6 Lines and Angles

Lesson at a Glance

1. All the collinear points lie on the same line.
2. The inclination of two rays originating from the same end point is called the angle.
3. The rays making an angle are called the arms of the angle and the end point is called the vertex.
4. Types of angles: Acute angle, right angle, obtuse angle, straight angle, reflex angle etc.
5. An acute angle is greater than $0°$ but less than $90°$.
6. A right angle is exactly equal to $90°$.
7. An obtuse angle is greater than $90°$ but less than $180°$.
8. A straight angle is exactly equal to $180°$.
9. A reflex angle is greater than $180°$ but less than $360°$.
10. The two angles whose sum is one right angle, *i.e.*, $90°$, are called complementary angles.
11. The complementary angle of an angle θ is $90° - \theta$.
12. The two angles whose sum is two right angles, *i.e.*, $180°$, are called supplementary angles.
13. The supplementary angle of an angle θ is $180° - \theta$.
14. If the sum of two adjacent angles is $180°$, then they form a linear pair of angles.
15. If two lines intersect each other, then two pairs of vertically opposite angles are formed.
16. Vertically opposite angles are equal.
17. If a transversal intersects two parallel lines, then
    (*i*) each pair of alternate interior angles is equal.
    (*ii*) each pair of vertically opposite angles is equal.
    (*iii*) each pair of alternate exterior angles is equal.
    (*iv*) each pair of corresponding angles is equal.

(*v*) sum of each pair of interior angles on the same side of the transversal is 180°.

18. Lines which are parallel to the same line are parallel to each other.

19. Lines which are perpendicular to the same line are parallel to each other.

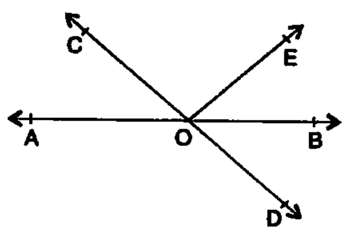
20. The sum of the angles of a triangle is 180°. This property is called *Angle Sum Property of a Triangle.*

21. An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

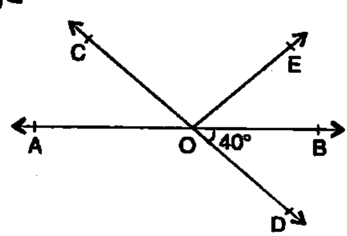
TEXTBOOK QUESTIONS SOLVED

Exercise 6.1 *(Pages – 96-97)*

1. *In figure given below, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.*



Sol. Ray OE stands on line AB.



$\therefore \quad \angle AOE + \angle EOB = 180°$ [Linear pair]

$\Rightarrow \quad (\angle AOC + \angle COE) + \angle EOB = 180°$

$\Rightarrow \quad (\angle AOC + \angle EOB) + \angle COE = 180°$ ...(*i*)

$\Rightarrow \quad 70° + \angle COE = 180°$

[$\because$ $\angle AOC + \angle BOE = 70°$ (given)]

$\Rightarrow \quad \angle COE = 110°$ ...(*ii*)

$\therefore$ Reflex $\angle COE = 360° - \angle COE = 360° - 110° = 250°$.

Also, $\angle AOC = \angle BOD = 40°$ ...(*iii*)

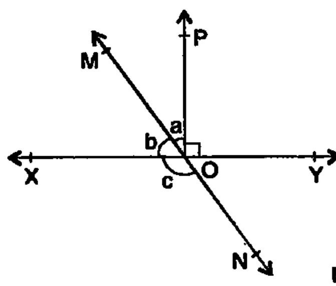
[Vertically opposite angles]

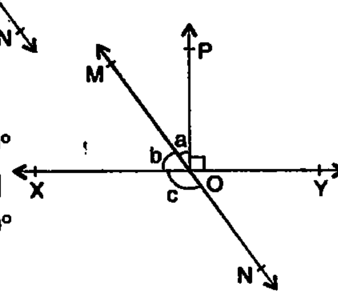
From (*i*), (*ii*), (*iii*), we get

$$40° + 110° + \angle BOE = 180°$$

$$\Rightarrow \quad \angle BOE = 180° - 150° = 30°.$$

2. *In figure given below, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.*





Sol. Ray OP stands on line XY.

