x^2 + 9x - 6 \\ \underline{-3x^2 + 9x - 6} \\ 0 \end{array}$$

$$\therefore x^3 - 6x^2 + 11x - 6 = (x^2 - 3x + 2)(x - 3)$$

$$\Rightarrow x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

$$\text{Hence, } x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

(iii) Let  $f(x) = x^3 + x^2 - 4x - 4$  be the given polynomial. The factors of the constant term  $-4$  are  $\pm 1, \pm 2, \pm 4$ .

We have,

$$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$$

and,

$$f(2) = (2)^3 + (2)^2 - 4(2) - 4 = 8 + 4 - 8 - 4 = 0$$

So,  $(x + 1)$  and  $(x - 2)$  are factors of  $f(x)$ .

$\Rightarrow (x + 1)(x - 2)$  is also a factor of  $f(x)$ .

$\Rightarrow x^2 - x - 2$  is a factor of  $f(x)$ .

Let us now divide  $f(x) = x^3 + x^2 - 4x - 4$  by  $x^2 - x - 2$  to get the other factors of  $f(x)$ .

By long division, we have

$$\begin{array}{r} x^2 - x - 2 \overline{) x^3 + x^2 - 4x - 4} \quad x + 2 \\ \underline{-(x^3 - x^2 + 2x)} \phantom{- 4} \\ 2x^2 - 2x - 4 \\ \underline{-(2x^2 - 2x - 4)} \\ 0 \end{array}$$

$$\therefore x^3 + x^2 - 4x - 4 = (x^2 - x - 2)(x + 2)$$

$$\Rightarrow x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$$

Hence,  $x^3 + x^2 - 4x - 4 = (x - 2)(x + 1)(x + 2)$

- (iv) Let  $f(x) = 3x^3 - x^2 - 3x + 1$  be the given polynomial. The factors of the constant term +1 are  $\pm 1$ . The factor of coefficient of  $x^3$  is 3. Hence,

possible rational roots of  $f(x)$  are:  $\pm \frac{1}{3}$ .

We have,

$$f(1) = 3(1)^3 - (1)^2 - 3(1) + 1 = 3 - 1 - 3 + 1 = 0$$

and

$$f(-1) = 3(-1)^3 - (-1)^2 - 3(-1) + 1 = -3 - 1 + 3 + 1 = 0$$

So,  $(x - 1)$  and  $(x + 1)$  are factors of  $f(x)$ .

$\Rightarrow (x - 1)(x + 1)$  is also a factor of  $f(x)$ .

$\Rightarrow x^2 - 1$  is a factor of  $f(x)$ .

Let us now divide  $f(x) = 3x^3 - x^2 - 3x + 1$  by  $x^2 - 1$  to get the other factors of  $f(x)$ .

By long division, we have

$$\begin{array}{r} x^2 - 1 \overline{) 3x^3 - x^2 - 3x + 1} \quad 3x - 1 \\ \underline{-(3x^3)} \phantom{- x^2 - 3x + 1} \\ -x^2 + 1 \\ \underline{-(-x^2 + 1)} \\ 0 \end{array}$$

$$\therefore 3x^3 - x^2 - 3x + 1 = (x^2 - 1)(3x - 1)$$

$$\Rightarrow 3x^3 - x^2 - 3x + 1 = (x-1)(x+1)(3x-1)$$

$$\text{Hence, } 3x^3 - x^2 - 3x + 1 = (x-1)(x+1)(3x-1)$$

25. Using suitable identity, evaluate the following:

(i)  $103^3$       (ii)  $101 \times 102$       (iii)  $999^2$

**Sol.** (i)  $103^3 = (100+3)^3$

Now, using identity  $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ , we have

$$\begin{aligned} (100+3)^3 &= (100)^3 + (3)^3 + 3(100)(3)(100+3) \\ &= 1000000 + 27 + 900(100+3) \\ &= 1000000 + 27 + 90000 + 2700 \\ &= 1092727 \end{aligned}$$

(ii)  $101 \times 102 = (100+1)(100+2)$

Now, using identity  $(x+a)(x+b) = x^2 + (a+b)x + ab$ , we have

$$\begin{aligned} (100+1)(100+2) &= (100)^2 + (1+2)100 + (1)(2) \\ &= 10000 + (3)100 + 2 = 10000 + 300 + 2 \\ &= 10302 \end{aligned}$$

(iii)  $(999)^2 = (1000-1)^2 = (1000)^2 - 2 \times (1000) \times 1 + 1^2$   
 $= 1000000 - 2000 + 1$   
 $= 998001$

