4 Linear Equations in Two Variables

EXERCISE 4.1

1. The linear equation $2x - 5y = 7$ has
   (*a*) a unique solution (*b*) two solutions
   (*c*) infinitely many solutions (*d*) no solution.

Sol. $2x - 5y = 7$ is a linear equation in two variables. A linear equation in two variables has infinitely many solutions.
Hence, (*c*) is the correct answer.

2. The equation $2x + 5y = 7$ has a unique solution if x, y are:
   (*a*) natural numbers (*b*) positive real numbers
   (*c*) real numbers (*d*) rational numbers.

Sol. The equation $2x + 5y = 7$ has a unique solution if x, y are natural numbers.
Hence, (*a*) is the correct answer.

3. If (2, 0) is a solution of the linear equation $2x + 3y = k$, then the value of k is:
   (*a*) 4 (*b*) 6 (*c*) 5 (*d*) 2

Sol. Substituting $x = 2$ and $y = 0$ in the given equation $2x + 3y = k$, we get
$2(2) + 3(0) = k \Rightarrow k = 4$
Therefore, the value of k is 4.
Hence, (*a*) is the correct answer.

4. Any solution of the linear equation $2x + 0y + 9 = 0$ in two variables is of the form:
   (*a*) $\left(-\frac{9}{2}, m\right)$ (*b*) $\left(n, -\frac{9}{2}\right)$ (*c*) $\left(0, -\frac{9}{2}\right)$ (*d*) $(-9, 0)$

Sol. The given linear equation is $2x + 0y + 9 = 0 \Rightarrow 2x = -9$

$$\therefore \quad x = -\frac{9}{2}$$

Since the coefficient of y is 0 in the given equation, the solution can be given as $\left(-\frac{9}{2}, m\right)$.

Hence, (*a*) is the correct answer.

5. The graph of the linear equation $2x + 3y = 6$ cuts the y-axis at the point

(*a*) (2, 0) (*b*) (0, 3) (*c*) (3, 0) (*d*) (0, 2)

Sol. The graph of the linear equation $2x + 3y = 6$ cuts the y-axis at the point where x-coordinate is zero.

Putting $x = 0$ in $2x + 3y = 6$, we get

$2(0) + 3y = 6 \Rightarrow 3y = 6 \Rightarrow y = 6 \div 3 = 2$

So, (0, 2) is the required point.

Hence, (*d*) is the correct answer.

6. The equation $x = 7$ in two variables can be written as

(*a*) $1.x + 1.y = 7$ (*b*) $1.x + 0.y = 7$

(*c*) $0.x + 1.y = 7$ (*d*) $0.x + 0.y = 7$

Sol. The equation $x = 7$ in two variables can be expressed as $1.x + 0.y = 7$.

Hence, (*b*) is the correct answer.

7. Any point on the x-axis is of the form

(*a*) (x, y) (*b*) $(0, y)$ (*c*) $(x, 0)$ (*d*) (x, x)

Sol. Any point on the x-axis has its ordinate 0.

So, any point on the x-axis is of the form $(x, 0)$.

Hence, (*c*) is the correct answer.

8. Any point on the line $y = x$ is of the form

(*a*) (a, a) (*b*) $(0, a)$ (*c*) $(a, 0)$ (*d*) $(a, -a)$

Sol. Any point on the line $y = x$ will have x and y coordinates same.

So, any point on the line $y = x$ is of the form (a, a).

Hence, (*a*) is the correct answer.

9. The equation of x-axis is of the form

(*a*) $x = 0$ (*b*) $y = 0$ (*c*) $x + y = 0$ (*d*) $x = y$

Sol. $y = 0$ is the equation of x-axis.

Hence, (*b*) is the correct answer.

10. The graph of $y = 6$ is a line

(*a*) parallel to x-axis at a distance 6 units from the origin

(*b*) parallel to y-axis at a distance 6 units from the origin

(*c*) making an intercept 6 on the x-axis.

(*d*) making an intercept 6 on both the axes.

Sol. The given equation $y = 6$ does not contain x. Its graph is a line parallel to x-axis.

