

7

Coordinate Geometry

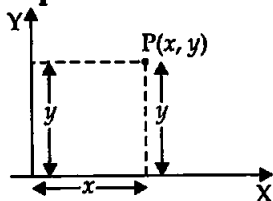
EXERCISE 7.1

Choose the correct answer from the given four options:

Q1. The distance of the point P(2, 3) from x-axis is

- (a) 2 (b) 3
(c) 1 (d) 5

Sol. (b): The perpendicular distance of P(2, 3) from x-axis is equal to the y coordinate so, it is 3 units. verifies ans. (b).



Q2. The distance between the points A(0, 6) and B(0, -2) is

- (a) 6 (b) 8 (c) 4 (d) 2

Sol. (b): $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(0 - 0)^2 + (-2 - 6)^2} = \sqrt{0 + (-8)^2} = \sqrt{64}$
 $\Rightarrow AB = 8$ units

Hence, verifies Ans (b).

Q3. The distance of the point P(-6, 8) from the origin is

- (a) 8 (b) $2\sqrt{7}$ (c) 10 (d) 6

Sol. (c): Coordinates of origin are O(0, 0) and P(-6, 8)

$\therefore (OP)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
 $= (-6 - 0)^2 + (8 - 0)^2 = 36 + 64$
 $OP = \sqrt{100}$
 $\Rightarrow OP = 10$ units. verifies ans. (c).

Q4. The distance between the points (0, 5) and (-5, 0) is

- (a) 5 (b) $5\sqrt{2}$ (c) $2\sqrt{5}$ (d) 10

Sol. (b): Let A(0, 5) and B(-5, 0) are the two points.

Then, $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
 $= (-5 - 0)^2 + (0 - 5)^2 = 25 + 25$
 $\Rightarrow AB^2 = 50$
 $\Rightarrow AB = 5\sqrt{2}$ units. verifies ans. (b).

Q5. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0), and B(5, 0). The length of its diagonal is

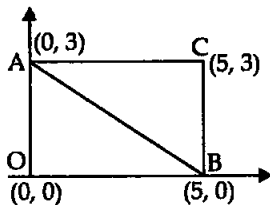
- (a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

Sol. (c): A (0, 3) and B(5, 0)

The length of diagonal = AB

$$\begin{aligned} AB^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (5 - 0)^2 + (0 - 3)^2 \\ &= 25 + 9 \end{aligned}$$

$\Rightarrow AB = \sqrt{34}$ verifies Ans. (c).



Q6. The perimeter of a triangle with vertices (0, 4), (0, 0), and (3, 0) is

- (a) 5 (b) 12 (c) 11 (d) $7 + \sqrt{5}$

Sol. (b): Perimeter of $\triangle ABC = AB + BC + AC$

Let A(0, 4), B(0, 0), C(3, 0) be the three vertices of $\triangle ABC$.

$$\begin{aligned} AB^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (0 - 0)^2 + (0 - 4)^2 = 0 + 16 \end{aligned}$$

$\Rightarrow AB = \sqrt{16} = 4$ cm

$$AC^2 = (3 - 0)^2 + (0 - 4)^2 = 9 + 16$$

$\Rightarrow AC^2 = 25$

$\Rightarrow AC = 5$ cm

$$BC^2 = (3 - 0)^2 + (0 - 0)^2 = 9 + 0$$

$\Rightarrow BC^2 = 9$

$\Rightarrow BC = 3$ cm

\therefore Perimeter = 4 cm + 5 cm + 3 cm = 12 cm

Hence, verifies Ans. (b).

Q7. The area of triangle with vertices A(3, 0), B(7, 0), and C(8, 4) is

- (a) 14 (b) 28 (c) 8 (d) 6

Sol. (c): Area (A) of $\triangle ABC$ whose vertices are A(3, 0), B(7, 0) and C(8, 4) is given by

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [3(0 - 4) + 7(4 - 0) + 8(0 - 0)] \\ &= \frac{1}{2} [-12 + 28 + 0] = \frac{1}{2} [16] = 8 \text{ sq. units} \end{aligned}$$

Hence, verifies the Ans. (c).

Q8. The points (-4, 0), (4, 0) and (0, 3) are the vertices of a

- (a) right triangle (b) isosceles triangle
(c) equilateral triangle (d) scalene triangle

Sol. (b): Let the vertices of $\triangle ABC$ are A(-4, 0), B(4, 0) and C(0, 3).

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$\Rightarrow AB^2 = [4 - (-4)]^2 + (0 - 0)^2 = 64 + 0 = 64$

$\Rightarrow AB = 8$ cm

$$AC^2 = [0 - (-4)]^2 + (3 - 0)^2 = 16 + 9 = 25$$

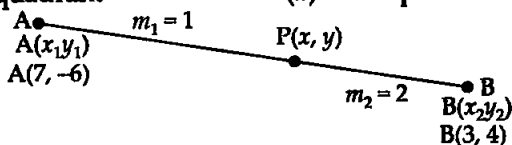
$$\begin{aligned} \Rightarrow AC^2 &= 25 \\ \Rightarrow AC &= 5 \text{ cm} \\ BC^2 &= (0-4)^2 + (3-0)^2 = 16+9=25 \\ \Rightarrow BC^2 &= 25 \\ \Rightarrow BC &= 5 \text{ cm} \\ \therefore AC &= BC = 5 \text{ cm and } AB = 8 \text{ cm} \end{aligned}$$

Hence, the triangle is an isosceles triangle. So, verifies ans. (b).

Q9. The point which divides the line segment joining the points (7, -6) and (3, 4) in ratio 1 : 2 internally lies in the

- (a) Ist quadrant (b) IInd quadrant
(c) IIIrd quadrant (d) IVth quadrant

Sol. (d):



$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{1(3) + 2(7)}{1+2} = \frac{3+14}{3}$$

$$y = \frac{1(4) + 2(-6)}{1+2} = \frac{4-12}{3}$$

$$\Rightarrow x = \frac{17}{3}$$

$$y = \frac{-8}{3}$$

$P\left(\frac{17}{3}, \frac{-8}{3}\right)$ verifies the Ans. (d).

Q10. The point which lies on the perpendicular bisector of the line segment joining the points A(-2, -5) and B(2, 5) is

- (a) (0, 0) (b) (0, 2) (c) (2, 0) (d) (-2, 0)

Sol. (a): The perpendicular bisector of AB will pass through the mid-point of AB. Mid-point of A(x_1, y_1) and B(x_2, y_2) is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

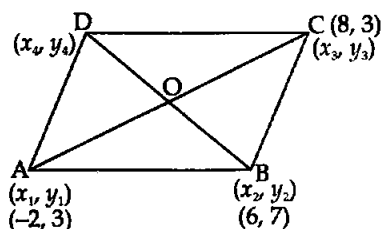
$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

So, the perpendicular bisector passes through (0, 0).

Q11. The fourth vertex D of a parallelogram ABCD whose three vertices are A(-2, 3), B(6, 7), and C(8, 3) is

- (a) (0, 1) (b) (0, -1) (c) (-1, 0) (d) (1, 0)

Sol. (b): We know that the diagonals AC and BD of parallelogram ABCD bisect each other.



OR

$$\left[\begin{array}{l} \text{The mid point} \\ \text{of diagonal AC} \end{array} \right] = \left[\begin{array}{l} \text{Mid point of} \\ \text{diagonal BD} \end{array} \right]$$

$$\Rightarrow \left(\frac{-2+8}{2}, \frac{3+3}{2} \right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2} \right)$$

$$\Rightarrow \left(\frac{6}{2}, \frac{6}{2} \right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2} \right)$$

$$\Rightarrow (3, 3) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2} \right)$$

Comparing both sides, we have

$$\frac{x_4+6}{2} = 3 \quad \text{and} \quad \frac{y_4+7}{2} = 3$$

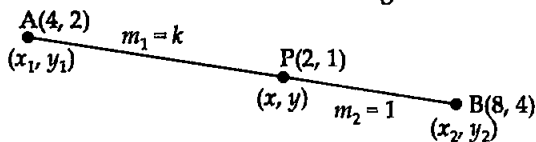
$$\Rightarrow x_4+6=6 \quad \Rightarrow y_4+7=6$$

$$\Rightarrow x_4=0 \quad \Rightarrow y_4=6-7=-1$$

\therefore The fourth vertex of parallelogram is $(0, -1)$ verifies ans. (b).

Q12. If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then

$$(a) AP = \frac{1}{3} AB \quad (b) AP = PB \quad (c) PB = \frac{1}{3} AB \quad (d) AP = \frac{1}{2} AB$$

Sol. (d):

$$\therefore x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow 2 = \frac{k(8) + 1(4)}{k+1}$$

$$1 = \frac{k(4) + 1(2)}{k+1}$$

$$\Rightarrow 8k + 4 = 2k + 2$$

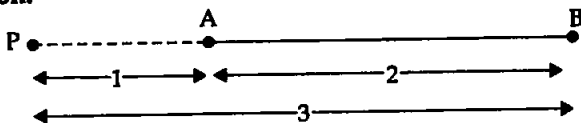
$$4k + 2 = k + 1$$

$$\Rightarrow 6k = -2$$

$$3k = -1$$

$$\Rightarrow k = \frac{-1}{3}$$

$$k = \frac{-1}{3}$$

Verification:

$$\begin{aligned} \therefore \quad & \frac{AP}{PB} = \frac{-1}{3} \\ \Rightarrow \quad & AP = -1 \quad \text{i.e., 1 part outside AB} \\ \text{and} \quad & PB = 3 \\ \therefore \quad & AP = 1x \text{ unit} \\ \text{and} \quad & AB = 3x - 1x = 2x \text{ units} \\ \text{So,} \quad & AP = \frac{1}{2} AB \\ \Rightarrow \quad & 1 = \frac{1}{2} \times 2 \Rightarrow 1 = 1, \text{ which is true} \end{aligned}$$

Hence, verifies the ans. (d).