26. Factorise the following:

(i)  $4x^2 + 20x + 25$

(ii)  $9y^2 - 66yz + 121z^2$

(iii)  $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

**Sol.** (i) We have,

$$\begin{aligned} 4x^2 + 20x + 25 &= (2x)^2 + 2(2x)(5) + (5)^2 \\ &= (2x+5)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2] \\ &= (2x+5)(2x+5) \end{aligned}$$

(ii) We have,

$$\begin{aligned} 9y^2 - 66yz + 121z^2 &= (-3y)^2 + 2(-3y)(11z) + (11z)^2 \\ &= (-3y+11z)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2] \\ &= (-3y+11z)(-3y+11z) \\ &= (3y-11z)(3y-11z) \end{aligned}$$

(iii)  $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Using identity  $a^2 - b^2 = (a+b)(a-b)$

$$\begin{aligned} &= \left[ \left(2x + \frac{1}{3}\right) + \left(x - \frac{1}{2}\right) \right] \left[ \left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right) \right] \\ &= \left(2x + \frac{1}{3} + x - \frac{1}{2}\right) \left(2x + \frac{1}{3} - x + \frac{1}{2}\right) = \left(3x - \frac{1}{6}\right) \left(x + \frac{5}{6}\right) \end{aligned}$$

27. Factorise the following:

(i)  $9x^2 - 12x + 3$

(ii)  $9x^2 - 12x + 4$

**Sol.** (i)  $9x^2 - 12x + 3 = 9x^2 - 9x - 3x + 3$   
 $= 9x(x - 1) - 3(x - 1)$   
 $= (9x - 3)(x - 1)$   
 $= 3(3x - 1)(x - 1)$

(ii) We have,

$$9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + (2)^2$$

$$= (3x - 2)^2 \quad [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= (3x - 2)(3x - 2)$$

28. Expand the following:

(i)  $(4a - b + 2c)^2$

(ii)  $(3a - 5b - c)^2$

(iii)  $(-x + 2y - 3z)^2$

**Sol.** (i) We have,

$$(4a - b + 2c)^2 = (4a)^2 + (-b)^2 + (2c)^2 + 2(4a)(-b) + 2(-b)(2c) + 2(2c)(4a)$$

$$[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= 16a^2 + b^2 + 4c^2 - 8ab - 4bc + 16ca$$

(ii) We have,

$$(3a - 5b - c)^2 = (3a)^2 + (-5b)^2 + (-c)^2 + 2(3a)(-5b)$$

$$+ 2(-5b)(-c) + 2(-c)(3a)$$

$$[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= 9a^2 + 25b^2 + c^2 - 30ab + 10bc - 6ca.$$

(iii)  $(-x + 2y - 3z)^2 = \{(-x) + 2y + (-3z)\}^2$   
 $= (-x)^2 + (2y)^2 + (-3z)^2 + 2(-x)(2y) + 2(2y)(-3z)$   
 $+ 2(-3z)(-x)$   
 $= x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6zx$

29. Factorise the following:

(i)  $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

(ii)  $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

(iii)  $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

**Sol.** (i) We have,

$$9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$$

$$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$$

$$= \{3x + 2y + (-4z)\}^2 \quad [\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z)$$

$$\begin{aligned}
 \text{(ii) } 25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz \\
 &= (-5x)^2 + (4y)^2 + (2z)^2 + 2(-5x)(4y) \\
 &\quad + 2(4y)(2z) + 2(2z)(-5x) \\
 &= (-5x + 4y + 2z)^2
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz \\
 &= (4x)^2 + (-2y)^2 + (3z)^2 + 2(4x)(-2y) + 2(-2y)(3z) + 2(3z)(4x) \\
 &= \{4x + (-2y) + 3z\}^2 \quad [\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\
 &= (4x - 2y + 3z)^2 \quad \quad \quad = (a + b + c)^2] \\
 &= (4x - 2y + 3z)(4x - 2y + 3z)
 \end{aligned}$$

30. If  $a + b + c = 9$  and  $ab + bc + ca = 26$ , find  $a^2 + b^2 + c^2$ .

Sol. We know that

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\Rightarrow (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$$

$$\Rightarrow 9^2 = (a^2 + b^2 + c^2) + 2(26)$$

[Putting the values of  $a + b + c$  and  $ab + bc + ca$ ]

$$\Rightarrow 81 = (a^2 + b^2 + c^2) + 52$$

$$\Rightarrow a^2 + b^2 + c^2 = 81 - 52 = 29$$

31. Expand the following:

$$\text{(i) } (3a - 2b)^3 \quad \text{(ii) } \left(\frac{1}{x} + \frac{y}{3}\right)^3 \quad \text{(iii) } \left(4 - \frac{1}{3x}\right)^3$$

