CHAPTER 1

NUMBER SYSTEMS

EXERCISE 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol :

Yes, zero is a rational number because it can be written in any one of the following forms :

$$0 = \frac{0}{1}, \frac{0}{2}, \frac{0}{-1}, \frac{0}{-2}, \frac{0}{-3}, \frac{0}{3} \text{ and so on}$$

This is in the form $\frac{p}{q}$, where $q \neq 0$.

2. Find six rational numbers between 3 and 4.

Sol :

Here may be infinite rational number between any two rational number. But here we have to determine only six number between 3 and 4.

To get six rational number between 3 and 4 in easiest way we write 3 and 4 as follows

$$3 = 3 \times \frac{6+1}{6+1} = \frac{3 \times 7}{7} = \frac{21}{7}$$

and $$4 = 4 \times \frac{6+1}{6+1} = \frac{4 \times 7}{7} = \frac{28}{7}$$

Six rational numbers between 3 and 4 are

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

There can be other set of rational numbers also. One other set is $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}, \frac{36}{10}$.

PRACTICE :

1. Find six rational numbers between 13 and 14.

Ans : $\frac{92}{7}, \frac{93}{7}, \frac{94}{7}, \frac{95}{7}, \frac{96}{7}$ and $\frac{97}{7}$

2. Find seven rational numbers between 22 and 23.

Ans : $\frac{177}{8}, \frac{178}{8}, \frac{179}{8}, \frac{180}{8}, \frac{181}{8}, \frac{182}{8}$ and $\frac{183}{8}$

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol :

Here may be infinite rational number between any two rational number. But here we have to determine only five number between $\frac{3}{5}$ and $\frac{4}{5}$.

To get five rational number between $\frac{3}{5}$ and $\frac{4}{5}$ in easiest way we write $\frac{3}{5}$ and $\frac{4}{5}$ as follows

$$\frac{3}{5} = \frac{3 \times (5+1)}{5 \times (5+1)} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$$

and $$\frac{4}{5} = \frac{4 \times (5+1)}{5 \times (5+1)} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

Five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$.

There can be other set of rational numbers also.

PRACTICE :

1. Find four rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$.

Ans : $\frac{26}{35}, \frac{27}{35}, \frac{28}{35}$ and $\frac{29}{35}$

2. Find six rational numbers between $\frac{4}{9}$ and $\frac{7}{8}$.

Ans : $\frac{33}{72}, \frac{34}{72}, \frac{35}{72}, \frac{36}{72}, \frac{37}{72}$ and $\frac{38}{72}$

4. State whether the following statements are true or false. Give reasons for your answers.

1. Every natural number is a whole number.
2. Every integer is a whole number.
3. Every rational number is a whole number.

Sol :

1. True, as the set of whole numbers contains all the natural numbers.
2. False, as negative integer, e.g., -3 is not a whole number.
3. False, as $\frac{2}{3}$ is a rational number but not a whole number.

EXERCISE 1.2

1. State whether the following statements are true or false. Justify your answers.

1. Every irrational number is a real number.
2. Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
3. Every real number is an irrational number.

Sol :

1. True, All irrational and rational numbers together

make up the collection of real numbers.

2. False, as $\frac{3}{2}$ on the number line cannot be a square root of a natural number.

   Also, a negative number cannot be a square root of natural number, as $\sqrt{m}$ represents a positive value.

3. False, as 2 is a real number but not an irrational number.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol :

No. Square roots of positive integers are not irrational For example,

$\sqrt{9} = 3$, is rational numbers.

$\sqrt{16} = 4$, is rational numbers.

$\sqrt{25} = 5$, is rational numbers.

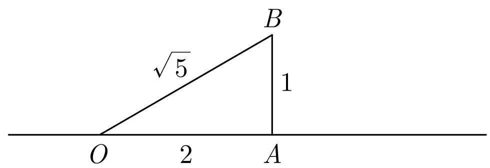
$\sqrt{36} = 6$, is rational numbers.

3. Show how $\sqrt{5}$ can be represented on the number line.

Sol :

Here $(\sqrt{5})^2 = 2^2 + 1^2$

Thus we can construct $\sqrt{5}$ as the length of hypotenuse of a right triangle whose sides are of length 2 and 1 units.