$\therefore \quad \angle XOP + \angle POY = 180°$ [Linear pair]

$\Rightarrow \quad \angle XOP + 90° = 180°$

$\Rightarrow \quad \angle XOP = 90°$

$\Rightarrow \quad \angle XOM + \angle MOP + = 90°$

$\Rightarrow \quad b + a = 90°$ ...(*i*)

Also, $a : b = 2 : 3 \;\Rightarrow\; \frac{a}{b} = \frac{2}{3} \;\Rightarrow\; a = \frac{2b}{3}$ ...(*ii*)

$\Rightarrow \quad b + \frac{2b}{3} = 90° \;\Rightarrow\; \frac{5b}{3} = 90°$ [From (*i*), (*ii*)]

$\Rightarrow \quad b = 54°$ ...(*iii*)

From (*i*), we get

$54° + a = 90° \;\Rightarrow\; a = 36°$

$\angle NOY = \angle XOM$ [Vertically opposite angles]

$\Rightarrow \quad \angle NOY = b = 54°$ ...(*iv*) [From (*iii*)]

Ray NO stands on line XY.

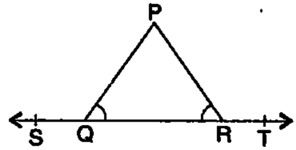
$\therefore \quad \angle XON + \angle NOY = 180°$ [Linear pair]

$\Rightarrow \quad c + 54° = 180°$ [From (*iv*)]

$\Rightarrow \quad c = 180° - 54° = 126°$

$\therefore \quad a = 36°, b = 54°, c = 126°.$

3. *In the given figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.*



Sol. ∠PQR = ∠PRQ ...(*i*) [Given]

Line segment PQ stands on ST.

∴ ∠PQS + ∠PQR = 180° ...(*ii*) [Linear pair]

Line segment PR stands on ST.

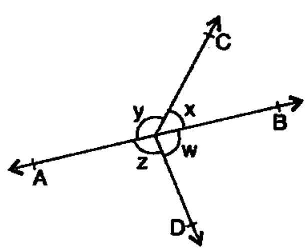
∴ ∠PRQ + ∠PRT = 180° ...(*iii*) [Linear pair]

From (*ii*) and (*iii*), we get

∠PQS + ∠PQR = ∠PRQ + ∠PRT

⇒ ∠PQS = ∠PRT. [Using (*i*)]

4. *In figure given below, if $x + y = w + z$, then prove that AOB is a line.*



Sol. We have $x + y + z + w = 360°$ ...(*i*)

Also $x + y = z + w$ ...(*ii*) [Given]

∴ $(x + y) + (x + y) = 360°$ [From (*i*), (*ii*)]

⇒ $2(x + y) = 360°$

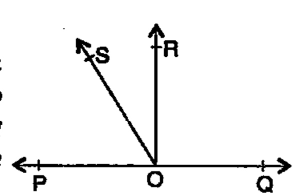
⇒ $x + y = 180°$ ...(*iii*)

As ray CO stands on line AB, such that

$x + y = 180°$ [From (*iii*)]

Hence, AOB is a straight line.

5. *In the adjoining figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that*



$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Sol. Ray OR is perpendicular to line PQ.

$\therefore$ $\angle POR = \angle QOR$ ...(*i*)

and $\angle POR + \angle QOR = 180°$ [Linear pair]

$\Rightarrow$ $2\angle POR = 180°$ ...(*ii*) [Using (*i*)]

$\because$ Also ray OS stands on line PQ.

$\therefore$ $\angle POS + \angle QOS = 180°$ [Linear pair]

$\Rightarrow$ $\angle POS + \angle QOS = 2\angle POR$ [From (*ii*)]

$\Rightarrow$ $\angle POS + \angle QOS = 2(\angle POS + \angle ROS)$

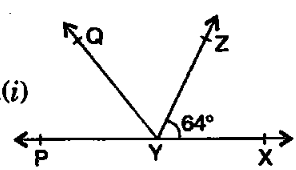
$\Rightarrow$ $\angle POS + \angle QOS = 2\angle POS + 2\angle ROS$

$\Rightarrow$ $2\angle ROS = \angle QOS - \angle POS$

$\Rightarrow$ $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

6. *It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.*

Sol. Ray YQ bisects $\angle PYZ$.



$\therefore$ $\frac{1}{2}\angle PYZ = \angle PYQ = \angle QYZ$ ...(*i*)

Ray YZ stands on line PX.