Sol. (i) We have

$$(3a - 2b)^3 = (3a)^3 - (2b)^3 - 3(3a)(2b)(3a - 2b)$$

$$[\because (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$$

$$= 27a^3 - 8b^3 - 18ab(3a - 2b)$$

$$= 27a^3 - 8b^3 - 54a^2b + 36ab^2$$

$$\text{(ii) } \therefore (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\therefore \left(\frac{1}{x} + \frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3 \times \frac{1}{x} \times \frac{y}{3} \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x} \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x} = \frac{1}{x^3} + \frac{y}{x^2} + \frac{y^2}{3x} + \frac{y^3}{27}$$

(iii) We have,

$$\begin{aligned} \left(4 - \frac{1}{3x}\right)^3 &= (4)^3 - \left(\frac{1}{3x}\right)^3 - 3(4)\left(\frac{1}{3x}\right)\left(4 - \frac{1}{3x}\right) \\ & \quad [\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)] \\ &= 64 - \frac{1}{27x^3} - \frac{4}{x}\left(4 - \frac{1}{3x}\right) \\ &= 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2} \end{aligned}$$

32. Factorise the following:

$$(i) 1 - 64a^3 - 12a + 48a^2 \quad (ii) 8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$$

Sol. (i) We have,

$$\begin{aligned} 1 - 64a^3 - 12a + 48a^2 &= (1)^3 - (4a)^3 - 3(1)(4a)(1 - 4a) \\ &= (1 - 4a)^3 [\because a^3 - b^3 - 3ab(a-b) = (a-b)^3] \\ &= (1 - 4a)(1 - 4a)(1 - 4a) \end{aligned}$$

$$\begin{aligned} (ii) 8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125} \\ &= (2p)^3 + 3 \times (2p)^2 \times \frac{1}{5} + 3 \times (2p) \times \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 \\ &= (2p)^3 + \left(\frac{1}{5}\right)^3 + 3 \times (2p) \times \frac{1}{5} \left[2p + \frac{1}{5}\right] \end{aligned}$$

Now, using  $a^3 + b^3 + 3ab(a+b) = (a+b)^3$

$$= \left(2p + \frac{1}{5}\right)^3 = \left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$$

33. Find the following products:

$$(i) \left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right) \quad (ii) (x^2 - 1)(x^4 + x^2 + 1)$$

Sol. (i) We have,

$$\begin{aligned} \left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right) &= \left(\frac{x}{2} + 2y\right)\left\{\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right\} \\ &= \left(\frac{x}{2}\right)^3 + (2y)^3 \quad [\because (a+b)(a^2 - ab + b^2) = a^3 + b^3] \\ &= \frac{x^3}{8} + 8y^3 \end{aligned}$$

(ii) We have,

$$\begin{aligned}(x^2 - 1)(x^4 + x^2 + 1) &= (x^2 - 1)\{(x^2)^2 + (x^2)(1) + (1)^2\} \\ &= (x^2)^3 - (1)^3 \\ &[\because (a - b)(a^2 + ab + b^2) = a^3 - b^3] \\ &= x^6 - 1\end{aligned}$$

34. Factorise:

(i)  $1 + 64x^3$

(ii)  $a^3 - 2\sqrt{2}b^3$

Sol. (i) We have,

$$\begin{aligned}1 + 64x^3 &= (1)^3 + (4x)^3 \\ &= (1 + 4x)\{(1)^2 - (1)(4x) + (4x)^2\} \\ &[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \\ &= (1 + 4x)(1 - 4x + 16x^2) \\ &= (1 + 4x)(16x^2 - 4x + 1) \\ &= (4x + 1)(16x^2 - 4x + 1)\end{aligned}$$

(ii) We have,

$$\begin{aligned}a^3 - 2\sqrt{2}b^3 &= (a)^3 - (\sqrt{2}b)^3 \\ &= (a - \sqrt{2}b)\{a^2 + (a)(\sqrt{2}b) + (\sqrt{2}b)^2\} \\ &[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\ &= (a - \sqrt{2}b)(a^2 + \sqrt{2}ab + 2b^2)\end{aligned}$$

