4 Linear Equations in Two Variables

Lesson at a Glance

1. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are not both zeroes, is called a linear equation in two variables x and y.
2. A linear equation in two variables has infinitely many solutions.
3. A solution of $ax + by + c = 0$ is an ordered pair (x, y).
4. The graph of a linear equation is obtained by joining any two of its solutions.
5. The graph of a linear equation in two variables is a straight line.
6. All the ordered pairs lie on the straight line.
7. The equation of x-axis is $y = 0$ and that of y-axis is $x = 0$.
8. A line parallel to x-axis is perpendicular to y-axis and parallel to y-axis is perpendicular to x-axis.
9. The equation of any line passing through the origin is of type $y = mx$.
10. Every point on the graph of a linear equation in two variables is a solution of the linear equation.

TEXTBOOK QUESTIONS SOLVED

Exercise 4.1 *(Page – 68)*

1. *The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.*

   (Take the cost of a notebook to be ₹ x and that of a pen to be ₹ y).

Sol. Let cost of a notebook = ₹ x and cost of a pen = ₹ y.

According to given condition,

$$x = 2y \Rightarrow x - 2y = 0.$$

2. *Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case:*

(i) $2x + 3y = 9.3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$ (iii) $-2x + 3y = 6$

(iv) $x = 3y$ (v) $2x = -5y$ (vi) $3x + 2 = 0$

(vii) $y - 2 = 0$ (viii) $5 = 2x$.

Sol. (i) $2x + 3y = 9.3\overline{5} \Rightarrow 2x + 3y - 9.3\overline{5} = 0$.

Here, $a = 2, b = 3, c = -9.3\overline{5}$.

(ii) $x - \frac{y}{5} - 10 = 0$, here $a = 1, b = \frac{-1}{5}$ and $c = -10$.

(iii) $-2x + 3y = 6 \Rightarrow -2x + 3y - 6 = 0$.

Here, $a = -2, b = 3, c = -6$.

(iv) $x = 3y \Rightarrow 1x - 3y + 0 = 0$.

Here, $a = 1, b = -3, c = 0$.

(v) $2x = -5y \Rightarrow 2x + 5y + 0 = 0$.

Here, $a = 2, b = 5, c = 0$.

(vi) $3x + 2 = 0 \Rightarrow 3x + 0y + 2 = 0$.

Here, $a = 3, b = 0, c = 2$.

(vii) $y - 2 = 0 \Rightarrow 0.x + 1.y - 2 = 0$.

Here, $a = 0, b = 1, c = -2$.

(viii) $5 = 2x \Rightarrow 2x + 0.y - 5 = 0$.

Here, $a = 2, b = 0, c = -5$.

Exercise 4.2 *(Page – 70)*

1. *Which one of the following options is true, and why?*

$y = 3x + 5$ has

(i) *a unique solution,* (ii) *only two solutions,*

(iii) *infinitely many solutions.*

Sol. (iii) As each linear equation in two variables has infinitely many solutions. Further, for every x there is a corresponding value of y and vice-versa.

2. *Write four solutions for each of the following equations:*

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$.

Sol. (i) Consider equation: $2x + y = 7 \Rightarrow y = 7 - 2x$

Let $x = 0$, then $y = 7$. Solution is $x = 0, y = 7$

Let $x = 1$, then $y = 5$. Solution is $x = 1, y = 5$

Let $x = 2$, then $y = 3$. Solution is $x = 2, y = 3$

Let $x = 3$, then $y = 1$. Solution is $x = 3, y = 1$.

(ii) Consider equation: $\pi x + y = 9 \Rightarrow y = 9 - \pi x$

Let $x = 0$, then $y = 9$. Solution is $x = 0, y = 9$

Let $x = 1$, then $y = 9 - \pi$. Solution is $x = 1, y = 9 - \pi$

Let $x = 2$, then $y = 9 - 2\pi$. Solution is $x = 2, y = 9 - 2\pi$

Let $x = 3$, then $y = 9 - 3\pi$. Solution is $x = 3, y = 9 - 3\pi$.

(iii) Consider equation: $x = 4y$.

