

Some Applications of Trigonometry

1. OBJECTIVE QUESTIONS

1. A vertical pole consists of two parts, the lower part being one third of the whole. At a point in the horizontal plane through the base of the pole and distance 20 meters from it, the upper part of the pole subtends an angle whose tangent is $\frac{1}{2}$. The possible heights of the pole are
- (a) 20 m and $20\sqrt{3}$ m (b) 20 m and 60 m
 (c) 16 m and 48 m (d) None of these

Ans : (b) 20 m and 60 m

Let AB be the vertical pole of height H .

$$\frac{H}{3} \cot \alpha = d$$

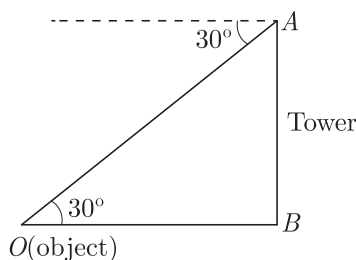
and $H \cot \beta = d$

or $\frac{H}{3d} = \tan \alpha$

and $\frac{H}{d} = \tan \beta$

2. If the angle of depression of an object from a 75 m high tower is 30° , then the distance of the object from the tower is
- (a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m
 (c) $75\sqrt{3}$ m (d) 150 m

Ans : (c) $75\sqrt{3}$ m



$$\tan 30^\circ = \frac{AB}{OB}$$

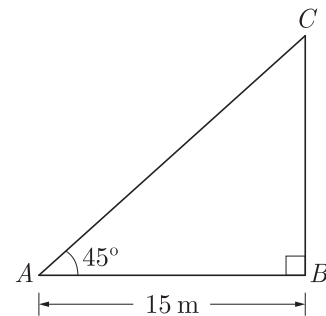
$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$

3. The height of a tree, if it casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45° , is
- (a) 10 m (b) 14 m
 (c) 8 m (d) 15 m

Ans : (d) 15 m

Let BC be the tree of height h meter.
 Let AB be the shadow of tree.



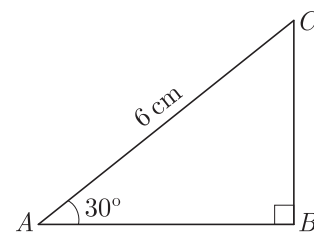
In ΔABC ,

$$\angle C = 90^\circ$$

$$\frac{BC}{BA} = \tan 45^\circ$$

$$BC = AB = 15 \text{ m}$$

4. In the adjoining figure, the length of BC is



(a) $2\sqrt{3}$ cm

(b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm

(d) 3 cm

Ans : (d) 3 cm

In ΔABC ,

$$\sin 30^\circ = \frac{BC}{AC}$$

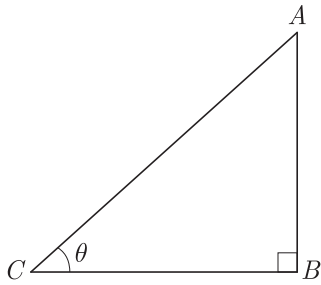
$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3 \text{ cm}$$

5. If the height and length of the shadow of a man are the same, then the angle of elevation of the sun is,
- (a) 45° (b) 60°
 (c) 90° (d) 120°

Ans : (a) 45°

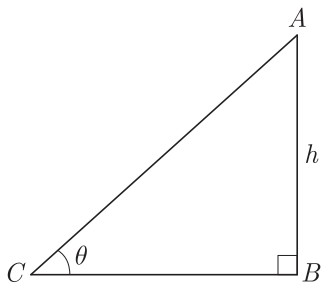
Let AB be the height of a man and BC be the shadow of a man.



In ΔABC ,
 $AB = BC$
 $\tan \theta = \frac{AB}{BC}$
 $\frac{AB}{AB} = \tan \theta$
 $\tan \theta = 1$
 $\theta = 45^\circ$

6. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then the angle of elevation of the sun is
 (a) 90° (b) 45°
 (c) 30° (d) 75°

Ans : (c) 30°
 Let AB be the rod of length h meter.
 Let BC be its shadow of length $\sqrt{3}h$ meter.



