CHAPTER 8

INTRODUCTION OF TRIGONOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

1. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is

(a) 0° (b) 90°

(c) 60° (d) 30°

h215

Ans :

[Board 2020 SQP Standard]

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ \quad ...(1)$$

and $\cos\beta = 0$

$$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ \quad ...(2)$$

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

2. If ΔABC is right angled at C, then the value of $\sec(A+B)$ is

(a) 0 (b) 1

(c) $\frac{2}{\sqrt{3}}$ (d) not defined

Ans : [Board 2020 SQP Standard]

h216

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

$$\angle A + \angle B = 180^\circ - \angle C$$

$$= 180^\circ - 90^\circ = 90^\circ$$

Now, $\sec(A+B) = \sec 90^\circ$ not defined

Thus (d) is correct option.

3. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90^\circ)$ then the value of $\tan\theta$ is

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

Ans : [Board 2020 SQP Standard]

h217

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$$

$$\tan\theta + 1 = \sqrt{2}$$

$$\tan\theta = \sqrt{2} - 1$$

Thus (a) is correct option.

4. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

h218

Ans :

We have $\cos A = \frac{4}{5}$

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$$\text{Perpendicular} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = 3$$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

Thus (b) is correct option.

5. If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

h219

Ans :

We have $\sin A = \frac{1}{2}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$$

Now, $\text{Base} = \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Hence, the required value of $\cot A$ is $\sqrt{3}$.
Thus (a) is correct option.

6. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

h221

Ans :

We have $\sin\theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Base} = \sqrt{b^2 - a^2}$$

So, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

Thus (c) is correct option.

7. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(a) $\cos\beta$ (b) $\cos 2\beta$

(c) $\sin\alpha$ (d) $\sin 2\alpha$

Ans :

Given, $\cos(\alpha + \beta) = 0 = \cos 90^\circ$ $[\cos 90^\circ = 0]$

$$\alpha + \beta = 90^\circ$$

$$\alpha = 90^\circ - \beta$$

h222

Now, $\sin(\alpha - \beta) = \sin(90^\circ - \beta - \beta)$

$$= \sin(90^\circ - 2\beta)$$

$= \cos 2\beta$

Thus (b) is correct option.

8. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) 0

h223

Ans :

We have $\cos 9\alpha = \sin\alpha$ where $9\alpha < 90^\circ$

$$\sin(90^\circ - 9\alpha) = \sin\alpha$$

$$90^\circ - 9\alpha = \alpha$$

$$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$$

$$\tan 5\alpha = \tan(5 \times 9^\circ)$$

$$= \tan 45^\circ = 1 \qquad [\tan 45^\circ = 1]$$

Thus (c) is correct option.

9. If ΔABC is right angled at C, then the value of $\cos(A + B)$ is

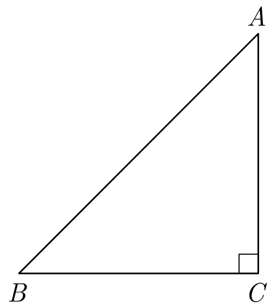
(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

h224

Ans :

We know that in ΔABC,



$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$A + B = 90^\circ$$

$$\cos(A + B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

10. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) 0° (b) 30°

(c) 60° (d) 90°

h225

Ans :

Given, $\sin\alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos\beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$$\alpha + \beta = 30^\circ + 60^\circ = 90^\circ$$

Thus (d) is correct option.

11. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

h226

Ans :

Given, $4\tan\theta = 3$

$$\tan\theta = \frac{3}{4} \qquad ...(i)$$

$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

12. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

h227

Ans :

Given, $\sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

$$\sin\theta = \sin(90^\circ - \theta)$$

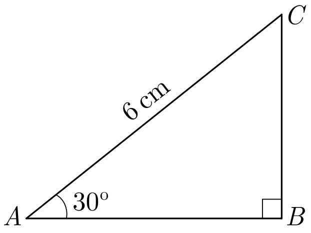
$$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Thus (c) is correct option.