Let $y = 0$, then $x = 0$. Solution is $x = 0, y = 0$

Let $y = 1$, then $x = 4$. Solution is $x = 4, y = 1$

Let $y = -1$, then $x = -4$. Solution is $x = -4, y = -1$

Let $y = 2$, then $x = 8$. Solution is $x = 8, y = 2$.

3. *Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:*

(i) $(0, 2)$ (ii) $(2, 0)$ (iii) $(4, 0)$

(iv) $(\sqrt{2}, 4\sqrt{2})$ (v) $(1, 1)$.

Sol. Consider the equation $x - 2y = 4$ ...(A)

(i) For (0, 2), substituting $x = 0, y = 2$ in (A), we get

$$0 - 4 = 4 \Rightarrow -4 = 4, \text{ not true.}$$

Hence, (0, 2) is not a solution.

(ii) For (2, 0), substituting $x = 2, y = 0$ in (A), we get

$$2 - 0 = 4 \Rightarrow 2 = 4, \text{ not true.}$$

Hence, (2, 0) is not a solution.

(iii) For (4, 0), substituting $x = 4, y = 0$ in (A), we get

$$4 - 0 = 4 \Rightarrow 4 = 4, \text{ true.}$$

Hence, (4, 0) is a solution.

(iv) For $(\sqrt{2}, 4\sqrt{2})$, substituting $x = \sqrt{2}, y = 4\sqrt{2}$ in (A), we get

$$\sqrt{2} - 8\sqrt{2} = 4 \Rightarrow -7\sqrt{2} = 4, \text{ not true.}$$

Hence, $(\sqrt{2}, 4\sqrt{2})$ is not a solution.

(v) For (1, 1), substituting $x = 1, y = 1$ in (A), we get

$1 - 2 = 4 \Rightarrow -1 = 4$, not true.

Hence, (1, 1) is not a solution.

4. *Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.*

Sol. If $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$, then

$4 + 3 = k \Rightarrow k = 7.$

Exercise 4.3 *(Pages – 74-75)*

1. *Draw the graph of each of the following linear equations in two variables:*

(i) $x + y = 4$ (ii) $x - y = 2$ (iii) $y = 3x$

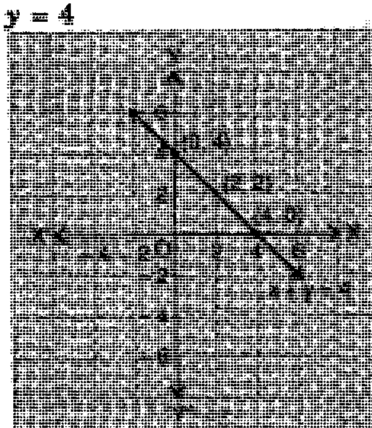
(iv) $3 = 2x + y$.

Sol. (i) Consider equation: $x + y = 4$

$\Rightarrow y = 4 - x$

Some points on graph are

x	0	2	4
y	4	2	0



Plotting the points on graph paper and joining them, we get the required graph (line) as shown in figure.

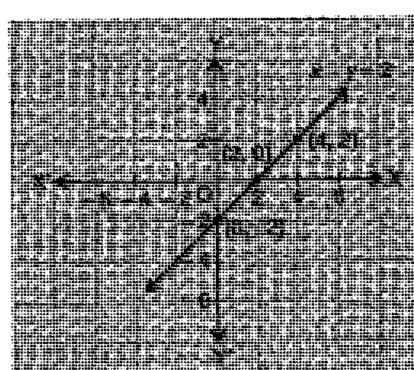
(ii) Consider equation:

$x - y = 2$

$\Rightarrow y = x - 2$

Some points on graph are

x	0	2	4
y	– 2	0	2

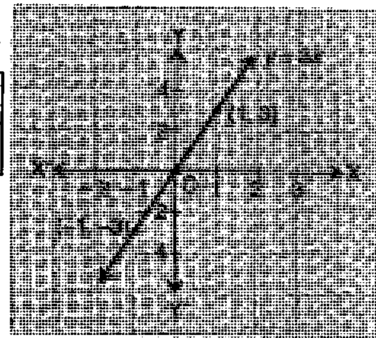


Plotting the points on graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.