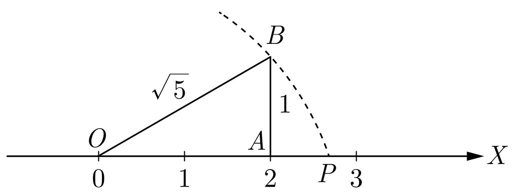
Let OX be a number line on which O represent 0 and A represent 2 unit length. Draw a line $AB \perp OA$ and mark point B on it so that $AB = 1$ unit.

Then $OB^2 = OA^2 + AB^2$

$= 2^2 + 1^2 = 5 = 4 + 1$

or $OB = \sqrt{5}$

Using a compass with centre O and radius OB we mark a point P on the number line corresponding to $\sqrt{5}$ on the number line.



Thus P represent the number $\sqrt{5}$ on number line.

PRACTICE :

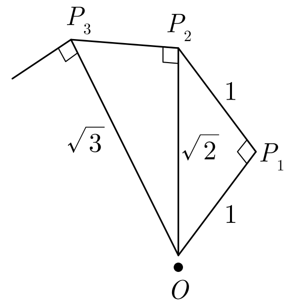
1. Show how $\sqrt{10}$ can be represented on the number line.

Ans : Proof

2. Show how $\sqrt{8}$ can be represented on the number line.

Ans : Proof

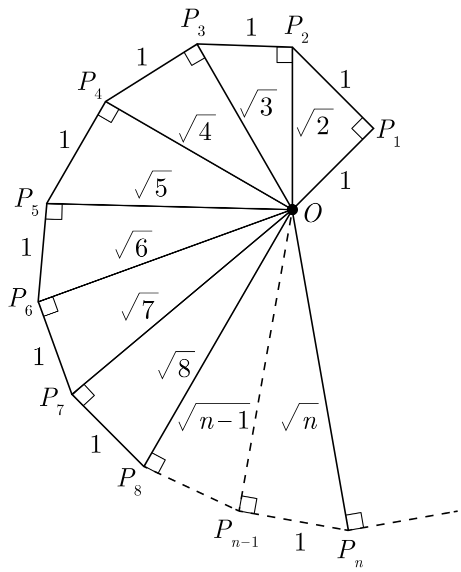
4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length (see Figure).



Constructing Square Root Spiral

Now, draw a line segment P_2P_3 perpendicular to OP_2 of unit length. Then, draw a line segment P_3P_4 perpendicular to OP_3 of unit length, Continuing in this manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3, ... , P_n, ... , and joined them to create a beautiful spiral depicting $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, ...

Sol :



Square Root Spiral

EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$

(v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Sol :

(i) $\frac{36}{100}$

$$\frac{36}{100} = 0.36,\text{ terminating decimal expansion.}$$

(ii) We have $\frac{1}{11}$

	0.09090909
11	100
	99
	100
	99
	100
	99
	100
	99
	1

$\frac{1}{11} = 0.090909..........$

$= 0.\overline{09}$,

non-terminating repeating decimal expansion.

(iii) $4\frac{1}{8}$

$$4\frac{1}{8} = \frac{33}{8}$$

	4.125
8	33
	32
	10
	8
	20
	16
	40
	40
	×

$4\frac{1}{8} = \frac{33}{8} = 4.125$, terminating decimal expansion.

(iv) $\frac{3}{13}$

	0.230769230769
13	30
	26
	40
	39
	100
	91
	90
	78
	120
	117
	30
	26
	40
	39
	100
	91
	90
	78
	120
	117
	3

$\frac{3}{13} = 0.230769230769..........$

$= 0.\overline{230769}$, non-terminating repeating decimal expansion.

(v) $\frac{2}{11}$

	0.1818
11	20
	11
	90
	88
	20
	11
	90
	88
	2

$\frac{2}{11} = 0.181818.........$

$= 0.\overline{18}$, non-terminating repeating decimal expansion.

(vi) $\frac{329}{400}$

	0.8225
400	3290
	3200
	900
	800
	1000
	800
	2000
	2000
	×

$\frac{329}{400} = 0.8225$, terminating decimal expansion.

