9 Circles

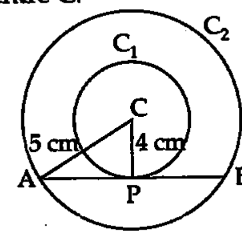
EXERCISE 9.1

Choose the correct answer from the given four options:

Q1. If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is the tangent to the other circle is

(*a*) 3 cm (*b*) 6 cm (*c*) 9 cm (*d*) 1 cm

Sol. (*b*): C_1, C_2 are concentric circles with their centre C.



Chord AB of circle C_2 touches C_1 at P

AB is tangent at P and PC is radius at P.

So, $CP \perp AB$.

$\Rightarrow \angle P = 90°$, CP = 4 cm and CA = 5 cm (Given)

$\therefore$ In right angle ΔPAC,

$$AP^2 = AC^2 - PC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$\Rightarrow$ AP = 3 cm

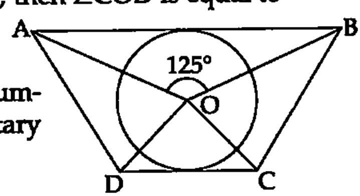
Perpendicular from centre to chord bisects the chord.

So, AB = 2AP = 2 × 3 = 6 cm. Hence, verifies option (*b*).

Q2. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to

(*a*) 62.5° (*b*) 45°

(*c*) 35° (*d*) 55°



Sol. (*d*): We know that a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

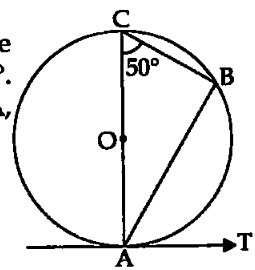
$$\therefore \quad \angle AOB + \angle COD = 180°$$
$$125° + \angle COD = 180°$$
$$\angle COD = 180° - 125° = 55°.$$

Hence, verifies option (*d*).

Q3. In the given figure, AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to

(*a*) 65° (*b*) 60°

(*c*) 50° (*d*) 40°



Sol. (*c*): AC is diameter.

$\Rightarrow \quad \angle B = 90°$ ($\angle$ in a semi-circle)

$\therefore \quad \angle BAC = 180° - \angle C - \angle B$ [Angle sum property of a triangle]

$\Rightarrow \quad \angle BAC = 180° - 50° - 90° = 180° - 140° = 40°$

Tangent AT at A and radius OA at A arc at 90°.

So, $\angle OAT = 90°$

$\therefore \quad \angle OAB + \angle BAT = 90°$

$\Rightarrow \quad 40° + \angle BAT = 90°$

$\Rightarrow \quad \angle BAT = 90° - 40°$

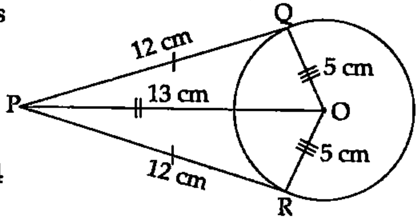
$\Rightarrow \quad \angle BAT = 50°.$

Hence, verifies option (*c*).

Q4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(*a*) 60 cm^2 (*b*) 65 cm^2 (*c*) 30 cm^2 (*d*) 32.5 cm^2

Sol. (*a*): PQ is tangent and QO is radius at contact point Q.



$\therefore \quad \angle PQO = 90°$

$\therefore$ By Pythagoras theorem,

$PQ^2 = OP^2 - OQ^2$

$= 13^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow \quad PQ = 12$ cm

$\therefore \quad \Delta OPQ \cong \Delta OPR$ [By SSS criterion of congruence]

$\therefore \quad$ Area of ΔOPQ = ar ΔOPR

Area of quadrilateral QORP = 2 ar (ΔOPR)

$= 2 \times \frac{1}{2}$ base $\times$ altitude

$= RP \times OR = 12 \times 5 = 60$ cm^2

Hence, verifies the option (*a*).

Q5. At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

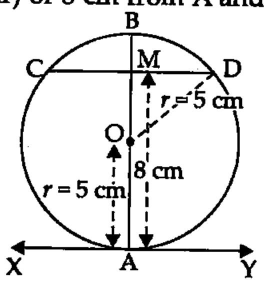
(*a*) 4 cm (*b*) 5 cm (*c*) 6 cm (*d*) 8 cm

Sol. (*d*): XAY is tangent and AO is radius at contact point A of circle.

AO = 5 cm

$\therefore \quad \angle OAY = 90°$

CD is another chord at distance (perpendicular) of 8 cm from A and CMD || XAY meets AB at M.



Join OD.

OD = 5 cm

OM = 8 − 5 = 3 cm

$\angle OMD = \angle OAY = 90°$

Now, in right angled ΔOMD,

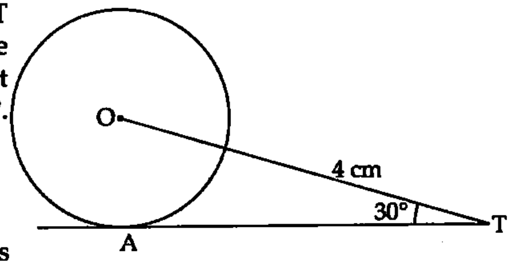
$MD^2 = OD^2 - MO^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow \quad MD = 4$ cm

Perpendicular from centre O of circle bisect the chord. So CD = 2MD = 2 × 4 = 8 cm.

Hence, length of chord CD = 8 cm, which verifies option (*d*).

Q6. In the given figure, AT is a tangent to the circle with centre 'O' such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to

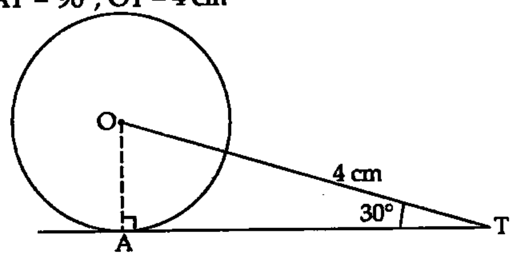


(*a*) 4 cm (*b*) 2 cm

(*c*) $2\sqrt{3}$ cm (*d*) $4\sqrt{3}$ cm

Sol. (*c*): Join OA. OA is radius and AT is tangent at contact point A.

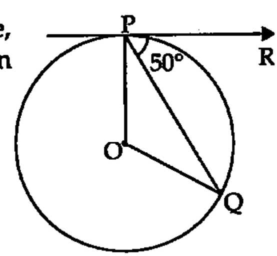
So, ∠OAT = 90°, OT = 4 cm [Given]



Now, $\frac{AT}{4} = \frac{\text{Base}}{\text{Hypotenuse}} = \cos 30° \Rightarrow AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm.

Hence, verifies the option (*c*).

Q7. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal to



(*a*) 100° (*b*) 80°

(*c*) 90° (*d*) 75°

Sol. (*a*): OP is radius and PR is tangent at P.

So, ∠OPR = 90°

⇒ ∠OPQ + 50° = 90°

⇒ ∠OPQ = 90° − 50°

⇒ ∠OPQ = 40°

In ΔOPQ,

OP = OQ [Radii of same circle]

∴ ∠Q = ∠OPQ = 40°

[Angles opposite to equal sides are equal]

But, ∠POQ = 180° − ∠P − ∠Q

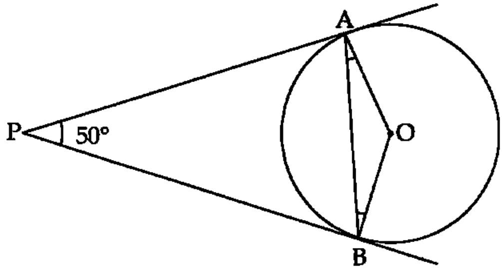
= 180° − 40° − 40° = 180° − 80° = 100°

⇒ ∠POQ = 100°.