Q13. If $P(-, 4)$ is the mid point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then the value of 'a' is
 (a) -4 (b) -12 (c) 12 (d) -6

Sol. (b): $P(x, y)$ is mid-point of QR then

$$\begin{aligned} \left(\frac{a}{3}, 4\right) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ \Rightarrow \quad \left(\frac{a}{3}, 4\right) &= \left(\frac{-6 - 2}{2}, \frac{5 + 3}{2}\right) \\ \Rightarrow \quad \frac{a}{3} &= \frac{-8}{2} \\ \Rightarrow \quad a &= -4 \times 3 = -12 \end{aligned}$$

Verifies the ans. (b).

Q14. The perpendicular bisector of the line segment joining the points $A(1, 5)$ and $B(4, 6)$ cuts y -axis at
 (a) $(0, 13)$ (b) $(0, -13)$ (c) $(0, 12)$ (d) $(13, 0)$

Sol. (a): The given points are $A(1, 5)$ and $B(4, 6)$.

The perpendicular bisector of the line segment joining the points $A(1, 5)$ and $B(4, 6)$ cuts the y -axis at $P(0, y)$.

$$\begin{aligned} \text{Now,} \quad & AP = BP \Rightarrow AP^2 = BP^2 \\ \therefore \quad & 1 + (y - 5)^2 = 16 + (y - 6)^2 \\ \Rightarrow \quad & 1 + y^2 - 10y + 25 = 16 + y^2 - 12y + 36 \\ \Rightarrow \quad & -10y + 26 = -12y + 52 \\ \Rightarrow \quad & 12y - 10y = 52 - 26 \\ \Rightarrow \quad & 2y = 26 \\ \Rightarrow \quad & y = 26 \div 2 = 13 \end{aligned}$$

So, the required point is $(0, 13)$.

Hence, (a) is the correct answer.

Q15. The coordinates of the point which is equidistant from the three vertices of the $\triangle AOB$ as shown in the figure is

- (a) (x, y) (b) (y, x)
 (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Sol. (a): In a right triangle, the mid-point of the hypotenuse is equidistant from the three vertices of triangle.

Mid-point of $A(2x, 0)$ and $B(0, 2y)$ is

$$= \left(\frac{2x + 0}{2}, \frac{0 + 2y}{2}\right) = (x, y)$$

Hence, (a) is the correct answer.

Q16. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

- (a) $\left(-\frac{3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$ (c) $\left(5, -\frac{1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

Sol. (d): Radius of circle = $\sqrt{\left(\frac{13}{2} - 0\right)^2 + (0 - 0)^2} = \frac{13}{2} = 6.5$ units

(a) Distance of point $\left(-\frac{3}{4}, 1\right)$ from $(0, 0)$ is

$$= \sqrt{\left(-\frac{3}{4} - 0\right)^2 + (1 - 0)^2} = \sqrt{\frac{9}{16} + 1} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25 \text{ units}$$

The distance $1.25 < 6.5$. So, the point $\left(-\frac{3}{4}, 1\right)$ lies in the interior of the circle.

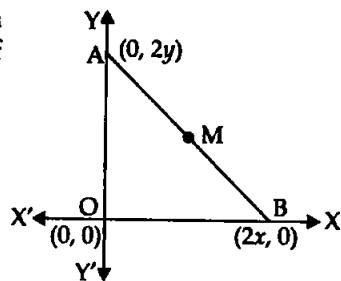
(b) Distance of point $\left(2, \frac{7}{3}\right)$ from $(0, 0)$ is

$$= \sqrt{(2 - 0)^2 + \left(\frac{7}{3} - 0\right)^2} = \sqrt{4 + \frac{49}{9}} = \sqrt{\frac{85}{9}} = \frac{9.2195}{3} = 3.0731 < 6.25$$

So, the point $\left(2, \frac{7}{3}\right)$ lies in the interior of the circle.

(c) Distance of point $\left(5, -\frac{1}{2}\right)$ from $(0, 0)$ is

$$= \sqrt{(5 - 0)^2 + \left(-\frac{1}{2} - 0\right)^2} = \sqrt{25 + \frac{1}{4}} = \sqrt{\frac{101}{4}} = \frac{10.0498}{2} = 5.0249 < 6.5$$



So, the point $\left(5, \frac{-1}{2}\right)$ lies in the interior of the circle.

(d) Distance of point $\left(-6, \frac{5}{2}\right)$ from $(0, 0)$ is

$$= \sqrt{(-6-0)^2 + \left(\frac{5}{2}-0\right)^2} = \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5 \text{ units}$$

So, $\left(-6, \frac{5}{2}\right)$ lies on the circle. It does not lie in the interior of the circle.

Hence, (d) is the correct answer.

Q17. A line intersects the y -axis and x -axis at points P and Q respectively. If $(2, -5)$ is the mid-point of PQ, then co-ordinates of P and Q are respectively.

(a) $(0, -5)$ and $(2, 0)$

(b) $(0, 10)$ and $(-4, 0)$

(c) $(0, 4)$ and $(-10, 0)$

(d) $(0, -10)$ and $(4, 0)$

Sol. (d): P lies on y -axis so co-ordinates of P are $(0, y)$.

Similarly, co-ordinates of Q lies on x -axis = $Q(x, 0)$

Mid-point of PQ is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M(2, -5), \text{ which is given}$$

$$\Rightarrow M\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = M(2, -5)$$

$$\Rightarrow \left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$$

Comparing both sides, we get

$$\frac{x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = -5$$

$$\Rightarrow x = 4 \quad \text{and} \quad y = -10$$

Hence, the co-ordinates of $P(0, -10)$ and $Q(4, 0)$ verifies ans. (d).

Q18. The area of the triangle with vertices $(a, b+c)$ $(b, c+a)$ and $(c, a+b)$ is

(a) $(a+b+c)^2$

(b) 0

(c) $a+b+c$

(d) abc

Sol. (b): If the vertices of ΔABC are

$$A(x_1, y_1) = A(a, b+c)$$

$$B(x_2, y_2) = B(b, c+a)$$

$$C(x_3, y_3) = C(c, a+b)$$

$$\text{Then, Area of } \Delta ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} [a(c+a - (a+b)) + b(a+b - (b+c)) + c(b+c - (c+a))]$$

$$= \frac{1}{2} [a(c-b) + b(a-c) + c(b-a)]$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} [ac - ab + ab - bc + bc - ac]$$

\Rightarrow Area of $\triangle ABC = 0$ So, verifies the option (b).

Q19. If the distance between the points $(4, p)$ and $(1, 0)$ is 5, then the value of p is

- (a) 4 only (b) ± 4 (c) -4 only (d) 0

Sol. (b): According to the question, the distance between $A(4, p)$ and $B(1, 0)$ is 5 units.

$$\begin{aligned} \therefore \quad & AB = 5 \text{ units} \\ \Rightarrow & (AB)^2 = (5)^2 \\ \Rightarrow & (4-1)^2 + (p-0)^2 = 25 \\ \Rightarrow & (3)^2 + (p)^2 = 25 \\ \Rightarrow & p^2 = 25 - 9 \\ \Rightarrow & p^2 = 16 \\ \Rightarrow & p = \pm 4 \quad \text{Hence, verifies the ans. (b).} \end{aligned}$$

Q20. If the points $A(1, 2)$, $O(0, 0)$ and $C(a, b)$ are collinear, then

- (a) $a = b$ (b) $a = 2b$ (c) $2a = b$ (d) $a = -b$

Sol. (c): Points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear if the area of $\triangle ABC$ is zero so, $A(1, 2)$, $B(0, 0)$, $C(a, b)$ will collinear if area $\triangle ABC = 0$

$$\begin{aligned} \text{or} \quad & \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0 \\ \Rightarrow & \frac{1}{2} [1(0 - b) + 0(b - 2) + a(2 - 0)] = 0 \\ \Rightarrow & \frac{1}{2} (-b + 2a) = 0 \\ \Rightarrow & \frac{-b}{2} + a = 0 \\ \Rightarrow & -b + 2a = 0 \\ \Rightarrow & 2a = b \end{aligned}$$

Hence, verifies the ans. (c).

EXERCISE 7.2

State whether the following statements are true or false. Justify your answer.

Q1. $\triangle ABC$ with vertices $A(-2, 0)$, $B(2, 0)$ and $C(0, 2)$ is similar to $\triangle DEF$ with vertices $D(-4, 0)$, $E(4, 0)$ and $F(0, 4)$.

Sol. True: $\triangle ABC \sim \triangle DEF$ if $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = k$

In $\triangle ABC$,

$$\begin{aligned} \Rightarrow \quad & AB^2 = [2 - (-2)]^2 + [0 - (0)]^2 = (4)^2 + 0 = (4)^2 \\ & AB = 4 \text{ units} \\ & BC^2 = (0 - 2)^2 + (2 - 0)^2 = 4 + 4 = 8 \end{aligned}$$

$$\Rightarrow \quad BC = 2\sqrt{2} \text{ units}$$

$$AC^2 = [0 - (-2)]^2 + (2 - 0)^2 = 2^2 + 2^2 = 4 + 4 = 8$$

$$\Rightarrow \quad AC = 2\sqrt{2} \text{ units}$$

In $\triangle DEF$,

$$DE^2 = [4 - (-4)]^2 + (0 - 0)^2 = (8)^2$$

$$\Rightarrow \quad DE = 8 \text{ units}$$

$$EF^2 = (0 - 4)^2 + (4 - 0)^2 = 4^2 + 4^2 = 16 + 16 = 32$$

$$\Rightarrow \quad EF = 4\sqrt{2} \text{ units}$$

$$DF^2 = [0 - (-4)]^2 + (4 - 0)^2 = 16 + 16 = 32$$

$$\Rightarrow \quad DF = 4\sqrt{2} \text{ units}$$

Now, $\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}$

$$\frac{BC}{EF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

$$\frac{AC}{DF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

$$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$$

Hence, $\triangle ABC \sim \triangle DEF$.

