CHAPTER 7

Coordinate Geometry

1. OBJECTIVE QUESTIONS

1. Ratio in which the line $3x+4y=7$ divides the line segment joining the points (1, 2) and $(-2,1)$ is
(a) 3 : 5
(b) 4 : 6
(c) 4 : 9
(d) None of these

Ans : (c) 4 : 9

$$\frac{3(1)+4(2)-7}{3(-2)+4(1)-7} = -\frac{4}{-9} = \frac{4}{9}$$

2. If the points $(a,0)$, $(0,b)$ and (1, 1) are collinear, then $\frac{1}{a}+\frac{1}{b}$ equals
(a) 1
(b) 2
(c) 0
(d) -1

Ans : (a) 1

Let the given points are $A(a,0)$, $B(0,b)$ and $C(1,1)$.
Since, A, B, C are collinear.

Hence, ar $(\Delta ABC) = 0$

$$\frac{1}{2}[a(b-1)+0(1-0)+1(0-b)] = 0$$
$$ab-a-b = 0$$
$$a+b = ab$$
$$\frac{a+b}{ab} = 1$$
$$\frac{1}{a}+\frac{1}{b} = 1$$

3. If the points $A(4,3)$ and $B(x,5)$ are on the circle with centre $O(2,3)$, then the value of x is
(a) 0
(b) 1
(c) 2
(d) 3

Ans : (c) 2

Since, A and B lie on the circle having centre O.

$$OA = OB$$
$$\sqrt{(4-2)^2+(3-3)^2} = \sqrt{(x-2)^2+(5-3)^2}$$
$$2 = \sqrt{(x-2)^2+4}$$
$$4 = (x-2)^2+4$$
$$(x-2)^2 = 0$$
$$x = 2$$

4. The ratio in which the point $(2,y)$ divides the join of $(-4,3)$ and (6, 3) sna hence the value of y is
(a) $2:3, y=3$
(b) $3:2, y=4$
(c) $3:2, y=3$
(d) $3:2, y=2$

Ans : (c) $3:2, y=3$

Let the required ratio be $k:1$

Then, $$2 = \frac{6k-4(1)}{k+1}$$

or $$k = \frac{3}{2}$$

The required ratio is $\frac{3}{2}:1$ or 3 : 2

Also, $$y = \frac{3(3)+2(3)}{3+2} = 3$$

5. The point on the X-axis which if equidistant from the points $A(-2,3)$ and $B(5,4)$ is
(a) (0, 2)
(b) (2, 0)
(c) (3, 0)
(d) $(-2,0)$

Ans : (b) (2, 0)

Let $P(x,0)$ be a point on X-axis such that,

$$AP = BP$$
$$AP^2 = BP^2$$
$$(x+2)^2+(0-3)^2 = (x-5)^2+(0+4)^2$$
$$x^2+4x+4+9 = x^2-10x+25+16$$
$$14x = 28$$
$$x = 2$$

Hence, required point $= (2,0)$

6. C is the mid-point of PQ, if P is $(4,x)$, C is $(y,-1)$ and Q is $(-2,4)$, then x and y respectively are
(a) -6 and 1
(b) -6 and 2
(c) 6 and -1
(d) 6 and -2

Ans : (a) -6 and 1

Since, $C(y,-1)$ is the mid-point of $P(4,x)$ and $Q(-2,4)$.

We have, $$\frac{4-2}{2} = y$$

and $$\frac{4+x}{2} = -1$$
$$y = 1$$

and $x = -6$

7. If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ equals

(a) 2 (b) -3

(c) -4 (d) None of these

Ans : (d) None of these

Let the given points are $A(0,0)$, $B(3,\sqrt{3})$ and $C(3,\lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\sqrt{(3-0)^2+(\sqrt{3}-0)^2} = \sqrt{(3-0)^2+(\lambda-0)^2}$$

$$9+3 = 9+\lambda^2$$

$$\lambda^2 = 3$$

$$\lambda = \pm\sqrt{3}$$

8. If the area of the triangle formed by the points $(x, 2x)$, $(-2, 6)$ and (3, 1) is 5 sq units, then x equals

(a) 2/3 (b) 3/5

(c) 3 (d) 5

Ans : (a) 2/3

We have, area = 5 sq units

$$\frac{1}{2}[x(6-1)-2(1-2x)+3(2x-6)] = \pm 5$$

$$5x-2+4x+6x-18 = \pm 10$$

$$15x = \pm 10+20$$

$$15x = 30 \text{ or } 10$$

$$x = \frac{30}{15} \text{ or } \frac{10}{15}$$

$$x = 2 \text{ or } \frac{2}{3}$$

9. The point which divides the line joining the points $A(1,2)$ and $B(-1,1)$ internally in the ratio $1:2$ is

(a) $\left(\frac{-1}{3},\frac{5}{3}\right)$ (b) $\left(\frac{1}{3},\frac{5}{3}\right)$