So, the graph of $y = 6$ is a line parallel to x-axis at a distance 6 units from the origin.

Hence, (*a*) is correct answer.

11. $x = 5, y = 2$ is a solution of the linear equation

(*a*) $x + 2y = 7$ (*b*) $5x + 2y = 7$ (*c*) $x + y = 7$ (*d*) $5x + y = 7$

Sol. $x = 5$, $y = 2$ is a solution of the linear equation $x + y = 7$, as $5 + 2 = 7$.
Hence, (*c*) is the correct answer.

12. If a linear equation has solutions $(-2, 2)$, $(0, 0)$ and $(2, -2)$, then it is of the form

(*a*) $y - x = 0$ (*b*) $x + y = 0$ (*c*) $-2x + y = 0$ (*d*) $-x + 2y = 0$

Sol. The points $(-2, 2)$ and $(2, -2)$ have x and y coordinates of opposite signs.
Also, any point on the graph of $x + y = 0$
i.e., $y = -x$ will have x and y coordinates of opposite signs. The point $(0,0)$ also satisfies $x + y = 0$.
Hence, (*b*) is the correct answer.

13. The positive solutions of the equation $ax + by + c = 0$ always lie in the

(*a*) Ist quadrant (*b*) 2nd quadrant
(*c*) 3rd quadrant (*d*) 4th quadrant

Sol. Quadrant I consists of all points (x, y) for which the x and y are positive.
So, the positive solution of the equation $ax + by + c = 0$ always lie in the Ist quadrant.
Hence, (*a*) is the correct answer.

14. The graph of the linear equation $2x + 3y = 6$ is a line which meets the x-axis at the point

(*a*) $(0, 2)$ (*b*) $(2, 0)$ (*c*) $(3, 0)$ (*d*) $(0, 3)$

Sol. The graph of the linear equation $2x + 3y = 6$ is a line which meets the x-axis at the point where $y = 0$.
Now, putting $y = 0$ in $2x + 3y = 6$, we get
$2x + 3(0) = 6 \Rightarrow 2x = 6 \Rightarrow x = 6 \div 2 = 3$
So, $(3, 0)$ is a point on the line $2x + 3y = 6$.
Hence, (*c*) is the correct answer.

15. The graph of the linear equation $y = x$ passes through the point

(*a*) $\left(\frac{3}{2}, \frac{-3}{2}\right)$ (*b*) $\left(0, \frac{3}{2}\right)$ (*c*) $(1, 1)$ (*d*) $\left(\frac{-1}{2}, \frac{1}{2}\right)$

Sol. We know that any point on the line $y = x$ will have x and y coordinates same.
So, the graph of the linear equation $y = x$ passes through the point $(1, 1)$.
Hence, (*c*) is the correct answer.

16. If we multiply or divide both sides of a linear equation with a non-zero number, then the solution of the linear equation:

(*a*) changes (*b*) remains the same
(*c*) changes in case of multiplication only
(*d*) changes in case of division only

Sol. If we multiply or divide both sides of a linear equation with a non-zero number, then the solution of the linear equation remains the same.

Hence, (*b*) is the correct answer.

17. How many linear equations in x and y can be satisfied by $x = 1$ and $y = 2$?

(*a*) Only one (*b*) Two (*c*) Infinitely many (*d*) Three

Sol. There are infinitely many linear equations which are satisfied by $x = 1$ and $y = 2$.

For example, a linear equation $x + y = 3$ is satisfied by $x = 1$ and $y = 2$. Others are $y = 2x$, $y - x = 1$, $2y - x = 3$ etc.

Hence, (*c*) is the correct answer.

18. The point of the form (*a*, *a*) always lies on:

(*a*) *x*-axis (*b*) *y*-axis

(*c*) on the line $y = x$ (*d*) on the line $x + y = 0$

Sol. The point of the form (*a*, *a*) have *x* and *y* coordinates same. So, the point of the form (*a*, *a*) always lies on the line $y = x$.

Hence, (*c*) is the correct answer.