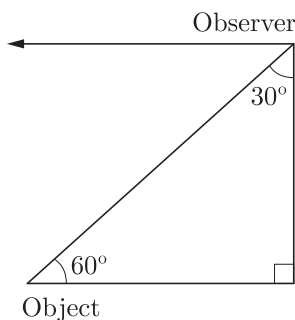
Let angle of elevation of the sun be ' θ '.
 In ΔABC ,

$$\frac{h}{\sqrt{3}h} = \tan \theta$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

7. In the given figure, the positions of the observer and the object are mentioned, the angle of depression is



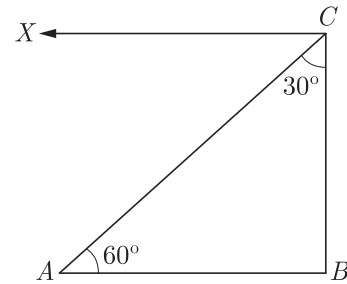
- (a) 30° (b) 90°

- (c) 60° (d) 45°

Ans : (c) 60°

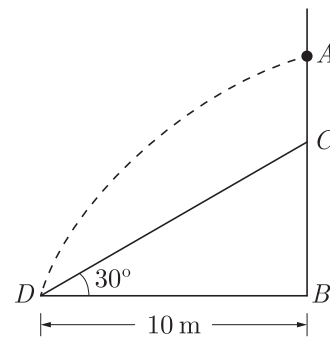
$$\angle XCA = \angle CAB = 60^\circ$$

Hence, angle of depression = 60°



8. A tree is broken by the wind. The top struck the ground at an angle of 30° and at distance of 10 m from its root. The whole height of the tree is $(\sqrt{3} = 1.732)$
 (a) $10\sqrt{3}$ m (b) $3\sqrt{10}$ m
 (c) $20\sqrt{3}$ m (d) $3\sqrt{20}$ m

Ans : (a) $10\sqrt{3}$ m
 Let AB be the tree of height x meter.
 Let AC be the broken part of tree.



$AC = CD$
 and $\angle CDB = 30^\circ$
 and $BD = 10$ m
 In ΔCDB , $\tan 30^\circ = \frac{CB}{DB} = \frac{CB}{10}$
 $\frac{1}{\sqrt{3}} = \frac{CB}{10}$
 $CB = \frac{10}{\sqrt{3}}$
 Also, $\cos 30^\circ = \frac{DB}{DC} = \frac{10}{DC}$
 $DC = \frac{20}{\sqrt{3}} = AC$
 Height of tree = $AC + CB$
 $= \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}}$
 $= 10\sqrt{3}$ m

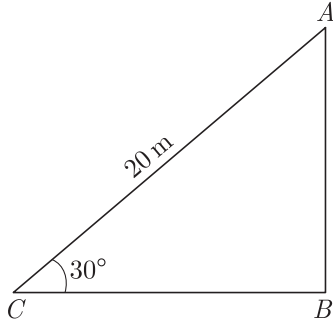
9. A circle artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the

angle made by the rope with the ground level is 30° , is

- (a) 5 m (b) 10 m
 (c) 15 m (d) 20 m

Ans : (b) 10 m

Let AB be the vertical pole and CA be the 20 m long rope such that its one end A is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.