Hence, verifies the option (*a*).

Q8. In the given figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to

(*a*) 25° (*b*) 30°
(*c*) 40° (*d*) 50°



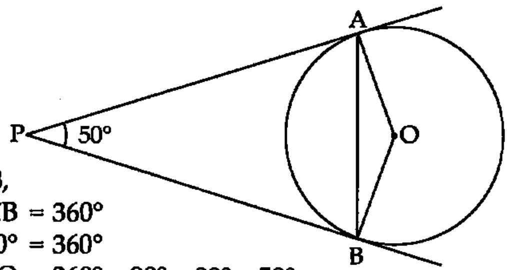
Sol. (*a*): In ΔOAB, we have

$OA = OB$ [Radii of same circle]

$\therefore \quad \angle OAB = \angle OBA$ [Angles opposite to equal sides are equal]

As OA and PA are radius and tangent respectively at contact point A.

So, $\angle OAP = 90°$. Similarly, $\angle OBP = 90°$



Now, in quadrilateral PAOB,

$$\angle P + \angle A + \angle O + \angle B = 360°$$
$$\Rightarrow \quad 50° + 90° + \angle O + 90° = 360°$$
$$\Rightarrow \quad \angle O = 360° - 90° - 90° - 50°$$
$$\Rightarrow \quad \angle O = 130°$$

Again, in ΔOAB,

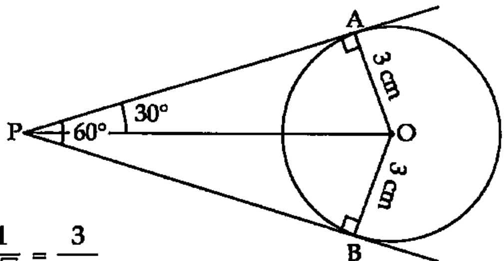
$$\angle O + \angle OAB + \angle OBA = 180°$$
$$\Rightarrow \quad 130° + \angle OAB + \angle OAB = 180° \quad [\because \angle OBA = \angle OAB]$$
$$\Rightarrow \quad 2\angle OAB = 180° - 130° = 50°$$
$$\Rightarrow \quad \angle OAB = 25°$$

Hence, $\angle OAB = 25°$ which verifies option (*a*).

Q9. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(*a*) $\frac{3}{2}\sqrt{3}$ cm (*b*) 6 cm (*c*) 3 cm (*d*) $3\sqrt{3}$ cm

Sol. (*d*): $\because$ OA and PA are the radius and the tangent respectively at contact point A of a circle of radius OA = 3 cm. So, $\angle PAO = 90°$.

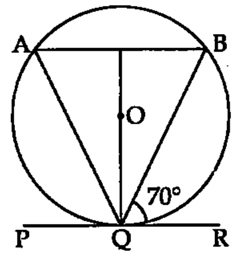


In right angled ΔPOA,

$$\tan 30° = \frac{OA}{PA} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3}{PA}$$

$\Rightarrow \quad PA = 3\sqrt{3}$ which verifies the option (*d*).

Q10. In the given figure, if PQR is the tangent to a circle at Q, whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to

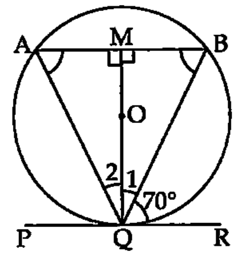


(*a*) 20° (*b*) 40°
(*c*) 35° (*d*) 45°

Sol. (*b*): AB || PQR [Given]

$\angle B = \angle BQR = 70°$ [Alternate interior angles]

and $\angle OQR = \angle AMQ$ [Alternate interior angles]



As PQR and OQ are tangent and radius at contact point Q

$\therefore \quad \angle OQR = 90°$

$\Rightarrow \quad \angle 1 + \angle 70° = 90°$

$\Rightarrow \quad \angle 1 = 90° - 70° = 20°$

$\therefore$ $\angle AMO = 90°$ and perpendicular from centre to chord bisect the chord

So, $\quad MA = MB$

$\angle QMA = \angle QMB$ [Each 90°]

$MQ = MQ$ [Common]

$\therefore \quad \Delta QMA \cong \Delta QMB$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle A = \angle B$

$\Rightarrow \quad \angle A = 70°$ [$\because \angle B = 70°$]

$\therefore \quad \angle A + \angle AMQ + \angle 2 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 70° + 90° + \angle 2 = 180°$

$\Rightarrow \quad \angle 2 = 180° - 160°$

$\Rightarrow \quad \angle 2 = 20°$

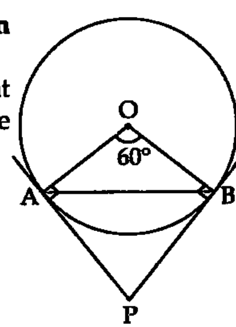
$\therefore \quad \angle AQB = \angle 1 + \angle 2 = 20° + 20° = 40°$

Hence, verifies option (*b*).

EXERCISE 9.2

Write True or False and justify your answer in each of the following:

Q1. If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is also 60°.



Sol. False: Chord AB subtends $\angle 60°$ at O.

$\therefore$ AP and OA are tangent and radius at A.

$\therefore \quad \angle OAP = 90°$

Similarly, $\quad \angle OBP = 90°$

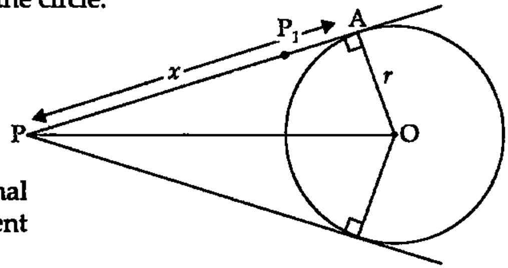
In quadrilateral OAPB,

$$\angle O + \angle P + \angle OAP + \angle OBP = 360^\circ$$
$$\Rightarrow \quad 60^\circ + \angle P + 90^\circ + 90^\circ = 360^\circ$$
$$\Rightarrow \quad \angle P = 360^\circ - 240^\circ$$
$$\Rightarrow \quad \angle P = 120^\circ$$

Hence, the given statement is false.

Q2. The length of tangent from an external point on a circle is always greater than the radius of the circle.

Sol. False: Consider any point P external to a circle away from O.



Now, draw tangent PA on the circle. Clearly,

PA > r [$\because$ P is external to circle and P is at sufficient distance]

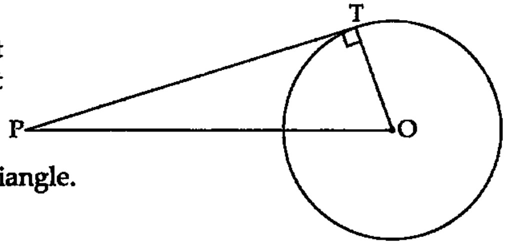
Now, again consider any point P_1 on the tangent AP very near to contact point A of tangent PA, $P_1A < AO$

So, it is clear that the length of the tangent PA and P_1A are greater and smaller respectively than radius OA.

Hence, the length of the tangent from an external point of a circle may or may not be greater than the radius of the circle. Hence, the given statement is false.