13. In the adjoining figure, the length of BC is



h228

(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm (d) 3 cm

Ans :

In ΔABC, $\sin 30^\circ = \frac{BC}{AC}$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3\,\text{cm}$$

Thus (d) is correct option.

14. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$

(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

Ans :

We know, $\sec^2\theta - \tan^2\theta = 1$

Substituting $\sec\theta = \frac{x}{p}$ and $\tan\theta = \frac{y}{q}$ in above equation we have

$$\left(\frac{x}{p}\right)^2 - \left(\frac{y}{q}\right)^2 = 1$$

h229

$$x^2q^2 - y^2p^2 = p^2q^2$$

Thus (d) is correct option.

15. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Ans :

h230

We have $\tan\theta = \frac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} - b}{a\frac{\sin\theta}{\cos\theta} + b} = \frac{a\tan\theta - b}{a\tan\theta + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Thus (d) is correct option.

16. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$

(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

Ans :

$$\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$$

$$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$$

$$= (\cos^2 A - \sin^2 A)(1)$$

$$= \cos^2 A - (1 - \cos^2 A)$$

$$= 2\cos^2 A - 1$$

h231

Thus (d) is correct option.

17. If $\sec 5A = \text{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

(a) 15° (b) 5°

(c) 20° (d) 10°

h232

Ans :

We have, $\sec 5A = \text{cosec}(A + 30^\circ)$

$$\sec 5A = \sec[90^\circ - (A - 30^\circ)]$$

$$\sec 5A = \sec(60^\circ - A)$$

$$5A = 60^\circ - A$$

$$6A = 60^\circ \Rightarrow A = 10^\circ$$

Thus (d) is correct option.

18. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0 (b) 1/2

(c) 1 (d) 3/2

h233

Ans :

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$$

Now, $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta$$

$$y = \sin\theta$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Thus (c) is correct option.

19. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2$ is equal to

(a) $\sqrt{mn}$ (b) $\sqrt{\frac{m}{n}}$

(c) $4\sqrt{mn}$ (d) None of these

h235

Ans :

Given, $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$$

$$= 4\tan\theta\sin\theta$$

$$= 4\sqrt{\tan^2\theta\sin^2\theta}$$

$$= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\sin^2\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{mn}$$

Thus (c) is correct option.

20. If $0 < \theta < \frac{\pi}{4}$, then the simplest form of $\sqrt{1 - 2\sin\theta\cos\theta}$ is

(a) $\sin\theta - \cos\theta$ (b) $\cos\theta - \sin\theta$

(c) $\cos\theta + \sin\theta$ (d) $\sin\theta\cos\theta$

h236

Ans :

$$\sqrt{1 - 2\sin\theta\cos\theta} = \sqrt{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$$

$$= \sqrt{(\cos\theta - \sin\theta)^2}$$

$$= \cos\theta - \sin\theta$$

For $0^\circ < \theta < 45^\circ$

	0	$\pi/6$	$\pi/4$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$
$\sin\theta$	0	1/2	$1/\sqrt{2}$

Here, we see that $\cos\theta > \sin\theta$, when $0 < \theta < \frac{\pi}{4}$, that's why we take $(\cos\theta - \sin\theta)^2$ instead of taking $(\sin\theta - \cos\theta)^2$.

Thus (b) is correct option.

21. If $f(x) = \cos^2 x + \sec^2 x$, then $f(x)$

(a) ≥ 1 (b) ≤ 1

(c) ≥ 2 (d) ≤ 2

h237

Ans : (c) ≥ 2

Given, $f(x) = \cos^2 x + \sec^2 x$

$$= \cos^2 x + \sec^2 x - 2 + 2$$

$$= \cos^2 x + \sec^2 x - 2\cos x \cdot \sec x + 2$$

$$= (\cos x - \sec x)^2 + 2$$

We know that, square of any expression is always greater than equal to zero.

$f(x) \geq 2$ Hence proved.

Thus (c) is correct option.

22. **Assertion :** The value of $\sin\theta = \frac{4}{3}$ is not possible.