In $\triangle ABC$, we have

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20}$$

$$AB = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

10. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan \theta = \frac{15}{8}$, then the height of kite is

- (a) 75 m (b) 78.05 m
 (c) 226 m (d) None of these

Ans : (a) 75 m

Given, Length of the string of the kite,

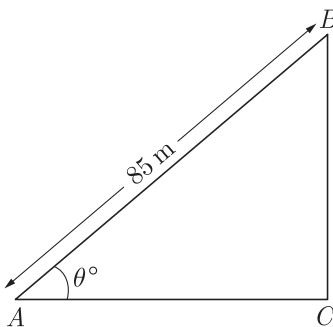
$$AB = 85 \text{ m}$$

and $\tan \theta = \frac{15}{8}$

$$\cot \theta = \frac{8}{15}$$

$$\operatorname{cosec}^2 \theta - 1 = \frac{64}{225}$$

$$\operatorname{cosec}^2 \theta = 1 + \frac{64}{225} = \frac{289}{225}$$



$$\operatorname{cosec} \theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$$

$$\sin \theta = \frac{15}{17}$$

In $\triangle ABC$, $\sin \theta = \frac{BC}{AB}$

$$\frac{15}{17} = \frac{BC}{85}$$

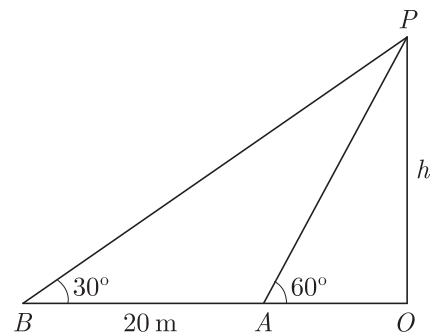
$$BC = 75 \text{ m}$$

Height of kite = 75 m

11. The angle of elevation of the top of a tower at point on the ground is 30° . If on walking 20 meters toward the tower, the angle of elevation become 60° , then the height of the tower is

- (a) 10 meter (b) $\frac{10}{\sqrt{3}}$ metre
 (c) $10\sqrt{3}$ metre (d) None of these

Ans : (c) $10\sqrt{3}$ metre



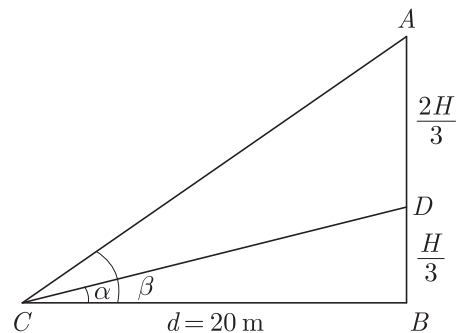
$$OA = h \cot 60^\circ$$

$$OB = h \cot 30^\circ$$

$$OB - OA = 20 = h(\cot 30^\circ - \cot 60^\circ)$$

$$h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)}$$

$$= \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$



$$\tan(\beta - \alpha) = \frac{1}{2} = \frac{\frac{H}{d} - \frac{H}{3d}}{1 + \frac{H^2}{3d^2}}$$

$$1 + \frac{H^2}{3d^2} = \frac{4H}{3d}$$

$$H^2 - 4dH + 3d^2 = 0$$

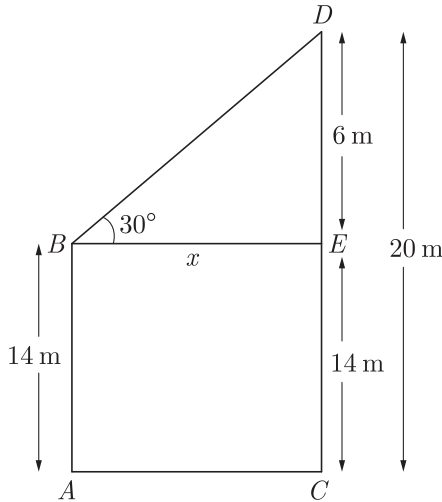
$$H^2 - 80H + 3(400) = 0$$

$$H = 20 \text{ or } 60 \text{ m}$$

12. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is
 (a) 12 m (b) 10 m
 (c) 8 m (d) 6 m

Ans : (a) 12 m

Here, $CD = 20 \text{ m}$ [height of big pole]
 $AB = 14 \text{ m}$ [height of small pole]



$$DE = CD - CE$$

$$DE = CD - AB \quad [AB = CE]$$

$$DE = 20 - 14 = 6 \text{ m}$$

In $\triangle BDE$, $\sin 30^\circ = \frac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD}$$