Q3. The length of the tangent from an external point P on a circle with centre O is always less than OP.

Sol. True:



PT and OT are the tangent and radius respectively at contact point T.

So, $\angle OTP = 90^\circ$

$\Rightarrow$ ΔOPT is right angled triangle.

Again, in ΔOPT

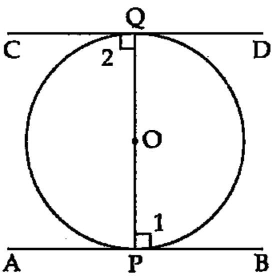
$\because \quad \angle T > \angle O$

$\therefore \quad OP > PT$ [Side opposite to greater angle is larger]

Hence, the given statement is true.

Q4. The angle between two tangents to a circle may be 0°.

Sol. True:

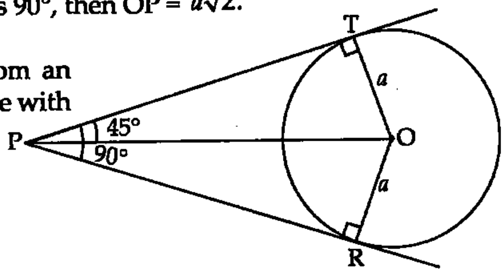


Consider the diameter POQ of a circle with centre O. The tangent at P and Q are drawn, as we know the radius and tangent at contact point are perpendicular so $\angle 1 = \angle 2 = 90^\circ$. These

are alternate angles so the tangent APB || CQD *i.e.,* angle between two tangents to a circle may be zero.
Hence, the given statement is true.

Q5. If the angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 90°, then OP = $a\sqrt{2}$.

Sol. True.



Consider a tangent PT from an external point P on a circle with radius 'a'.
OT and PT are radius and tangent respectively at contact point T.

$\therefore \quad \angle T = 90°$

As $\quad \Delta OPT \cong \Delta OPR \quad$ [By SSS criterion of congruence]

$\therefore \quad \angle OPT = \angle OPR = \frac{90°}{2} = 45°$

$\therefore$ In right angle ΔOPT,

$$\sin 45° = \frac{OT}{OP}$$

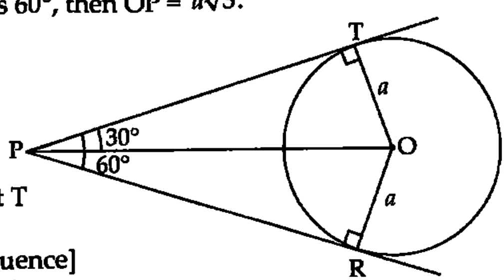
$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{a}{OP}$$

$$\Rightarrow \quad OP = \sqrt{2}a.$$

Hence, the given statement is true.

Q6. If the angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 60°, then OP = $a\sqrt{3}$.

Sol. False: PT and OT are tangent and radius respectively at contact point T.



$\therefore \quad \angle OTP = 90°$

$\Rightarrow \Delta OTP$ is right angle Δ at T

As $\quad \Delta OPT \cong \Delta OPR$

[By SSS criterion of congruence]

$\Rightarrow \quad \angle OPT = \angle OPR = \frac{1}{2} \times 60° = 30°$

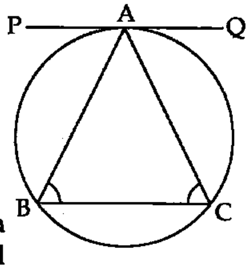
$\therefore$ In right angle ΔOPT,

$$\sin 30° = \frac{OT}{OP} \Rightarrow \frac{1}{2} = \frac{a}{OP} \Rightarrow OP = 2a$$

Hence, the given statement is false.

Q7. The tangent to the circumcircle of an isosceles ΔABC at A, in which AB = AC, is parallel to BC.

Sol. True.



A ΔABC, inscribed in a circle in which $AB = AC$.

PAQ is tangent at A.

AB is chord.

$\therefore \quad \angle PAB = \angle C \quad ...(i)$

$\because$ Angle $\angle PAB$ formed by chord (AB) with tangent is equal to the angle $\angle C$ formed by chord AC in alternate segment.

In ΔABC,

$AB = AC$ [Given]

$\therefore \quad \angle B = \angle C$ [$\because$ Angles opposite to equal sides are equal] ...(*ii*)

From (*i*) and (*ii*), $\angle B = \angle PAB$

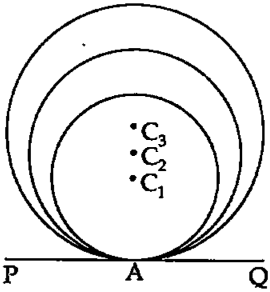
These are alternate interior angles.

So, PAQ || BC

Hence, the given statement is true.

Q8. If a number of circles touch a given line segment PQ at a point A, then their centres lies on the perpendicular bisector of PQ.

Sol. False:



C_1A and PAQ are radius and tangent at contact point A.

$\therefore \quad \angle C_1AP = 90° \Rightarrow C_1A \perp PQ$

Similarly, $\angle C_2AP = 90° \Rightarrow C_2A \perp PQ$

$\angle C_3AP = 90° \Rightarrow C_3A \perp PQ$

We know that perpendicular on any point of a segment PQ may be only one.

So, point segments C_1A, C_2A, C_3A, C_4A, ... will be on a line.

$\Rightarrow C_1A, C_2A, C_3A, C_4A$ will lie on a line, which is perpendicular on PQ at A.

As A is not mid point of PQ. So, the perpendicular AB will not be perpendicular bisector of PQ.

Hence, the given statement is false.

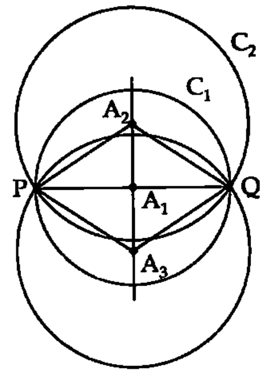
Q9. If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Sol. True: Centre of any circle passing through the end points P and Q of a line segment are equidistant from P and Q.

$\therefore \quad A_1P = A_1Q$

$A_2P = A_2Q$

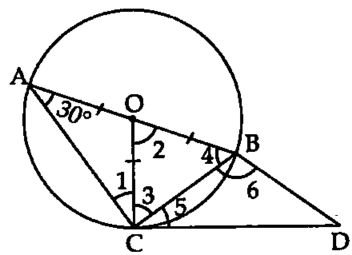
$A_3P = A_3Q$



as we know that any point on perpendicular bisector of a segment is equidistant from the end points of the segment. Hence, A_1, A_2, A_3 points are the centres of circles passing through the end points P and Q of a segment PQ or the centres of circles lie on the perpendicular bisector of PQ.

Q10. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.

Sol. True:



CD is a tangent at contact point C. AOB is diameter which meets tangent produced at D.

Chord AC makes ∠A = 30° with diameter AB.

To prove: BD = BC

Proof: In ΔOAC,

OA = OC = r [Radii of same circle]

∠1 = ∠A [∠s opp. to equal sides are equal]

⇒ ∠1 = 30° [∵ ∠A = 30°]

Exterior ∠BOC = ∠2 = ∠1 + ∠A = (30° + 30°) = 60°

Now, in ΔOCB,

OC = OB [Radii of same circle]

∴ ∠3 = ∠4 [Angles opposite to equal sides are equal]

∠3 + ∠4 + ∠COB = 180°

⇒ ∠3 + ∠3 + 60° = 180° [Angle sum property of triangle]

⇒ 2 ∠3 = 180° – 60° = 120°

⇒ ∠3 = 60° = ∠4

∠6 + ∠4 = 180° [Linear pair axiom]

⇒ ∠6 = 180° – ∠4

= 180° – 60°

⇒ ∠6 = 120°

∵ Tangent CD and radius CO are at contact point C.