(c) $(-1,5)$ (d) (1, 5)

Ans : (b) $\left(\frac{1}{3},\frac{5}{3}\right)$

10. If $x-2y+k=0$ is a median of the triangle whose vertices are at points $A(-1,3)$, $B(0,4)$ and $C(-5,2)$, then the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

Ans : (d) 8

Coordinate of the centroid G of ΔABC

$$= \left(\frac{-1+0-5}{2},\frac{3+4+2}{3}\right)$$

$$= (-2,3)$$

Since, G lies on the median,

$$x-2y+k = 0$$

So, G satisfy the equation,

$$x-2y+k = 0$$

Hence, $-2-6+k = 0$

$$k = 8$$

11. The centroid of the triangle whose vertices are $(3,-7)$, $(-8,6)$ and (5, 10) is

(a) (0, 9) (b) (0, 3)

(c) (1, 3) (d) (3, 5)

Ans : (b) (0, 3)

Centroid is $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

i.e. $$\left(\frac{3+(-8)+5}{3},\frac{-7+6+10}{3}\right) = \left(\frac{0}{3},\frac{9}{3}\right)$$

$$= (0,3)$$

12. The points $A(-4,-1)$, $B(-2,-4)$, $C(4,0)$ and $D(2,3)$ are the vertices of a

(a) Parallelogram (b) Rectangle

(c) Rhombus (d) Square

Ans : (b) Rectangle

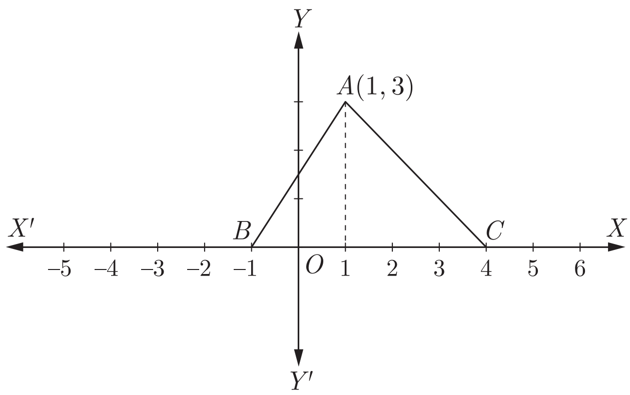
13. If the point $P(p,q)$ is equidistant from the points $A(a+b,b-a)$ and $B(a-b,a+b)$, then

(a) $ap=by$ (b) $bp=ay$

(c) $ap+bq=0$ (d) $bp+aq=0$

Ans : (b) $bp=ay$

14. In the given figure, the area of ΔABC (in sq units) is



(a) 15 (b) 10

(c) 7.5 (d) 2.5

Ans : (c) 7.5

From the given graph, it is clear that $A(1,3)$, $B(-1,0)$ and $C(4,0)$

Area of ΔABC

$$= \frac{1}{2}|[1(0-0)+(-1)(0-3)+4(3-0)]|$$

$$= \frac{1}{2}|[0+3+12]| = \frac{15}{2} = 7.5 \text{ sq units}$$

15. If the vertices of a triangle have integral coordinates, the triangle connot be

(a) right angled triangle (b) isosceles triangle

(c) equilateral triangle (d) none of these

Ans : (c) equilateral triangle

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ be the vertices of a ΔABC, where x_i, y_i, $i=1,2,3$ are intergers. Then, the area of ΔABC.

$$\Delta = \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

Area of ΔABC = A rational number [Since, x_i, y_i are integers]

If possible, let the ΔABC be an equilateral triangle, then its area is given by

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \cdot (AB)^2$$

[Since $AB = BC = CA$]

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \text{ (a positive integer)}$$

[Since, vertices are integers, Hence, AB^2 is a positive integer]

Area of ΔABC = An irrational number

This is a contradiction to the fact that the area is a rational number. Hence, the triangle cannot be equilateral.