Q2. Point $P(-4, 2)$ lies on the line segment joining the points $A(-4, 6)$ and $B(-4, -6)$.

Sol. True: We observe that x -coordinate is same i.e., equal to (-4) so line is parallel to y -axis. y -coordinate of P i.e., 2 lies between 6 and -6 of A and B respectively. Hence, P lies between and on AB .

OR

Point $P(-4, 2)$ will lie on the line AB if area of $\triangle ABP$ is zero.

\therefore i.e., $\text{ar}(\triangle ABP) = 0$

$$\Rightarrow \quad \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2} [-4(-6 - 2) - 4(2 - 6) - 4(6 + 6)] = 0$$

$$\Rightarrow \quad [-4(-8) - 4(-4) - 4(12)] = 0$$

$$\Rightarrow \quad 32 + 16 - 48 = 0$$

$$\Rightarrow \quad 48 - 48 = 0, \text{ which is true.}$$

Hence, point P lies on the line joining A and B .

Q3. The points $(0, 5)$, $(0, -9)$ and $(3, 6)$ are collinear.

Sol. False: Three points A , B , and C will be collinear if the area of $\triangle ABC = 0$

$$\Rightarrow \quad \frac{1}{2} [0(-9 - 6) + 0(6 - 5) + 3(5 - (-9))] = 0$$

$$\Rightarrow 0 + 0 + 3(14) = 0$$

$$\Rightarrow 42 \neq 0, \text{ which is false.}$$

Hence, the given points are not collinear.

Q4. Point P(0, 2) is the point of intersection of y -axis and perpendicular bisector of line segment joining the points A(-1, 1) and B(3, 3).

Sol. False: As the point P(0, 2) is the point of intersection of y -axis and perpendicular bisector of the line joining the points A(-1, 1) and B(3, 3), then point P must be equidistant from A and B. So, we must write $PA = PB$.

$$PA = \sqrt{(-1-0)^2 + (1-2)^2} = \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$PB = \sqrt{(3-0)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$\therefore PA \neq PB$$

Hence, the given statement is false.

Q5. Points A(3, 1), B(12, -2) and C(0, 2) cannot be the vertices of a triangle.

Sol. True: Points A, B, C can form a triangle if the sum of any two sides is greater than the third side.

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow AB^2 = (12-3)^2 + (-2-1)^2 = 81 + 9 = 90$$

$$\Rightarrow AB = 3\sqrt{10} \text{ units}$$

$$BC^2 = (0-12)^2 + [2-(-2)]^2 = 144 + 16 = 160$$

$$\Rightarrow BC = 4\sqrt{10} \text{ units}$$

$$AC^2 = (0-3)^2 + (2-1)^2 = 9 + 1 = 10 \Rightarrow AC = \sqrt{10} \text{ units}$$

$$\therefore AC = \sqrt{10} \text{ units, } AB = 3\sqrt{10} \text{ units and } BC = 4\sqrt{10} \text{ units}$$

$$\text{Now, } AB + AC = \sqrt{10} + 3\sqrt{10} = 4\sqrt{10} \text{ units} = BC$$

So, A, B, C points cannot form a Δ .

Q6. Points A(4, 3), B(6, 4), C(5, -6) and D(-3, 5) are the vertices of a parallelogram.

Sol. False: The diagonals of parallelogram bisect each other so, ABCD will be a parallelogram if

mid-point of diagonal AC = mid-point of diagonal BD

$$\Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{x'_1 + x'_2}{2}, \frac{y'_1 + y'_2}{2} \right)$$

$$\Rightarrow \left(\frac{4+5}{2}, \frac{-6+3}{2} \right) = \left(\frac{6-3}{2}, \frac{4+5}{2} \right)$$

$$\Rightarrow \left(\frac{9}{2}, \frac{-3}{2} \right) \neq \left(\frac{3}{2}, \frac{9}{2} \right)$$

Hence, ABCD is not a parallelogram.

Q7. A circle has its centre at the origin and a point P(5, 0) lies on it. The point Q(6, 8) lies outside the circle.

Sol. True: If the distance of Q from the centre O(0, 0) is greater than the radius then point Q lies in the exterior of the circle. Point P(5, 0) lies on the circle and centre is at O(0, 0) so radius = OP

$$OP^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$= (5 - 0)^2 + (0 - 0)^2$$

$$\Rightarrow OP^2 = 5^2$$

$$\Rightarrow OP = 5 \text{ units}$$

Now, $OQ^2 = (6 - 0)^2 + (8 - 0)^2 = 36 + 64 = 100$

$$\Rightarrow OQ = 10 \text{ units}$$

$$\therefore OQ > OP \text{ (radius)}$$

So, point Q lies exterior to circle.

Q8. The point A(2, 7) lies on the perpendicular bisector of line segment joining the points P(6, 5) and Q(0, -4).

Sol. False: Any point (A) on perpendicular bisector will be equidistant from P and Q so

$$PA = QA$$

$$PA^2 = QA^2$$

or

$$\Rightarrow (2 - 6)^2 + [7 - (5)]^2 = (2 - 0)^2 + [7 - (-4)]^2$$

$$\Rightarrow (-4)^2 + (2)^2 = 2^2 + (11)^2$$

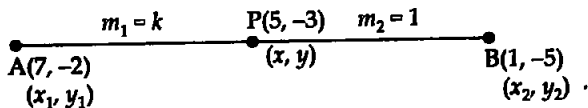
$$\Rightarrow 16 + 4 = 4 + 121$$

$$\Rightarrow 20 \neq 125$$

So, A does not lie on the perpendicular bisector of PQ.

Q9. Point P(5, -3) is one of the two points of trisection of the line segment joining the points A(7, -2) and B(1, -5).

Sol. True



Let point P divides the line AB in ratio $k : 1$ then

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{k(1) + 1(7)}{(k + 1)}, \quad y = \frac{k(-5) + 1(-2)}{k + 1}$$

$$\Rightarrow 5 = \frac{k + 7}{k + 1}, \quad -3 = \frac{-5k - 2}{k + 1}$$

$$\Rightarrow 5k + 5 = k + 7, \quad -5k - 2 = -3k - 3$$

$$\Rightarrow 4k = 7 - 5, \quad -2k = -3 + 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}, \quad k = \frac{-1}{-2} = \frac{1}{2}$$

So, P divides AB in 1 : 2 ratio.

Hence, P is one point of trisection of AB.

Q10. Points A(-6, 10), B(-4, 6) and C(3, -8) are collinear such that

$$AB = \frac{2}{9} AC.$$

Sol. True: Points A, B and C will be collinear if ar (ΔABC) = 0

$$\text{ar } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2}[-6\{6 - (-8)\} - 4\{-8 - 10\} + 3\{10 - 6\}] = 0$$

$$\Rightarrow -6(14) - 4(-18) + 3(4) = 0$$

$$\Rightarrow -84 + 72 + 12 = 0$$

$$\Rightarrow -84 + 84 = 0, \text{ which is true}$$

So, points A, B and C are collinear.

$$\begin{aligned} AC^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (3 + 6)^2 + (-8 - 10)^2 = 81 + 324 \end{aligned}$$

$$\Rightarrow AC = \sqrt{405} = 9\sqrt{5} \text{ units}$$

$$\begin{aligned} AB^2 &= [-4 - (-6)]^2 + (6 - 10)^2 \\ &= (-4 + 6)^2 + (-4)^2 \\ &= (2)^2 + (-4)^2 = 4 + 16 \end{aligned}$$

$$\Rightarrow AB^2 = 20$$

$$\Rightarrow AB = 2\sqrt{5} \text{ units}$$

Now, $AB = \frac{2}{9} AC$

$$\begin{aligned} \text{R.H.S.} &= \frac{2}{9} \times 9\sqrt{5} \\ &= 2\sqrt{5} \\ &= AB \end{aligned}$$

Hence, $AB = \frac{2}{9} AC$ is true.

Q11. The point P(-2, 4) lies on a circle of radius 6 and centre (3, 5).

Sol. False: The point P(-2, 4) lies on a circle if distance between P and centre is equal to the radius so distance of P from centre O(3, 5) will be

$$OP^2 = (-2 - 3)^2 + (4 - 5)^2$$

$$\Rightarrow OP^2 = 25 + (-1)^2$$

$$\Rightarrow OP = \sqrt{26} \neq \text{radius } 6$$

So, P does not lie on the circle. It will lie inside the circle.

Q12. The points A(-1, -2), B(4, 3), C(2, 5) and D(-3, 0) in that order form a rectangle.

Sol. True: ABCD will form a rectangle if

- (i) it is a parallelogram. (ii) diagonals are equal.

For parallelogram: Diagonals bisect each other.

i.e., Mid point of AC = Mid point of BD is

$$\text{i.e., } \left(\frac{-1+2}{2}, \frac{-2+5}{2} \right) = \left(\frac{4-3}{2}, \frac{3+0}{2} \right)$$

$$\Rightarrow \left(\frac{1}{2}, \frac{3}{2} \right) = \left(\frac{1}{2}, \frac{3}{2} \right)$$

Hence, ABCD is a parallelogram.

$$\text{Now, Diagonal AC} = \sqrt{(2+1)^2 + (5+1)^2} = \sqrt{9+49}$$

$$\Rightarrow AC = \sqrt{58} \text{ units}$$

$$\text{and Diagonal BD} = \sqrt{(-3-4)^2 + (0-3)^2}$$

$$\Rightarrow BD = \sqrt{49+9} \text{ units}$$

$$\Rightarrow BD = \sqrt{58} \text{ units}$$

$$\therefore \text{Diagonal AC} = \text{Diagonal BD}$$

Hence, ABCD is a rectangle.