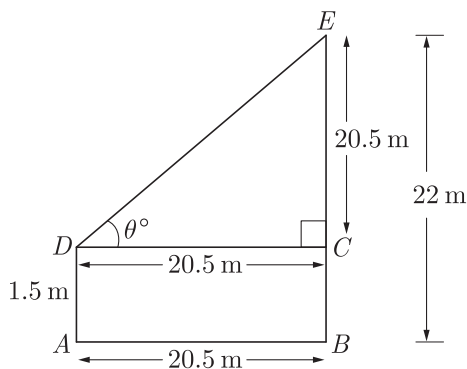
$$BD = 12 \text{ m}$$

Length of wire = 12 m

13. An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of observer is
 (a) 30° (b) 45°
 (c) 60° (d) 90°

Ans : (b) 45°

Let $BE = 22 \text{ m}$ be the height of the tower and $AD = 1.5 \text{ m}$ be the height of the observer. The point D be the observer's eye. Draw $DC \parallel AB$



Then, $AB = 20.5 \text{ m} = DC$

and $EC = BE - BC = BE - AD$
 $= 22 - 1.5 = 20.5 \text{ m} [BC = AD]$

Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle DCE$,

$$\tan \theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$

$$\tan \theta = 1$$

$$\tan \theta = \tan 45^\circ$$

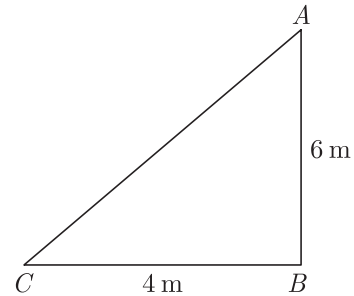
$$\theta = 45^\circ$$

14. A tree 6 m tall cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

- (a) 75 m (b) 100 m
 (c) 150 m (d) 50 m

Ans : (a) 75 m

Let AB be height of tree and BC its shadow.

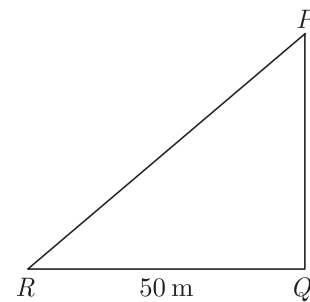


Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal

$$\triangle ABC \sim \triangle PQR$$

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

$$\frac{6}{4} = \frac{PQ}{50}$$



$$PQ = \frac{50 \times 6}{4} = 75 \text{ m}$$

15. From the top of a 7m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° , then the height of the tower is

- (a) 14.124 m (b) 17.124 m
 (c) 19.124 m (d) 15.124 m

Ans : (c) 19.124 m

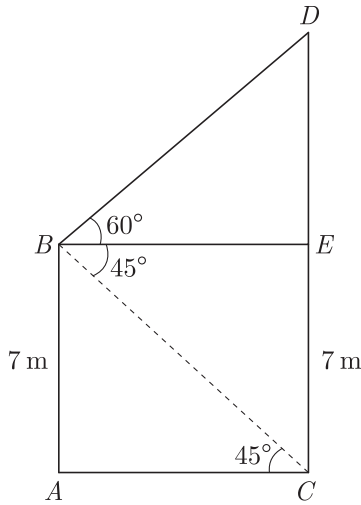
Let AB be the building and CD be the tower.

Draw $BE \perp CD$

Then, $CE = AB = 7$ m

$\angle EBD = 60^\circ$

and $\angle ACB = \angle CBE = 45^\circ$



From $\triangle ACB$, we have

$$\cot 45^\circ = \frac{AC}{AB}$$

$$\frac{AC}{7} = 1$$

$$AC = 7 \text{ m}$$

$$BE = AC = 7 \text{ m}$$

From $\triangle EBD$, we have

$$\tan 60^\circ = \frac{DE}{BE}$$

$$\frac{DE}{7} = \sqrt{3}$$

$$DE = 7\sqrt{3} \text{ m}$$

$$\begin{aligned} \text{Height of the tower} &= (7 + 7\sqrt{3}) = 7(\sqrt{3} + 1) \\ &= 7(1.732 + 1) = 7 \times 2.732 \\ &= 19.124 \text{ m} \end{aligned}$$