35. Find the following product:

$$(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$$

Sol. We have,

$$\begin{aligned}(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz) \\ &= \{2x + (-y) + 3z\} \{(2x)^2 + (-y)^2 + (3z)^2 - (2x)(-y) - (-y)(3z) - (3z)(2x)\} \\ &= (2x)^3 + (-y)^3 + (3z)^3 - 3(2x)(-y)(3z) \\ &[\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc] \\ &= 8x^3 - y^3 + 27z^3 + 18xyz\end{aligned}$$

36. Factorise:

(i)  $a^3 - 8b^3 - 64c^3 - 24abc$

(ii)  $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

Sol. (i) We have,

$$\begin{aligned}a^3 - 8b^3 - 64c^3 - 24abc \\ &= \{(a)^3 + (-2b)^3 + (-4c)^3 - 3(a)(-2b)(-4c)\} \\ &= \{a + (-2b) + (-4c)\} \{a^2 + (-2b)^2 + (-4c)^2 - a(-2b) \\ &\quad - (-2b)(-4c) - (-4c)a\}\end{aligned}$$



$$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= (a - 2b - 4c)(a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ca)$$

(ii) We have,

$$2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$$

$$= \{(\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}a)(2b)(-3c)\}$$

$$= \{\sqrt{2}a + 2b + (-3c)\} \{(\sqrt{2}a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{2}a)(2b) - (2b)(-3c) - (-3c)(\sqrt{2}a)\}$$

$$= (\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ca)$$

37. Without actually calculating the cubes, find the value of:

(i)  $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$       (ii)  $(0.2)^3 - (0.3)^3 + (0.1)^3$

Sol. (i) Let  $a = \frac{1}{2}$ ,  $b = \frac{1}{3}$ ,  $c = -\frac{5}{6}$

$$\therefore a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6}$$

$$= \frac{3+2-5}{6} = \frac{0}{6} = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\therefore \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3$$

$$= 3 \times \frac{1}{2} \times \frac{1}{3} \times \left(-\frac{5}{6}\right) = -\frac{5}{12}$$

(ii) We have,

$$(0.2)^3 - (0.3)^3 + (0.1)^3 = (0.2)^3 + (-0.3)^3 + (0.1)^3$$

Let  $a = 0.2$ ,  $b = -0.3$  and  $c = 0.1$ . Then,

$$a + b + c = 0.2 + (-0.3) + 0.1$$

$$= 0.2 - 0.3 + 0.1 = 0$$

$$\therefore a + b + c = 0$$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow (0.2)^3 + (-0.3)^3 + (0.1)^3 = 3(0.2)(-0.3)(0.1) = -0.018$$

$$\text{Hence, } (0.2)^3 + (-0.3)^3 + (0.1)^3 = -0.018$$

38. Without finding the cubes, factorise

$$(x-2y)^3 + (2y-3z)^3 + (3z-x)^3$$

Sol. Let  $x-2y = a$ ,  $2y-3z = b$  and  $3z-x = c$

$$\therefore a + b + c = x - 2y + 2y - 3z + 3z - x = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\begin{aligned} \text{Hence, } (x-2y)^3 + (2y-3z)^3 + (3z-x)^3 \\ = 3(x-2y)(2y-3z)(3z-x) \end{aligned}$$

39. Find the value of

(i)  $x^3 + y^3 - 12xy + 64$ , when  $x + y = -4$

(ii)  $x^3 + 8y^3 - 36xy - 216$ , when  $x = 2y + 6$

Sol. (i)  $x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3x \times y \times 4$   
 $= (x + y + 4)(x^2 + y^2 + 4^2 - xy - 4y - 4x)$   
 $\quad \quad \quad [\because x + y = -4]$   
 $= (0)(x^2 + y^2 + 4^2 - xy - 4y - 4x) = 0$

(ii)  $x^3 - 8y^3 - 36xy - 216 = x^3 + (-2y)^3 + (-6)^3 - 3x(-2y)(-6)$   
 $= (x - 2y - 6)$   
 $\quad \quad \quad [x^2 + (-2y)^2 + (-6)^2 - x(-2y) - (-2y)(-6) - (-6)x]$   
 $= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$   
 $= (0)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x) = 0$   
 $\quad \quad \quad [\because x = 2y + 6]$

40. Give possible experiments for the length and breadth of the rectangle whose area is given by  $4a^2 + 4a - 3$ .

Sol. Area :  $4a^2 + 4a - 3$

Using the method of splitting the middle term, we first find two numbers whose sum is +4 and product is  $4 \times (-3) = -12$ .