$\therefore$ $\angle PYZ + \angle ZYX = 180°$ [Linear pair]

$\Rightarrow$ $2\angle PYQ + 64° = 180°$ [From (*i*)]

$\Rightarrow$ $2\angle PYQ = 180° - 64° = 116°$

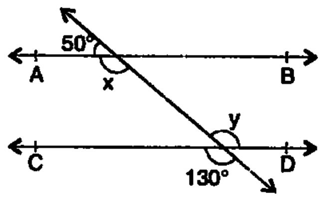
$\Rightarrow$ $\angle PYQ = 58°$ ...(*ii*)

$\therefore$ Reflex $\angle QYP = 360° - \angle PYQ = 360° - 58° = 302°$.

Also $\angle XYQ = \angle XYZ + \angle QYZ = 64° + 58° = 122°$. [From (*i*), (*ii*)]

Exercise 6.2 *(Pages – 103-104)*

1. *In figure given below, find the values of x and y and then show that AB || CD.*



Sol. $y = 130°$ [Vertically opposite angles]

Further, $50° + x = 180°$ [Linear pair]

$\Rightarrow$ $x = 130°$

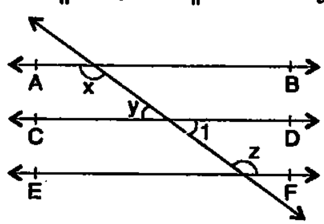
Hence, $x = y = 130°$

Transversal intersects lines AB and CD.

Such that $x = y$. [Alternate interior angles]

Hence, AB $\parallel$ CD.

2. *In figure, if AB $\parallel$ CD, CD $\parallel$ EF and $y : z = 3 : 7$, find x.*

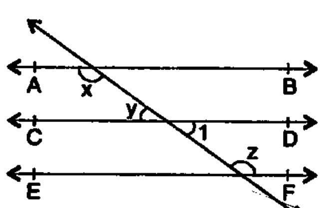


Sol. AB $\parallel$ CD and CD $\parallel$ EF ...(*i*) [Given]

$\angle 1 = y$ [Vertically opposite angles]

$\angle 1 + z = 180°$ [CD $\parallel$ EF and $\angle 1$, $\angle z$ are on the same side of the transversal]

$\Rightarrow$ $y + z = 180°$ ...(*ii*)



Given: $y : z = 3 : 7 \Rightarrow \frac{y}{z} = \frac{3}{7} \Rightarrow y = \frac{3z}{7}$

$\therefore \quad \frac{3z}{7} + z = 180° \Rightarrow \frac{10z}{7} = 180°$ [From (*ii*)]

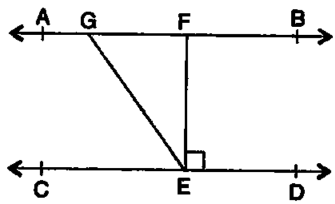
$\Rightarrow \quad z = 126°$

$\therefore \quad y = 180° - 126° = 54°$ [From (*i*)]

Now, AB || CD and transversal intersects these lines.

$\therefore \quad x + y = 180° \Rightarrow x + 54° = 180° \Rightarrow x = 180° - 54° = 126°$.

3. *In figure, AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.*



Sol. AB || CD and GE is transversal.

$\therefore \quad \angle AGE = \angle GED$ [Alternate angles]

$\Rightarrow \quad AGE = 126°$.

Further, $\angle GED = \angle GEF + \angle FED$

$\Rightarrow \quad 126° = \angle GEF + 90°$ [∵ EF ⊥ CD]

$\Rightarrow \quad \angle GEF = 126° - 90° = 36°$.

Again, AB || CD and GE is transversal.

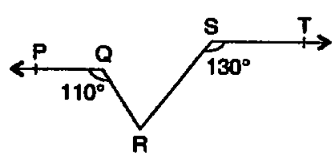
$\therefore \quad \angle FGE + \angle GED = 180°$ [Sum of interior angles on the same side of transversal is 180°.]

$\Rightarrow \quad \angle FGE + 126° = 180°$

$\Rightarrow \quad \angle FGE = 180° - 126° = 54°$.

4. *In figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.*

[**Hint:** Draw a line parallel to ST through point R.]



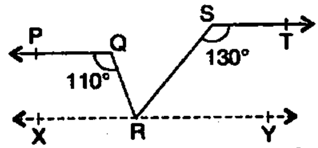
Sol. Construction: Through R draw a line XRY parallel to PQ.

Proof: PQ || XRY and QR is transversal.