∴ ∠OCD = 90°

⇒ ∠3 + ∠5 = 90°

⇒ 60° + ∠5 = 90°

⇒ ∠5 = 30°

Now, in ΔBCD, we have

$\angle D + \angle 5 + \angle 6 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle D = 180° - \angle 5 - \angle 6$

$= 180° - 30° - 120° = 180° - 150°$

$\Rightarrow \angle D = 30°$

$\therefore \angle D = \angle 5 = 30°$

$\Rightarrow BC = BD$

[Sides opposite to equal $\angle$s of a triangle are equal]

Hence, verifies the given statement true.

EXERCISE 9.3

Q1. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

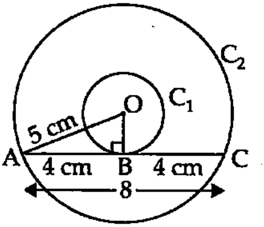
Sol. Given: Two concentric circles C_1 and C_2 with centre O.

Chord AC of circle C_2 is tangent of circle C_1 at B.

We know that tangent AC and radius BO at point B are perpendicular.

$\therefore$ Perpendicular from centre to chord bisects the chord.

$\therefore \quad AB = CB = \frac{AC}{2} = \frac{8}{2} = 4$ cm



In right angle ΔABO,

$OB^2 = OA^2 - AB^2$

[By Pythagoras theorem]

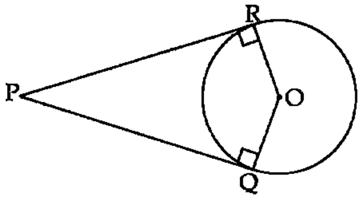
$= 5^2 - 4^2 = 25 - 16 = 9$

$\Rightarrow OB = 3$ cm

Hence, radius of circle C_1 is 3 cm.

Q2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Sol. Given: Tangents PR and PQ from an external point P to a circle with centre O.



To prove: Quadrilateral QORP is cyclic.

Proof: RO and RP are the radius and tangent respectively at contact point R.

$\therefore \quad \angle PRO = 90°$

Similarly, $\angle PQO = 90°$

In quadrilateral QORP, we have

$\angle P + \angle R + \angle O + \angle Q = 360°$

$\Rightarrow \angle P + \angle 90° + \angle O + \angle 90° = 360°$

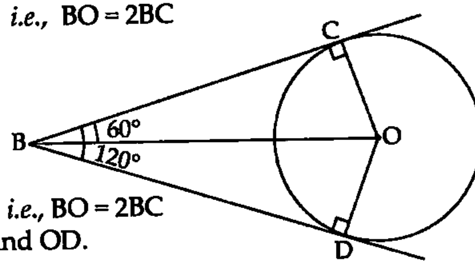
$\Rightarrow \angle P + \angle O = 360° - 180° = 180°$

These are opposite angles of quadrilateral QORP and are supplementary.

$\therefore$ Quadrilateral QORP is cyclic. Hence, proved.

Q3. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that $\angle DBC = 120°$, prove that

$$BC + BD = BO, \text{ i.e., } BO = 2BC$$



Sol. Given: A circle with centre O.
Tangents BC and BD are drawn from an external point B such that $\angle DBC = 120°$.

To prove: $BC + BD = BO$, *i.e.*, $BO = 2BC$

Construction: Join OB, OC and OD.

Proof: In ΔOBC and ΔOBD, we have

$OB = OB$ [Common]

$OC = OD$ [Radii of same circle]

$BC = BD$ [Tangents from an external point are equal in length] ...(*i*)

$\therefore \quad \Delta OBC \cong \Delta OBD$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle OBC = \angle OBD$ (CPCT)

$\therefore \quad \angle OBC = \frac{1}{2}\angle DBC = \frac{1}{2} \times 120°$ [$\because \angle CBD = 120°$ given]

$\Rightarrow \quad \angle OBC = 60°$

OC and BC are radius and tangent respectively at contact point C.

So, $\quad \angle OCB = 90°$

Now, in right angle ΔOCB, $\angle OBC = 60°$

$\therefore \quad \cos 60° = \frac{BC}{BO}$

$\Rightarrow \quad \frac{1}{2} = \frac{BC}{BO}$

$\Rightarrow \quad OB = 2BC$

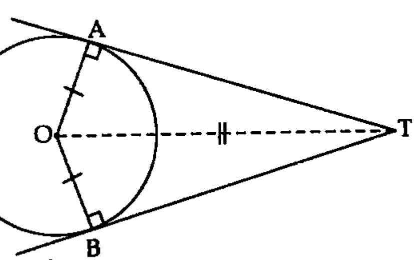
Hence, proved (*ii*) part.

$\Rightarrow \quad OB = BC + BC$

$\Rightarrow \quad OB = BC + BD$ [$\because BC = BD$ from (*i*)]

Hence, proved.

Q4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.



Sol. Given: Two intersecting lines AT and BT intersect at T.
A circle with centre O touches the above lines at A and B.

To prove: OT bisects the $\angle ATB$.

Construction: Join OA and OB.

Proof: OA is radius and AT is tangent at A.

∴ $\angle OAT = 90°$

Similarly, $\angle OBT = 90°$

In ΔOTA and ΔOTB, we have

$\angle OAT = \angle OBT = 90°$

$OT = OT$ [Common]

$OA = OB$ [Radii of same circle]

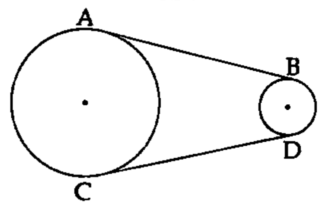
∴ $\Delta OTA \cong \Delta OTB$ [By RHS criterion of congruence]

⇒ $\angle OTA = \angle OTB$ [CPCT]

⇒ Centre of circle 'O' lies on the angle bisector of $\angle ATB$.

Hence, proved.

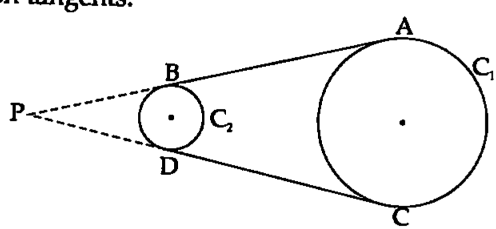
Q5. In the given figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.



Sol. Given: Circles C_1 and C_2 of radii r_1 and r_2 respectively and $r_1 < r_2$. AB and CD are two common tangents.

To prove: AB = CD

Construction: Produce AB and CD upto point P where both tangents meet.



Proof: Tangents from an external point to a circle are equal.