PRACTICE :

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{1}{13}$ (ii) $5\frac{2}{3}$

(iii) $14\frac{3}{10}$ (iv) $\frac{476}{3}$

(v) $\frac{15752}{25}$ (vi) $\frac{11}{17}$

(vii) $\frac{2}{7}$ (viii) $\frac{62608}{32}$

Ans : (i) $0.\overline{076923}$ non-terminating repeating
(ii) $5.\overline{6}$ non-terminating repeating
(iii) 14.3 terminating
(iv) $158.\overline{6}$ non-terminating repeating
(v) 630.08 terminating
(vi) $0.\overline{6470588235294117}$ non-terminating repeating
(vii) $0.\overline{285714}$ non-terminating repeating
(viii) 1956.5 terminating

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are without actually doing the long division? If so, how?
[**Hint :** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Sol :

Yes. All the above will have repeating decimals which are permutations of 1, 4, 2, 8, 5, 7. For example, here is $\frac{1}{7}$

	0.1428571...
7	10
	7
	30
	28
	20 ←
	14
	60
	56
	40
	35
	50
	49
	10
	7
	30

To find $\frac{2}{7}$, locate when the remainder becomes 2 and the respective quotient (here it is 2) then, write the new quotient beginning from there (the arros drawn in the figure above using the repeating digits, 1, 4, 2, 8, 5, 7. So $\frac{2}{7} = 0.\overline{285714}$.

Alternative

Yes, we can predict the required decimal expansions.

We are given, $\frac{1}{7} = 0.\overline{142857}$

On dividing 1 by 7, we find that the remainders repeat after six divisions, therefore, the quotient has a repeating block of six digits in the decimal expansion of $\frac{1}{7}$. So, to obtain decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$; we multiply 142857 by 2, 3, 4, 5 and 6 respectively, to get the integer part and in the decimal part, we take block of six repeating digits in each case. Hence, we get

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

and $$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

PRACTICE :

1. You know that $\frac{1}{13} = 0.\overline{076923}$. Can you predict what the decimal expansions of $\frac{2}{13}, \frac{3}{13}, \frac{4}{13}, \frac{5}{13}, \frac{6}{13}$ are without actually doing the long division? If so, how?

Ans : Do yourself

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

Sol :

(i) $0.\overline{6}$

Let $x = 0.\overline{6}$

or $x = 0.666..........$ ...(1)

Multiplying eq (1) by 10 we have

$10x = 6.666..........$ ...(2)

Subtracting eq (1) from (2) we have

$9x = 6$

Hence, $x = \frac{2}{3}$

(ii) $0.4\overline{7}$

Let $x = 0.4\overline{7}$

or $x = 0.4777..........$ ...(1)

Multiplying eq (1) by 10 we have

$10x = 4.777..........$ ...(2)

Again multiplying eq (2) by 10, we get

$100x = 47.777..........$ ...(3)

Subtracting equation (2) from equation (3), we get

$90x = 43$

Hence, $x = \frac{43}{90}$

(iii) $0.\overline{001}$

Let $x = 0.\overline{001}$

or $x = 0.001001..........$ ...(1)

Multiplying eq (1) by 1000 we have

$1000x = 1.001001..........$ ...(2)

Subtracting (1) from (2) we have

$999x = 1$

Hence, $x = \frac{1}{999}$

PRACTICE :

1. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) $0.\overline{3}$ (ii) $0.2\overline{7}$
(iii) $0.2\overline{35}$ (iv) $0.\overline{237}$
(v) $0.\overline{132}$ (vi) $1.\overline{27}$
(vii) $7.\overline{478}$ (viii) $15.7\overline{12}$

Ans : (i) $\frac{1}{3}$ (ii) $\frac{25}{9}$ (iii) $\frac{233}{990}$ (iv) $\frac{47}{198}$ (v) $\frac{131}{990}$ (vi) $\frac{14}{11}$ (vii) $\frac{7471}{999}$ (viii) $\frac{5185}{330}$

4. Express 0.99999 .... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates, discuss why the answer makes sense.

Sol :

Let $x = 0.99999..........$ ...(1)

Multiplying equation (1) by 10 we have

$10x = 9.99999..........$ ...(2)

Subtracting equation (1) from (2) we have

$9x = 9$ Hence, $x = 1$

Yes, we are surprised by our answer. This answer makes sense as $0.\overline{9}$ is much close to 1, i.e. we can make the difference between 1 and 0.9999.....as small as we wish by taking enough 9's

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol :

```
      0.0588235294117647...
17 ) 1.0000000000000000
     00
     100  <-
      85
      150
      136
       140
       136
         40
         34
         60
         51
         90
         85
         50
         34
         160
         153
          70
          68
          20
          17
          30
          17
          130
          119
          110
          102
           80
           68
          120
          119
            1 <- Repeating
```

Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$

Hence, the required number of digits in the repeating block is 16.