19. The point of the form $(a, -a)$ always lie on the line

(*a*) $x = a$ (*b*) $y = -a$ (*c*) $y = x$ (*d*) $x + y = 0$

Sol. The point of the form $(a, -a)$ have *x* and *y* coordinates of opposite signs.

So, the point of the form $(a, -a)$ always lie on the line $y = -x$, *i.e*, $x + y = 0$.

Hence, (*d*) is the correct answer.

EXERCISE 4.2

Write whether the following statements are true or false. Justify your answer.

1. The point (0, 3) lies on the graph of the linear equation $3x + 4y = 12$.

Sol. Substituting $x = 0$ and $y = 3$ in the equation, we get

$$3(0) + 4(3) = 12 \Rightarrow 12 = 12, \text{ which is true.}$$

The point (0, 3) satisfies the equation $3x + 4y = 12$.

Hence, the given statement is true.

2. The graph of the linear equation $x + 2y = 7$ passes through the point (0, 7).

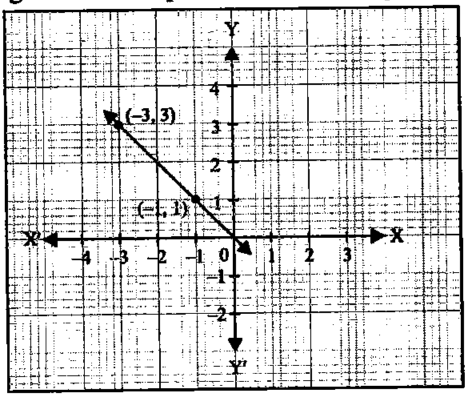
Sol. Substituting $x = 0$ and $y = 7$ in the given equation $x + 2y = 7$, we get

$0 + 2(7) = 7 \Rightarrow 14 = 7$, which is false.

The point (0, 7) does not satisfy the equation.

Hence, the given statement is false.

3. The graph given below represents the linear equation $x + y = 0$.

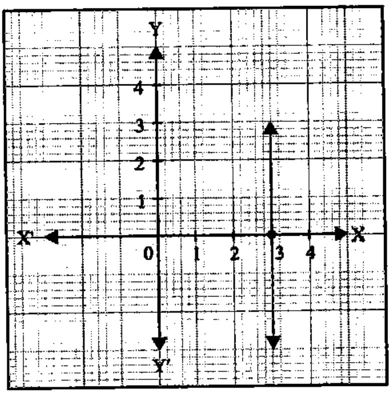


Sol. The given equation is $x + y = 0$, *i.e.*, $y = -x$.

Any point on the graph of $y = -x$, will have x and y coordinates of opposite signs.

As the points $(-1, 1)$ and $(-3, 3)$ have x and y coordinates of opposite signs, so these points satisfy the given equation and the two points determine a unique line, hence the given statement is true.

4. The graph given below represents the linear equation $x = 3$. (See fig.)



Sol. We know that the graph of the equation $x = a$ is a line parallel to the y-axis and to the right of y-axis, if $a > 0$.

The given statement is true, since the graph is a line parallel to y-axis at a distance of 3 units to the right of it.

5. The coordinates of points in the table:

x	0	1	2	3	4
y	2	3	4	–5	6

represent some of the solutions of the equation $x - y + 2 = 0$.

Sol. The points (0, 2), (1, 3), (2, 4) and (4, 6) satisfy the given equation $x - y + 2 = 0$. Each of these points is the solution of the equation $x - y + 2 = 0$. But, the point (3, – 5) does not satisfy the given equation as $3-(-5)+2=0$, i.e., $3+5+2=0$ or $10 = 0$, which is false.

Hence, the given statement is false, since the point (3, – 5) does not satisfy the given equation.

6. Every point on the graph of a linear equation in two variables does not represent a solution of the linear equation.

Sol. As every point on the graph of a linear equation in two variables represent a solution of the equation, so the given statement is false.

7. The graph of every linear equation in two variables need not be a line.

Sol. As the graph of a linear equation in two variables is always a line, so the given statement is false.

