

# 7



# Triangles

## Lesson at a Glance

1. Congruent figures are of same size and same shape.
2. When placing a figure of two congruent figures on the other one, they cover each other completely.
3. Two circles of the same radii are congruent.
4. Two squares of the same sides are congruent.
5. Two photographs of same sizes but different pictures are not congruent.
6. Types of triangles: Equilateral, isosceles, acute-angled, obtuse-angled, right-angle, scalene triangles.
7. If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent (SSS congruence rule).
8. If two sides and the included angle of one triangle are equal to the two sides and included angle of another triangle, then the two triangles are congruent (SAS congruence rule).
9. If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the two triangles are congruent (ASA congruence rule).
10. If two angles and one side (other than included side) of one triangle are equal to two angles and one side (other than included side) of another triangle, then the two triangles are congruent (AAS congruence rule).
11. In two right-angled triangles, if hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent (RHS congruence rule).
12. Sum of angles of a triangle is  $180^\circ$ . This property is called *Angle Sum Property of a Triangle*.
13. A triangle in which all the three sides are equal is called an equilateral triangle.

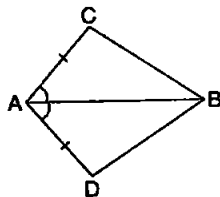
14. All the three angles of an equilateral triangle are equal, measuring  $60^\circ$  each.
15. A triangle in which two sides are equal is called an isosceles triangle.
16. Angles opposite to equal sides of a triangle are equal.
17. Sides opposite to equal angles of a triangle are equal.
18. The angle opposite to longer side of a triangle is larger.
19. The side opposite to the larger angle of a triangle is longer.
20. The sum of the two sides of a triangle is greater than the third side.
21. The difference of the two sides of a triangle is less than the third side.
22. All the three sides of a scalene triangle are of different lengths.

## TEXTBOOK QUESTIONS SOLVED

### Exercise 7.1 (Pages – 118-120)

1. In quadrilateral  $ABCD$ ,  $AC = AD$  and  $AB$  bisects  $\angle A$  (see figure). Show that  $\triangle ABC \cong \triangle ABD$ .

What can you say about  $BC$  and  $BD$ ?



- Sol.** Consider triangles  $ABC$  and  $ABD$ ,

We have  $AC = AD$

[Given]

$AB = AB$

[Common]

and  $\angle CAB = \angle DAB$

[ $\because$   $AB$  bisects  $\angle CAD$ ]

$\therefore \triangle ABC \cong \triangle ABD$

[SAS rule]

Therefore,  $BC = BD$ .

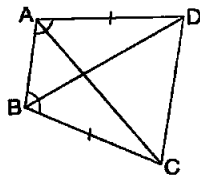
[CPCT]

2.  $ABCD$  is a quadrilateral in which  $AD = BC$  and  $\angle DAB = \angle CBA$  (see figure). Prove that

(i)  $\triangle ABD \cong \triangle BAC$

(ii)  $BD = AC$

(iii)  $\angle ABD = \angle BAC$ .



- Sol.** (i) Consider triangles  $ABD$  and  $BAC$ ,

We have  $AD = BC$  [Given]

$AB = BA$  [Common]

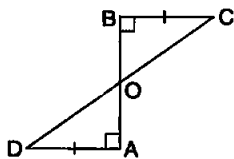
and  $\angle DAB = \angle CBA$  [Given]

$\therefore \triangle ABD \cong \triangle BAC$  [SAS rule]

(ii)  $BD = AC$ , [CPCT]

(iii)  $\angle ABD = \angle BAC$ . [CPCT]

3.  $AD$  and  $BC$  are equal perpendiculars to a line segment  $AB$  (see figure). Show that  $CD$  bisects  $AB$ .



Sol.  $AD$  and  $BC$  are perpendiculars to  $AB$ .

$\Rightarrow AD \parallel BC$  and  $CD$  is transversal.

$\therefore \angle BCD = \angle ADC$ , i.e.,  $\angle BCO = \angle ADO$  [Alternate angles]

Consider triangles  $BCO$  and  $ADO$ ,

We have  $BC = AD$  [Given]

$\angle OBC = \angle OAD$  [90° each]

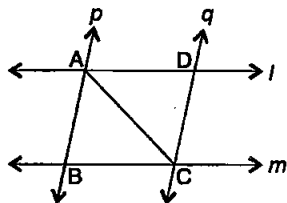
and  $\angle BCO = \angle ADO$  [Proved above]

$\therefore \triangle OBC \cong \triangle OAD$  [ASA rule]

Therefore,  $OB = OA$  [CPCT]

Hence,  $CD$  bisects  $AB$ .