(*iii*) Consider equation: $y = 3x$

Some points on graph are

x	0	1	– 1
y	0	3	– 3



Plotting the points on graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.

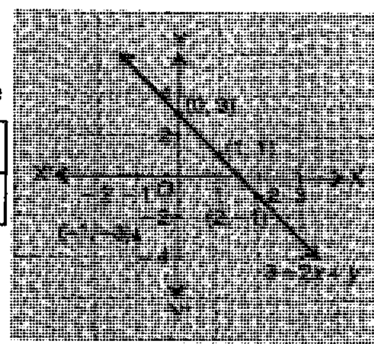
(*iv*) Consider equation:

$$3 = 2x + y$$

$$\Rightarrow \quad y = 3 - 2x$$

Some points on graph are

x	0	1	2
y	3	1	– 1



Plotting the points on graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.

2. *Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?*

Sol. Let equation of a line be $x + y + c = 0$.

If (2, 14) lies on it,

then $2 + 14 + c = 0 \quad \Rightarrow \quad c = -16$

$\therefore$ Equation of a line is $x + y - 16 = 0$.

Let equation of another line be $x - 3y + c = 0$.

If (2, 14) lies on it, then $2 - 42 + c = 0 \quad \Rightarrow \quad c = 40$.

$\therefore$ Equation of another line is $x - 3y + 40 = 0$.

There can be infinitely many lines passing through the point (2, 14), because through a point infinitely many lines can be drawn.

3. *If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.*

Sol. If point (3, 4) lies on the graph of the equation $3y = ax + 7$, then the point satisfies this equation.

$$\therefore \quad 12 = 3a + 7 \quad \Rightarrow \quad 3a = 5 \quad \Rightarrow \quad a = \frac{5}{3}.$$

4. *The taxi fare in a city is as follows: For the first kilometre, the fare is ₹ 8 and for the subsequent distance it is ₹ 5 per km. Taking the distance covered as x km and total fare as ₹ y, write a linear equation for this information, and draw its graph.*

Sol. Fare for one kilometre = ₹ 8.

Fare for subsequent kilometre = ₹ 5.

Total distance covered = x km; total fare = ₹ y.

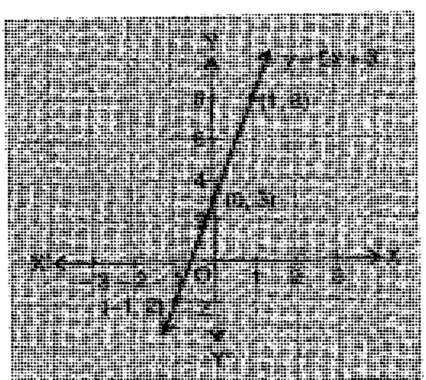
$\therefore \quad y = 8 + 5(x - 1)$

$\Rightarrow \quad y = 5x + 3$ is the required linear equation.

Some points on graph are

x	0	– 1	1
y	3	– 2	8

Plotting the points on the graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.



5. *From the choices given below, choose the equation whose graphs are given in Fig. (1) and Fig. (2).*

For Fig. (1)

(i) $y = x$

(ii) $x + y = 0$

(iii) $y = 2x$

(iv) $2 + 3y = 7x$

For Fig. (2)

(i) $y = x + 2$

(ii) $y = x - 2$

(iii) $y = -x + 2$

(iv) $x + 2y = 6$

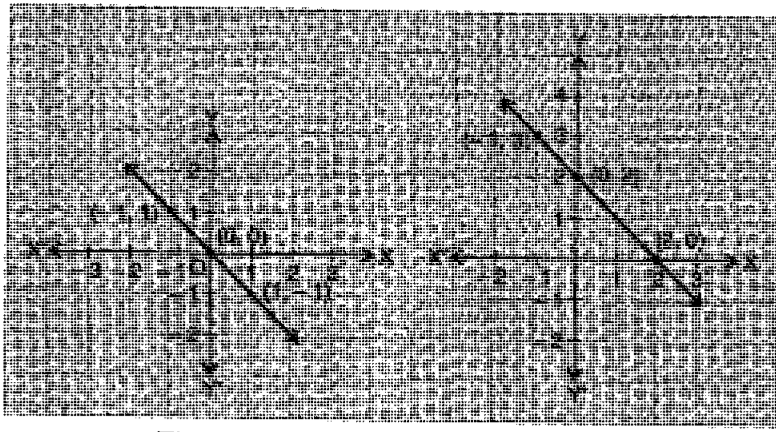


Fig. (1) Fig. (2)

Sol. (1) To choose the right equation, we test that the points on the graph satisfy which linear equation from given choices. We notice (– 1, 1), (0, 0) and (1, – 1) satisfy equation $x + y = 0$.