Reason : Hypotenuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

h238

$$\sin\theta = \frac{P}{H} = \frac{4}{3}$$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. **Assertion :** $\sin^2 67^\circ + \cos^2 67^\circ = 1$

Reason : For any value of $\theta, \sin^2\theta + \cos^2\theta = 1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

h239

We have $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2 67^\circ + \cos^2 67^\circ = 1$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

1. FILL IN THE BLANK

1. Maximum value for sine of any angle is ....

Ans :

h241

1

2. Triangle in which we study trigonometric ratios is called ....

Ans :

h242

Right Triangle

3. Cosine of 90° is ....

Ans :

h243

Zero

4. Sum of .... of sine and cosine of angle is one.

Ans :

h244

Square

5. Reciprocal of $\sin\theta$ is ....

Ans :

h245

$\operatorname{cosec}\theta$

6. The value of $\sin A$ or $\cos A$ never exceeds ....

Ans :

h246

1

7. sine of $(90^\circ - \theta)$ is ....

Ans :

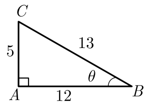
h247

$\cos\theta$

8. If $\sin\theta = \frac{5}{13}$, then the value of $\tan\theta$ is ....

Ans : [Board 2020 OD Basic]

From $\sin\theta = \frac{5}{13}$ we can draw the figure as given below.



h248

Now, $$\tan\theta = \frac{AC}{BC} = \frac{5}{12}$$

9. The value of the $(\tan^2 60^\circ + \sin^2 45^\circ)$ is ....

Ans : [Board 2020 OD Basic]

h250

$$\tan^2 60^\circ + \sin^2 45^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3 + \frac{1}{2} = \frac{7}{2}$$

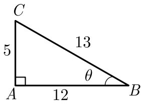
10. If $\cot\theta = \frac{12}{5}$, then the value of $\sin\theta$ is ....

Ans : [Board 2020 Delhi Basic]

Given, $$\cot\theta = \frac{12}{5} \Rightarrow \tan\theta = \frac{5}{12}$$

From $\tan\theta = \frac{5}{12}$ we can draw the figure as given below.

h252



So, $\sin\theta = \frac{AC}{CB} = \frac{5}{13}$

11. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is ....

Ans : [Board 2020 Delhi Basic]

h253

We have $\tan(A+B) = \sqrt{3}$

$= \tan 60°$

Hence, $A + B = 60°$ ...(1)

Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$= \tan 30°$

$A - B = 30°$ ...(2)

Adding equation (1) and (2) we get

$2A = 90° \Rightarrow A = 45°$

12. The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right) =$ ....

Ans : [Board 2020 Delhi Standard]

h262

$\sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$

$= \sin^2\theta + \cos^2\theta = 1$

13. The value of $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) =$ ....

Ans : [Board 2020 Delhi Standard]

h263

$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$= \sec^2\theta(1-\sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta = 1$

VERY SHORT ANSWER QUESTIONS

14. Prove that

$(1+\tan A - \sec A) \times (1 + \tan A + \sec A) = 2\tan A$

Ans : [Board 2020 Delhi Basic]

h266

LHS $= (1+\tan A - \sec A) \times (1+\tan A + \sec A)$

$= (1+\tan A)^2 - \sec^2 A$

$= 1 + \tan^2 A + 2\tan A - \sec^2 A$

$= \sec^2 A + 2\tan A - \sec^2 A$

$= 2\tan A =$ RHS

15. If $\tan A = \cot B$, then find the value of $(A+B)$.

Ans : [Board 2020 OD Standard]

h267

We have $\tan A = \cot B$

$\tan A = \tan(90° - B)$

$A = 90° - B$

Thus $A + B = 90°$

16. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then prove that $x^2 + y^2 = 25$.