EXERCISE 7.3

Q1. Name the type of triangle formed by the points A(-5, 6), B(-4, -2) and C(7, 5).

Sol. A(-5, 6), B(-4, -2), C(7, 5)

$$\begin{aligned} \Rightarrow AB^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (-4 + 5)^2 + (-2 - 6)^2 \\ &= (1)^2 + (-8)^2 = 1 + 64 = 65 \end{aligned}$$

$$\Rightarrow AB = \sqrt{65} \text{ units}$$

$$AC^2 = (7 + 5)^2 + (5 - 6)^2$$

$$\Rightarrow AC^2 = (12)^2 + (-1)^2 \Rightarrow AC^2 = 144 + 1$$

$$\Rightarrow AC = \sqrt{145} \text{ units}$$

$$BC^2 = (7 + 4)^2 + (5 + 2)^2 = 11^2 + 7^2 = 121 + 49$$

$$\Rightarrow BC = \sqrt{170} \text{ units}$$

As $AB \neq BC \neq AC$ so scalene triangle.

$\therefore AC^2 + AB^2 = 145 + 65 = 210 \neq BC^2$, so it is not a right angled Δ
So, a scalene Δ will be formed.

Q2. Find the points on the x -axis which are at a distance of $2\sqrt{5}$ from point (7, -4). How many such points are there?

Sol. Let point P(x, 0) be a point on x -axis, and A be the point (7, -4).

$$\text{So, } AP = 2\sqrt{5} \quad \text{[Given]}$$

$$\Rightarrow AP^2 = 4 \times 5 = 20$$

$$\Rightarrow (x - 7)^2 + [0 - (-4)]^2 = 20$$

$$\Rightarrow x^2 + 49 - 14x + 16 = 20$$

$$\begin{aligned} \Rightarrow x^2 - 14x - 20 + 65 &= 0 \\ \Rightarrow x^2 - 14x + 45 &= 0 \\ \Rightarrow x^2 - 9x - 5x + 45 &= 0 \\ \Rightarrow x(x-9) - 5(x-9) &= 0 \\ \Rightarrow (x-9)(x-5) &= 0 \\ \Rightarrow x-9=0 &\quad \text{or} \quad x-5=0 \\ \Rightarrow x=9 &\quad \text{or} \quad x=5 \end{aligned}$$

Hence, there are two such points on x -axis whose distance from $(7, -4)$ is $2\sqrt{5}$. Hence, required points are $(9, 0)$, $(5, 0)$.

Q3. What type of quadrilateral do the points $A(2, -2)$, $B(7, 3)$, $C(11, -1)$ and $D(6, -6)$ taken in that order, form?

Sol. (i) A quadrilateral is a parallelogram, if mid points of diagonals AC and BD are same.

(ii) A parallelogram is not a rectangle, if diagonals $AC \neq BD$.

(iii) A parallelogram may be a rhombus if $AB = BC$.

(iv) If in a parallelogram diagonals are equal, then it is rectangle.

In a rectangle if the sides $AB = BC$, then the rectangle is a square.

For parallelogram with vertices $A(2, -2)$, $B(7, 3)$, $C(11, -1)$, $D(6, -6)$.

mid point of AC = mid point of BD

$$\begin{aligned} \Rightarrow \left(\frac{2+11}{2}, \frac{-2-1}{2} \right) &= \left(\frac{7+6}{2}, \frac{3-6}{2} \right) \\ \Rightarrow \left(\frac{13}{2}, \frac{-3}{2} \right) &= \left(\frac{13}{2}, \frac{-3}{2} \right), \text{ which is true.} \end{aligned}$$

Hence, $ABCD$ is a parallelogram.

Now, we will check whether $AC = BD$

or

$$AC^2 = BD^2$$

\Rightarrow

$$(11-2)^2 + (-1+2)^2 = (6-7)^2 + (-6-3)^2$$

\Rightarrow

$$(9)^2 + (1)^2 = (-1)^2 + (-9)^2$$

\Rightarrow

$$81 + 1 = 1 + 81$$

\Rightarrow

$$82 = 82, \text{ which is true.}$$

As the diagonals are equal so it is a rectangle or square.

Now, we will check whether adjacent sides $AB = BC$

or

$$AB^2 = BC^2$$

\Rightarrow

$$(7-2)^2 + (3+2)^2 = (11-7)^2 + (-1-3)^2$$

\Rightarrow

$$5^2 + 5^2 = (4)^2 + (-4)^2$$

\Rightarrow

$$25 + 25 = 16 + 16$$

\Rightarrow

$$50 \neq 32, \text{ which is false.}$$

So, $ABCD$ is not a square. Hence, $ABCD$ is a rectangle.

Q4. Find the value of a , if the distance between the points $A(-3, -14)$ and $B(a, -5)$ is 9 units.

Sol. Consider $A(-3, -14)$ and $B(a, -5)$.

According to the question, $AB = 9$

$$\Rightarrow AB^2 = 81$$

$$\Rightarrow (a+3)^2 + (-5+14)^2 = 81$$

$$\Rightarrow a^2 + 9 + 6a + (9)^2 = 81$$

$$\Rightarrow a^2 + 6a + 9 = 81 - 81$$

$$\Rightarrow (a+3)^2 = 0$$

$$\Rightarrow a+3 = 0$$

$$\Rightarrow a = -3$$

Q5. Find a point which is equidistant from the points $A(-5, 4)$ and $B(-1, 6)$. How many such points are there?

Sol. Let $P(x, y)$ is equidistant from $A(-5, 4)$ and $B(-1, 6)$, then

$$PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x+5)^2 + (y-4)^2 = (x+1)^2 + (y-6)^2$$

$$\Rightarrow x^2 + 25 + 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$$

$$\Rightarrow 41 + 10x - 8y = 37 + 2x - 12y$$

$$\Rightarrow 8x + 4y + 4 = 0$$

$$\Rightarrow 2x + 1y + 1 = 0 \quad (1)$$

The above equation shows that infinite points are equidistant from AB , because all the points on perpendicular bisector of AB will be equidistant from AB .

\Rightarrow One such point which is equidistant from A and B is the mid-point M of AB i.e.,

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{-5-1}{2}, \frac{4+6}{2}\right)$$

$$M\left(\frac{-6}{2}, \frac{10}{2}\right)$$

$$M(-3, 5)$$

So, $(-3, 5)$ is equidistant from points A and B .

Q6. Find the coordinates of the point Q on the x -axis which lies on the perpendicular bisector of the line segment joining the points $A(-5, -2)$ and $B(4, -2)$. Name the type of triangle formed by the points Q, A and B .

Sol. Let $Q(x, 0)$ be a point on x -axis which lies on the perpendicular bisector of AB .

$$\therefore QA = QB$$

$$\Rightarrow QA^2 = QB^2$$

$$\Rightarrow (-5-x)^2 + (-2-0)^2 = (4-x)^2 + (-2-0)^2$$

$$\Rightarrow (x+5)^2 + (-2)^2 = (4-x)^2 + (-2)^2$$

$$\Rightarrow x^2 + 25 + 10x + 4 = 16 + x^2 - 8x + 4$$

$$\begin{aligned} \Rightarrow 10x + 8x &= 16 - 25 \\ \Rightarrow 18x &= -9 \\ \Rightarrow x &= \frac{-9}{18} = \frac{-1}{2} \end{aligned}$$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

$$\begin{aligned} \text{Now, } QA^2 &= \left[-5 + \frac{1}{2}\right]^2 + [-2 - 0]^2 \\ &= \left(\frac{-9}{2}\right)^2 + \frac{4}{1} \end{aligned}$$

$$\Rightarrow QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81 + 16}{4} = \frac{97}{4}$$

$$\Rightarrow QA = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2} \text{ units}$$

$$\text{Now, } QB^2 = \left(4 + \frac{1}{2}\right)^2 + (-2 - 0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$$

$$\Rightarrow QB^2 = \frac{81}{4} + \frac{4}{1} = \frac{81 + 16}{4} = \frac{97}{4}$$

$$\Rightarrow QB = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2} \text{ units}$$

$$\text{and } AB = \sqrt{(4 + 5)^2 + [-2 - (-2)]^2} = \sqrt{(9)^2} = 9 \text{ units}$$

$$\Rightarrow AB = 9 \text{ units}$$

$$\text{As } QA = QB$$

So, ΔQAB is an isosceles Δ .

Q7. Find the value of m if the points $(5, 1)$, $(-2, -3)$ and $(8, 2m)$ are collinear.

Sol. Points A, B, C will be collinear if the area of $\Delta ABC = 0$.

$$\text{i.e., } \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2}[5(-3 - 2m) - 2(2m - 1) + 8(1 + 3)] = 0$$

$$\Rightarrow -15 - 10m - 4m + 2 + 32 = 0$$

$$\Rightarrow -14m - 15 + 34 = 0$$

$$\Rightarrow -14m + 19 = 0$$

$$\Rightarrow -14m = -19$$

$$\Rightarrow m = \frac{19}{14}$$

Hence, the required value of $m = \frac{19}{14}$.

Q8. If the point A(2, -4) is equidistant from P(3, 8) and Q(-10, y), then find the values of y. Also find distance PQ.

Sol. According to the question,

$$PA = QA$$

$$\Rightarrow PA^2 = QA^2$$

$$\Rightarrow (3-2)^2 + (8+4)^2 = (-10-2)^2 + (y+4)^2$$

$$\Rightarrow 1^2 + 12^2 = (-12)^2 + y^2 + 16 + 8y$$

$$\Rightarrow y^2 + 8y + 16 - 1 = 0$$

$$\Rightarrow y^2 + 8y + 15 = 0$$

$$\Rightarrow y^2 + 5y + 3y + 15 = 0$$

$$\Rightarrow y(y+5) + 3(y+5) = 0$$

$$\Rightarrow (y+5)(y+3) = 0$$

$$\Rightarrow y+5 = 0 \quad \text{or} \quad y+3 = 0$$

$$\Rightarrow y = -5 \quad \text{or} \quad y = -3$$

So, the co-ordinates are P(3, 8), $Q_1(-10, -3)$, $Q_2(-10, -5)$.