4.  $l$  and  $m$  are two parallel lines intersected by another pair of parallel lines  $p$  and  $q$  (see figure). Show that  $\triangle ABC \cong \triangle CDA$ .



Sol. Since  $l \parallel m$  and  $AC$  is transversal.

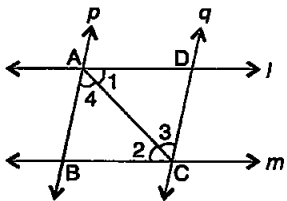
So,  $\angle 1 = \angle 2$  ... (i)

[Alternate angles]

Again,  $p \parallel q$  and  $AC$  is transversal.

So,  $\angle 3 = \angle 4$  ... (ii)

[Alternate angles]



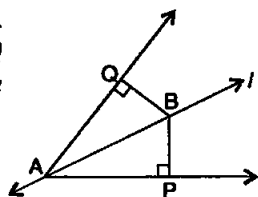
Now, in  $\triangle ADC$  and  $\triangle ABC$ ,

$AC = CA$  [Common]

$\angle 1 = \angle 2$  and  $\angle 3 = \angle 4$  [From (i) and (ii)]

$\therefore \triangle ABC \cong \triangle CDA$ . [ASA rule]

5. Line  $l$  is the bisector of an angle  $\angle A$  and  $B$  is any point on  $l$ .  $BP$  and  $BQ$  are perpendiculars from  $B$  to the arms of  $\angle A$  (see figure). Show that:



(i)  $\triangle APB \cong \triangle AQB$

(ii)  $BP = BQ$  or  $B$  is equidistant from the arms of  $\angle A$ .

**Sol.**  $\therefore$  Line  $l$  is the bisector of  $\angle QAP$ .

$\therefore \angle QAB = \angle PAB$  ... (i)

$AB = BA$  ... (ii) [Common]

and  $\angle BQA = \angle BPA$  ... (iii) [ $90^\circ$  each]

(i) In triangles  $AQB$  and  $APB$ ,

$\angle QAB = \angle PAB$ ;  $AB = BA$  and  $\angle BQA = \angle BPA$ .

[From (i), (ii), (iii)]

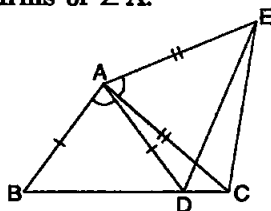
$\therefore \triangle APB \cong \triangle AQB$ .

[AAS rule]

(ii) Therefore,  $BP = BQ$  [CPCT]

i.e.,  $B$  is equidistant from the arms of  $\angle A$ .

6. In figure,  $AC = AE$ ,  $AB = AD$  and  $\angle BAD = \angle EAC$ . Show that  $BC = DE$ .



**Sol.** As  $\angle BAD = \angle CAE$  [Given]

$\Rightarrow \angle BAD + \angle DAC$

$= \angle DAC + \angle CAE$

[ $\angle DAC$  is added to both sides]

$\Rightarrow \angle BAC = \angle DAE$  ... (i)

Consider triangles  $BAC$  and  $DAE$ ,

We have  $AB = AD$  [Given]

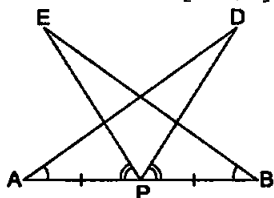
$AC = AE$  [Given]

and  $\angle BAC = \angle DAE$  [From (i)]

$\therefore \triangle BAC \cong \triangle DAE$  [SAS rule]

$\Rightarrow BC = DE$ . [CPCT]

7.  $AB$  is a line segment and  $P$  is its mid-point.  $D$  and  $E$  are points on the same side of  $AB$  such that  $\angle BAD = \angle ABE$  and  $\angle EPA = \angle DPB$  (see figure). Show that



$$(i) \triangle DAP \cong \triangle EBP$$

$$(ii) AD = BE.$$

**Sol.** Since  $\angle EPA = \angle DPB$  [Given]

$$\Rightarrow \angle EPA + \angle EPD = \angle EPD + \angle DPB$$

[Adding  $\angle EPD$  to both sides]

$$\Rightarrow \angle APD = \angle BPE \quad \dots(i)$$

Also,  $AP = BP$   $\dots(ii)$  [Given]

and  $\angle DAP = \angle EBP$   $\dots(iii)$  [Given]