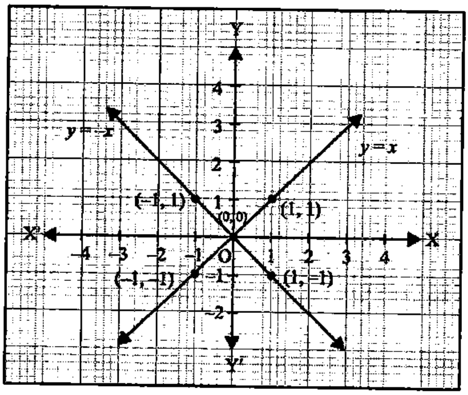
EXERCISE 4.3

1. Draw the graph of linear equations $y=x$ and $y=-x$ on the same cartesian plane. What do you observe?

Sol. Any point on the graph of $y=x$ will have x and y coordinates same. The line passes through the points (0, 0), (1, 1) and (– 1, – 1).

Again, any point on the graph of $y=-x$ will have x and y coordinates of opposite signs. The line passes through the points (1, – 1) and (– 1, 1). Also, (0, 0) satisfies $y=-x$.

The graph of linear equations $y=x$ and $y=-x$ on the same cartesian plane is shown in the figure given below.



We observe that the graph of these equations passes through (0, 0).

2. Determine the point on the graph of the linear equation $2x + 5y = 19$, whose ordinate is $1\frac{1}{2}$ times its abscissa.

Sol. Let the coordinates of the point be (2, 3).
Now, for $x = 2$ and $y = 3$,

$$2x + 5y = 2(2) + 5(3) = 4 + 15 = 19$$

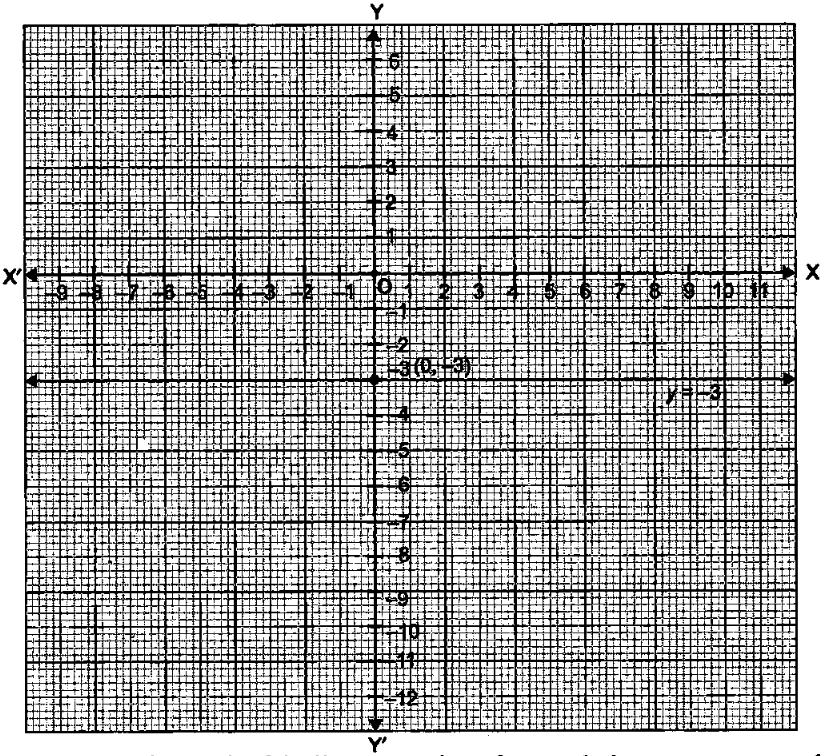
Therefore, the point (2, 3) is a solution of the equation $2x + 5y = 19$.
Abscissa of the point is 2 and ordinate is 3.

Now, $$2 \times 1\frac{1}{2} = 2 \times \frac{3}{2} = 3$$

So, ordinate of the point (2, 3) is $1\frac{1}{2}$ times its abscissa.