Now, $PQ_1^2 = (3+10)^2 + (8+3)^2 = 13^2 + 11^2$

$$\Rightarrow PQ_1^2 = 169 + 121$$

$$\Rightarrow PQ_1 = \sqrt{290} \text{ units}$$

and $PQ_2^2 = (3+10)^2 + (8+5)^2 = 13^2 + 13^2$

$$= 13^2[1+1]$$

$$\Rightarrow PQ_2^2 = 13^2 \times 2$$

$$\Rightarrow PQ_2 = 13\sqrt{2} \text{ units}$$

Hence, $y = -3, -5$, and $PQ = \sqrt{290}$ units and $13\sqrt{2}$ units.

Q9. Find the area of the triangle whose vertices are (-8, 4), (-6, 6) and (-3, 9).

Sol. Vertices of ΔABC are A(-8, 4), B(-6, 6) and C(-3, 9).

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2} [-8(6-9) - 6(9-4) - 3(4-6)]$$

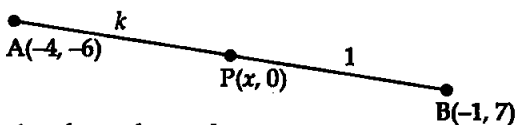
$$= \frac{1}{2} [-8(-3) - 6(5) - 3(-2)]$$

$$= \frac{1}{2} [24 - 30 + 6] = 0$$

Hence, the area of given triangle is zero.

Q10. In what ratio does the x-axis divides the line segment joining the points (-4, -6) and (-1, 7)? Find the coordinates of the point of division.

Sol. Point P(x, 0) on x-axis intersects the line joining the points A(-4, -6) and B(-1, 7). Let P divides the line in the ratio k : 1.



Using the section formula, we have

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \quad (I)$$

$$\Rightarrow \frac{0}{1} = \frac{k(7) + 1(-6)}{k + 1}$$

$$\Rightarrow 7k - 6 = 0$$

$$\Rightarrow k = \frac{6}{7}$$

$$\Rightarrow m_1 = 6 \text{ and } m_2 = 7$$

Again, using the section formula, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{6(-1) + 7(-4)}{6 + 7} = \frac{-6 - 28}{13}$$

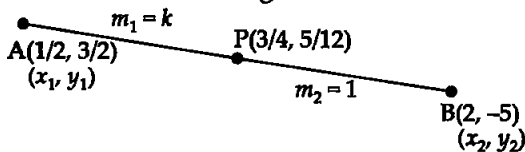
$$\Rightarrow x = \frac{-34}{13}$$

Now, $y = \frac{6(7) + 7(-6)}{6 + 7} = \frac{42 - 42}{13} = 0$ [From (I)]

\therefore Hence, the required point of intersection is $\left(\frac{-34}{13}, 0\right)$.

Q11. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{3}, \frac{3}{2}\right)$ and $B(2, -5)$.

Sol. Let point P divides the line segment AB in the ratio $k : 1$, then



The coordinates of P, by section formula are

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\therefore x = \frac{k(2) + 1\left(\frac{1}{2}\right)}{k + 1}$$

$$\Rightarrow \frac{3}{4} = \frac{2k + \frac{1}{2}}{k+1}$$

$$\Rightarrow 8k + 2 = 3k + 3$$

$$\Rightarrow 8k - 3k = 3 - 3$$

$$\Rightarrow 5k = 1$$

$$\Rightarrow k = \frac{1}{5}$$

$$\Rightarrow m_1 = 1 \text{ and } m_2 = 5$$

Now,

$$\Rightarrow y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$$

$$\Rightarrow \therefore y\text{-coordinate of P is } \left(\frac{5}{12}\right) y = \frac{1(-5) + 5\left(\frac{3}{2}\right)}{1+5} = \frac{-5 + \frac{15}{2}}{6}$$

$$= \frac{-10 + 15}{6}$$

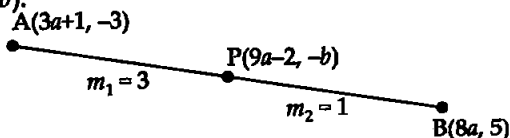
$$\Rightarrow y = \frac{5}{2} \times \frac{1}{6} = \frac{5}{12}$$

y -coordinate of P is $\left(\frac{5}{12}\right)$.

Hence, P divides AB in ratio 1 : 5.

Q12. If point $P(9a - 2, -b)$ divides the line segment joining the points $A(3a + 1, -3)$ and $B(8a, 5)$ in the ratio 3 : 1, then find the values of a and b .

Sol. Point $P(9a - 2, -b)$ divides the line segment joining the points $A(3a + 1, -3)$ and $B(8a, 5)$ in the ratio 3 : 1. But, the coordinates of P are $(9a - 2, -b)$.



Using section formula, we have

$$9a - 2 = \frac{3(8a) + 1(3a + 1)}{3 + 1} \quad \left| \quad -b = \frac{3(+5) + 1(-3)}{3 + 1} \right.$$

$$= \frac{24a + 3a + 1}{4} \quad \Rightarrow \quad -b = \frac{+15 - 3}{4} = \frac{12}{4}$$

$$\Rightarrow 36a - 8 = 27a + 1 \quad \Rightarrow \quad b = -3$$

$$\Rightarrow 36a - 27a = 8 + 1$$

$$\Rightarrow 9a = 9$$

$$\Rightarrow a = \frac{9}{9} = 1$$

Hence, $a = +1$ and $b = -3$

Q13. If (a, b) is mid-point of the line segment joining points $A(10, -6)$ and $B(k, 4)$ and $a - 2b = 18$, then find the value of k and the distance AB .

Sol. Let $P(a, b)$ is the mid-point of the line-segment joining the points $A(10, -6)$ and $B(k, 4)$. Therefore, $P(a, b)$ divides the line segment joining the points $A(10, -6)$ and $B(k, 4)$ in the ratio $1 : 1$.

$$\Rightarrow a = \frac{10+k}{2} \quad (\text{I}) \quad \text{and} \quad b = \frac{-6+4}{2}$$

$$\Rightarrow b = \frac{-2}{2}$$

$$\Rightarrow b = -1 \quad (\text{II})$$

$$\begin{aligned} \text{But,} & a - 2b = 18 \\ \Rightarrow & a - 2(-1) = 18 \end{aligned} \quad (\text{III}) \quad [\text{Given}]$$

$$\Rightarrow a = 18 - 2 \Rightarrow a = 16 \quad [\text{Using (II)}]$$

$$\text{But,} \quad a = \frac{10+k}{2} \quad [\text{From (I)}]$$

$$\Rightarrow 16 = \frac{10+k}{2}$$

$$\Rightarrow 10 + k = 32$$

$$\Rightarrow k = 32 - 10$$

$$\Rightarrow k = 22$$

Now, the co-ordinates of A and B are given by $A(10, -6)$ and $B(22, 4)$.

$$\begin{aligned} \therefore AB^2 &= (22 - 10)^2 + (4 + 6)^2 \\ &= 12^2 + 10^2 = 144 + 100 \end{aligned}$$

$$\Rightarrow AB^2 = 244$$

$$\Rightarrow AB = 2\sqrt{61} \text{ units}$$

Hence, the required value of $k = 22$, $a = 16$, $b = -1$ and $AB = 2\sqrt{61}$ units.

Q14. If the centre of circle is $(2a, a - 7)$ then find the values of a if the circle passes through the point $(11, -9)$ and has diameter $10\sqrt{2}$ units.

Sol. Let $C(2a, a - 7)$ be the centre of the circle and it passes through the point $P(11, -9)$.

$$\therefore PQ = 10\sqrt{2}$$

$$\Rightarrow CP = 5\sqrt{2}$$

$$\Rightarrow CP^2 = (5\sqrt{2})^2 = 50$$

$$\Rightarrow (2a - 11)^2 + (a - 7 + 9)^2 = 50$$

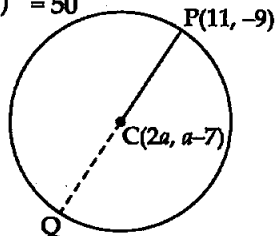
$$\Rightarrow (2a)^2 + (11)^2 - 2(2a)(11) + (a + 2)^2 = 50$$

$$\Rightarrow 4a^2 + 121 - 44a + (a)^2 + (2)^2 + 2(a)(2) = 50$$

$$\Rightarrow 5a^2 - 40a + 125 = 50$$

$$\Rightarrow a^2 - 8a + 25 = 10$$

$$\Rightarrow a^2 - 8a + 25 - 10 = 0$$

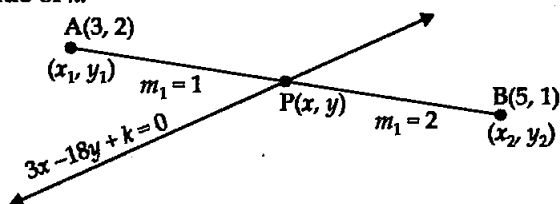


$$\begin{aligned}
 \Rightarrow & a^2 - 8a + 15 = 0 \\
 \Rightarrow & a^2 - 5a - 3a + 15 = 0 \\
 \Rightarrow & a(a-5) - 3(a-5) = 0 \\
 \Rightarrow & (a-5)(a-3) = 0 \\
 \Rightarrow & a-5 = 0 \quad \text{or } a-3 = 0 \\
 \Rightarrow & a = 5 \quad \text{or } a = 3
 \end{aligned}$$

Hence, the required values of a are 5 and 3.