(i) Consider triangles DAP and EBP,

$$\angle APD = \angle BPE, AP = BP \text{ and } \angle DAP = \angle EBP.$$

[From (i), (ii), (iii)]

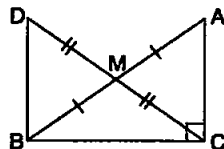
$$\triangle DAP \cong \triangle EBP.$$

[SAS rule]

$$(ii) AD = BE.$$

[CPCT]

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that  $DM = CM$ . Point D is joined to point B (see figure). Show that:



$$(i) \triangle AMC \cong \triangle BMD$$

(ii)  $\angle DBC$  is a right angle.

$$(iii) \triangle DBC \cong \triangle ACB$$

$$(iv) CM = \frac{1}{2}AB.$$

**Sol.** (i) Consider triangles AMC and DMB,

$$\text{We have } AM = BM \quad \text{[Given]}$$

$$CM = DM \quad \text{[Given]}$$

and  $\angle AMC = \angle BMD$  [Vertically opposite angles]

$$\therefore \triangle AMC \cong \triangle BMD \quad \text{[SAS rule]}$$

(ii) As  $\angle BAC = \angle DBA$  [CPCT, from part (i)]

and AB is the transversal.

So,  $DB \parallel AC$  ... (i)

We have  $AC \perp BC$  ... (ii) [Given]

So,  $DB \perp BC$  [From (i) and (ii)]

i.e.,  $\angle DBC$  is a right angle, i.e.,  $\angle DBC = 90^\circ$ .

(iii) Consider triangles  $ABC$  and  $DCB$ .

We have  $AC = DB$  [Since  $\triangle AMC \cong \triangle BMD$ ; result (i)]

$BC = CB$  [Common]

and  $\angle ACB = \angle DBC$  [Each  $90^\circ$ ]

$\therefore \triangle DBC \cong \triangle ACB$  [SAS rule]

(iv) As  $DC = AB$  [CPCT from part (iii)]

$\Rightarrow 2CM = AB$  [ $\because$  M is mid-point of DC]

$\Rightarrow CM = \frac{1}{2} AB$ .

### Exercise 7.2 (Pages – 123-124)

1. In an isosceles triangle  $ABC$ , with  $AB = AC$ , the bisectors of  $\angle B$  and  $\angle C$  intersect each other at  $O$ . Join  $A$  to  $O$ . Show that:

(i)  $OB = OC$  (ii)  $AO$  bisects  $\angle A$ .

**Sol.** (i) In  $\triangle ABC$ ,  $AB = AC$  [Given]

$\Rightarrow \angle ABC = \angle ACB$   
[Angles opposite to equal sides are equal.]

$\Rightarrow \frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$

$\Rightarrow \angle OBC = \angle OCB$

[ $\because$   $OB$  and  $OC$  are bisectors of  $\angle ABC$  and  $\angle ACB$  respectively.]

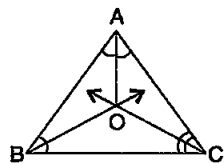
$\Rightarrow OC = OB$  ... (i)

[Sides opposite to equal angles of a triangle are equal.]

(ii) Consider triangles  $AOB$  and  $AOC$ ,

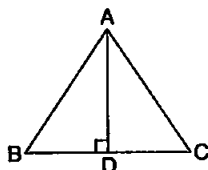
We have  $OC = OB$  [From (i)]

$AO = OA$  [Common]



and  $AB = AC$  [Given]  
 $\therefore \triangle AOB \cong \triangle AOC$  [SSS rule]  
 $\Rightarrow \angle OAB = \angle OAC$  [CPCT]  
 $\Rightarrow AO$  bisects  $\angle BAC$ .

2. In  $\triangle ABC$ ,  $AD$  is the perpendicular bisector of  $BC$  (see figure). Show that  $\triangle ABC$  is an isosceles triangle in which  $AB = AC$ .