16. The angles of elevation of the top of a tower from the points P and Q at distance of a and b respectively from the base and in the same straight line with it, are complementary. The height of the tower is

- (a) ab (b) \sqrt{ab}
 (c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

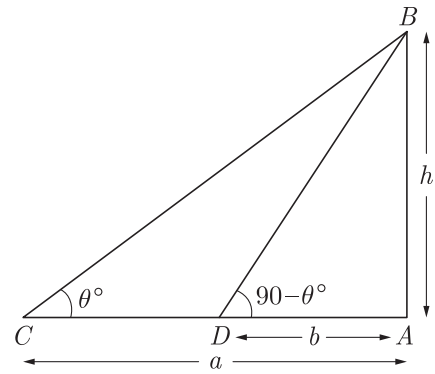
Ans : (b) \sqrt{ab}

Let AB be the tower. Let C and D be two points at distance a and b respectively from the base of the tower.

In $\triangle ABC$,

$$\tan \theta = \frac{AB}{AC}$$

$$\tan \theta = \frac{h}{a} \quad \dots(1)$$



In $\triangle ABD$,

$$\tan(90^\circ - \theta) = \frac{AB}{AD}$$

$$\cot \theta = \frac{h}{b} \quad \dots(2)$$

From Eqs. (1) and (2), we have

$$\tan \theta \times \cot \theta = \frac{h}{a} \times \frac{h}{b}$$

$$1 = \frac{h^2}{ab}$$

$$h = \sqrt{ab}$$

17. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively, then the height of the tower is

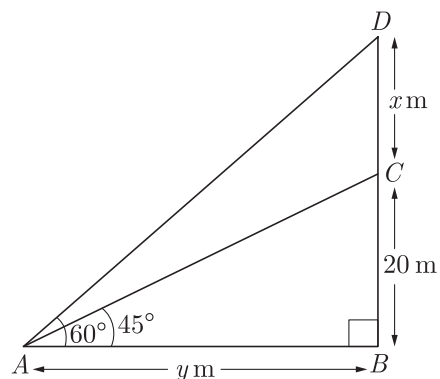
- (a) 14.64 m (b) 28.64 m
 (c) 38.64 m (d) 19.64 m

Ans : (a) 14.64 m

Let the height of the building be BC , $BC = 20$ m and height of the tower be CD . Let the point A be at a distance y m from the foot of the building.

Now, in $\triangle ABC$,

$$\frac{BC}{AB} = \tan 45^\circ = 1$$



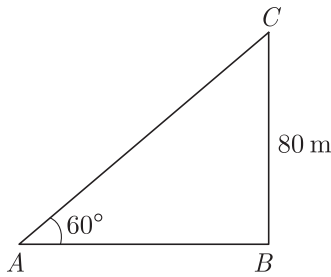
$$\frac{20}{y} = 1$$

$$y = 20 \text{ m}$$

i.e. $AB = 20$ m

Now, in $\triangle ABC$,

$$\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$$



$$AC = \frac{80 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{160\sqrt{3}}{3} = 92.37 \text{ m}$$

Hence, the length of the string is 92.37 m.

21. A spherical balloon of radius r subtends an angle θ at the eye of the observer. If the angle of elevation of its centre is ϕ , then the height of the centre of balloon is

- (a) $r \sin \phi / 2 \cos \theta$ (b) $r \sin \phi \operatorname{cosec} \theta$
 (c) $r \sin \phi \operatorname{cosec} \theta / 2$ (d) None of these

Ans : (c) $r \sin \phi \operatorname{cosec} \theta / 2$

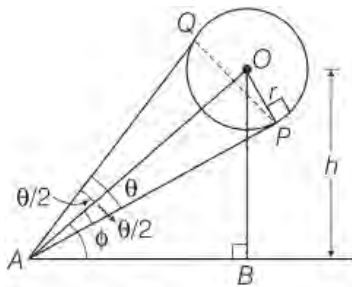
Let A be the eye of the observer and O be the centre of the balloon.