Q15. The line segment joining the points A(3, 2) and B(5, 1) is divided at the point P in the ratio of 1 : 2 and it lies on the line $3x - 18y + k = 0$. Find the value of k .

Sol.



P divides AB in the ratio 1 : 2. Then, the coordinates of $P(x, y)$ are given by

$$\begin{aligned}
 x &= \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2} \\
 \Rightarrow x &= \frac{1(5) + 2(3)}{1 + 2} = \frac{5 + 6}{3} & \Rightarrow y &= \frac{1(1) + 2(2)}{1 + 2} = \frac{1 + 4}{3} \\
 \Rightarrow x &= \frac{11}{3} & \Rightarrow y &= \frac{5}{3}
 \end{aligned}$$

But, $P\left(\frac{11}{3}, \frac{5}{3}\right)$ lies on the line $3x - 18y + k = 0$

$$\begin{aligned}
 \therefore & 3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0 \\
 \Rightarrow & \frac{33}{3} - \frac{90}{3} + k = 0 \\
 \Rightarrow & 33 - 90 + 3k = 0 \\
 \Rightarrow & 3k = 90 - 33 \\
 \Rightarrow & 3k = 57 \\
 \Rightarrow & k = \frac{57}{3} \\
 \Rightarrow & k = 19
 \end{aligned}$$

Hence, the required value of $k = 19$.

Q16. If $D\left(\frac{-1}{2}, \frac{5}{2}\right)$, $E(7, 3)$ and $F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the mid-points of sides of ΔABC , find the area of ΔABC .

Sol. In $\triangle ABC$, D is mid point of BC, E is mid point of AC, and F is mid point of AB.

$$\therefore \triangle DEF \cong \triangle AFE \cong \triangle FBD \\ \cong \triangle EDC$$

So, area of $\triangle ABC = 4$ (area of $\triangle DEF$)
The mid-points of sides of $\triangle ABC$ are

given by $D\left(\frac{-1}{2}, \frac{5}{2}\right)$, $E(7, 3)$, and $F\left(\frac{7}{2}, \frac{7}{2}\right)$.

$$\therefore \text{Area } \triangle DEF = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ = \frac{1}{2} \left[-\frac{1}{2} \left(3 - \frac{7}{2} \right) + 7 \left(\frac{7}{2} - \frac{5}{2} \right) + \frac{7}{2} \left(\frac{5}{2} - 3 \right) \right] \\ = \frac{1}{2} \left[-\frac{1}{2} \left(\frac{-1}{2} \right) + 7(1) + \frac{7}{2} \left(\frac{-1}{2} \right) \right]$$

or

$$= \frac{1}{2} \left[\frac{1}{4} + 7 - \frac{7}{4} \right] \\ = \frac{1}{2} \left[\frac{1 + 28 - 7}{4} \right] \\ = \frac{1}{2} \left(\frac{29 - 7}{4} \right) \\ = \frac{22}{8} = \frac{11}{4}$$

$$\therefore \text{Area of } \triangle ABC = 4 \times \text{Area } \triangle DEF \\ = 4 \times \frac{11}{4} \\ = 11 \text{ square units}$$

Hence, the required area of $\triangle ABC$ is 11 square units.

Q17. The points $A(2, 9)$, $B(a, 5)$ and $C(5, 5)$ are the vertices of a $\triangle ABC$ right angled at B. Find the values of a and hence the area of $\triangle ABC$.

Sol. $\triangle ABC$ is right angled at B.

\therefore By Pythagoras theorem,

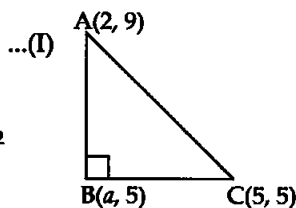
$$AB^2 + BC^2 = AC^2$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow AB^2 = (a - 2)^2 + (5 - 9)^2$$

$$\Rightarrow AB^2 = (a)^2 + (2)^2 - 2(a)(2) + (-4)^2 \\ = a^2 + 4 - 4a + 16$$

$$\Rightarrow AB^2 = a^2 - 4a + 20$$



$$BC^2 = (a-5)^2 + (5-5)^2$$

$$= (a)^2 + (5)^2 - 2(a)(5) + 0^2$$

$$\Rightarrow BC^2 = a^2 + 25 - 10a$$

$$AC^2 = (5-2)^2 + (5-9)^2$$

$$= 3^2 + (-4)^2$$

$$= 9 + 16 = 25$$

$$\Rightarrow AC = \sqrt{25} = 5 \text{ units}$$

$$\therefore a^2 - 4a + 20 + a^2 + 25 - 10a = (5)^2 \quad [\text{From (1)}]$$

$$\Rightarrow 2a^2 - 14a + 45 - 25 = 0$$

$$\Rightarrow 2a^2 - 14a + 20 = 0$$

$$\Rightarrow a^2 - 7a + 10 = 0$$

$$\Rightarrow a^2 - 5a - 2a + 10 = 0$$

$$\Rightarrow a(a-5) - 2(a-5) = 0$$

$$\Rightarrow (a-5)(a-2) = 0$$

$$\Rightarrow a-5 = 0 \quad \text{or} \quad a-2 = 0$$

$$\Rightarrow a = 5 \quad \text{or} \quad a = 2$$

If $a = 5$ then $B(5, 5)$ and $C(5, 5)$ and $BC = 0$, which is not possible.

Hence, $a = 2$.

Now,
$$AB^2 = a^2 - 4a + 20$$

$$= (2)^2 - 4(2) + 20$$

$$= 4 - 8 + 20$$

$$\Rightarrow AB^2 = 24 - 8$$

$$\Rightarrow AB^2 = 16$$

$$\Rightarrow AB = 4 \text{ units}$$

And,
$$BC^2 = a^2 + 25 - 10a$$

$$= (2)^2 + 25 - 10(2)$$

$$= 4 + 25 - 20 = 29 - 20 = 9$$

$$[\because a = 2]$$

$$\Rightarrow BC^2 = 9$$

$$\Rightarrow BC = 3 \text{ units}$$

$$\therefore \text{Area of right angled triangle ABC} = \frac{1}{2} \text{ base} \times \text{altitude}$$

$$= \frac{1}{2} BC \times AB$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ square units}$$

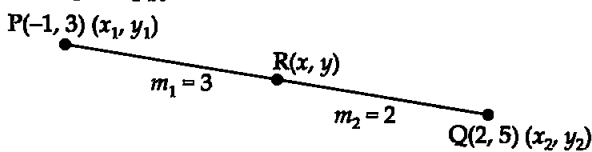
Hence, the value of $a = 2$ and area of $\triangle ABC$ is 6 sq. units.

Q18. Find the coordinates of the point R on the line segment joining

the points $P(-1, 3)$ and $Q(2, 5)$ such that $PR = \frac{3}{5} PQ$.

Sol. $PR = \frac{3}{5}PQ$ [Given]

$$\Rightarrow \frac{5}{3} = \frac{PQ}{PR}$$



$$\Rightarrow \frac{5}{3} = \frac{PR + RQ}{PR}$$

$$\Rightarrow \frac{5}{3} = \frac{PR}{PR} + \frac{RQ}{PR}$$

$$\Rightarrow \frac{QR}{PR} = \frac{5}{3} - 1 = \frac{5-3}{3}$$

$$\Rightarrow \frac{QR}{PR} = \frac{2}{3}$$

or $\frac{PR}{QR} = \frac{3}{2}$ or $PR : QR = 3 : 2$

$\therefore m_1 = 3$ and $m_2 = 2$

Now, the coordinates of point R are given by

$$x = \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$$

$$\Rightarrow x = \frac{3(2) + 2(-1)}{3+2} = \frac{6-2}{5} \quad \Rightarrow \quad y = \frac{3(5) + 2(3)}{3+2} = \frac{15+6}{5}$$

$$\Rightarrow x = \frac{4}{5} \quad \Rightarrow \quad y = \frac{21}{5}$$

Hence, the required coordinates of R are $\left(\frac{4}{5}, \frac{21}{5}\right)$.

Q19. Find the value of k if the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear.

Sol. Points A, B, and C will be collinear if area of $\triangle ABC = 0$

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2}[(k+1)\{2k+3-5k\} + 3k\{5k-2k\} + (5k-1)\{2k-(2k+3)\}] = 0$$

$$\Rightarrow (k+1)(-3k+3) + 3k(3k) + (5k-1)(2k-2k-3) = 0$$

$$\Rightarrow -3(k+1)(k-1) + 3(3k^2) - 3(5k-1) = 0$$

Divide by 3 on both sides, we have

$$[(k+1)(-k+1) + 3k^2 + (5k-1)(-1)] = 0$$

$$\Rightarrow 1 - k^2 + 3k^2 - 5k + 1 = 0$$

$$\Rightarrow 2k^2 - 5k + 2 = 0$$

$$\Rightarrow 2k^2 - 4k - 1k + 2 = 0$$

$$\Rightarrow 2k(k-2) - 1(k-2) = 0$$

$$\Rightarrow (k-2)(2k-1) = 0$$

$$\Rightarrow k-2 = 0 \quad \text{or} \quad 2k-1 = 0$$

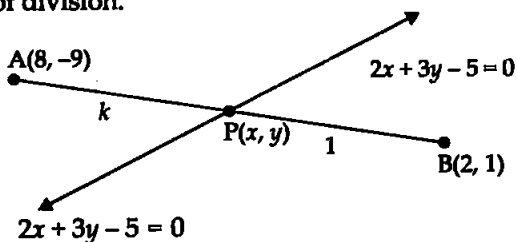
$$\Rightarrow k = 2 \quad \text{or} \quad 2k = 1$$

$$\Rightarrow k = 2 \quad \text{or} \quad k = \frac{1}{2}$$

Hence, the required value of k are 2 and $\frac{1}{2}$.