**Sol.** Since  $AD$  is perpendicular bisector of  $BC$ ,

$\therefore BD = CD$  and  $\angle ADB = \angle ADC = 90^\circ$  ... (i)

Consider triangles  $ADB$  and  $ADC$ ,

We have  $BD = DC$

and  $\angle ADB = \angle ADC$  [From (i)]

$AD = AD$  [Common]

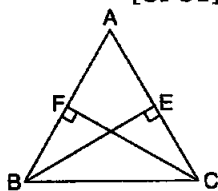
$\therefore \triangle ADB \cong \triangle ADC$  [SAS rule]

$\therefore AB = AC$ .

Therefore,  $\triangle ABC$  is an isosceles triangle with  $AB = AC$ .

[CPCT]

3.  $ABC$  is an isosceles triangle in which altitudes  $BE$  and  $CF$  are drawn to equal sides  $AC$  and  $AB$  respectively (see figure). Show that these altitudes are equal.



**Sol.** In  $\triangle ABC$ ,  $AB = AC$  [Given]

$\Rightarrow \angle ACB = \angle ABC$  ... (i)

[Angles opposite to equal sides of a triangle are equal.]

Consider triangles  $BFC$  and  $BCE$ ,

We have  $\angle FBC = \angle ECB$  [From (i)]

$BC = CB$  [Common]

$\angle BFC = \angle CEB$  [90° each]

$\therefore \triangle BCF \cong \triangle CBE$  [AAS rule]

$\Rightarrow CF = BE$ .

Hence, altitudes to the equal sides of a triangle are equal.

4.  $ABC$  is a triangle in which altitudes  $BE$  and  $CF$  to sides  $AC$  and  $AB$  are equal (see figure). Show that:

(i)  $\triangle ABE \cong \triangle ACF$

(ii)  $AB = AC$ , i.e.,  $ABC$  is an isosceles triangle.

**Sol.** (i) Consider triangles  $ABE$  and  $ACF$ ,

We have  $BE = CF$  [Given]

$\angle A = \angle A$  [Common]

$\angle AEB = \angle AFC$  [ $90^\circ$  each]

$\therefore \triangle ABE \cong \triangle ACF$  [AAS rule]

(ii) Since  $\triangle ABE \cong \triangle ACF$  [Proved above]

Hence,  $AB = AC$

i.e.,  $\triangle ABC$  is isosceles.

5.  $ABC$  and  $DBC$  are two isosceles triangles on same base  $BC$  (see figure). Show that  $\angle ABD = \angle ACD$ .

**Sol. Construction:** Join  $A$  and  $D$ .

**Proof:** Consider triangles  $ABD$  and  $ACD$ ,

We have  $AB = AC$

$BD = CD$

$AD = AD$

$\therefore \triangle ABD \cong \triangle ACD$

$\therefore \angle ABD = \angle ACD$ .

6.  $\triangle ABC$  is an isosceles triangle in which  $AB = AC$ . Side  $BA$  is produced to  $D$  such that  $AD = AB$  (see figure). Show that  $\angle BCD$  is a right angle.

**Sol.** As  $AB = AC$  [Given]

$\Rightarrow \angle 1 = \angle 2$  ... (i)

[Angles opposite to equal sides of a triangle are equal.]

and  $AC = AD$  ( $\because AB = AD$ ) [Given]

$\Rightarrow \angle 3 = \angle 4$  ... (ii)

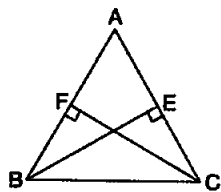
[Angles opposite to equal sides of a triangle are equal.]

Also, in  $\triangle DBC$ ,

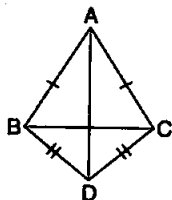
$\angle DBC + \angle BCD + \angle CDB = 180^\circ$

[Sum of angles of a triangle is  $180^\circ$ .]

$\Rightarrow \angle 1 + (\angle 2 + \angle 3) + \angle 4 = 180^\circ$



[CPCT]



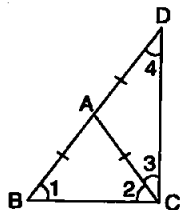
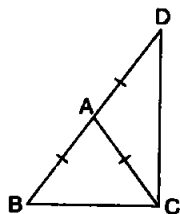
[Given]

[Given]

[Common]

[SSS rule]

[CPCT]





$$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 = 180^\circ \quad [\text{Using (i), (ii)}]$$

$$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ$$

$$\Rightarrow \angle 2 + \angle 3 = 90^\circ \Rightarrow \angle BCD = 90^\circ.$$

7.  $ABC$  is a right angled triangle in which  $\angle A = 90^\circ$  and  $AB = AC$ . Find  $\angle B$  and  $\angle C$ .