Hence, graph in Fig. (1) is of linear equation $x + y = 0$.

(2) To choose the right equation, we test that the points on the graph satisfy which linear equation from given choices. We notice (– 1, 3), (0, 2) and (2, 0) satisfy the equation $y = -x + 2$.

Hence, graph in Fig. (2) is of linear equation $y = -x + 2$.

6. *If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is*

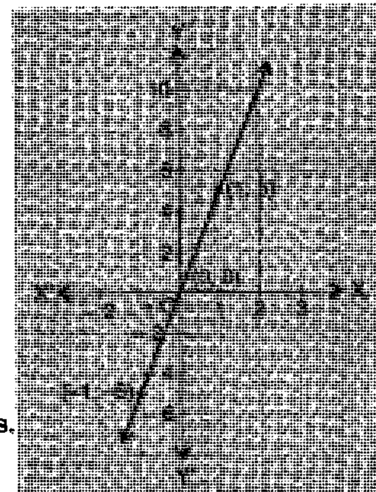
(i) 2 units *(ii) 0 unit.*

Sol. Let y units be the work done by a body on application of a constant force of 5 units and the distance travelled be x units.

Therefore, we have $y = 5x$ as the required equation.

Some points on graph are:

x	0	1	–1
y	0	5	–5



Plotting these points on the graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.

From the graph

(i) When $x = 2, y = 10$,

$\therefore$ work done = 10 units.

(ii) When $x = 0, y = 0$,

$\therefore$ work done = 0 unit.

7. *Yamini and Fatima, two students of Class IX of a school, together contributed ₹ 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹ x and ₹ y). Draw the graph of the same.*

Sol. Let Yamini contributed ₹ x and Fatima contributed ₹ y.

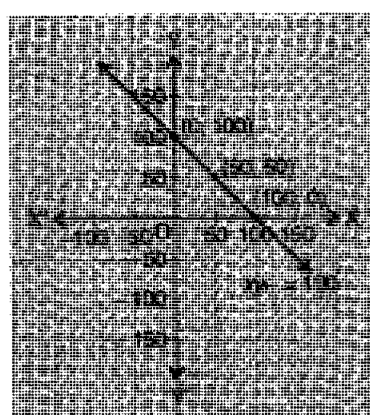
According to the given condition,

$$x + y = 100$$

This is the required linear equation.

Some points on graph are:

x	0	100	50
y	100	0	50



Plotting these points on the graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.

8. *In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it* *is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:*

$$F = \left(\frac{9}{5}\right)C + 32$$

(*i*) *Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.*

(*ii*) *If the temperature is 30°C, what is the temperature in Fahrenheit?*

(*iii*) *If the temerature is 95°F, what is the temperature in Celsius?*

(*iv*) *If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?*

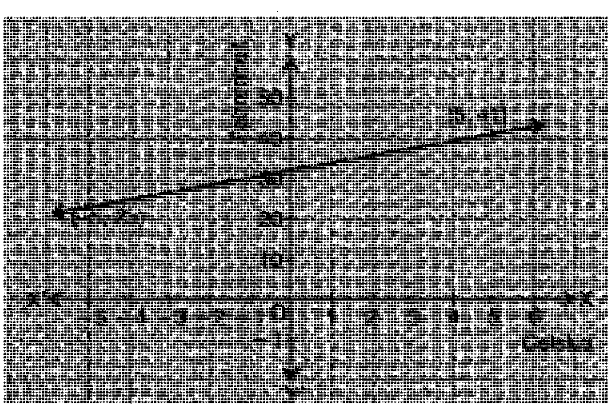
(*v*) *Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.*

Sol. Given equation is $F = \frac{9}{5}C + 32$

(*i*) Some points on graph are

x	C	0	5	– 5
y	F	32	41	23

Plotting these points on the graph paper and joining them, we get the required line as shown below.