Q20. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points $(8, -9)$ and $(2, 1)$. Also find the coordinates of the point of division.

Sol.



Let the line given by equation I divides AB at $P(x, y)$ in the ratio $k : 1$. Then, using the section formula, the coordinates of P are given by

$$x = \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$$

$$\Rightarrow x = \frac{k(2) + 1(8)}{(k+1)} \quad \text{and} \quad y = \frac{k(1) + 1(-9)}{k+1}$$

$$\Rightarrow x = \frac{2k+8}{k+1} \quad \text{and} \quad y = \frac{k-9}{k+1}$$

$\Rightarrow P(x, y) = \left(\frac{2k+8}{k+1}, \frac{k-9}{k+1} \right)$ lies on line I so P must satisfy equation (I)

So substitute $x = \frac{2k+8}{k+1}$ and $y = \frac{k-9}{k+1}$ in equation I

$$\Rightarrow 2\left(\frac{2k+8}{k+1}\right) + 3\left(\frac{k-9}{k+1}\right) - 5 = 0$$

On multiplying by $(k + 1)$ in above equation both sides, we get

$$2(2k + 8) + 3(k - 9) - 5(k + 1) = 0$$

$$\Rightarrow 4k + 16 + 3k - 27 - 5k - 5 = 0$$

$$\Rightarrow 2k - 16 = 0$$

$$\Rightarrow k = \frac{16}{2} = 8$$

\therefore Point of intersection is given by $P\left(\frac{2k + 8}{k + 1}, \frac{k - 9}{k + 1}\right)$

$$= P\left(\frac{2 \times 8 + 8}{8 + 1}, \frac{8 - 9}{8 + 1}\right)$$

$$= P\left(\frac{16 + 8}{9}, \frac{-1}{9}\right)$$

$$= P\left(\frac{24}{9}, \frac{-1}{9}\right)$$

$$= P\left(\frac{8}{3}, \frac{-1}{9}\right)$$

Hence, line of eqn. (I) divides AB in ratio 8 : 1 at $P\left(\frac{8}{3}, \frac{-1}{9}\right)$.

EXERCISE 7.4

Q1. If $(-4, 3)$ and $(4, 3)$ are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

Sol. Let $A(-4, 3)$, $B(4, 3)$ and $C(x, y)$ are the three vertices of $\triangle ABC$.

As the triangle is equilateral,

so $AC = BC = AB$

or $AC^2 = BC^2 = AB^2$ (I)

Now, $AB^2 = (4 + 4)^2 + (3 - 3)^2$

$\Rightarrow AB^2 = (8)^2 = 64$

$\Rightarrow AB = 8$ units (II)

$$\begin{aligned} AC^2 &= (x + 4)^2 + (y - 3)^2 \\ &= (x)^2 + (4)^2 + 2(x)(4) + (y)^2 + (3)^2 - 2(y)(3) \\ &= x^2 + y^2 + 8x - 6y + 16 + 9 \end{aligned}$$

$\Rightarrow AC^2 = x^2 + y^2 + 8x - 6y + 25$ (III)

$$\begin{aligned} BC^2 &= (x - 4)^2 + (y - 3)^2 \\ &= (x)^2 + (4)^2 - 2(x)(4) + (y)^2 + (3)^2 - 2(y)(3) \\ &= x^2 + y^2 - 8x - 6y + 16 + 9 \end{aligned}$$

$\Rightarrow BC^2 = x^2 + y^2 - 8x - 6y + 25$ (IV)

Now, $AC^2 = AB^2$ [From (I)]
 $\Rightarrow x^2 + y^2 + 8x - 6y + 25 = 64$ [From (III), (II)]
 $\Rightarrow x^2 + y^2 + 8x - 6y = 64 - 25$
 $\Rightarrow x^2 + y^2 + 8x - 6y = 39$ (V)
 Again, $BC^2 = AB^2$ [From (I)]
 $\Rightarrow x^2 + y^2 - 8x - 6y + 25 = 64$ [From (II), (IV)]
 $\Rightarrow x^2 + y^2 - 8x - 6y = 64 - 25$
 $\Rightarrow x^2 + y^2 - 8x - 6y = 39$ (VI)

Subtracting (V) from (VI), we have

$$\begin{array}{r} x^2 + y^2 - 8x - 6y = 39 \quad \text{(VI)} \\ x^2 + y^2 + 8x - 6y = 39 \quad \text{(V)} \\ \hline -16x = 0 \end{array}$$

$\Rightarrow x = 0$

Putting $x = 0$ in (V), we have

$$\begin{aligned} (0)^2 + y^2 + 8(0) - 6y &= 39 \\ \Rightarrow y^2 - 6y - 39 &= 0 \\ D &= b^2 - 4ac \quad (a = 1, b = -6, c = -39) \\ &= (-6)^2 - 4(1)(-39) = 36 + 156 \\ \Rightarrow D &= 192 \\ \Rightarrow \sqrt{D} &= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3} \\ \Rightarrow \sqrt{D} &= 8\sqrt{3} \\ \therefore y &= \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm 8\sqrt{3}}{2 \times 1} = \frac{2(3 \pm 4\sqrt{3})}{2} \\ \Rightarrow y_1 &= 3 + 4\sqrt{3} \quad \text{and} \quad y_2 = 3 - 4\sqrt{3} \end{aligned}$$

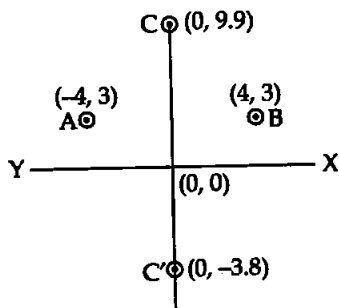
Hence, the third vertex of ΔABC may be $C(0, 3 + 4\sqrt{3})$ and $C'(0, 3 - 4\sqrt{3})$.

Now, $C(0, 3 + 4\sqrt{3})$

$$\begin{aligned} &= C(0, 3 + 4 \times 1.732) \\ &= C(0, 3 + 6.9) \\ &= C(0, 9.9) \end{aligned}$$

and $C'(0, 3 - 4\sqrt{3})$

$$\begin{aligned} &= C'(0, 3 - 4 \times 1.732) \\ &= C'(0, 3 - 6.9) \\ &= C'(0, -3.9) \end{aligned}$$



So, the required point so that origin lies inside it is $(0, 3 - 4\sqrt{3})$.

Q2. A(6, 1), B(8, 2) and C(9, 4) are three vertices of a parallelogram ABCD. If E is the mid point of DC, then find the area of $\triangle ADE$.

Sol. ABCD is a parallelogram so

[Mid point of diagonal BD] = [Mid point of diagonal AC]

$$\therefore \text{Mid point of BD} = \left(\frac{x_4 + 8}{2}, \frac{y_4 + 2}{2} \right)$$

$$\text{and Mid point of AC} = \left(\frac{6 + 9}{2}, \frac{1 + 4}{2} \right)$$

$$\Rightarrow \frac{x_4 + 8}{2} = \frac{15}{2} \quad \text{and} \quad \frac{y_4 + 2}{2} = \frac{5}{2}$$

$$\Rightarrow x_4 = 15 - 8 \quad \text{and} \quad y_4 = 5 - 2$$

$$\Rightarrow x_4 = 7 \quad \text{and} \quad y_4 = 3$$

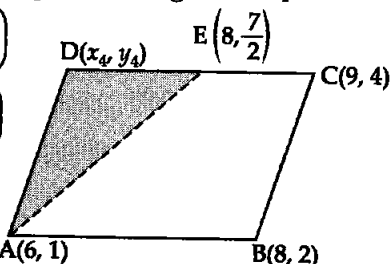
\therefore

$$D = (7, 3)$$

$$\text{Mid point of DC is } E \left(\frac{x_4 + 9}{2}, \frac{y_4 + 4}{2} \right)$$

$$= E \left(\frac{7 + 9}{2}, \frac{3 + 4}{2} \right)$$

$$= E \left(\frac{16}{2}, \frac{7}{2} \right) = E \left(8, \frac{7}{2} \right)$$



$$\text{Now, Area of } \triangle ADE = \frac{1}{2} \left[6 \left(3 - \frac{7}{2} \right) + 7 \left(\frac{7}{2} - 1 \right) + 8(1 - 3) \right]$$

$$= \frac{1}{2} \left[6 \left(\frac{-1}{2} \right) + 7 \left(\frac{5}{2} \right) + 8(-2) \right]$$

$$= \frac{1}{2} \left(3 + \frac{35}{2} - 16 \right) = \frac{1}{2} \left(\frac{-6 + 35 - 32}{2} \right)$$

$$= \frac{1}{2} \times \frac{(-3)}{2} = \frac{-3}{4} \text{ sq units} = \frac{3}{4} \text{ sq. units}$$

[In magnitude]

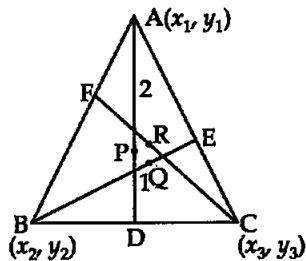
Hence, the area of $\triangle ADE$ is $\frac{3}{4}$ sq. units.

Q3. The points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of $\triangle ABC$.

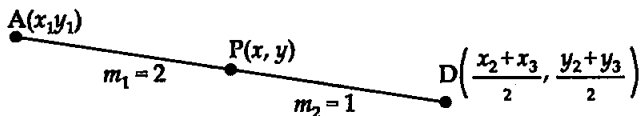
- The median from A meets BC at D. Find the coordinates of the point D.
- Find the coordinates of the point P on AD such that AP : PD = 2 : 1.
- Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.
- What are the coordinates of the centroid of the $\triangle ABC$?