$\therefore \quad \angle PQR = \angle QRY = 110°$ ...(*i*) [Alternate angles]

Also, PQ || ST [Given]

and PQ || RY [Construction]



∴ ST ∥ XRY and SR is transversal.

∴ $\angle TSR + \angle SRY = 180°$

[Sum of interior angles on the same side of transversal]

⇒ $130° + \angle SRY = 180°$

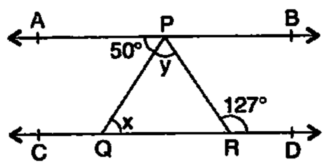
⇒ $\angle SRY = 180° - 130° = 50°$ ...(*ii*)

Also, $\angle QRY = \angle QRS + \angle SRY$

⇒ $110° = \angle QRS + 50°$ [From (*i*) and (*ii*)]

⇒ $\angle QRS = 110° - 50° = 60°$.

5. *In figure, if AB ∥ CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.*



Sol. AB ∥ CD and PQ is transversal.

∴ $\angle PQR = \angle APQ \Rightarrow x = 50°$

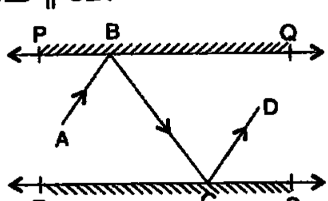
Again AB ∥ CD and PR is transversal.

∴ $\angle APR = \angle PRD$ [Alternate angles]

⇒ $\angle APQ + \angle QPR = \angle PRD$.

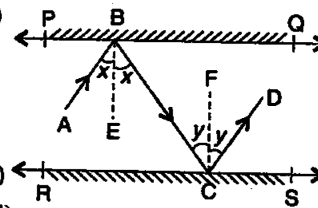
⇒ $50° + y = 127° \Rightarrow y = 127° - 50° = 77°$.

6. *In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB ∥ CD.*



Sol. Construction: Draw BE perpendicular to PQ and CF perpendicular to RS.

Proof: As BE ⊥ PQ and CF ⊥ RS and PQ || RS.



⇒ BE || CF ...(*i*)

Also, we know

Angle of incidence = angle of reflection.

i.e., $\angle ABE = \angle EBC = x$. ...(*ii*)

and $\angle BCF = \angle FCD = y$. ...(*iii*)

From (*i*), BE || CF and BC is transversal.

∴ $\angle EBC = \angle BCF$ [Alternate angles]

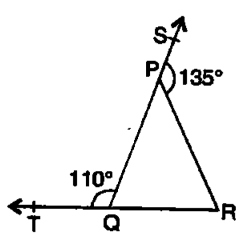
⇒ $x = y \Rightarrow 2x = 2y$

⇒ $\angle ABC = \angle BCD$ [From (*ii*) and (*iii*)]

But these are alternate angles. Hence, AB || CD.

Exercise 6.3 *(Pages – 107-108)*

1. *In figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.*



Sol. $\angle PQT = \angle QPR + \angle PRQ$

[Exterior angle of a triangle is equal to sum of interior opposite angles]

⇒ $110° = \angle QPR + \angle PRQ$ ...(*i*)

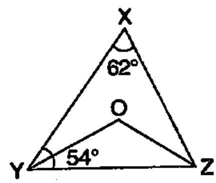
Also, $\angle SPR + \angle QPR = 180°$ [Linear pair]

⇒ $135° + \angle QPR = 180° \Rightarrow \angle QPR = 180° - 135° = 45°$.

Substituting the value of ∠ QPR in (*i*), we get

$110° = 45° + \angle PRQ \Rightarrow \angle PRQ = 110° - 45° = 65°$.

2. *In figure, X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of ΔXYZ, find ∠OZY and ∠YOZ.*



Sol. In ΔXYZ, $\angle X + \angle XYZ + \angle YZX = 180°$ [Angle sum property]

$\Rightarrow \quad 62° + 54° + \angle YZX = 180°$

$\Rightarrow \quad \angle YZX = 180° - 116° = 64°$ ...(*i*)

Also, $\angle YZX = 2\angle OZY$ [∵ OZ is bisector of ∠YZX]

$\Rightarrow \quad 2\angle OZY = 64° \Rightarrow \angle OZY = 32°$ ...(*ii*) [From (*i*)]

Also, $\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2} \times 54° = 27°$ ...(*iii*)

[OY is bisector of ∠XYZ]

In ΔOYZ, $\angle OYZ + \angle OZY + \angle YOZ = 180°$

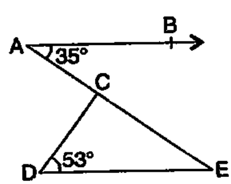
[Sum of angles of a triangle is 180°.]