For circle C_1, $PB = PD$ ...(*i*)

and for circle C_2, $PA = PC$ ...(*ii*)

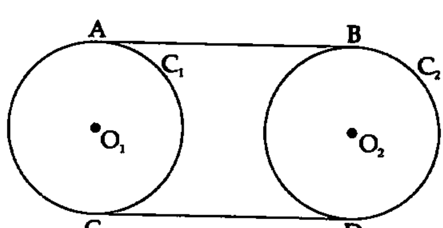
Subtracting (*i*) from (*ii*), we have

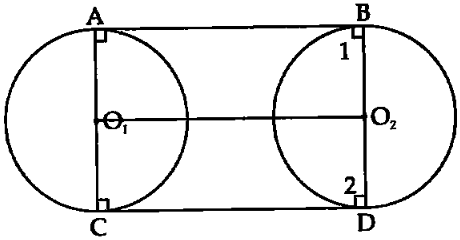
$PA - PB = PC - PD$

⇒ $AB = CD$.

Hence, proved.

Q6. In Question 5 above, if radii of the two circles are equal, prove that AB = CD.





Sol. Given: Two circles of equal radii, two common tangents, AB and CD on circles C_1 and C_2.

To prove: AB = CD

Construction: Join O_1A, O_1C and O_2B and O_2D. Also, join O_1O_2.

Proof: Since tangent at any point of a circle is perpendicular to the radius to the point of contact.

$\therefore$ $$\angle O_1AB = \angle O_2BA = 90°$$

As $O_1A = O_2B$, so O_1ABO_2 is a rectangle.

Since opposite sides of a rectangle are equal,

$\therefore$ $$AB = O_1O_2 \qquad ...(i)$$

Similarly, we can prove that O_1CDO_2 is a rectangle.

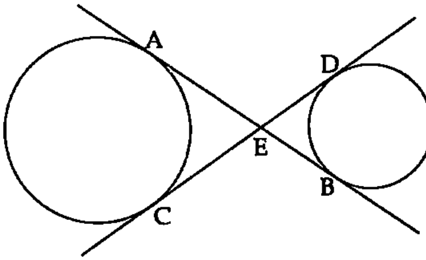
$\therefore$ $$O_1O_2 = CD \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$$AB = CD.$$

Hence, proved.

Q7. In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



Sol. Given: Two non-intersecting circles are shown in the figure. Two intersecting tangents AB and CD intersect at E. E point is between the circles and outside also.

To prove: AB = CD

Proof: We know that tangents drawn from an external point (E) to a circle are equal. Point E is outside of both the circles.

So, $$EA = EC \qquad ...(i)$$

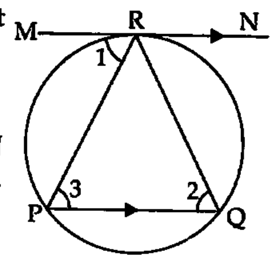
$$EB = ED \qquad ...(ii)$$

$\Rightarrow$ $$EA + EB = EC + ED \qquad \text{[Adding (i) and (ii)]}$$

$\Rightarrow$ $$AB = CD$$

Hence, proved.

Q8. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.



Sol. Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN

To prove: R bisects the arc PRQ.

Construction: Join RP and RQ.

Proof: Chord RP subtends ∠1 with tangent MN and ∠2 in alternate segment of circle so ∠1 = ∠2.

MRN || PQ

∴ ∠1 = ∠3 [Alternate interior angles]

⇒ ∠2 = ∠3

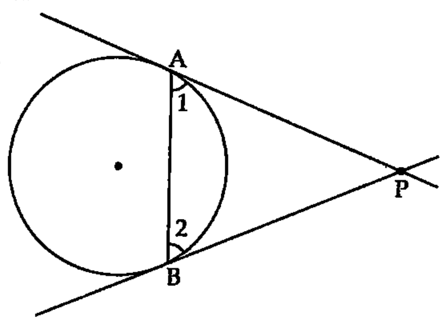
⇒ PR = RQ [Sides opp. to equal ∠s in ΔRPQ]

∵ Equal chords subtend equal arcs in a circle so

arc PR = arc RQ

or R bisect the arc PRQ. Hence, proved.

Q9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Sol. Given: A chord AB of a circle, tangents AP and BP at A and B respectively are drawn.

To prove: ∠PAB = ∠PBA

Proof: We know that tangents drawn from an external point P to a circle are equal so PA = PB.

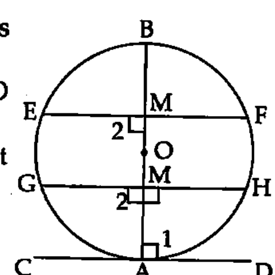
⇒ ∠2 = ∠1

[Angles opposite to equal sides of a triangle are equal]

Hence, tangents PA and PB make equal angles with chord AB.

Hence, proved.

Q10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.



Sol. Given: A circle with centre O and AOB is diameter.

CAD is a tangent at A. Chord EF || tangent CAD

To prove: AB bisects any chord EF || CAD.

Proof: OA radius is perpendicular to tangent CAD.

∴ ∠1 = 90°

CAD || EF [Given]

∴ ∠1 = ∠2 = 90° [Alternate interior angles]

Point M is on diameter which passes through centre O.

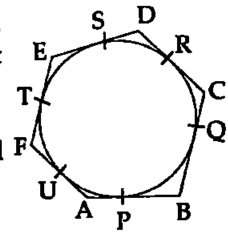
∵ Perpendicular drawn from centre to chord bisect the chord.

Hence, AB bisects any chord EF || CAD.

EXERCISE 9.4

Q1. If a hexagon ABCDEF circumscribe a circle, then prove that

$$AB + CD + EF = BC + DE + FA$$



Sol. Given: A circle inscribed in a hexagon ABCDEF. Sides, AB, BC, CD, DE and DF touches the circle at P, Q, R, S, T and U respectively.

To prove: AB + CD + EF = BC + DE + FA

Proof: We know that tangents from an external point to a circle are equal.

Here, vertices of hexagon are outside the circle so

$$AP = AU$$
$$BP = BQ$$
$$CQ = CR$$
$$DR = DS$$
$$ES = ET$$
$$FT = FU$$

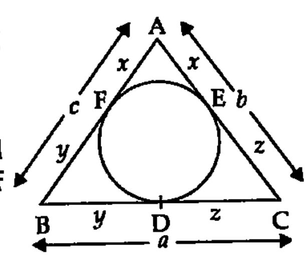
$$LHS = AB + CD + EF = (AP + PB) + (DR + CR) + (ET + TF)$$

By using above results, we have

$$LHS = AB + CD + EF = AU + BQ + DS + CQ + ES + FU$$
$$= AU + FU + BQ + CQ + DS + ES$$
$$= AF + BC + DE.$$

Hence, proved.

Q2. Let s denotes the semi-perimeter of a ΔABC in which $BC = a$, $CA = b$, $AB = c$. If a circle touches the sides BC, CA, AB at D, E, F respectively, prove that $BD = s - b$.



Sol. Given: A circle inscribed in ΔABC touches the sides BC, CA and AB at D, E, F respectively.

To prove: $BD = s - b$

Proof: Tangents drawn from an external point to the circle are equal. Vertices of ΔABC are in the exterior of circle. So,

$$AF = AE = x$$
$$BF = BD = y$$
$$CD = CE = z$$

Now, $AB + BC + CA = c + a + b$

$\Rightarrow AF + BF + BD + DC + AE + CE = a + b + c$

$\Rightarrow x + y + y + z + x + z = a + b + c$

$\Rightarrow 2x + 2y + 2z = a + b + c$

$\Rightarrow 2(x + y + z) = a + b + c$

$\Rightarrow x + y + z = \dfrac{a + b + c}{2}$

$\Rightarrow \quad x + y + z = s$ [Given]

$\Rightarrow \quad y = s - (x + z) \Rightarrow y = s - x - z$

$\Rightarrow \quad y = s - (AE + EC)$

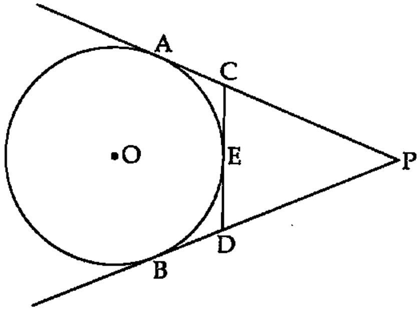
$\Rightarrow \quad = s - AC$

$\Rightarrow \quad BD = s - b$

Hence, proved.