6. Look at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Sol :

To represent rational number $\frac{p}{q}$ $(q \neq 0)$ in terminating decimal form it is necessary to choose denominator q such that its prime factorization must has only powers of 2 or powers of 5.
For example
(i) $\frac{7}{16}$ is a terminating decimal because $16 = 2^4$
(ii) $\frac{11}{25}$ is a terminating decimal because $25 = 5^2$

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Sol :

As we know that irrational numbers in the decimal expansion are always non-terminating and non-recurring.
Therefore,

$$\sqrt{3} = 1.73205080756 ....$$

$$\frac{1}{\sqrt{5}} = 0.44721359549 ....$$

$$\sqrt{10} = 3.16227766016 ....$$

Student may have their own answers.
For example

0.01001000100001 ....

0.202002000200002 ....

0.003000300003 ....

PRACTICE :
1. Write three numbers whose decimal expansions are non-terminating non-recurring.
Ans : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{7}}$

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol :

$\frac{5}{7}$ in decimal representation is as follows :

```
     0.714285..........
   ----------------
 7 | 5.0      ——→ A
   | 49
   |----
   |  10
   |   7
   |----
   |   30
   |   28
   |----
   |    20
   |    14
   |----
   |     60
   |     56
   |----
   |      40
   |      35
   |----
   |       5 ——→ B
```

Remainder at stage B is same as that of remainder of stage A.

Hence $\frac{5}{7} = 0.\overline{714285}$

Now $\frac{9}{11}$ in decimal representation is as follows:

```
      0.81
    ---------
 11 | 9.0  ——→ C
    |  88
    |-----
    |  20
    |  11
    |-----
    |   9 ——→ D
```

Remainder at stage D is same as that of remainder at stage C.

Hence $\frac{9}{11} = 0.\overline{81}$

Now we can have infinite many irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$.
Any three of these are :

0.75075007500075000075..........,

0.767076700767000..........

and 0.80800800080000..........

9. Classify the following numbers as rational or irrational:
(i) $\sqrt{23}$ (ii) $\sqrt{225}$
(iii) 0.3796 (iv) 7.478478 ....
(v) 1.101001000100001 ....

Sol :

(i) $\sqrt{23}$ is irrational number because 23 is prime number and prime number is not a perfect square root.
(ii) $\sqrt{225}$ is rational number because $\sqrt{225} = 15$, rational number.
(iii) 0.3796, terminating decimal, so rational number.
(iv) 7.478478 .... $= 7.\overline{478}$, non-terminating repeating (recurring), so rational number.
(v) 1.101001000100001 .... non-terminating non-repeating, so irrational number.

PRACTICE :

1. Classify the following numbers as rational or irrational:

(i) $\sqrt{17}$ (ii) $\sqrt{625}$

(iii) $\sqrt{29}$ (v) $\sqrt{529}$

(vi) 0.514 (vii) 0.8349

(vii) 2.4563563563....... (viii) 4.34343434.....

(ix) 2.202002000200002 ....

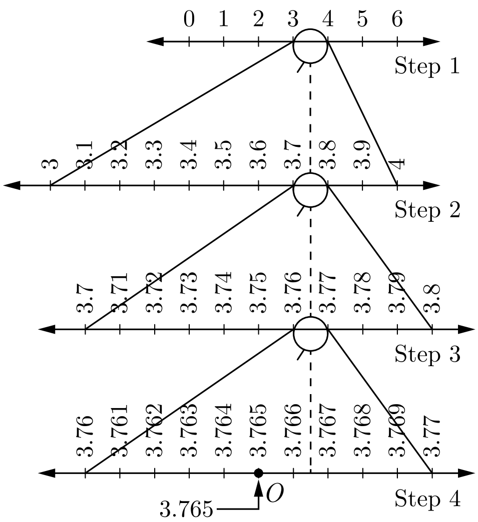
Ans : (i) Irrational (ii) Rational
(iii) Irrational (iv) Rational
(v) Rational (vi) Rational
(vii) Rational (viii) Rational
(xi) Irrational

EXERCISE 1.4

1. Visualise 3.765 on the number line, using successive magnification.

Sol :

1. We notice the number 3.7 lies between 3 and 4. So, first we locate numbers 3 and 4 on number line and divide the portion into ten equal parts and locate 3.7 and 3.8 [Refer step 2]
2. Further 3.76 lies between 3.7 and 3.8. So, we magnify 3.7 and 3.8 and divide the portion into ten equal parts and locate 3.76 and 3.77. [Refer step 3]
3. Further 3.765 lies between 3.76 and 3.77. So, we magnify 3.76 and 3.77 and divide the portion into ten equal parts and locate 3.765. [Refer step 4]



Point O in step 4 represents the number 3.765 on the number line.