$\Rightarrow \quad 27° + 32° + \angle YOZ = 180°$

$\Rightarrow \quad \angle YOZ = 180° - 59° = 121°$

Thus, $\angle OZY = 32°$ and $\angle YOZ = 121°$.

3. *In figure, if AB ∥ DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.*



Sol. AB ∥ DE and AE is transversal.

$\therefore \quad \angle DEC = \angle BAC = 35°$ ...(*i*)

Now, in triangle CDE,

$\angle CDE = 53°$ ...(*ii*) [Given]

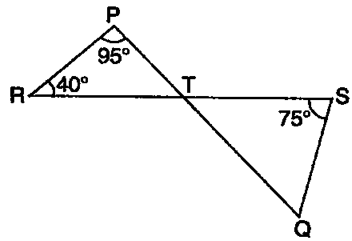
and $\angle DCE + \angle DEC + \angle CDE = 180°$

[Sum of angles of a triangle is 180°.]

$\Rightarrow \quad \angle DCE + 35° + 53° = 180°$ [From (*i*) and (*ii*)]

$\Rightarrow \quad \angle DCE = 180° - 88° = 92°$.

4. *In figure, if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, find ∠SQT.*



Sol. In ΔPRT, ∠P + ∠R + ∠PTR = 180°

[Sum of angles of a triangle is 180°.]

⇒ 95° + 40° + ∠PTR = 180°

⇒ ∠PTR = 180° – 135° = 45° ...(*i*)

Also, ∠STQ = ∠PTR

[Vertically opposite angles]

⇒ ∠STQ = 45°. ...(*ii*) [From (*i*)]

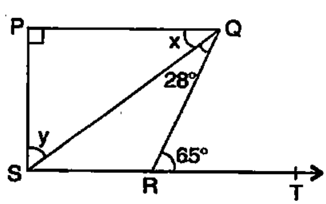
In ΔTSQ, ∠STQ + ∠S + ∠TQS = 180°

[Sum of angles of a triangle is 180°.]

⇒ 45° + 75° + ∠TQS = 180°

⇒ ∠TQS = 180° – 120° = 60°.

5. *In figure, if PQ ⊥ PS, PQ ∥ SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.*



Sol. PQ ∥ SR and QR is transversal.

∴ ∠PQR = ∠QRT [Alternate angles]

⇒ ∠PQS + ∠SQR = ∠QRT

⇒ x + 28° = 65° ⇒ x = 65° – 28° = 37° ...(*i*)

In right-angled triangle SPQ,

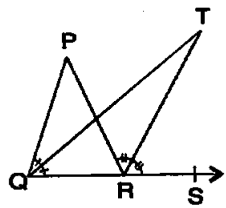
∠P + y + x = 180°

[Sum of angles of a triangle is 180°.]

$\Rightarrow \quad 90° + y + 37° = 180° \quad \Rightarrow \quad y = 180° - 127° = 53°.$

6. *In figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that*

$$\angle QTR = \frac{1}{2}\angle QPR.$$



Sol. $\angle PRT = \angle TRS = \frac{1}{2}\angle PRS$ ...(*i*)

[∵ TR is bisector of ∠PRS]

and $\angle PQT = \angle TQR = \frac{1}{2}\angle PQR$ ...(*ii*)

[∵ TQ is bisector of ∠PQR]

Also, $\angle PRS = \angle QPR + \angle PQR$ ...(*iii*)

[Exterior angle of a triangle is equal to sum of interior opposite angles.]

and $\angle TRS = \angle QTR + \angle TQR$ ...(*iv*)

[Reason same as above]

From (*i*), (*iii*) and (*iv*),

$\angle QPR + \angle PQR = 2(\angle QTR + \angle TQR)$

$\Rightarrow \angle QPR + \angle PQR = 2\angle QTR + 2\angle TQR$

$\Rightarrow \angle QPR + \angle PQR = 2\angle QTR + \angle PQR$ [From (*ii*)]

$\Rightarrow \angle QPR = 2\angle QTR$

$\Rightarrow \angle QTR = \frac{1}{2}\angle QPR.$

□□