**Sol.** As  $AB = AC$

[Given]

$$\Rightarrow \angle B = \angle C \quad \dots(i) \quad [\text{Angles opposite to equal sides of a triangle are equal.}]$$

In  $\triangle ABC$ ,  $\angle A + \angle B + \angle C = 180^\circ$

[Sum of angles of a triangle is  $180^\circ$ .]

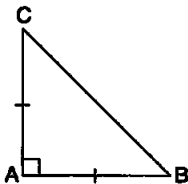
$$\Rightarrow 90^\circ + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 90^\circ$$

[From (i)]

$$\Rightarrow \angle B = 45^\circ$$

$$\therefore \angle B = \angle C = 45^\circ.$$



8. Show that the angles of an equilateral triangle are  $60^\circ$  each.

**Sol.** As  $\triangle ABC$  is equilateral.

$$\therefore AB = BC = CA$$

$$\text{Now } AB = BC \Rightarrow \angle C = \angle A \quad \dots(i)$$

[Angles opposite to equal sides of a triangle are equal.]

$$\text{Similarly, } BC = AC \Rightarrow \angle A = \angle B \quad \dots(ii)$$

[Reason same as above]

$$\Rightarrow \angle A = \angle B = \angle C \quad \dots(iii) \quad [\text{From (i) and (ii)}]$$

In  $\triangle ABC$ ,  $\angle A + \angle B + \angle C = 180^\circ$

[Sum of angles of a triangle is  $180^\circ$ .]

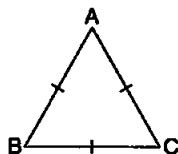
$$\Rightarrow \angle A + \angle A + \angle A = 180^\circ$$

[From (iii)]

$$\Rightarrow 3\angle A = 180^\circ$$

$$\Rightarrow \angle A = 60^\circ$$

$$\therefore \angle A = \angle B = \angle C = 60^\circ.$$



### Exercise 7.3 (Page – 128)

1.  $\triangle ABC$  and  $\triangle DBC$  are two isosceles triangles on the same base  $BC$  and vertices  $A$  and  $D$  are on the same side of  $BC$  (see

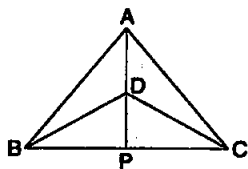


figure). If  $AD$  is extended to intersect  $BC$  at  $P$ , show that

- (i)  $\triangle ABD \cong \triangle ACD$
- (ii)  $\triangle ABP \cong \triangle ACP$
- (iii)  $AP$  bisects  $\angle A$  as well as  $\angle D$ .
- (iv)  $AP$  is the perpendicular bisector of  $BC$ .

**Sol.** (i) Consider triangles  $ABD$  and  $ACD$ ,

We have  $AB = AC$  [Given]

$BD = CD$  [Given]

$AD = DA$  [Common]

So,  $\triangle ABD \cong \triangle ACD$  [SSS rule]

$\therefore \angle BAD = \angle CAD$  and  $\angle ABD = \angle ACD$   
 ... (i) [CPCT]

(ii) Consider triangles  $ABP$  and  $ACP$ ,

We have  $AB = AC$  [Given]

$AP = PA$  [Common]

and  $\angle BAP = \angle CAP$  [From (i)]

$\therefore \triangle ABP \cong \triangle ACP$  [SAS rule]

$\Rightarrow BP = PC, \angle BPA = \angle CPA$

(iii)  $\angle BAP = \angle CAP$  [From result (ii)]

$\Rightarrow AP$  bisects  $\angle A$ .

Also,  $\angle BAD + \angle ABD = \angle CAD + \angle ACD$  [From (i)]

$\Rightarrow \angle BDP = \angle CDP$  [Exterior angle property]

So,  $DP$  bisects  $\angle D$ .

Hence,  $AP$  bisects  $\angle A$  as well as  $\angle D$ .

(iv) Also,  $\angle BPA + \angle CPA = 180^\circ$  [Linear pair]

$\Rightarrow 2\angle BPA = 180^\circ$

$\Rightarrow \angle BPA = 90^\circ$  [From result (ii)]

As  $BP = CP$  [From result (ii)]

and  $AP \perp BC$  [Proved above]

$\Rightarrow AP$  is perpendicular bisector of  $BC$ .