PRACTICE :

1. Visualise 2.458 on the number line, using successive magnification.

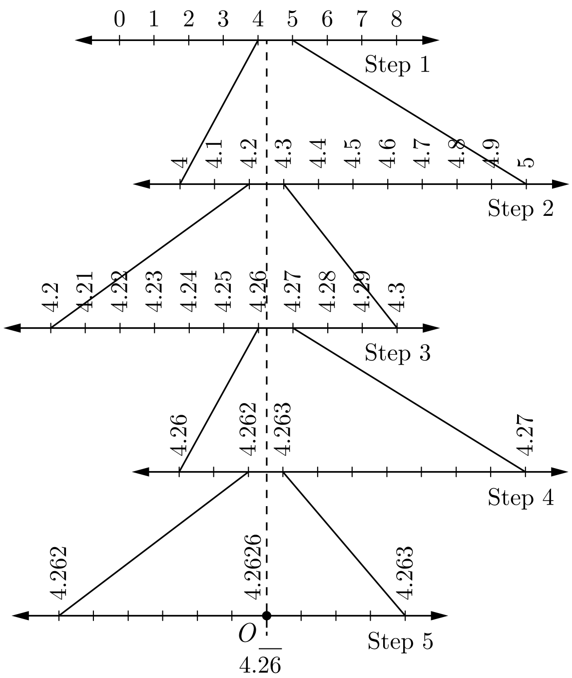
Ans : Do it yourself

2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Sol :

We have $4.\overline{26} = 4.2626262626$ ....

1. Visualise 4 and 5 as $4.\overline{26}$ lies between 4 and 5 and divide portion in ten equal parts and locate 4.2. [Refer step 2]
2. Visualise 4.2 and 4.3 as 4.26 lies between 4.2 and 4.3 and divide portion in ten equal parts and locate 4.26. [Refer step 3]
3. Visualise 4.26 and 4.27 as 4.262 lies between 4.26 and 4.27 and divide portion in ten equal parts and locate 4.262. [Refer step 4]
4. Visualise 4.262 and 4.263 as 4.2626 lies between 4.262 and 4.263 and divide portion in ten equal parts and locate 4.2626. [Refer step 5]



Point O in step 5 represents the number $4.\overline{26}$ on the number line.

PRACTICE :

1. Visualise $3.\overline{52}$ on the number line, up to 4 decimal places.

Ans : Do it yourself

EXERCISE 1.5

1. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Sol :

(i) $2-\sqrt{5}$ is an irrational number, as difference of a rational and an irrational number is irrational.

(ii) $(3+\sqrt{23})-\sqrt{23} = 3+\sqrt{23}-\sqrt{23} = 3$,

8 is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$, is a rational number.

(iv) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ is an irrational number, as divisors of an irrational number by a non-zero rational number is irrational.

(v) 2π, irrational number, as π is an irrational number and multiplication of a rational and an irrational number is irrational.

PRACTICE :

1. Classify the following numbers as rational or irrational:

(i)$(5+\sqrt{14})-\sqrt{14}$ (ii) $5+\sqrt{3}$

(iii) $(5+\sqrt{17})+\sqrt{17}$ (iv) $2-\sqrt{5}$

(v) $\frac{3\sqrt{5}}{8\sqrt{5}}$ (vi) $\frac{7\sqrt{3}}{8\sqrt{5}}$

(vii) $\frac{1}{\sqrt{6}}$ (viii) $\frac{\sqrt{5}}{2}$

(ix) 5π (x) $2+3\pi$

Ans : (i) Rational (ii) Irrational (iii) Irrational (iv) Irrational (v) Rational (vi) Irrational (vii) Irrational (viii) Irrational (ix) Irrational (x) Irrational