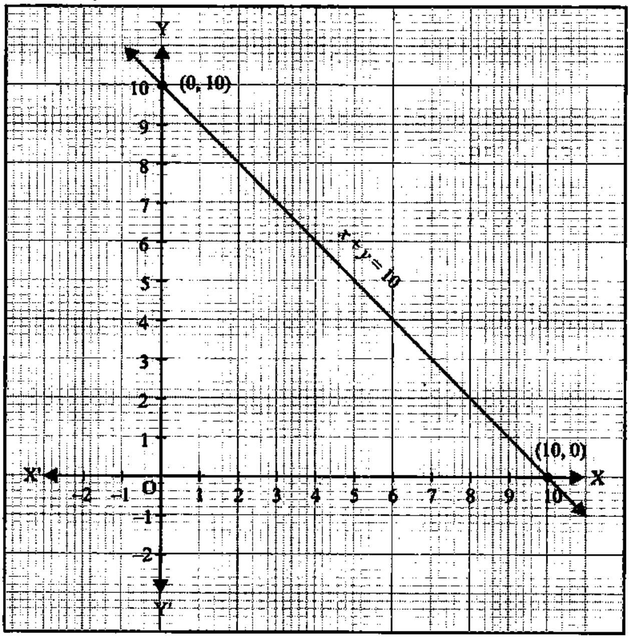
3. Draw the graph of the equation represented by a straight line which is parallel to the x-axis and at a distance 3 units below it.

Sol. The graph of the equation $y = -3$ is a line parallel to the x-axis and at a distance 3 units below it. So, graph of the equation $y = -3$ is a line parallel to x-axis and passing through the point (0, – 3) as shown in the figure given below:



4. Draw the graph of the linear equation whose solutions are represented by the points having the sum of coordinates as 10 units.

Sol. A linear equation whose solutions are represented by the points having the sum of coordinates as 10 units is $x + y = 10$.
When $x = 0, y = 10$ and when $x = 10, y = 0$.
Now, plot these two points $(0, 10)$ and $(10, 0)$ on a graph paper and join them to obtain a straight line.
The graph of $x + y = 10$ is a straight line as shown in the figure given below.



5. Write the linear equation such that each point on its graph has an ordinate 3 times its abscissa.

Sol. A linear equation such that each point on its graph has an ordinate 3 times its abscissa is $y = 3x$.

6. If the point (3, 4) lies on the graph of $3y = ax + 7$, then find the value of a.

Sol. The point (3, 4) lies on the graph of $3y = ax + 7$.
Substituting $x = 3$ and $y = 4$ in the given equation $3y = ax + 7$, we get

$$\therefore \quad 3 \times 4 = a \times 3 + 7$$

$$\Rightarrow \quad 12 = 3a + 7 \quad \Rightarrow 3a = 5 \quad \Rightarrow a = \frac{5}{3}$$

7. How many solution(s) of the equation $2x + 1 = x - 3$ are there on the
(*i*) number line (*ii*) Cartesian plane?

Sol. (*i*) The number of solution(s) of the equation $2x+1=x-3$ which are on the number line is one.

$2x+1=x-3 \Rightarrow 2x-x=-3-1 \Rightarrow x=-4$

$\therefore$ $x=-4$ is the solution of the given equation.

(*ii*) The number of solution(s) of the equation $2x+1=x-3$ which are on the cartesian plane are infinitely many solutions.

8. Find the solution of the linear equation $x+2y=8$ which represents a point on

(*i*) x-axis, (*ii*) y-axis.

Sol. We know that the point which lies on x-axis has its ordinate 0.

Putting $y=0$ in the equation $x+2y=8$, we get

$x+2(0)=8 \quad \Rightarrow x=8$

A point which lies on y-axis has its abscissa 0.

Putting $x=0$ in the equation $x+2y=8$, we get

$0+2y=8 \quad \Rightarrow y=4$

9. For what value of c, the linear equation $2x+cy=8$ has equal values of x and y for its solution?

Sol. The value of c for which the linear equation $2x+cy=8$ has equal values of x and y

i.e., $x=y$ for its solution is

$2x+cy=8 \Rightarrow 2x+cx=8$ $[\because y=x]$

$\Rightarrow \quad cx=8-2x$

$\therefore \quad c=\dfrac{8-2x}{x}, x\neq 0$

10. Let y varies directly as x. If $y=12$ when $x=4$, then write a linear equation. What is the value of y, when $x=5$?

Sol. y varies directly as x.