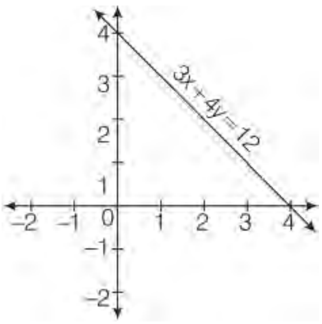
16. Find the length of the longest side of the triangle formed by the line $3x + 4y = 12$ with the coordinate axes

(a) 9 (b) 16

(c) 5 (d) 7

Ans : (d) 7

The graph of given linear equation is shown below:



Here, vertices of the triangle formed are (4, 0), (0, 3) and (0, 0). Clearly, the longest side is the hypotenuse joining (4, 0) and (0, 3).

$$\text{Its length} = \sqrt{4^2 + 3^2} \quad \text{[By Pythagoras Theorem]}$$

$$= \sqrt{16 + 9} = \sqrt{25} = 5 \text{ units}$$

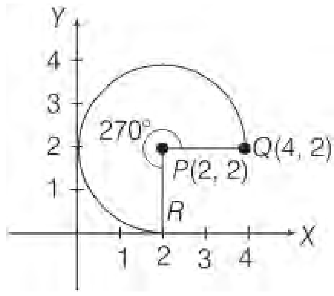
17. Join two points $P(2, 2)$ and $Q(4, 2)$ in a plane. Fixe the point P and rotate the line PQ in anti-clockwise direction at an angle of 270°. The area formed by this figure, is

(a) 9 sq units (b) 9.5 sq units

(c) 9.42 sq units (d) 9.45 sq units

Ans : (c) 9.42 sq units

When we rotate the line PQ in anti-clockwise direction at an angle of 270°, then the new coordinates of point Q will be at R, which touches the X-axis at (2, 0).



Hence, the coordinates of R point are (2, 0).

Now, $PQ = \sqrt{(4-2)^2 + (2-2)^2}$

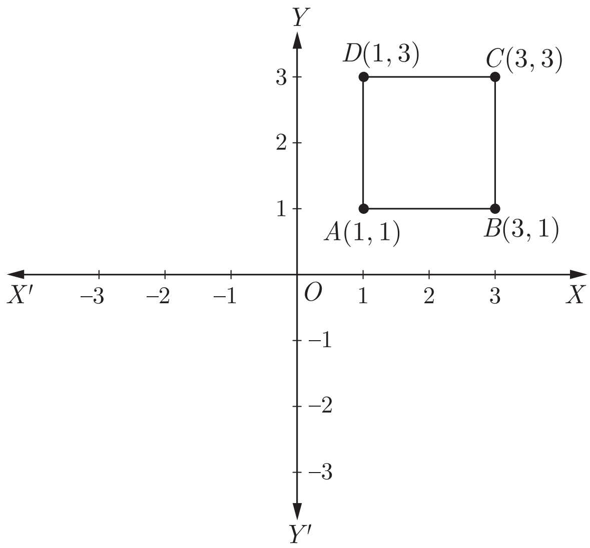
$$= \sqrt{2^2 + 0} = 2 \text{ units}$$

$$\left[\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$\text{Area of the figure} = \frac{\pi r^2}{2} + \frac{\pi r^2}{4} = \frac{3}{4}\pi r^2$$

$$= \frac{3}{4} \times 3.14 \times 4 = 9.42 \text{ sq units}$$

18. A figure is shown adjacent :



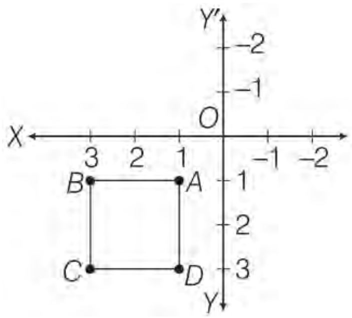
If we rotate this graph about O at an angle of 180° in anti-clockwise direction, then the point of intersection of diagonals is

(a) (1, 1) (b) (2, 1)

(c) (1, 2) (d) (2, 2)

Ans : (d) (2, 2)

When we rotate the given graph at an angle of 180°, then the new graph obtained is shown below



Thus, the new coordinates will be remain same.

i.e. $A(1,1)$, $B(3,1)$, $C(3,3)$ and $D(1,3)$.
We know that in a square, the diagonals bisect each other.

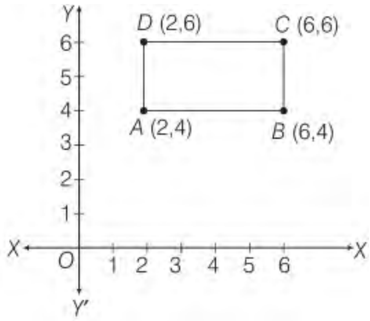
$$\text{Mid-point of } BD = \left(\frac{3+1}{2}, \frac{1+3}{2}\right)$$
$$= \left(\frac{4}{2}, \frac{4}{2}\right) = (2,2)$$

19. Suppose there are four points $A(2,4)$, $B(6,4)$, $C(6,6)$ and $D(2,6)$, which lie in the first quadrant.
If we rotate only the axes at an angle of 90° in anti-clockwise direction, then the figure obtained by joining the adjacent points is.