**2.**  $AD$  is an altitude of an isosceles triangle  $ABC$  in which  $AB = AC$ . Show that

(i)  $AD$  bisects  $BC$

(ii)  $AD$  bisects  $\angle A$ .

**Sol.** Consider triangles ABD and ACD,

We have  $AB = AC$  [Given]

$AD = AD$  [Common]

$\angle ADB = \angle ADC$  [ $90^\circ$  each]

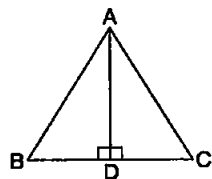
$\therefore \triangle ABD \cong \triangle ACD$  [RHS rule]

(i)  $\therefore BD = CD$  [CPCT]

$\Rightarrow AD$  bisects  $BC$ .

(ii) and  $\angle BAD = \angle CAD$  [CPCT]

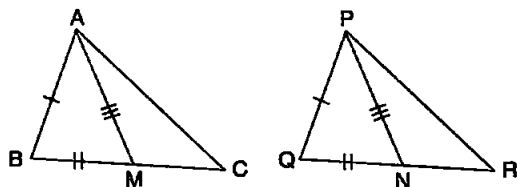
$AD$  bisects  $\angle BAC$ , i.e.,  $\angle A$ .



3. Two sides  $AB$  and  $BC$  and median  $AM$  of one triangle  $ABC$  are respectively equal to sides  $PQ$  and  $QR$  and median  $PN$  of  $\triangle PQR$  (see figure). Show that:

(i)  $\triangle ABM \cong \triangle PQN$

(ii)  $\triangle ABC \cong \triangle PQR$ .



**Sol.** (i)  $M$  and  $N$  are mid-points of  $BC$  and  $QR$  respectively, as  $AM$  and  $PN$  are medians.

$$\therefore BM = \frac{1}{2}BC \text{ and } QN = \frac{1}{2}QR \quad \dots(i)$$

Also,  $BC = QR$  [Given]

$$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR \Rightarrow BM = QN \quad \dots(ii) \text{ [From (i)]}$$

Consider triangles  $ABM$  and  $PQN$ ,

We have  $AB = PQ$  [Given]

$AM = PN$  [Given]

and  $BM = QN$  [From (ii)]

$\therefore \triangle ABM \cong \triangle PQN$  [SSS rule]

(ii) From result (i),

$$\angle ABM = \angle PQN \Rightarrow \angle ABC = \angle PQR \quad \dots(iii)$$

Consider triangles ABC and PQR,

We have  $AB = PQ$  [Given]

$BC = QR$  [Given]

and  $\angle ABC = \angle PQR$  [From (iii)]

$\therefore \triangle ABC \cong \triangle PQR$ . [SAS rule]

4. *BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.*

**Sol.** Consider triangles BFC and CEB,

We have  $BE = CF$  [Given]

$BC = CB$  [Common]

and  $\angle BFC = \angle CEB$  [ $90^\circ$  each]

$\therefore \triangle BFC \cong \triangle CEB$  [RHS rule]

$\Rightarrow \angle FBC = \angle ECB$  [CPCT]

i.e.,  $\angle ABC = \angle ACB$

$\Rightarrow AC = AB$

[Sides opposite to equal angles of a triangles are equal.]

$\Rightarrow \triangle ABC$  is an isosceles triangle.

5. *ABC is an isosceles triangle with  $AB = AC$ . Draw  $AP \perp BC$  to show that  $\angle B = \angle C$ .*

**Sol.** Consider triangles APB and APC,

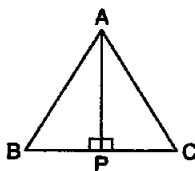
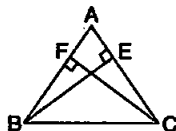
We have  $AB = AC$  [Given]

$AP = PA$  [Common]

$\angle APB = \angle APC$  [ $90^\circ$  each]

$\therefore \triangle ABP \cong \triangle ACP$  [RHS rule]

$\Rightarrow \angle B = \angle C$ . [CPCT]



### Exercise 7.4 (Pages – 132-133)

1. *Show that in a right angled triangle, the hypotenuse is the longest side.*

**Sol.** Triangle ABC is right angled at A.

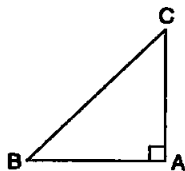
$\therefore \angle A > \angle B$  and  $\angle A > \angle C$ .