Given, $OP = r$

$$\angle PAQ = \theta$$

and $\angle OAB = \phi$

Let height of the centre of balloon be $OB = h$



In right angled $\triangle OAP$,

$$\angle OPA = 90^\circ$$

[radius is perpendicular to the tangent]

$$\sin \frac{\theta}{2} = \frac{OP}{OA} = \frac{r}{OA} \quad \dots(1)$$

and in right angled $\triangle OBA$,

$$\sin \phi = \frac{OB}{OA} = \frac{h}{OA} \quad \dots(2)$$

From Eqs. (1) and (2), we get

$$\frac{\sin \phi}{\sin \frac{\theta}{2}} = \frac{\frac{h}{OA}}{\frac{r}{OA}} = \frac{h}{r}$$

$$h = \frac{r \sin \phi}{\sin \frac{\theta}{2}}$$

$$h = r \sin \phi \operatorname{cosec} \frac{\theta}{2}$$

Hence, the height of the centre of balloons is $r \sin \phi \operatorname{cosec} \frac{\theta}{2}$

2. FILL IN THE BLANK

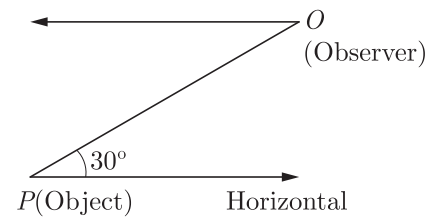
1. The of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

Ans : angle of elevation

2. The of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

Ans : angle of depression

3. In the adjoining figure, the positions of observer and object are marked. The angle of depression is



Ans : 30°

4. The is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Ans : line of sight

5. are used to find height or length of an object or distance between two distant objects.

Ans : Trigonometric ratios

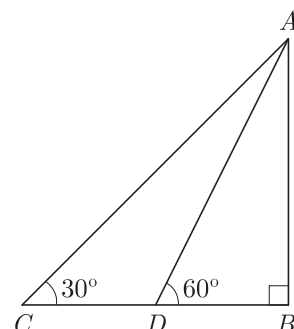
6. The top of a building from a fixed point is observed at an angle of elevation 60° and the distance from the foot of the building to the point is 100 m. then the height of the building is

Ans : $100\sqrt{3}$

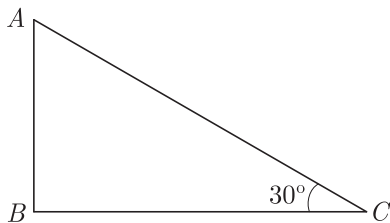
3. TRUE/FALSE

DIRECTION : (Q 1 to 4) Read the following statements and write your answer as true or false.

A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60° . Now, based on the above information, mark the given statements as true or false.



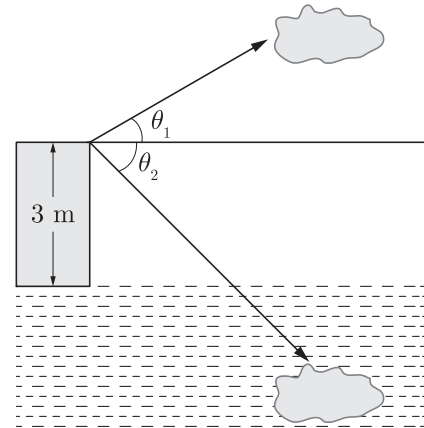
- Distance between the cars is 57.6 m.
Ans : True
- First car is at a distance of 38.90 m from the tower.
Ans : False
- Second car is at a distance of 86.50 m from the tower.
Ans : True
- Car at point C is at a distance of 200 m away from the top of the tower.
Ans : False
- When the length of the shadow of a pole is equal to the height of the pole, then the angle of elevation of source of light is 90° .
Ans : False
- When we lower our head to look at the object, the angle formed by the line of sight with horizontal is known as angle of depression.
Ans : True
- If two towers of height h_1 and h_2 , and $\frac{h_1}{x} = \tan 60^\circ$; $\frac{h_2}{x} = \tan 30^\circ$ respectively at the mid-point of the line joining their feet then, $h_1 : h_2 = 3 : 1$.
Ans : True
- The angle of elevation of the top of a tower is 30° in Figure. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.