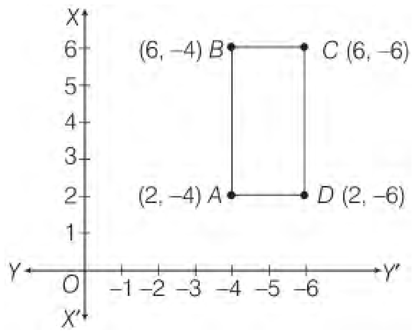
(a) square (b) rectangle
(c) rhombus (d) none of these

Ans : (b) rectangle

Given, points are $A(2,4)$, $B(6,4)$, $C(6,6)$ and $D(2,6)$.
We plot on a graph paper, as shown below:



When we rotate the axes at an angle of 90° in anti-clockwise direction, the new axes are shown below,



Here, we see that, in first quadrant, y-coordinates will be negative.
The new coordinates of A, B, C and D are respectively $A(2,-4)$, $B(6,-4)$, $C(6,-6)$ and $D(2,-6)$.

Now,
$$AB = \sqrt{(6-2)^2+(-4+4)^2}$$
$$= \sqrt{4^2+0^2} = 4 \text{ units}$$
$$BC = \sqrt{(6-6)^2+(-6+4)^2}$$
$$= \sqrt{(0)^2+(-2)^2} = 2 \text{ units}$$
$$CD = \sqrt{(2-6)^2+(-6+6)^2}$$
$$= \sqrt{(-4)^2+0^2} = 4 \text{ units}$$
and
$$DA = \sqrt{(2-2)^2+(-6+4)^2}$$
$$= \sqrt{0^2+(-2)^2} = 2 \text{ units}$$
Hence, $AB = CD$ and $BC = DA$

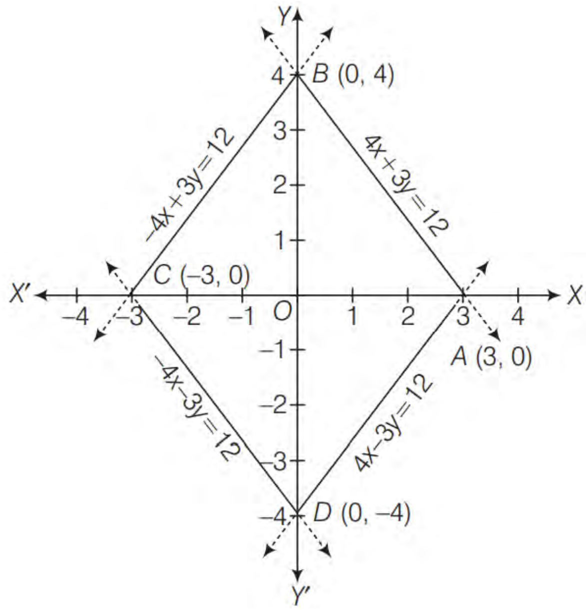
Now, diagonals,
$$AC = \sqrt{(6-2)^2+(-6+4)^2}$$
$$= \sqrt{4^2+(-2)^2}$$
$$= \sqrt{16+4} = \sqrt{20}$$
$$= 2\sqrt{5} \text{ units}$$
and
$$BD = \sqrt{(2-6)^2+(-6+4)^2}$$
$$= \sqrt{(-4)^2+(-2)^2}$$
$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ units}$$
$$AC = BD$$
Hence, $ABCD$ forms a rectangle.

20. Area of the region formed by $4|x|+3|y|=12$, is

(a) 18 sq units (b) 20 sq units
(c) 24 sq units (d) 36 sq units

Ans : (c) 24 sq units

Here, $4|x|+3|y|=12$ implies the following lines
$$4x+3y = 12 \quad ...(1)$$
$$4x-3y = 12 \quad ...(2)$$
$$-4x+3y = 12 \quad ...(3)$$
$$-4x-3y = 12 \quad ...(4)$$
These lines form the following figure:



Clearly, the vertices of figure so formed are $A(3,0)$, $B(0,4)$, $C(-3,0)$ and $D(0,-4)$.

Here,
$$AB = \sqrt{3^2+(-4)^2} = 5$$
$$BC = \sqrt{3^2+4^2} = 5$$
$$CD = \sqrt{3^2+(-4)^2} = 5$$
and
$$DA = \sqrt{3^2+4^2} = 5$$
Hence, $ABCD$ is a rhombus.