Ans : [Board 2020 OD Basic]

h269

We have $x = 3\sin\theta + 4\cos\theta$

and $y = 3\cos\theta - 4\sin\theta$

$x^2 + y^2$

$= (3\sin\theta + 4\cos\theta)^2 + (3\cos\theta - 4\sin\theta)^2$

$= (9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta) +$

$+ (9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta)$

$= 9(\sin^2\theta + \cos^2\theta) + 16(\sin^2\theta + \cos^2\theta)$

$= 9 + 16 = 25$

17. Evaluate $\sin^2 60° - 2\tan 45° - \cos^2 30°$

Ans : [Board 2019 OD]

h270

$\sin^2 60° - 2\tan 45° - \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} - 2 - \frac{3}{4} = -2$

18. If $\sin\theta + \sin^2\theta = 1$ then prove that $\cos^2\theta + \cos^4\theta = 1$.

Ans : [Board 2020 OD Basic]

We have

$$\sin\theta + \sin^2\theta = 1$$

$$\sin\theta + (1-\cos^2\theta) = 1$$

$$\sin\theta - \cos^2\theta = 0$$

$$\sin\theta = \cos^2\theta$$

Squaring both sides, we get

$$\sin^2\theta = \cos^4\theta$$

$$1-\cos^2\theta = \cos^4\theta$$

$$\cos^4\theta + \cos^2\theta = 1$$

Hence Proved

19. In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A.

Ans : [Board Term-1 2016]

In a triangle

$$A+B+C = 180^\circ$$

$$B+C = 180^\circ - A$$

Thus

$$\cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180^\circ - A}{2}\right]$$

$$= \cos\left[90 - \frac{A}{2}\right]$$

$$= \sin\frac{A}{2}$$

20. If $\sec\theta\cdot\sin\theta = 0$, then find the value of θ.

Ans : [Board Term-1 2016]

We have

$$\sec\theta\cdot\sin\theta = 0$$

$$\frac{1}{\cos\theta}\cdot\sin\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = 0 = \tan 0^\circ$$

Thus $\theta = 0^\circ$

21. If $\tan 2A = \cot(A+60^\circ)$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1 2016]

We have

$$\tan 2A = \cot(A+60^\circ)$$

$$\cot(90^\circ - 2A) = \cot(A+60^\circ)$$

$$90^\circ - 2A = A + 60^\circ$$

$$3A = 30^\circ \Rightarrow A = 10^\circ$$

22. If $\tan(3x+30^\circ) = 1$ then find the value of x.

Ans : [Board Te

We have

$$\tan(3x+30^\circ) = 1 = \tan 45^\circ$$

$$3x + 30^\circ = 45^\circ$$

$$x = 5^\circ$$

23. What happens to value of $\cos\theta$ when θ increases from 0° to 90°.

Ans : [Board Term-1 2015]

$\cos\theta$ decreases from 1 to θ.

24. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A+B$.

Ans : [Board Term-1 2016]

We have

$$\sin A = \cos B$$

$$\sin A = \sin(90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

25. If $\cos A = \frac{2}{5}$, find the value of $4 + 4\tan^2 A$.

Ans : [Board SQP 2018]

$$4 + 4\tan^2 A = 4(1+\tan^2 A)$$

$$4\sec^2 A = \frac{4}{\cos^2 A} = \frac{4}{\left(\frac{2}{5}\right)^2} = 4\times\frac{25}{4} = 25$$

26. If $k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$, then find the value of k.

Ans : [Board Term-1 2015]

We have

$$k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$$

$$= \sec^2\theta(1-\sin^2\theta)$$

$$= \sec^2\theta.\cos^2\theta$$

$$= \sec^2\theta\times\frac{1}{\sec^2\theta}$$

$$k+1 = 1 \Rightarrow k = 1-1 = 0$$

Thus $k = 0$

27. Find the value of $\sin^2 41^\circ + \sin^2 49^\circ$

Ans : [Board Term-1 2012, NCERT]