Q3. From an external point P, two tangents PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of ΔPCD.

Sol. Given: A circle with centre O. PA, PB are tangents from an external point P. A tangent CD at E intersect AP and PB at C and D respectively.



To find: Perimeter of ΔPCD.

Method: Tangents drawn from an external point to a circle are equal.

$\therefore \quad PA = PB = 10$ cm [Given]

$CA = CE$

$DE = DB$

Perimeter of $\Delta PCD = PC + PD + CD$

$= PC + PD + CE + DE$

$= PC + CE + PD + DE$

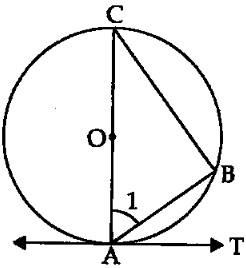
$= PC + CA + PD + DB$

$= PA + PB$

$= 10 + 10 = 20$ cm

$\therefore$ Perimeter of $\Delta PCD = 20$ cm.

Q4. If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB.



Sol. Given: Chord AB, diameter AOC and tangent at A of a circle with centre O.

To prove: $\angle BAT = \angle ACB$

Proof: Radius OA and tangent AT at A are perpendicular.

$\therefore \quad \angle OAT = 90°$

$\Rightarrow \quad \angle BAT = 90° - \angle 1$ ...(*i*)

AOC is diameter.

$\therefore \quad \angle B = 90°$

$\Rightarrow \quad \angle C + \angle 1 = 90°$

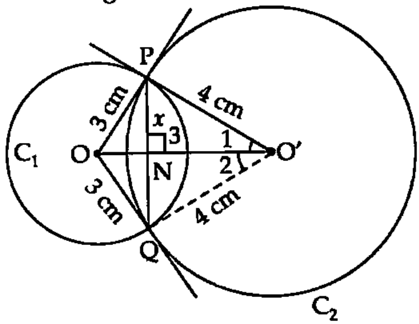
$\Rightarrow \quad \angle C = 90° - \angle 1$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\angle BAT = \angle ACB. \text{ Hence, proved.}$$

Q5. Two circles with centres O and O′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O′P are tangents to the two circles. Find the length of common chord PQ.

Sol. PO′ is tangent on circle C_1 at P. OP is tangent on circle C_2 at P. As radius OP and tangent PO′ are at a point of contact P



$\therefore \quad \angle P = 90°$

So, by Pythagoras theorem in right angled $\Delta OPO'$,

$OO'^2 = OP^2 + PO'^2 = 3^2 + 4^2 = 9 + 16$
$= 25$ cm

$\Rightarrow \quad OO' = 5$ cm

$\Delta OO'P \cong \Delta OO'Q$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle 1 = \angle 2$

$\Delta O'NP \cong \Delta O'NQ$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle 3 = \angle O'NQ$ [CPCT]

$\Rightarrow \quad \angle 3 = \angle O'NQ = 90°$ [Linear Pair axiom]

Let ON = y, then NO′ = $(5 - y)$

Let PN = x

By Pythagoras theorem in ΔPNO and $\Delta PNO'$, we have

$$x^2 + y^2 = 3^2 \quad \text{...}(i)$$

$$x^2 + (5 - y)^2 = 4^2$$

$$x^2 + 25 + y^2 - 10y = 16 \quad \text{...}(ii)$$

$$x^2 + y^2 = 9 \quad \text{[From } (i)\text{]}$$

$$- \qquad - \qquad -$$

$$25 - 10y = 7 \quad \text{[Subtract } (i) \text{ from } (ii)\text{]}$$

$\Rightarrow \quad -10y = 7 - 25$

$\Rightarrow \quad -10y = -18$

$\Rightarrow \quad y = 1.8$

But, $\quad x^2 + y^2 = 3^2$ [From (*i*)]

$\Rightarrow \quad x^2 + (1.8)^2 = 3^2$

$\Rightarrow \quad x^2 = 9 - 3.24$

$\Rightarrow \quad x^2 = 5.76$

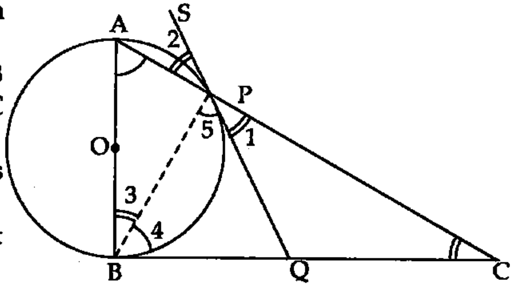
$\Rightarrow \quad x = 2.4$

$\therefore$ The perpendicular drawn from the centre bisects the chord.

$\therefore \quad PQ = 2PN = 2x$
$= 2 \times 2.4$

$\Rightarrow \quad PQ = 4.8$ cm

Q6. In a right triangle ABC in which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.



Sol. Given: ΔABC in which

∠B = 90°

Circle with diameter AB intersect the hypotenuse AC at P.

A tangent SPQ at P is drawn to meet BC at Q.

To prove: Q is mid point of BC.

Construction: Join PB.

Proof: SPQ is tangent and AP is chord at contact point P.

∴ ∠2 = ∠3 [Angles in alternate segment of circle]

∠2 = ∠1 [Vertically opposite angles]

⇒ ∠3 = ∠1 ...(*i*) [From above two relations]

∠ABC = 90° [Given]

OB is radius so, BC will be tangent at B.

∴ ∠3 = 90° – ∠4 ...(*ii*)

∠APB = 90° [∠ in a semi circle]

⇒ ∠C = 90° – ∠4 ...(*iii*)

From (*ii*) and (*iii*), ∠C = ∠3

Using (*i*), ∠C = ∠1

⇒ CQ = QP ...(*iv*) [Sides opp. to = ∠s in ΔQPC]

∠4 = 90° – ∠3

∠5 = 90° – ∠1 [From fig.]

∠3 = ∠1

∴ ∠4 = ∠5

⇒ PQ = BQ ...(*v*) [Sides opp. to equal angles in ΔQPB]

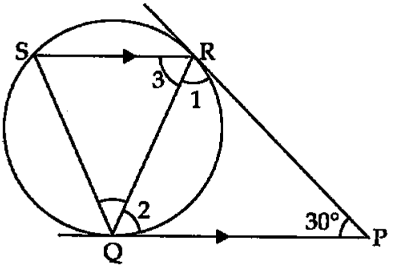
From (*iv*) and (*v*),

BQ = CQ

Therefore, Q is mid-point of BC. Hence, proved.

Q7. In the given figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to tangent PQ. Find the ∠RQS.

[Hint: Draw a line through Q and perpendicular to QP.]



Sol. In ΔPRQ, PQ and PR are tangents from an external point P to circle.