Now,  $+6 - 2 = +4$  and  $(+6) \times (-2) = -12$

We split the middle term  $4a$  as  $4a = +6a - 2a$ ,

so that  $4a^2 + 4a - 3 = 4a^2 + 6a - 2a - 3$   
 $= 2a(2a + 3) - 1(2a + 3)$   
 $= (2a - 1)(2a + 3)$

Now, area of rectangle =  $4a^2 + 4a - 3$

Also, area of rectangle = length  $\times$  breadth and  $4a^2 + 4a - 3 = (2a - 1)(2a + 3)$

So, the possible expressions for the length and breadth of the rectangle are length =  $(2a - 1)$  and breadth =  $(2a + 3)$  or, length =  $(2a + 3)$  and breadth =  $(2a - 1)$ .

<b>EXERCISE 2.4</b>
---------------------

1. If the polynomials  $ax^3 + 4z^2 + 3z - 4$  and  $z^3 - 4z + a$  leave the same remainder when divided by  $z - 3$ , find the value of  $a$ .

**Sol.** Let  $p(z) = ax^3 + 4z^2 + 3z - 4$

and  $q(z) = z^3 - 4z + a$

As these two polynomials leave the same remainder, when divided by  $z - 3$ , then  $p(3) = q(3)$ .

$$\begin{aligned} \therefore p(3) &= a(3)^3 + 4(3)^2 + 3(3) - 4 \\ &= 27a + 36 + 9 - 4 \end{aligned}$$

or  $p(3) = 27a + 41$

and  $q(3) = (3)^3 - 4(3) + a$   
 $= 27 - 12 + a = 15 + a$

Now,  $p(3) = q(3)$

$$\Rightarrow 27a + 41 = 15 + a$$

$$\Rightarrow 26a = -26; a = -1$$

Hence, the required value of  $a = -1$ .

2. The polynomial  $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$  when divided by  $x + 1$  leaves remainder 19. Also, find the remainder when  $p(x)$  is divided by  $x + 2$ .

**Sol.** We know that if  $p(x)$  is divided by  $x + a$ , then the remainder  $= p(-a)$ .

Now,  $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$  is divided by  $x + 1$ , then the remainder  $= p(-1)$

$$\begin{aligned} \text{Now, } p(-1) &= (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7 \\ &= 1 - 2(-1) + 3(1) + a + 3a - 7 \\ &= 1 + 2 + 3 + 4a - 7 \\ &= -1 + 4a \end{aligned}$$

Also, remainder  $= 19$

$$\therefore -1 + 4a = 19$$

$$\Rightarrow 4a = 20; a = 20 \div 4 = 5$$

Again, when  $p(x)$  is divided by  $x + 2$ , then

$$\begin{aligned} \text{remainder} &= p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - a(-2) + 3a - 7 \\ &= 16 + 16 + 12 + 2a + 3a - 7 \\ &= 37 + 5a \\ &= 37 + 5(5) = 37 + 25 = 62 \end{aligned}$$

3. If both  $(x - 2)$  and  $\left(x - \frac{1}{2}\right)$  are factors of  $px^2 + 5x + r$ , show that  $p = r$ .

**Sol.** Let  $p(x) = px^2 + 5x + r$ .

As  $(x-2)$  is a factor of  $p(x)$

$$\text{So, } p(2) = 0 \Rightarrow p(2)^2 + 5(2) + r = 0$$

$$\Rightarrow 4p + 10 + r = 0 \quad \dots(1)$$

Again,  $\left(x - \frac{1}{2}\right)$  is a factor of  $p(x)$ ,

$$\therefore p\left(\frac{1}{2}\right) = 0$$

$$\begin{aligned} \text{Now, } p\left(\frac{1}{2}\right) &= p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r \\ &= \frac{1}{4}p + \frac{5}{2} + r \end{aligned}$$

$$\therefore p\left(\frac{1}{2}\right) = 0 \Rightarrow \frac{1}{4}p + \frac{5}{2} + r = 0 \quad \dots(2)$$

From (1), we have  $4p + r = -10$

From (2), we have  $p + 10 + 4r = 0$

$$\Rightarrow p + 4r = -10$$

$$\therefore 4p + r = p + 4r \quad [\because \text{Each} = -10]$$

$$\therefore 3p = 3r \Rightarrow p = r$$

Hence, proved.