We have

$$\sin^2 41 + \sin^2 49 = \sin^2(90^\circ - 49^\circ) + \sin^2 49^\circ$$

$$= \cos^2 49 + \sin^2 49^o$$

$$= 1$$

h155

TWO MARKS QUESTIONS

28. Prove that $1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$

Ans : [Board 2020 OD Standard]

$$1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$$

h273

$$= 1 + \frac{(1+\operatorname{cosec}\alpha)(\operatorname{cosec}\alpha - 1)}{1+\operatorname{cosec}\alpha}$$

$$= 1 + \operatorname{cosec}\alpha - 1$$

$$= \operatorname{cosec}\alpha \qquad \text{Hence Proved}$$

29. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Ans : [Board 2018]

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

h275

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \tan A \frac{[1-2(1-\cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A \frac{[1-2+2\cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$

$$= \tan A \qquad \text{Hence Proved}$$

30. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Ans : [Board 2020 OD Standard]

$$\tan^4\theta + \tan^2\theta = \tan^2\theta(1+\tan^2\theta)$$

h276

$$= \tan^2\theta \times \sec^2\theta$$

$$= (\sec^2\theta - 1)\sec^2\theta$$

$$= \sec^4\theta - \sec^2\theta \qquad \text{Hence Proved}$$

31. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$.

h278

Ans :

$$\text{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = \text{RHS} \qquad \text{Hence Proved}$$

32. Prove that : $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Ans : [Board 2020 OD Basic]

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$$

h279

$$= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$$

$$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$$

$$= \cos^2\theta - \sin^2\theta \qquad \text{Hence Proved}$$

33. Prove that $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = 1$.

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

h280

$$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \sin^2\theta + \cos^2\theta = 1 = \text{RHS}$$

34. Prove that : $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

h281

$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = 2\sec^2\theta = \text{RHS}$$

35. Prove that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$.

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$$

$$= \operatorname{cosec}\theta\left[\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}\right]$$

$$= \operatorname{cosec}\theta\left[\frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}\right]$$

$$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}\right)$$

h283

$$= \frac{2\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$$

$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{RHS}$$ Hence Proved

36. If $5\tan\theta = 3$, then what is the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$?

Ans : [Board 2020 Delhi Basic]

We have $5\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{5}$

h285

Dividing numerator and denominator by $\cos\theta$ we have

$$\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 3}{4\frac{\sin\theta}{\cos\theta} + 3} = \frac{5\tan\theta - 3}{4\tan\theta + 3}$$

$$= \frac{5 \times \frac{3}{5} - 3}{4 \times \frac{3}{5} + 3} = \frac{3 - 3}{\frac{12}{5} + 3} = 0$$

37. Evaluate :

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

Ans : [Board Term-1 2016]

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + 2 - 1}{(1)^2}$$

$$= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1}$$

h114

$$= 1 + 3 + 2 - 1 = 5$$

38. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}$, $0 \le A + B < 90^0$ and $A > B$, then find A and B.

Ans : [Board Term-1 2016]

h115

We have $\sin(A+B) = 1 = \sin 90^\circ$

$$A + B = 90^\circ \quad ...(1)$$

and $\sin(A-B) = \frac{1}{2} = \sin 30^\circ$

$$A - B = 30^\circ \quad ...(2)$$

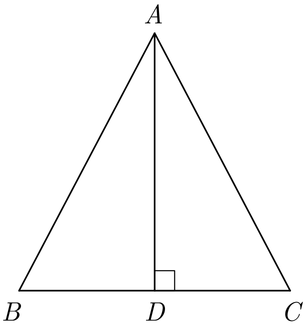
Solving eq. (1) and (2), we obtain

$$A = 60^\circ \text{ and } B = 30^\circ$$

39. Find $\operatorname{cosec} 30^\circ$ and $\cos 60^\circ$ geometrically.

Ans : [Board Term-1 2015]

Let a triangle ABC with each side equal to $2a$ as shown below.



h116

In ΔABC, $\angle A = \angle B = \angle C = 60^\circ$

Now we draw AD perpendicular to BC, then

$$\Delta BDA \cong \Delta CDA$$

$$BD = CD$$

$$\angle BAD = CAD = 30^\circ \quad \text{by } CPCT$$

$$AD = \sqrt{3}a$$

In ΔBDA, $\operatorname{cosec} 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