Now,
$$\text{Area} = \frac{1}{2} \times AC \times BD$$
[Area of rhombus $= \frac{1}{2}$ (Product of diagonals)]
$$= \frac{1}{2} \times 6 \times 8$$
[$AC = 6$ units and $BD = 8$ units]
$$= 24 \text{ sq units}$$

21. The circumcentre of the triangle, whose vertices are $(0, 0)$, $(3, \sqrt{3})$ and $(0, 2\sqrt{3})$, is

(a) $(1, \sqrt{3})$ (b) $(\sqrt{3}, \sqrt{3})$
(c) $(2\sqrt{3}, 1)$ (d) $(2, \sqrt{3})$

Ans : (a) $(1, \sqrt{3})$

Let $O(0, 0)$, $A(3, \sqrt{3})$ and $B(0, 2\sqrt{3})$. Then,

$$OA = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{12}$$

$$OB = \sqrt{0^2 + (2\sqrt{3})^2} = \sqrt{12}$$

and $$AB = \sqrt{(0-3)^2 + (2\sqrt{3} - \sqrt{3})^2}$$

$$= \sqrt{9 + (\sqrt{3})^2} = \sqrt{12}$$

$$OA = OB = AB$$

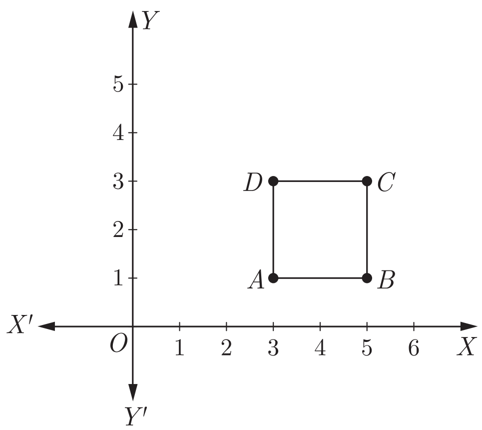
ΔABC is an equilateral triangle.

Now, circumcenter of triangle coincides with centroid of triangle.

Circumcentre of triangle is $\left(\frac{0+3+0}{3}, \frac{0+\sqrt{3}+2\sqrt{3}}{3}\right)$

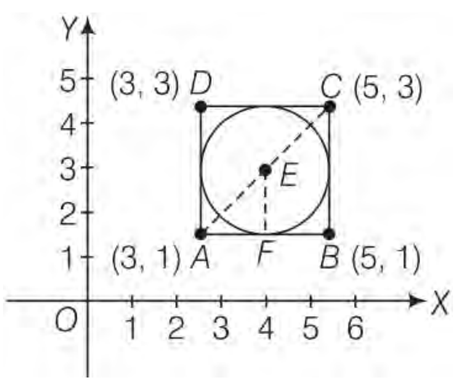
$$= (1, \sqrt{3})$$

22. A circle is inscribed in a square given below. The area between the square and inscribed circle is



(a) 0.8 sq unit (b) 1 sq unit
(c) 0.86 sq unit (d) 1.8 sq unit

Ans : (c) 0.86 sq unit



Clearly, the intersection points of two diagonals of square is the centre of the inscribed circle. Here, mid-point of A and C is, $E\left(\frac{3+5}{2}, \frac{1+3}{2}\right)$ i.e. $E(4, 2)$.

Also, we know that inscribed circle touches the square at the mid-points of its sides.

Here, mid-point of A and B is $F\left(\frac{3+5}{2}, \frac{1+1}{2}\right)$ i.e. $F(4, 1)$.

Now, radius of circle, $$EF = \sqrt{(4-4)^2 + (2-1)^2}$$

$$= \sqrt{0+1} = 1$$

and side of square, $$AB = \sqrt{(5-3)^2 + (1-4)^2}$$

$$= \sqrt{2^2} = 2$$

Now, area of square, $$A_1 = (2)^2 = 4 \text{ sq units}$$

and area of a circle, $$A_2 = \pi r^2$$

$$= 3.14 \times 1 \; [r = EF = 1]$$

$$= 3.14 \text{ sq units}$$

Area between the squire and inscribed circle

$$= A_1 - A_2 = 4 - 3.14$$

$$= 0.86 \text{ sq units.}$$

2. FILL IN THE BLANK

1. The point which divide the line segment joining the points $(5, 4)$ and $(-6, -7)$ in the ratio $1:3$ internally lies in the .... quadrant.