Ans : False

- If the height of a tower and the distance of the point of observation from its foot, both are increased by 10%, then the angle of elevation of its top remains unchanged.
Ans : True
- The angle of elevation of the top of a building from the top and bottom of a tree are x and y respectively, If the height of the tree is h metre, then the height of the building is $\frac{h \cot x}{\cot x - \cot y}$.
Ans : True
- The angle for which sine and cosine have equal values is 90° .
Ans : False

- If a man standing on a platform, 3 metres above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection (Figure)

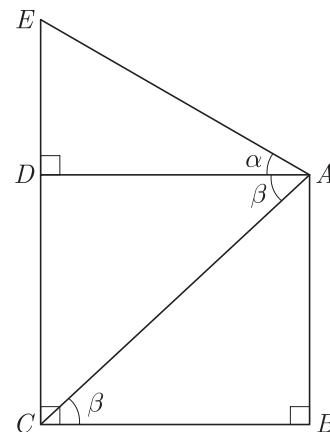


Ans : False

4. MATCHING QUESTIONS

DIRECTION : Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

- From a window, h metres high above the ground, of a house in a street, the angles of elevation and depression of the top and bottom of another house on the opposite side of the street are α and β respectively, then match the column.



	Column-I		Column-II
(A)	DB	(p)	$h(1 + \tan \alpha \cot \beta)$
(B)	DE	(q)	$h \sin \beta$
(C)	CE	(r)	$h \tan \cot \beta$
(D)	AD	(s)	$h \cot \beta$

Ans : (A) – s, (B) – r, (C) – p, (D) – q

2.

$$\tan \theta = \frac{AB}{BD} = \frac{2}{10} = \frac{1}{5}$$

	Column-I		Column-II
(A)		(p)	60°
(B)		(q)	10
(C)		(r)	$\frac{1}{5}$
(D)		(s)	3

Ans : (A) - q, (B) - s, (C) - p, (D) - r

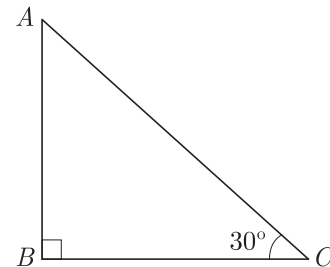
- (A) $\tan 45^\circ = \frac{AB}{BC}$
 $BC = 10$
- (B) $\tan 60^\circ = \frac{AB}{BC} = \frac{AB}{\sqrt{3}}$
 $AB = \sqrt{3} \times \sqrt{3} = 3$
- (C) $\cos \theta = \frac{20}{40} = \frac{1}{2} = \cos 60^\circ$
 $\theta = 60^\circ$
- (D) In $\triangle ABC$,
 $\tan 45^\circ = \frac{AB}{BC}$
 $AB = 2$

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- Assertion (A) is true but reason (R) is false.
- Assertion (A) is false but reason (R) is true.

1. **Assertion :** In the figure, if $BC = 20$ m, then height AB is 11.56 m.



Reason : $\tan \theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$ where θ is the angle $\angle ACB$.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Both the assertion and reason are correct, reason is the correct explanation of the assertion.

$$\tan 30^\circ = \frac{AB}{BC} = \frac{AB}{20}$$

$$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73} = 11.56 \text{ m}$$

2. **Assertion :** If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45° .

Reason : According to pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Both Assertion and Reason are correct, but Reason is not the correct explanation of the Assertion.