40. Evaluate : $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ}$

Ans : [Board Term-1 2013]

h118

We have $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$$

41. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \text{cosec}^2\theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{2}\sin\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^{\circ}$$

h120

Thus $\theta = 45^{\circ}$

Now $\sec^2\theta - \text{cosec}^2\theta = \sec^2 45^{\circ} - \text{cosec}^2 45^{\circ}$

$$= (\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 0$$

42. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

Ans : [Board Term-1 2012]

We have $4\cos\theta = 11\sin\theta$

or, $\cos\theta = \frac{11}{4}\sin\theta$

h121

Now $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11 \times \frac{11}{4}\sin\theta - 7\sin\theta}{11 \times \frac{11}{4}\sin\theta + 7\sin\theta}$

$$= \frac{\sin\theta(\frac{121}{4} - 7)}{\sin\theta(\frac{121}{4} + 7)}$$

$$= \frac{121-28}{121+28} = \frac{93}{149}$$

43. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$ $0^{\circ} < A+B \le 90^{\circ}$, then find A and B.

Ans : [Board Term-1 2012]

We have $\tan(A+B) = \sqrt{3} = \tan 60^{\circ}$

$$A + B = 60^{\circ} \quad ...(1)$$

Also $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^{\circ}$

$$A - B = 30^{\circ} \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^{\circ}$$

$$A = \frac{90^{\circ}}{2} = 45^{\circ}$$

h122

Substituting this value of A in equation (1), we get

$$B = 60^{\circ} - A = 60^{\circ} - 45^{\circ} = 15^{\circ}$$

Hence, $A = 45^{\circ}$ and $B = 15^{\circ}$

44. If $\cos(A-B) = \frac{\sqrt{3}}{2}$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B, where $(A+B)$ and $(A-B)$ are acute angles.

Ans : [Board Term-1 2012]

We have $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^{\circ}$

$$A - B = 30^{\circ} \quad ...(1)$$

Also $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60^{\circ}$

$$A + B = 60^{\circ} \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^{\circ}$$

$$A = 45^{\circ}$$

h123

Substituting this value of A in equation (1), we get $B = 15^{\circ}$

45. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Ans : [Board Term-1 2012, Set-25]

We have $2\sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^{\circ}$$

h125

$$2\theta = 60^{\circ}$$

Hence, $\cos 2\theta = \cos 60^{\circ} = \frac{1}{2}$.

46. Find the value of $\sin 30^{\circ}\cos 60^{\circ} + \cos 30^{\circ}\sin 60^{\circ}$ is it equal to $\sin 90^{\circ}$ or $\cos 90^{\circ}$?

Ans : [Board Term-1 2016]

$$\sin 30^{\circ}\cos 60^{\circ} + \cos 30^{\circ}\sin 60^{\circ} = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^{\circ} = 1$ but not equal to $\cos 90^{\circ}$ as $\cos 90^{\circ} = 0$.

h126

47. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^{\circ} < \theta < 90^{\circ}$, find the value of θ.

Ans : [Boar Term-1, 2012]

We have

h128

$$\sqrt{3}\sin\theta - \cos\theta = 0 \text{ and } 0^{\circ} < \theta < 90^{\circ}$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^{\circ} \quad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\theta = 30^\circ$$

48. Evaluate : $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ}$

Ans : [Board Term-1 2012]

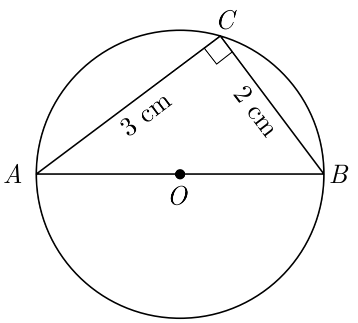
We have $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \dfrac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{1}{2} = \frac{\sqrt{6}+2}{4}$$

h129

49. In the given figure, AOB is a diameter of a circle with centre O, find $\tan A \tan B$.



Ans : [Board Term-1 2012]