$\therefore$ $PR = PQ$

$\Rightarrow$ $\angle 2 = \angle 1$ [$\angle$s opp. to equal sides in ΔPRQ are equal]

$\angle 1 + \angle 2 + \angle RPQ = 180°$ [Int. $\angle$s of Δ]

$\Rightarrow$ $\angle 1 + \angle 1 + 30° = 180°$

$\Rightarrow$ $2\angle 1 = 180° - 30°$

$\Rightarrow$ $\angle 1 = \dfrac{150°}{2}$

$\therefore$ $\angle 1 = \angle 2 = 75°$

Tangent PQ || SR [Given]

$\therefore$ $\angle 2 = \angle 3 = 75°$ [Alternate interior angles]

PQ is tangent at Q and QR is chord at Q.

$\therefore$ $\angle S = \angle 2 = 75°$ [$\angle$s in alternate segment of circle]

In ΔSRQ,

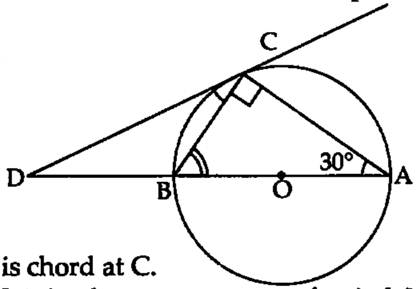
$\angle S + \angle 3 + \angle SQR = 180°$ [Angle sum property of a triangle]

$\Rightarrow$ $75° + 75° + \angle SQR = 180°$

$\Rightarrow$ $\angle SQR = 180° - 150°$

$\Rightarrow$ $\angle SQR = 30°$

Q8. AB is a diameter and AC is chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects extended AB at a point D. Prove that BC = BD.



Sol. Given: A circle with centre O.

A tangent CD at C.

Diameter AB is produced to D.

BC and AC chords are joined, $\angle BAC = 30°$.

To prove: BC = BD

Proof: DC is tangent at C and, CB is chord at C.

$\therefore$ $\angle DCB = \angle BAC$ [$\angle$s in alternate segment of a circle]

$\Rightarrow$ $\angle DCB = 30°$ ...(*i*) [$\because$ $\angle BAC = 30°$ (Given)]

AOB is diameter. [Given]

$\therefore$ $\angle BCA = 90°$ [Angle in a semi circle]

$\therefore$ $\angle ABC = 180° - 90° - 30° = 60°$

In ΔBDC,

Exterior $\angle ABC = \angle D + \angle BCD$

$\Rightarrow$ $60° = \angle D + 30°$

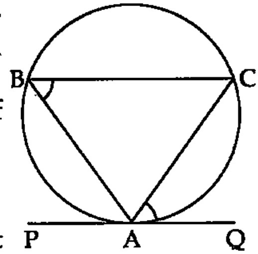
$\Rightarrow$ $\angle D = 30°$ ...(*ii*)

$\therefore$ $\angle DCB = \angle D = 30°$ [From (*i*), (*ii*)]

$\Rightarrow$ $BD = BC$ [$\because$ Sides opposite to equal angles are equal in a triangle]

Hence, proved.

Q9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.



Sol. Given: arc BAC in which A is mid point of arc BAC.
PAQ is tangent at A.
To prove: BC || PAQ
Proof: PAQ is tangent and CAB is an arc at contact point A.

∴ $\angle CAQ = \angle B$ ...(*i*)
[Angles in alternate segment of a circle]

A is mid point of arc BAC.

∴ min. arc AB = min. arc AC

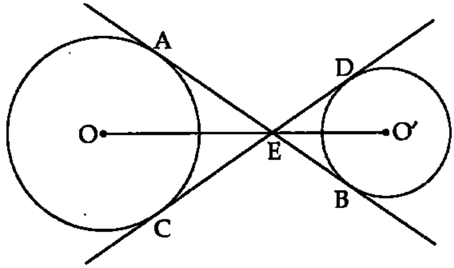
⇒ Chord AB = Chord AC [Equal arcs subtend equal chords]

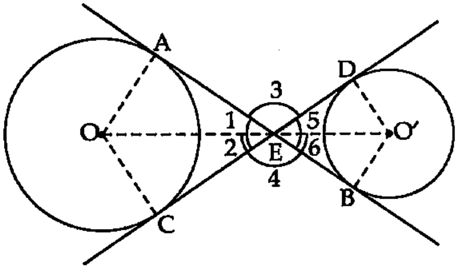
⇒ $\angle C = \angle B$ ...(*ii*) [Angles opp. to equal sides in ΔABC are equal]

⇒ $\angle C = \angle CAQ$ [From (*i*) and (*ii*)]

These are alternate interior angles and are equal.
∴ BC || PAQ.
Hence, proved.

Q10. In the given figure, the common tangents, AB and CD to two circles with centres O and O′ intersect at E. Prove that the points O, E and O′ are collinear.





Sol. Given: Two circles (non intersecting) with their centres O and O′.
Two common tangents AB and CD intersect at E between the circles.
To prove: O, E, O′ points are collinear.
Construction: Join OA, OC, O′D, O′B and EO and EO′

Proof: In ΔAEO and ΔCEO,

$OE = OE$ [Common]

$OA = OC$ [Radii of same circle]

$EA = EC$ [Tangents from an external point to a circle are equal in length]

$\therefore \quad \angle OEA \cong \angle OEC$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle OEA = \angle OEC$ [CPCT]

$\therefore \quad \angle 1 = \angle 2$ [CPCT]

Similarly, $\angle 5 = \angle 6$

and $\angle 3 = \angle 4$ [Vertically opposite angles]

Since sum of angles at a point = 360°

$\therefore \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

$\Rightarrow \quad 2(\angle 1 + \angle 3 + \angle 5) = 360°$

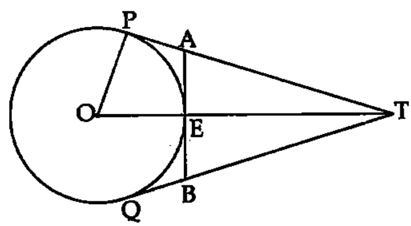
$\Rightarrow \quad \angle 1 + \angle 3 + \angle 5 = 180°$

$\Rightarrow \quad \angle OEO' = 180°$

$\therefore$ OEO′ is a straight line.

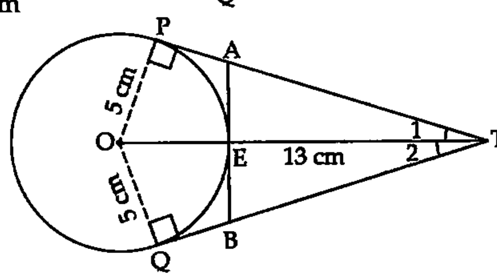
Hence, O, E and O′ are collinear.

Q11. In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



Sol. $OP = OQ = 5$ cm

$OT = 13$ cm



OP and PT are radius and tangent respectively at contact point P.

$\therefore \quad \angle OPT = 90°$

So, by Pythagoras theorem, in right angled ΔOPT,

$PT^2 = OT^2 - OP^2 = 13^2 - 5^2$

$= 169 - 25 = 144$

$\Rightarrow \quad PT = 12$ cm.

AP and AE are two tangents from an external point A to a circle.

$\therefore \quad AP = AE$

AEB is tangent and OE is radius at contact point E.