4. Without actual division, prove that  $2x^4 - 5x^3 + 2x^2 - x + 2$  is divisible by  $x^2 - 3x + 2$ .

**Sol.** We have,

$$\begin{aligned} x^2 - 3x + 2 &= x^2 - x - 2x + 2 \\ &= x(x-1) - 2(x-1) \\ &= (x-1)(x-2) \end{aligned}$$

$$\text{Let } p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$$

$$\text{Now, } p(1) = 2(1)^4 - 5(1)^3 + 2(1)^2 - 1 + 2 = 2 - 5 + 2 - 1 + 2 = 0$$

Therefore,  $(x-1)$  divides  $p(x)$

$$\begin{aligned} \text{and } p(2) &= 2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2 \\ &= 32 - 40 + 8 - 2 + 2 = 0 \end{aligned}$$

Therefore,  $(x-2)$  divides  $p(x)$ .

$$\text{So, } (x-1)(x-2) = x^2 - 3x + 2 \text{ divides } 2x^4 - 5x^3 + 2x^2 - x + 2$$

5. Simplify  $(2x-5y)^3 - (2x+5y)^3$ .

**Sol.** We have,

$$(2x-5y)^3 - (2x+5y)^3$$

$$= \{(2x-5y) - (2x+5y)\} \{(2x-5y)^2 + (2x-5y)(2x+5y) + (2x+5y)^2\}$$

$$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$\begin{aligned}
 &= (2x - 5y - 2x - 5y)(4x^2 + 25y^2 - 20xy + 4x^2 - 25y^2 + 4x^2 + 25y^2 + 20xy) \\
 &= (-10y)(2x^2 + 25y^2) \\
 &= -120x^2y - 250y^3
 \end{aligned}$$

6. Multiply  $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$  by  $(-z + x - 2y)$ .

**Sol.** We have,

$$\begin{aligned}
 &(-z + x - 2y)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz) \\
 &= \{x + (-2y) + (-z)\} \{(x)^2 + (-2y)^2 + (-z)^2 - (x)(-2y) - (-2y)(-z) - (-z)(x)\} \\
 &= x^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z) \\
 &\quad [\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc] \\
 &= x^3 - 8y^3 - z^3 - 6xyz
 \end{aligned}$$

7. If  $a, b, c$  are all non-zero and  $a + b + c = 0$ , prove that

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3.$$

**Sol.** We have  $a, b, c$  are all non-zero and  $a + b + c = 0$ , therefore

$$a^3 + b^3 + c^3 = 3abc$$

$$\text{Now, } \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc} = 3$$

8. If  $a + b + c = 5$  and  $ab + bc + ca = 10$ , then prove that  $a^3 + b^3 + c^3 - 3abc = -25$

**Sol.** We know that,

$$\begin{aligned}
 a^3 + b^3 + c^3 - 3abc &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
 &= (a + b + c)[a^2 + b^2 + c^2 - (ab + bc + ca)] \\
 &= 5 \{a^2 + b^2 + c^2 - (ab + bc + ca)\} \\
 &= 5(a^2 + b^2 + c^2 - 10)
 \end{aligned}$$

Now,  $a + b + c = 5$

Squaring both sides, we get

$$\begin{aligned}
 &(a + b + c)^2 = 5^2 \\
 \Rightarrow &a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25 \\
 \therefore &a^2 + b^2 + c^2 + 2(10) = 25 \\
 \Rightarrow &a^2 + b^2 + c^2 = 25 - 20 = 5
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } a^3 + b^3 + c^3 - 3abc &= 5(a^2 + b^2 + c^2 - 10) \\
 &= 5(5 - 10) = 5(-5) = -25
 \end{aligned}$$

Hence, proved.

9. Prove that  $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

Sol.

$$\begin{aligned}
 (a + b + c)^3 &= [a + (b + c)]^3 \\
 &= a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3 \\
 &= a^3 + 3a^2b + 3a^2c + 3a(b^2 + 2bc + c^2) \\
 &\quad + (b^3 + 3b^2c + 3bc^2 + c^3) \\
 &= a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 \\
 &\quad + b^3 + 3b^2c + 3bc^2 + c^3 \\
 &= a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2c + 3b^2a \\
 &\quad + 3c^2a + 3c^2b + 6abc \\
 &= a^3 + b^3 + c^3 + 3a^2(b + c) + 3b^2(c + a) \\
 &\quad + 3c^2(a + b) + 6abc
 \end{aligned}$$

Hence, above result can be put in the form

$$(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$$

$$\therefore (a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$$