$\Rightarrow \quad y\propto x,$

$\therefore \quad y=kx$

Substituting $y=12$ when $x=4$, we get

$12=k\times 4 \Rightarrow k=12\div 4=3$

Hence, the required equation is $y=3x$.

The value of y when $x=5$ is $y=3\times 5=15$.

EXERCISE 4.4

1. Show that the points A(1, 2), B(–1, –16) and C(0, –7) lie on the graph of the linear equation $y=9x-7$.

Sol. For A(1, 2), we have $2=9(1)-7=9-7=2$

For B(–1, –16), we have $-16=9(-1)-7=-9-7=-16$

For C(0, –7), we have $-7=9(0)-7=0-7=-7$

We see that the line $y = 9x - 7$ is satisfied by the points A(1, 2), B(– 1, – 16) and C(0, – 7). Therefore, A(1, 2), B(– 1, – 16) and C(0, – 7) are solutions of the linear equation $y = 9x - 7$ and therefore, lie on the graph of the linear equation $y = 9x - 7$.

2. The following observed values of x and y are thought to satisfy a linear equation.

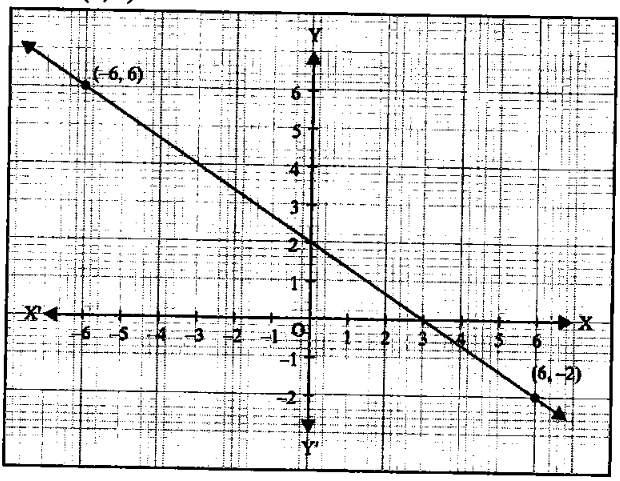
x	6	– 6
y	–2	6

Write the linear equation.

Draw the graph using the values of x, y given in the above table. At what points, the graph of the linear equation cuts the x-axis and the y-axis?

Sol. The linear equation is $2x + 3y = 6$. Both the points (6, –2) and (–6, 6) satisfy the given linear equation.

Plot the points (6, –2) and (–6, 6) on a graph paper. Now, join these two points and obtain a line. We see that the graph cuts the x-axis at (3, 0) and y-axis at (0, 2).



3. Draw the graph of the linear equation $3x + 4y = 6$. At what points, the graph cuts the x-axis and y-axis?

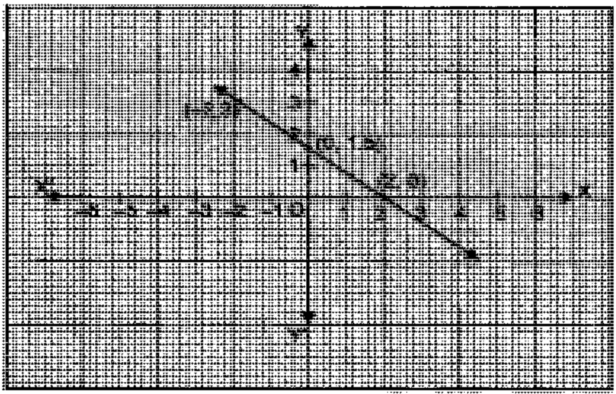
Sol. The solutions of the linear equation

$$3x + 4y = 6$$

can be expressed in the form of a table as follows by writing the values of y below the corresponding values of x :

x	2	–2	0
y	0	3	1.5

Now, plot the points (2, 0), (– 2, 3) and (0, 1.5) on a graph paper. Now, join the points and obtain a line.



We see that the graph cuts the x-axis at (2, 0) and y-axis at (0, 1.5).