2. Simplify each of the following expressions :

(i) $(3+\sqrt{3})(2+\sqrt{2})$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol :

(i) $(3+\sqrt{3})(2+\sqrt{2}) = 3\times 2+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$

$= 9-3 = 6$

(iii) $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+2\cdot\sqrt{5}\cdot\sqrt{2}+(\sqrt{2})^2$

$= 5+2\sqrt{10}+2$

$= 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2$

$= 5-2 = 3$

PRACTICE :

1. Simplify each of the following expressions :

(i) $(7+\sqrt{3})(4+\sqrt{5})$

(ii) $(2+\sqrt{2})(2-\sqrt{2})$

(iii) $(\sqrt{3}-\sqrt{11})^2$

(iv) $(\sqrt{13}-\sqrt{7})(\sqrt{13}+\sqrt{7})$

(v) $(\sqrt{18}+\sqrt{9})(\sqrt{18}-\sqrt{9})$

(vi) $(7+\sqrt{2})(7-\sqrt{2})$

(vii) $(4+\sqrt{6})(3+\sqrt{6})$

Ans : (i) $28+4\sqrt{3}+7\sqrt{5}+\sqrt{15}$ (ii) 2 (iii) $14+2\sqrt{33}$ (iv) 6 (v) 9 (vi) 47 (vii) $18+7\sqrt{6}$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

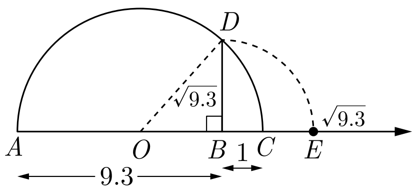
Sol :

On measuring c with any device, we get only approximate measurement. Therefore, π is an irrational.

4. Represent $\sqrt{9.3}$ on the number line.

Sol :

Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B such that $AB = 9.3$ units. From B mark a distane of 1 unit and call the new point as C. Find the mid-point of AC and call that point as O. Draw a semi-circle with centre O and radius $OC = 5.15$ units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D. Then $BD = \sqrt{9.3}$.



Mathematically Justification

$$OA = OC = OD = 5.15$$

[radius of semi-circle]

$$OB = AB - OA$$

$$= 9.3 - 5.15$$

$$OB = 4.15$$

In rt. $\angle d$ ΔOBD;

$$OB^2 + BD^2 = OD^2$$

[Using Pythagoras theorem]

$$(4.15)^2 + BD^2 = (5.15)^2$$

$$\begin{aligned} BD^2 &= (5.15)^2 - (4.15)^2 \\ &= (5.15 + 4.15)(5.15 - 4.15) \\ &\quad [\text{Using } a^2 - b^2 = (a+b)(a-b)] \\ &= (9.3)(1) \\ BD^2 &= 9.3 \\ BD &= \sqrt{9.3} \end{aligned}$$

If we want to know the position of $\sqrt{9.3}$ on the number line then we treat line BC as number line with B as zero, C as 1 and so on. Draw an arc with B as centre and radius BD which intersects the number line in E. Then E represents $\sqrt{9.3}$.

PRACTICE :

1. Represent $\sqrt{6.3}$ on the number line.

Ans : Do Yourself

2. Represent $\sqrt{7.5}$ on the number line.

Ans : Do Yourself

5. Rationalise the denominators of the following :

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Sol :

(i) $\frac{1}{\sqrt{7}}$

Multiplying the numerator and denominator by $\sqrt{7}$ we have

$$\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Multiplying the numerator and denominator by $(\sqrt{7}+\sqrt{6})$ we have

$$\begin{aligned} \frac{1}{\sqrt{7}-\sqrt{6}} &= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} \\ &= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6} \end{aligned}$$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Multiplying the numerator and denominator by $(\sqrt{5}-\sqrt{2})$ we have

$$\begin{aligned} \frac{1}{\sqrt{5}+\sqrt{2}} &= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} \\ &= \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3} \end{aligned}$$

(iv) $\frac{1}{\sqrt{7}-2}$

Multiplying the numerator and denominator by $(\sqrt{7}+2)$ we have

$$\begin{aligned} \frac{1}{\sqrt{7}-2} &= \frac{\sqrt{7}+2}{(\sqrt{7}-2)(\sqrt{7}+2)} \\ &= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3} \end{aligned}$$

PRACTICE :