$\Rightarrow BC > AC$  and  $BC > AB$

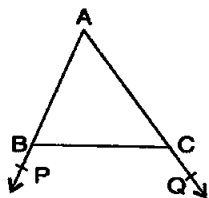
[Because the side opposite to the greater angle is longer.]

$\Rightarrow BC$  is the longest side.

i.e., The hypotenuse is the longest side in an right angled triangle.



2. In figure, sides  $AB$  and  $AC$  of  $\triangle ABC$  are extended to points  $P$  and  $Q$  respectively. Also,  $\angle PBC < \angle QCB$ . Show that  $AC > AB$ .



**Sol.** Given:  $\angle PBC < \angle QCB$ .

$$\Rightarrow 180^\circ - \angle ABC < 180^\circ - \angle ACB.$$

$$\Rightarrow -\angle ABC < -\angle ACB.$$

$$\Rightarrow \angle ABC > \angle ACB$$

$$\Rightarrow AC > AB.$$

[Linear pair]

[Multiplying by  $(-1)$ ]

[Greater angle has longer side opposite to it.]

3. In figure,  $\angle B < \angle A$  and  $\angle C < \angle D$ . Show that  $AD < BC$ .

**Sol.** As  $\angle B < \angle A$  [Given]

$$\Rightarrow OA < OB \quad \dots(i)$$

[Greater angle has longer side opposite to it.]

$$\angle C < \angle D$$

$$\Rightarrow OD < OC \quad \dots(ii)$$

[Greater angle has longer side opposite to it.]

$$\Rightarrow OA + OD < OB + OC$$

[Adding corresponding sides of (i) and (ii)]

$$\Rightarrow AD < BC.$$

4.  $AB$  and  $CD$  are respectively the smallest and longest sides of a quadrilateral  $ABCD$  (see figure). Show that  $\angle A > \angle C$  and  $\angle B > \angle D$ .

**Sol. Construction:** Join  $B$  and  $D$ .

In  $\triangle ABD$ ,  $AB < AD$  [Given]

$$\Rightarrow \angle ADB < \angle ABD \quad \dots(i)$$

[Longer side has greater angle opposite to it.]

In  $\triangle BCD$ ,

$BC < CD$  [Given]

$$\Rightarrow \angle BDC < \angle DBC \quad \dots(ii)$$

[Longer side has greater angle opposite to it.]

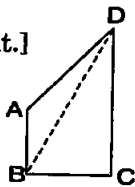
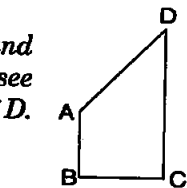
Adding corresponding sides of (i) and (ii), we get

$$\angle ADB + \angle BDC < \angle ABD + \angle DBC$$

$$\Rightarrow \angle ADC < \angle ABC \Rightarrow \angle D < \angle B$$

Similarly, by joining  $A$  and  $C$ , we can show  $\angle A > \angle C$ .

(Students try themselves)



5. In figure,  $PR > PQ$  and  $PS$  bisects  $\angle QPR$ . Prove that  $\angle PSR > \angle PSQ$ .

**Sol.** In  $\triangle PQR$ ,  $PR > PQ$  [Given]

$$\Rightarrow \angle Q > \angle R \quad \dots(i)$$

[Angle opposite to longer side is greater.]

$$\text{Also, } \angle QPS = \angle RPS \quad \dots(ii)$$

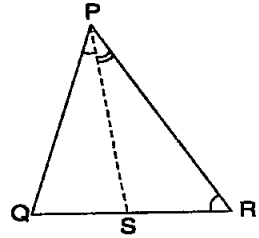
[ $\because$   $PS$  is bisector of  $\angle QPR$ ]

$$\Rightarrow \angle Q + \angle QPS > \angle R + \angle RPS \quad \dots(iii) \text{ [Adding (i) and (ii)]}$$

Also,  $\angle PSR = \angle Q + \angle QPS \quad \dots(iv)$  [Exterior angle is equal to sum of interior opposite angles.]

$$\text{and } \angle PSQ = \angle R + \angle RPS \quad \dots(v)$$

$$\therefore \angle PSR > \angle PSQ. \quad \text{[From (iii), (iv) and (v)]}$$

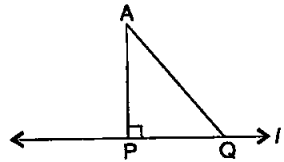


6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

**Sol.**  $AP$  is perpendicular line segment on line  $l$  and  $AQ$  is any other line segment.

$\angle APQ > \angle AQP$  [In a triangle, if an angle is  $90^\circ$ , other two angles are less than  $90^\circ$ .]