Sol. (i) Median from A meets BC at D i.e., D is the mid-point of BC.

So, the coordinates of D are given by $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$



(ii)



The coordinates of the point P on AD such that AP : PD = 2 : 1 are given by

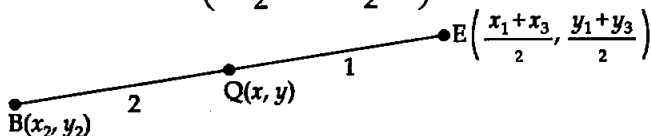
$$x = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2 + 1}, \quad y = \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2 + 1}$$

$$\Rightarrow x = \frac{x_2 + x_3 + x_1}{3}, \quad y = \frac{y_2 + y_3 + y_1}{3}$$

$\therefore P\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ is the required point.

(iii) (a) Median BE meets the side AC at its mid-point E.

\therefore Coordinates of E are $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$.



Now, the coordinates of Q such that BE is median and BQ : QE = 2 : 1 are given by

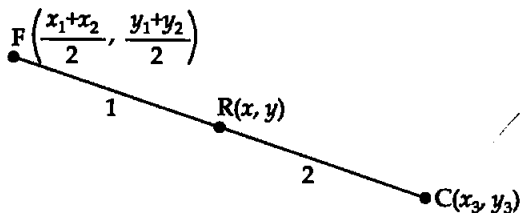
$$x = \frac{2\left(\frac{x_1 + x_3}{2}\right) + 1(x_2)}{2 + 1}, \quad y = \frac{2\left(\frac{y_1 + y_3}{2}\right) + 1(y_2)}{2 + 1}$$

$$\Rightarrow x = \frac{x_1 + x_3 + x_2}{3}, \quad y = \frac{y_1 + y_3 + y_2}{3}$$

\therefore The coordinates of point Q on median BE such that BQ : QE = 2 : 1 are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

(b) Median CF meets the side AB at its mid-point F.

∴ Coordinate of F are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Now, the coordinates of R such that CF is median and $CR : RF = 2 : 1$ are given by

$$x = \frac{1(x_3) + 2\left(\frac{x_1 + x_2}{2}\right)}{1 + 2}, \quad y = \frac{1(y_3) + 2\left(\frac{y_1 + y_2}{2}\right)}{1 + 2}$$

$$\Rightarrow x = \frac{x_3 + x_1 + x_2}{3}, \quad y = \frac{y_3 + y_1 + y_2}{3}$$

So, the coordinates of point R on the median CF such that $CR : RF = 2 : 1$ are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

(iv) Coordinates of centroid G of ΔABC are

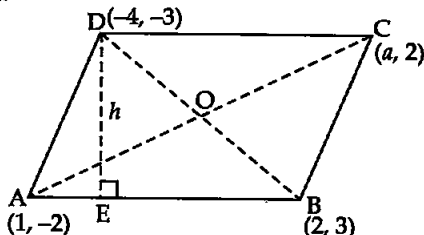
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

It is observed that coordinates of P, Q, R and G are same.

Hence, the medians intersect at the same point *i.e.*, centroid which divides the medians in the ratio 2 : 1.

Q4. If the points $A(1, -2)$, $B(2, 3)$, $C(a, 2)$ and $D(-4, -3)$ form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

Sol. As ABCD is a parallelogram and diagonals of parallelogram bisect each other.



OR

The mid points of diagonals of parallelogram will coincide i.e.,
Mid-point of diagonal AC = Mid-point of diagonal BD

$$\Rightarrow \left(\frac{1+a}{2}, \frac{-2+2}{2} \right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2} \right)$$

$$\Rightarrow \left(\frac{1+a}{2}, 0 \right) = \left(\frac{-2}{2}, 0 \right)$$

$$\Rightarrow \frac{1+a}{2} = \frac{-2}{2}$$

$$\Rightarrow a = -2 - 1 = -3$$

Hence, the value of a is -3 .

Now, Area of $\triangle ABD = \frac{1}{2}$ base \times altitude

$$\Rightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = \frac{1}{2} AB \times h$$

$$\Rightarrow \frac{1}{2} [1(3 - (-3)) + 2(-3 - (-2)) - 4(-2 - 3)] = -\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \frac{1}{2} [(3+3) + 2(-3+2) - 4(-5)] = \frac{h}{2} \sqrt{(2-1)^2 + (3+2)^2}$$

$$\Rightarrow \frac{1}{2} [6 + 2(-1) + 20] = \frac{h}{2} \sqrt{(1)^2 + (5)^2}$$

$$\Rightarrow \frac{1}{2} [6 - 2 + 20] = \frac{h}{2} \sqrt{1+25}$$

$$\Rightarrow \frac{1}{2} [26 - 2] = \frac{h}{2} \sqrt{26}$$

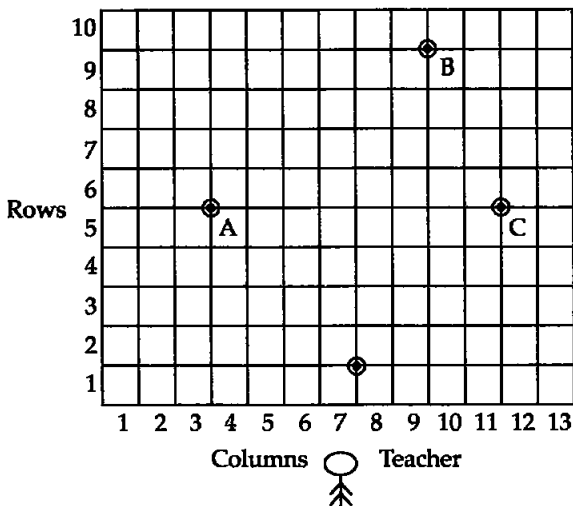
$$\Rightarrow h\sqrt{26} = 24$$

$$\Rightarrow h = \frac{24}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} = \frac{24\sqrt{26}}{26}$$

$$\Rightarrow h = \frac{12}{13} \sqrt{26} \text{ units}$$

Hence, the perpendicular distance between parallel sides AB and CD is $\frac{12\sqrt{26}}{13}$ units.

Q5. Student of a school are standing in rows and columns in their playground for a drill practice. A, B, C, D are the positions of four students as shown in the figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B C and D? If so, what should be his position?



Sol. Coordinates of A, B, C and D from graph are A(3, 5), B(7, 9), C(11, 5), and D(7, 1).

To find the shape of $\square ABCD$:

$$AB^2 = (7 - 3)^2 + (9 - 5)^2 = 4^2 + 4^2 = 4^2(1 + 1)$$

$$\Rightarrow AB = 4\sqrt{2} \text{ units}$$

$$BC^2 = (11 - 7)^2 + (5 - 9)^2 = (4)^2 + (-4)^2 = 4^2(1 + 1)$$

$$\Rightarrow BC = 4\sqrt{2} \text{ units}$$

$$CD^2 = (7 - 11)^2 + (1 - 5)^2 = (-4)^2 + (-4)^2 = 4^2 + 4^2$$

$$\Rightarrow CD = 4\sqrt{2} \text{ units}$$

$$DA^2 = (7 - 3)^2 + (1 - 5)^2 = 4^2 + (-4)^2 = 4^2 + 4^2$$

$$\Rightarrow DA = \sqrt{4^2(1 + 1)} = 4\sqrt{2} \text{ units}$$

$$\therefore AB = BC = CD = DA = 4\sqrt{2} \text{ units.}$$

So, ABCD will be either square or rhombus.

$$\text{Now, Diagonal } AC = \sqrt{(11 - 3)^2 + (5 - 5)^2}$$

$$\Rightarrow AC = \sqrt{(8)^2 + (0)^2}$$

$$\Rightarrow AC = 8 \text{ units}$$

$$\text{and diagonal } BD = \sqrt{(7 - 7)^2 + (1 - 9)^2} = \sqrt{(0)^2 + (8)^2} = \sqrt{0 + (8)^2} = \sqrt{8^2}$$

$$\Rightarrow BD = 8 \text{ units}$$

$$\therefore \text{Diagonal } AC = \text{Diagonal } BD$$

So, the given quadrilateral ABCD is a square. The point which is equidistant from point A, B, C, D of a square ABCD will be at the intersecting point of diagonals and diagonals bisect each other.

Hence, the required point O equidistant from A, B, C, D is mid point of any diagonal = $\left(\frac{7+7}{2}, \frac{9+1}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$.

Hence, the required point is (7, 5).

Q6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8) school at (13, 14) and office at (13, 26) and coordinates are in km.

Sol. Consider the coordinates of house H(2, 4), bank B(5, 8), school S(13, 14) and office O(13, 26).

$$\text{Distance } HB^2 = (5-2)^2 + (8-4)^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$\Rightarrow HB = 5 \text{ km}$$

$$\text{Distance } BS^2 = (13-5)^2 + (14-8)^2 = 8^2 + 6^2 = 64 + 36$$

$$\Rightarrow BS^2 = 100$$

$$\Rightarrow BS = 10 \text{ km}$$

$$\text{Distance } SO^2 = (13-13)^2 + (26-14)^2 = 0^2 + 12^2 = 12^2$$

$$\Rightarrow SO = 12 \text{ km}$$

Total distance travelled by Ayush from house to bank to school and then to office

$$= HB + BS + SO$$

$$= 5 + 10 + 12 = 27 \text{ km}$$

Direct distance from house to office = HO

$$\Rightarrow HO^2 = (13-2)^2 + (26-4)^2 = (11)^2 + (22)^2$$

$$\Rightarrow HO^2 = 121 + 484$$

$$\Rightarrow HO = \sqrt{605} = 24.6 \text{ km}$$

So, extra distance travelled by Ayush = 27 km - 24.6 km = 2.4 km.

Hence, extra distance travelled by Ayush = 2.4 km

□□□