4. The linear equation that converts Fahrenheit (F) to Celsius (°C) is given by the relation:

$$C = \frac{5F - 160}{9}$$

(*i*) If the temperature is 86° F, what is the temperature in Celsius?

(*ii*) If the temperature is 35° C, what is the temperature in Fahrenheit?

(*iii*) If the temperature is 0° C, what is the temperature in Fahrenheit and if the temperature is 0° F, what is the temperature in Celsius?

(*iv*) What is the numerical value of temperature which is same in both the scales?

Sol.

$$C = \frac{5F - 160}{9}$$

(*i*) Putting F = 86°, we get $C = \frac{5(86) - 160}{9} = \frac{430 - 160}{9} = \frac{270}{9} = 30°$

Hence, the temperature in celsius is 30° C.

(*ii*) Putting C = 35°, we get $35° = \frac{5F - 160}{9} \Rightarrow 315° = 5F - 160$

$\Rightarrow \quad 5F = 315 + 160 = 475$

$\therefore \quad F = \frac{475}{5} = 95°$

Hence, the temperature in Fahrenheit is 95 F.

(*iii*) Putting C = 0°, we get

$$0 = \frac{5F - 160}{9} \Rightarrow 0 = 5F - 160$$

$\Rightarrow \quad 5F = 160$

$\therefore \quad F = \frac{160}{5} = 32°$

Now, putting F = 0°, we get

$$C = \frac{5F-160}{9} \Rightarrow C = \frac{5(0)-160}{9} = \left(-\frac{160}{9}\right)^{\circ}$$

If the temperature is 0° C, the temperature in Fahrenheit is 32° and if the temperature is 0 F, then the temperature in Celsius is $\left(-\frac{160}{9}\right)^{\circ}$ C.

(*iv*) Putting C = F, in the given relation, we get

$$F = \frac{5F-160}{9} \Rightarrow 9F = 5F - 160$$

$\Rightarrow$ $4F = -160$

$\therefore$ $F = \frac{-160}{4} = -40^{\circ}$

Hence, the numerical value of the temperature which is same in both the scales is -40.

The linear equation that converts Kelvin (x) to Fahrenheit (y) is given by the relation:

$$y = \frac{9}{5}(x - 273) + 32$$

5. If the temperature of a liquid can be measured in Kelvin units as $x°$ K or in Fahrenheit units as $y°$ F, the relation between the two systems of measurement of temperature is given by the linear equation

$$y = \frac{9}{5}(x - 273) + 32$$

(*i*) Find the temperature of the liquid in Fahrenheit if the temperature of the liquid is 313° K.

(*ii*) If the temperature is 158° F, then find the temperature in Kelvin.

Sol.

$$y = \frac{9}{5}(x - 273) + 32$$

(*i*) When the temperature of the liquid is $x = 313°$ K

$$y = \frac{9}{5}(313 - 273) + 32 = \frac{9}{5} \times 40 + 32 = 72° + 32° = 104° \text{ F}$$

(*ii*) When the temperature of the liquid is $y = 158°$ F

$$158 = \frac{9}{5}(x - 273) + 32 \Rightarrow \frac{9}{5}(x - 273) = 158 - 32$$

$\Rightarrow$ $x - 273 = 126 \times \frac{5}{9} = 70$

$\Rightarrow$ $x - 273 = 70 \Rightarrow x = 273 + 70 = 343°$ K

6. The force exerted to pull a cart is directly proportional to the acceleration produced in the body.

Express the statement as a linear equation in two variables and draw the graph of the same by taking the constant mass equal to 6 kg. Read from the graph, the force required when the acceleration produced in the body is

(*i*) 5 m/s^2 (*ii*) 6 m/s^2.

Sol. We have $y \propto x \Rightarrow y = mx$

where y denotes the force, x denotes the acceleration and m denotes the constant mass.

Taking $m = 6$ kg, we get $y = 6x$.

Now, we form a table as follows by writing the values of y below the corresponding values of x.

x	0	1	2
y	0	6	12

Plot the points (0, 0), (1, 6) and (2, 12) on a graph paper and join any two points and obtain a line.