Ans : first

2. Point $(-4, 6)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ in the ratio ....

Ans : 2 : 7

3. If the coordinates of the points P, Q, R and S are such that $PQ = QR = RS = SP$ and $PQ \neq QS$, then quadrilateral $DEFG$ is a ....

Ans : rhombus

4. (1, 2), $(4, y)$, $(x, 6)$ and (3, 5) are the vertices of a parallelogram taken in order, then the value of x and y are ....

Ans : (6, 3)

5. Points (1, 5), (2, 3) and $(-2, -11)$ are ....

Ans : Non-collinear

6. All the points equidistant from two given points A and B lie on the .... of the line segment AB.

Ans : perpendicular bisector

7. $(5, -2)$ (6, 4) and $(7, -2)$ are the vertices of an .... triangle.

Ans : isosceles

8. The distance ofa point from the y-axis is called its ....

Ans : abscissa

9. If $x - y = 2$ then point (x, y) is equidistant from (7, 1) and (..........)

Ans : (3, 5)

10. If the co-ordinates of the points A, B, C and D are such that $AB = BC = CD = DA$ and $AC = BD$, then

quadrilateral $ABCD$ is a ....
Ans : square

11. Distance between (2, 3) and (4, 1) is ....
Ans : $2\sqrt{2}$

12. The distance of a point from the x-axis is called its ....
Ans : ordinate

13. The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2,5)$, $B(6,9)$ and $C(8,5)$ is ....
Ans : $(0,1)$

14. Point on the X-axis which is equidistant from $(2,-5)$ and $(-2,9)$ is ....
Ans : $(-7,0)$

15. If the coordinates of the points D, E, F and G are such that $DE = FG, EF = GD$ and $DF = EG$, then quadrilateral $DEFG$ is a ....
Ans : rectangle

16. The value of the expression $\sqrt{x^2+y^2}$ is the distance of the point $P(x,y)$from the ....
Ans : origin

17. Area of a rhombus if its vertices are (3, 0), (4, 5), $(-1,4)$ and $(-2,-1)$ taken in order is ....
Ans : 24. sq. units

18. The distance of the point (p,q) from (a,b) is ....
Ans : $\sqrt{(a-p)^2+(b-q)^2}$

19. Area of a triangle formed by the points $A(5,2)$, $B(4,7)$ and $C(7,-4)$ is ....
Ans : 2 sq. units

20. If the area of the triangle formed by the vertices $A(x_1,y_1)\, B(x_2,y_2)$ and $C(x_3,y_3)$is zero, then the points A, B and C are ....
Ans : collinear

21. Relation between x and y if the points (x,y), (1, 2) and (7, 0) are collinear is ....
Ans : $x+3y=7$

22. A point of the form $(b,0)$ lies on ....
Ans : x-axis

23. The distance of the point (x_1,y_1) from the origin is ....
Ans : $\sqrt{x_1^2+y_1^2}$

24. A point of the form $(0,a)$ lies on ....
Ans : y-axis

25. Points (3, 2), $(-2,-3)$ and (2, 3) form a .... triangle.
Ans : right angle

3. TRUE/FALSE

1. The distance of a point from the y-axis is its ordinate.
Ans : False

2. Area of the triangle formed by the points $P(-1.5,3)$. $Q(6,-2)$ and $R(-3,4)$ is 0.
Ans : True

3. The abscissa of point in the third quadrant is always negative.
Ans : True

4. The ratio in which the point (3, 5) divides the join of (1, 3) and (4, 6) is 2 : 1.
Ans : True

5. There exists only one point equidistant from two given points.
Ans : False

6. The distance between $P(x_1,y_1)$ and $Q(x_2,y_2)$ is $\sqrt{(x_2+x_1)^2+(y_2+y_1)^2}$.
Ans : False

7. The centroid of a triangle divides each median in the ratio $2:1$.
Ans : True

8. The coordinates of the point $P(x,y)$ which divides the line segment joining the points $A(x_1,y_1)$ and $B(x_2,y_2)$ $\left(\frac{m_1x_2-m_2x_1}{m_1+m_2}, \frac{m_1y_2-m_2y_1}{m_1+m_2}\right)$.
Ans : False

9. The mid-point of the line segment joining the points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
Ans : True

10. The area of the triangle formed by the points (x_1,y_1), (x_2,y_2) and (x_3,y_3) is the numerical value of the expression $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$.
Ans : True