$$\Rightarrow AQ > AP.$$



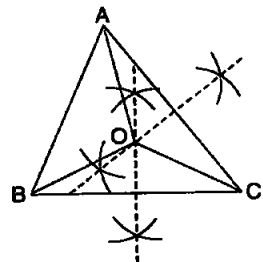
Hence, any line segment  $AQ$ , from point  $A$  to the line  $l$ , is greater than the perpendicular line segment  $AP$ , from point  $A$  to the line  $l$ .

Hence,  $AP$  is the shortest segment of all the segments.

### Exercise 7.5 (Page – 133)

1.  $ABC$  is a triangle. Locate a point in the interior of  $\triangle ABC$  which is equidistant from all the vertices of  $\triangle ABC$ .

**Sol.** We know that any point on the perpendicular bisector of the line segment is equidistant from the end points of the line segment. Hence,



- (i) Draw the perpendicular bisector of line segment BC.
- (ii) Draw the perpendicular bisector of line segment AC.

Let these perpendicular bisectors meet at O.

Then we have  $OB = OC = OA$ , i.e., O is a point equidistant from the vertices of a triangle ABC.

2. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

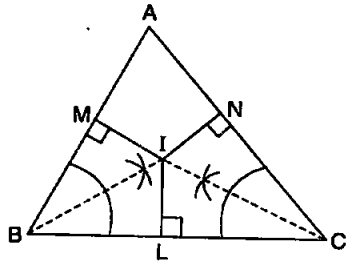
**Sol.** We know that any point on the bisector of an angle is equidistant from the arms of an angle.

(i) Draw the bisector of  $\angle B$ .

(ii) Draw the bisector of  $\angle C$ .

Let these bisectors meet at I.

IL, IM and IN are perpendiculars drawn on the sides BC, AB and AC respectively.



As I lies on the bisector of  $\angle ABC \Rightarrow IL = IM$

and I lies on the bisector of  $\angle ACB \Rightarrow IL = IN$ .

Then  $IL = IM = IN$ .

Hence, I is equidistant from BC, AC and AB.

3. In a huge park, people are concentrated at three points (see figure):

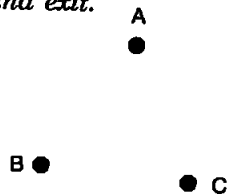
A: where there are different slides and swings for children.

B: near which a man-made lake is situated.

C: which is near to a large parking and exit.

Where should an ice cream parlour be set up so that maximum number of persons can approach it?

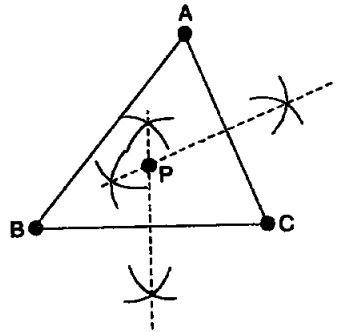
[Hint: The parlour should be equidistant from A, B and C.]



**Sol.** Position should be equidistant from A, B and C.

We know that any point on the perpendicular bisector of the line segment is equidistant from the end points of the line segment. Hence,

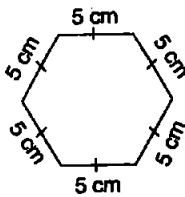
- (i) Draw the perpendicular bisector of line segment BC.
- (ii) Draw the perpendicular bisector of line segment AC.



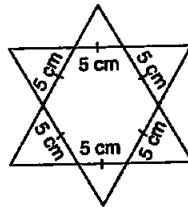
Let these perpendicular bisectors meet at P.

Then we have  $PB = PC = PA$ , i.e., P is a point equidistant from the vertices of a triangle ABC. Hence parlour should be situated at P.

4. Complete the hexagonal and star shaped Rangolies [see figure (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



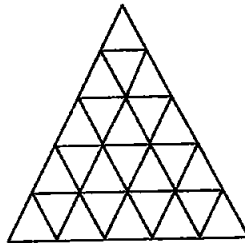
(i)



(ii)

Sol. Try yourself.

e.g.,



Side of bigger equilateral triangle is 5 cm and side of each smaller equilateral triangle is 1 cm.

□□