(*ii*) When temperature is 30°C, *i.e.*, C = 30

Substituting this value of C in the given equation, we get

$$F = \frac{9}{5} \times 30 + 32 = 54 + 32 = 86$$

Hence, the required temperature is 86°F.

(*iii*) When temperature is 95°F, *i.e.*, F = 95

Substituting this value of F in the given equation, we get

$$95 = \frac{9}{5}C + 32 \quad \Rightarrow \quad \frac{9}{5}C = 95 - 32 = 63$$

$$\Rightarrow \quad C = \frac{63 \times 5}{9} = 35$$

Hence, the required temperature is 35°C.

(*iv*) When C = 0, $F = \frac{9}{5} \times 0 + 32 = 32$

When F = 0, $0 = \frac{9}{5}C + 32$

$$\Rightarrow \quad C = -\frac{32 \times 5}{9} = -17.8 \text{ (approx.)}.$$

Hence, the required temperatures are 32°F and – 17.8°C (approx.).

(*v*) When F = C numerically, from given equation, we get

$$F = \frac{9}{5}F + 32 \quad \Rightarrow \quad F - \frac{9}{5}F = 32$$

$$\Rightarrow \quad -\frac{4}{5}F = 32 \quad \Rightarrow \quad F = -40$$

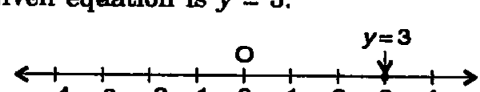
Hence, there is a temperature which is numerically the same in both Fahrenheit and Celsius, *i.e.*, – 40°F = – 40°C.

Exercise 4.4 *(Page – 77)*

1. *Give the geometric representations of y = 3 as an equation:*

(*i*) *in one variable* (*ii*) *in two variables.*

Sol. (*i*) Given equation is $y = 3$.



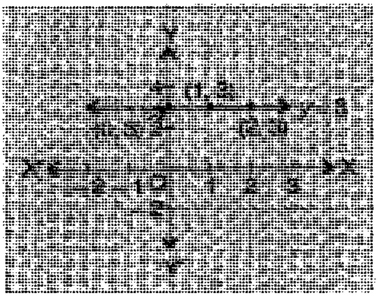
(ii) $y = 3$ in two variables is

$$0.x + 1.y - 3 = 0.$$

$\Rightarrow$ y is 3 and x can take any value.

Therefore, some points on graph are (0, 3), (1, 3), (2, 3), etc.

Plotting these points on the graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.

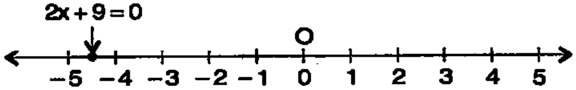


2. *Give the geometric representation of $2x + 9 = 0$ as an equation:*

(i) *in one variable* (ii) *in two variables.*

Sol. (i) Given equation is $2x + 9 = 0$

$$\Rightarrow \qquad x = -\frac{9}{2}.$$

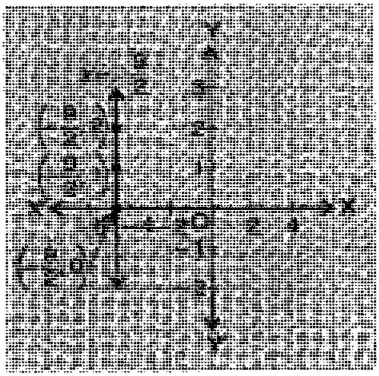


(ii) $2x + 9 = 0$ in two variables is $2x + 0y + 9 = 0$.

$\Rightarrow$ $x = -\dfrac{9}{2}$ and y can take any value.

Therefore, some points on graph are

$\left(-\dfrac{9}{2}, 0\right)$, $\left(-\dfrac{9}{2}, 1\right)$, $\left(-\dfrac{9}{2}, 2\right)$, etc.



Plotting these points on the graph paper and joining them, we get the required graph (line) as shown in the adjoining figure.