11. The points (0, 5), (0, -9), and (3, 6) are collinear.
Ans : False

12. The distance of the point $P(3,2)$ from the y- axis in 2 units.
Ans : False

13. The distance of the point (5, 3) from the X-axis is 5 units.
Ans : False

14. Any point on the x-axis is of the form $(x, 0)$.
Ans : True

15. Points (1, 7), (4, 2), $(-1,-1)$ and $(-4,4)$ are the vertices of a square.
Ans : True

16. The points $A(-1,-2), B(4,3), C(2,5)$ and $D(-3,0)$ in that order form a rectangle.
Ans : True

17. Coordinates of the point which divides the join of $(-1,7)$ and $(4,-3)$ in the ratio 2 : 3 is (1, 3).
Ans : True

18. The abscissa and ordinate of a point in IV quadrant have same sign.
Ans : False

19. Ratio in which the line segment joining the points $(-3,10)$ and $(6,-8)$ is divided by $(-1,6)$ is 3 : 7.
Ans : False

20. $\triangle ABC$ with vertices $A(-2,0), B(2,0)$ and $C(0,2)$ is similar to $\triangle DEF$ with vertices $D(-4,0)$, $E(4,0)$ and $F(0,4)$.
Ans : True

21. The distance of a point (2, 3) from Y-axis is y-units.
Ans : False

4. MATCHING QUESTIONS

DIRECTION : Each question contains statements given in two Columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II.

1. Column-II gives distance between pair of points given in Column-I.

	Column-I		Column-II
(A)	$(-5,7),(-1,3)$	(p)	$\sqrt{17}$
(B)	(5, 6), (1, 3)	(q)	$\sqrt{8}$
(C)	$(\sqrt{3}+1,1)$, $(0,\sqrt{3})$	(r)	$\sqrt{6}$
(D)	$(0,0)(-\sqrt{3},\sqrt{3})$	(s)	$4\sqrt{2}$

Ans : (A) – s, (B) – p, (C) – q, (D) – r

2. Column-II gives the coordinates of the point P that divides the line segment joining the points given in Column-I.

	Column-I		Column-II
(A)	$A(-1,3)$ and $B(-5,6)$ internally in the ratio 1 : 2	(p)	(7, 3)
(B)	$A(-2,1)$ and $B(1,4)$ internally in the ratio 2 : 1	(q)	(0, 3)
(C)	$A(-1,7)$ and $B(4,-3)$ internally in the ratio 2 : 3	(r)	(1, 3)
(D)	$A(4,-3)$ and $B(8,5)$ internally in the ratio 3 : 1	(s)	(1, 0)

Ans : (A) – s, (B) – q, (C) – r, (D) – p

3. Column-II gives the area of triangles whose vertices are given in Column-I.

	Column-I		Column-II
(A)	(2, 3), $(-1,0)$, $(2,-4)$	(p)	40
(B)	$(-5,-1)$, $(3,-5)$, (5, 2)	(q)	24
(C)	$(1,-1)$, $(-4,6)$, $(-3,-5)$	(r)	32
(D)	(0, 0), (8, 0), (0, 10)	(s)	10.5

Ans : (A) – s, (B) – r, (C) – q, (D) – p

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1. **Assertion :** The point (0, 4) lies on y-axis.
Reason : The x co-ordinate on the point on y-axis is zero.
Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
The x co-ordinate of the point (0, 4) is zero.
Point (0, 4) lies on y-axis.

2. **Assertion :** The value of y is 6, for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10.
Reason : Distance between two given points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ is given 6,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Ans : (d) Assertion (A) is false but reason (R) is true.

$$PQ = 10$$
$$PQ^2 = 100$$
$$(10-2)^2+(y+3)^2 = 100$$

$$(y+3)^2 = 100 - 64 = 36$$
$$y+3 = \pm 6$$
$$y = -3 \pm 6$$
$$y = 3, -9$$

3. Assertion : If $A(2a, 4a)$ and $B(2a, 6a)$ are two vertices of a equilateral triangle ABC then the vertex C is given by $(2a + a\sqrt{3}, 5a)$.
Reason : In equilateral triangle all the coordinates of three vertices can be rational.

Ans : (c) Assertion (A) is true but reason (R) is false.
Let, $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are all rational coordinates,

$$\text{ar}\,(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
$$= \frac{\sqrt{3}}{4}[(x_1 - x_2)^2 + (y_1 - y_2)^2]$$
$$\text{LHS} = \text{rational}$$
$$\text{RHS} = \text{irrational}$$

Hence, $(x_1, y_1)(x_2, y_2)$ and (x_3, y_3) cannot be all rational.