1. Rationalise the denominators of the following :

(i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{2}{\sqrt{8}}$

(iii) $\frac{1}{\sqrt{6}-\sqrt{5}}$ (iv) $\frac{1}{\sqrt{6}+\sqrt{5}}$

(v) $\frac{1}{2-\sqrt{3}}$ (iv) $\frac{1}{\sqrt{7}+2}$

Ans : (i) $\frac{\sqrt{5}}{5}$ (ii) $\frac{\sqrt{8}}{4}$ (iii) $\sqrt{6}+\sqrt{5}$ (iv) $\sqrt{6}-\sqrt{5}$ (v) $2+\sqrt{3}$ (vi) $\frac{\sqrt{7}-2}{11}$

EXERCISE 1.6

1. Find :

(i) $64^{1/2}$ (ii) $32^{1/5}3$ (iii) $125^{1/3}$

Sol :

(i) $64^{1/2} = (8^2)^{1/2} = (8)^{2\times 1/2} = 8$

(ii) $32^{1/5} = (2^5)^{1/5} = (2)^{5\times 1/5} = 2$

(iii) $125^{1/3} = (5^3)^{1/3} = (5)^{3\times 1/3} = 5$

PRACTICE :

1. Find :

(i) $81^{1/2}$ (ii) $243^{1/5}$

(iii) $343^{1/3}$ (iv) $625^{1/4}$

(v) $256^{1/8}$ (vi) $729^{1/6}$

Ans : (i) 9 (ii) 3 (iii) 7 (iv) 5 (v) 2 (vi) 3

2. Find :

(i) $9^{3/2}$ (ii) $32^{2/5}$

(iii) $16^{3/4}$ (iv) $125^{-1/3}$

Sol :

(i) $9^{3/2} = (3^2)^{3/2}$

$= (3)^{2\times 3/2} = (3)^3 = 27$

(ii) $32^{2/5} = (2^5)^{2/5}$

$= 2^{5\times 2/5} = (2)^2 = 4$

(iii) $16^{3/4} = (2^4)^{3/4}$

$= (2)^{4\times 3/4} = (2)^3 = 8$

(iv) $125^{-1/3} = (5^3)^{-1/3}$

$= (5)^{3\times(-1/3)} = (5)^{-1} = \frac{1}{5}$

PRACTICE :

1. Find :

(i) $27^{2/3}$ (ii) $49^{3/2}$

(iii) $64^{5/6}$ (iv) $16^{5/2}$

(v) $625^{3/4}$ (vi) $81^{3/4}$

(vii) $64^{-1/3}$ (viii) $64^{-2/3}$

Ans : (i) 9 (ii) 343 (iii) 32 (iv) 1024 (v) 125 (vi) 27 (vii) $\frac{1}{4}$ (viii) $\frac{1}{16}$

3. Simplify :

(i) $2^{2/3} \cdot 2^{1/5}$ (ii) $\left(\frac{1}{3^3}\right)^7$

(iii) $\frac{11^{1/2}}{11^{1/4}}$ (iv) $7^{1/2} \cdot 8^{1/2}$

Sol :

(i) $2^{2/3} \cdot 2^{1/5} = 2^{2/3+1/5} = 2^{13/15}$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1}{(3^3)^7} = \frac{1}{3^{3\times 7}} = \frac{1}{3^{21}} = 3^{-21}$

(iii) $\frac{11^{1/2}}{11^{1/4}} = 11^{1/2-1/4} = 11^{1/4}$

(iv) $7^{1/2} \cdot 8^{1/2} = (7.8)^{1/2} = 56^{1/2}$

PRACTICE :

1. Simplify :

(i) $3^{2/3} \cdot 3^{1/3}$ (ii) $2^{1/3} \cdot 2^{2/5}$

(iii) $(6^{1/3})^6$ (iv) $5^{1/2} \cdot 4^{1/2}$

(v) $(9^{1/3})^{1/2}$ (vi) $3^{3/2} \cdot 4^{3/2}$

(vii) $\frac{13^{3/4}}{13^{2/3}}$ (viii) $\frac{15^{1/2}}{15^{1/3}}$

Ans : (i) 3 (ii) $2^{7/15}$ (iii) 36 (iv) $20^{1/2}$ (v) $3^{1/3}$ (vi) $12^{3/2}$ (vii) $13^{1/12}$ (viii) $15^{1/6}$