So, $AB \perp OT$ ...(i)

So, by Pythagoras theorem, in right angled ΔAET,

$AE^2 = AT^2 - ET^2$

$\Rightarrow \quad AE^2 = (PT - PA)^2 - [TO - OE]^2$

$$= (12 - AE)^2 - (13 - 5)^2$$

$\Rightarrow \quad AE^2 = (12)^2 + (AE)^2 - 2(12)(AE) - (8)^2$

$\Rightarrow AE^2 - AE^2 + 24AE = 144 - 64$

$\Rightarrow \quad 24AE = 80$

$\Rightarrow \quad AE = \frac{80}{24}$ cm

$\Rightarrow \quad AE = \frac{10}{3}$ cm

In ΔTPO and ΔTQO,

$OT = OT$ [Common]

$PT = QT$ [Tangents from T]

$OP = OQ$ [Radii of same circle]

$\therefore \quad \Delta TPO \cong \Delta TQO$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle 1 = \angle 2$ ...(*ii*) [CPCT]

In ΔETA and ΔETB,

$ET = ET$ [Common]

$\angle TEA = \angle TEB = 90°$ [From (*i*)]

$\angle 1 = \angle 2$ [CPCT] [From (*ii*)]

$\therefore \quad \Delta ETA \cong \Delta ETB$ [By ASA criterion of congruence]

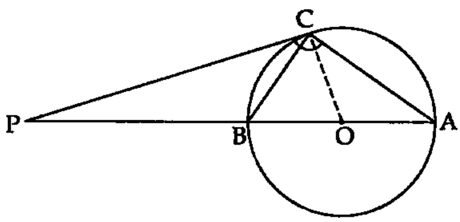
$\Rightarrow \quad AE = BE$ [CPCT]

$\Rightarrow \quad AB = 2AE = 2 \times \frac{10}{3}$

$\Rightarrow \quad AB = \frac{20}{3}$ cm.

Hence, the required length is $\frac{20}{3}$ cm.

Q12. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110°$, find $\angle CBA$. [**Hint:** Join C with centre O].



Sol. OC and CP are radius and tangent respectively at contact point C.

So, $\quad \angle OCP = 90°$

$\angle OCA = \angle ACP - \angle OCP$

$\Rightarrow \quad \angle OCA = 110° - 90°$

$\Rightarrow \quad \angle OCA = 20°$

In ΔOAC,

$OA = OC$ [Radii of same circle]

$\therefore \quad \angle OCA = \angle A = 20°$ [$\because$ Angles opposite to equal sides are equal]

CP and CB are tangent and chord of a circle.

$\therefore$ $\angle CBP = \angle A$ [Angles in alternate segments are equal]

In ΔCAP,

$\angle P + \angle A + \angle ACP = 180°$ [Angle sum property of a triangle]

$\Rightarrow$ $\angle P + 20° + 110° = 180°$

$\Rightarrow$ $\angle P = 180° - 130°$

$\Rightarrow$ $\angle P = 50°$

In ΔBPC,

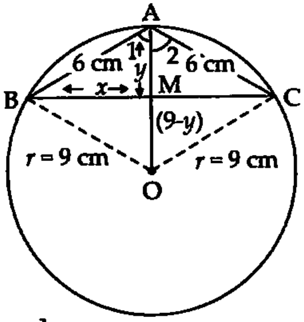
Exterior angle $\angle CBA = \angle P + \angle BCP$

$\Rightarrow$ $\angle CBA = 50° + 20°$

$\Rightarrow$ $\angle CBA = 70°$

Q13. If an isosceles ΔABC in which $AB = AC = 6$ cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

Sol. In figure, ΔABC has $AB = AC = 6$ cm.



In ΔOAB and ΔOAC,

$AB = AC$ [Given]

$OA = OA$ [Common]

$OB = OC$ [Radii of same circle]

$\therefore$ $\Delta OAB \cong \Delta OAC$

[By SSS criterion of congruence]

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]

In ΔAMC and ΔAMB,

$\angle 1 = \angle 2$ [Proved above]

$AM = AM$ [Common]

$AB = AC$ [Given]

$\therefore$ $\Delta AMB \cong \Delta AMC$ [By SAS criterion of congruence]

$\Rightarrow$ $\angle AMB = \angle AMC = 90°$ [CPCT and Linear pair axiom]

Now, Area of $\Delta ABC = \frac{1}{2} BC \times AM$

Let $BM = x$ and $AM = y$,

then $MO = OA - AM$

$\Rightarrow$ $MO = OA - AM$

$\Rightarrow$ $MO = 9 - y$

In right angled ΔBMA and ΔBMO,

$x^2 + y^2 = 6^2$ ...(*i*) [By Pythagoras theorem]

$x^2 + (9 - y)^2 = 9^2$

$x^2 + (9)^2 + (y)^2 - 2(9)(y) = 81$

$\Rightarrow$ $x^2 + 81 + y^2 - 18y = 81$

$\Rightarrow$ $x^2 + y^2 - 18y = 0$ ...(*ii*)

Now, subtract (*i*) from (*ii*)

$$\begin{array}{rl} x^2 + y^2 - 18y &= 0 \\ x^2 + y^2 \quad\quad &= 36 \\ - \quad - \quad\quad\quad & \quad - \\ \hline -18y &= -36 \end{array}$$

$\Rightarrow \quad y = \dfrac{-36}{-18}$

$\Rightarrow \quad y = 2 \text{ cm} \Rightarrow AM = 2 \text{ cm}$

But, $\quad x^2 + y^2 = 36$ [From (*i*)]

$\Rightarrow \quad x^2 + (-2) = 36$

$\Rightarrow \quad x^2 = 36 - 4 = 32$

$\Rightarrow \quad x = \sqrt{32} = 4\sqrt{2} \text{ cm}$

$\therefore \quad BC = 2x = 2 \times 4\sqrt{2} = 8\sqrt{2} \text{ cm}$

($\because$ Perpendicular from centre to chord bisects the chord)

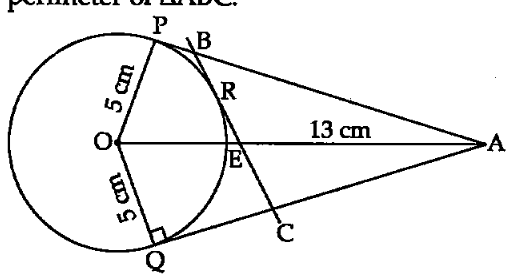
$\therefore \quad \text{Area of } \Delta ABC = \dfrac{1}{2} \times 2 \times 8\sqrt{2}$

$\Rightarrow \quad \text{Area of } \Delta ABC = 8\sqrt{2} \text{ cm}^2$

Q14. A is a point at a distance 13 cm from the centre 'O' of a circle of radius 5 cm. AP and AQ are the tangents to circle at P and Q. If a tangent BC is drawn at point R lying on minor arc PQ to intersect AP at B and AQ at C. Find the perimeter of ΔABC.

Sol. $OA = 13$ cm

$OP = OQ = 5$ cm



OP and PA are radius and tangent respectively at contact point P.

$\therefore \quad \angle OPA = 90°$

In right angled ΔOPA by Pythagoras theorem

$$PA^2 = OA^2 - OP^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$\Rightarrow \quad PA = 12$ cm

Points A, B and C are exterior to the circle and tangents drawn from an external point to a circle are equal so

$PA = QA$

$BP = BR$

$CR = CQ$

Perimeter of $\Delta ABC = AB + BC + AC$

$= AB + BR + RC + AC$ [From figure]

$= AB + BP + CQ + AC = AP + AQ$

$= AP + AP = 2AP = 2 \times 12 = 24$ cm

So, the perimeter of ΔABC = 24 cm.

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