4. Assertion : The point $(-1, 6)$ divides the line segment joining the points $(-3, 10)$ and (6, -8) in the ratio $2:7$ internally.
Reason : Three points A, B and C are collinear if area of $\Delta ABC = 0$.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
Using section formula, we have

$$-1 = \frac{k \times 6 + 1 \times (-3)}{k+1}$$
$$-k - 1 = 6k - 3$$
$$7k = 2$$
$$k = \frac{2}{7}$$

Ratio be $2:7$ internally.

Also, if $ar(\Delta ABC) = 0$
A, B and C all these points are collinear.

5. Assertion : Mid-point of a line segment divides line in the ratio 1 : 1.
Reason : If area of triangle is zero that means points are collinear.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
Both statements are individually correct.

6. Assertion : Centroid of a triangle formed by the points $(a, b), (b, c)$ and (c, a) is at origin, Then $a + b + c = 0$.
Reason : Centroid of a ΔABC with vertices $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ is given by
$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
Centroid of a triangle with vertices $(a, b), (b, c)$ and (c, a) is $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right)$

$$\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right) = (0, 0)$$
$$a + b + c = 0$$

7. Assertion : The points $(k, 2-2k)$, $(-k+1, 2k)$ and $(-4-k, 6-2k)$ are collinear if $k = \frac{1}{2}$.

Reason : Three points A, B and C are collinear in same straight line, if $AB + BC = AC$.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
Both Assertion and Reason are correct. Reason is correct explanation.

8. Assertion : The area of the triangle with vertices $(-5, -1)$, $(3, -5)$, $(5, 2)$, is 32 square units.
Reason : The point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $k:1$ externally then

$$x = \frac{kx_2 + x_1}{k+1},$$
$$y = \frac{ky_2 + y_1}{k+1}$$

Ans : (c) Assertion (A) is true but reason (R) is false.
Area of triangle

$$= \frac{1}{2}[-5(-5-2) + 3(2+1) + 5(-1+5)]$$
$$= \frac{1}{2}[35 + 9 + 20] = \frac{1}{2} \times 64 = 32$$

and section formula (externally), we have

$$x = \frac{kx_2 - x_1}{k-1},$$
$$y = \frac{ky_2 - y_1}{k-1}$$

CHAPTER 8

Introduction to Trigonometry

1. OBJECTIVE QUESTIONS

1. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$
(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

Ans : (d) $x^2q^2 - y^2p^2 = p^2q^2$

We know, $\sec^2\theta - \tan^2\theta = 1$

and $\sec\theta = \frac{x}{p}$

$\tan\theta = \frac{y}{q}$

$x^2q^2 - y^2p^2 = p^2q^2$

2. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$
(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Ans : (d) $\frac{a^2-b^2}{a^2+b^2}$

$\tan\theta = \frac{a}{b}$

$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2-b^2}{a^2+b^2}$

3. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ ... \tan 89^\circ$ is
(a) 0 (b) 1
(c) ∞ (d) None of these

Ans : (b) 1

Given, $\tan 1^\circ \tan 2^\circ \tan 3^\circ ... \tan 89^\circ$

$= \tan(90^\circ - 89^\circ)\tan(90^\circ - 88^\circ)$

$\tan(90^\circ - 87^\circ)...\tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \cot 88^\circ \cot 87^\circ ... \tan 87^\circ$

$\tan 88^\circ \tan 89^\circ$

$= (\cot 89^\circ \tan 89^\circ)(\cot 88^\circ \tan 88^\circ)$

$(\cot 87^\circ \tan 87^\circ)...(\cot 44^\circ \tan 44^\circ)\tan 45^\circ$

$= 1 \times 1 \times 1 ... 1 \times 1 = 1$

4. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$
(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

Ans : (d) $2\cos^2 A - 1$

$(\cos^4 A - \sin^4 A) = (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A)(1)$

$= \cos^2 A - (1 - \cos^2 A)$

$= 2\cos^2 A - 1$

5. If $\sec 5A = \operatorname{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is
(a) 15° (b) 5°
(c) 20° (d) 10°

Ans : (d) 10°

We have, $\sec 5A = \operatorname{cosec}(A + 30^\circ)$

$\sec 5A = \sec[90^\circ - (A - 30^\circ)]$

$[\sec(90^\circ - \theta) = \operatorname{cosec}\theta]$

$\sec 5A = \sec(60^\circ - A)$

$5A = 60^\circ - A$

$6A = 60^\circ$

$A = 10^\circ$

6. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to
(a) 0 (b) 1/2
(c) 1 (d) 3/2

Ans : (c) 1

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

Now, $x\sin\theta = y\cos\theta$

$\cos\theta\sin\theta = y\cos\theta$

$y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$