In ΔABC, $\angle C$ is a angle in a semi-circle, thus

$$\angle C = 90^\circ$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and $$\tan B = \frac{AC}{BC} = \frac{3}{2}$$

$$\tan A \tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

h130

50. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Ans :

We have $$\sin\phi = \frac{1}{2}$$

$$\phi = 30^\circ$$

h131

Now substituting this value of θ in LHS we have

$$3\cos\phi - 4\cos^3\phi = 3\cos 30^\circ - 4\cos^3 30^\circ$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$ Hence Proved

51. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Ans : [Board Term-1 2015]

We have $$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$$

and $$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$$

h156

52. Prove that : $\dfrac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$

Ans : [Board Term-1 2015]

$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\cos^2\theta}$$

$$= 1$$

h157

53. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Ans : [Board Term-1 2015]

We have

h158

$$\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1)$$

$[[1 + \tan^2\theta = \sec^2\theta]]$

$$= \sec^2\theta(\tan^2\theta)$$

$$= (1 + \tan^2\theta)\tan^2\theta$$

$$= \tan^2\theta + \tan^4\theta$$

Hence Proved.

54. Find the value of θ, if,

$\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4; \theta \le 90^\circ$

Ans : [Board Term-1 2015]

We have $$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$$

h159

$$\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$

$$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} = 4$$

$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

Thus $\theta = 60^\circ$.

55. Prove that : $-1+\dfrac{\sin A\sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$

Ans : [Board Term-1 2012]

$$-1+\frac{\sin A\sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$$

h160

$$\frac{\sin A\sin(90^\circ - A)}{\cot(90^\circ - A)} = 1-\sin^2 A$$

$$\frac{\sin A\cos A}{\tan A} = \cos^2 A$$

$$\frac{\sin A\cos A}{\frac{\sin A}{\cos A}} = \cos^2 A$$

$$\frac{\cos A}{\sin A}\sin A\cos A = \cos^2 A$$

$$\cos^2 A = \cos^2 A \quad \text{Hence Proved.}$$

56. Prove that : $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}$$

h161

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A - \cot A \qquad \text{Hence Proved.}$$

57. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

Ans : [Board Term-1 2013]

We have $\quad \sin\theta - \cos\theta = \dfrac{1}{2}$

h162

Squaring both sides, we get

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{1}{4}$$

$$1 - 2\sin\theta\cos\theta = \frac{1}{4}$$

$$2\sin\theta\cos\theta = 1-\frac{1}{4} = \frac{3}{4}$$

Again, $(\sin\theta+\cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$= 1 + 2\sin\theta\cos\theta$$

$$= 1+\frac{3}{4} = \frac{7}{4}$$

Thus $\quad \sin\theta + \cos\theta = \sqrt{\dfrac{7}{4}} = \dfrac{\sqrt{7}}{2}$

58. If θ be an acute angle and $5\operatorname{cosec}\theta = 7$, then evaluate $\sin\theta + \cos^2\theta - 1$.

Ans : [Board Term-1 2012]

We have $\quad 5\operatorname{cosec}\theta = 7$

h163

$$\operatorname{cosec}\theta = \frac{7}{5}$$

$$\sin\theta = \frac{5}{7} \qquad \left[\operatorname{cosec}\theta = \tfrac{1}{\sin\theta}\right]$$

$$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1-\cos^2\theta)$$

$$= \sin\theta - \sin^2\theta \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{5}{7} - \left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$$

59. If $\sin A = \dfrac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$.

Ans : [Board Term-1 2012]

Using $\cot^2\theta = -1 + \operatorname{cosec}^2\theta$ we have

h164

$$2\cot^2 A - 1 = 2(\operatorname{cosec}^2 A - 1) - 1$$

$$= \frac{2}{\sin^2 A} - 3$$

$$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3 = \frac{8}{3} - 3 = \frac{-1}{3}$$

Thus $\quad 2\cot^2 A - 1 = \dfrac{-1}{3}$

THREE MARKS QUESTIONS

60. Show that : $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}=1$

Ans : [Board 2020 OD Standard]

$$\text{LHS} = \frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$$

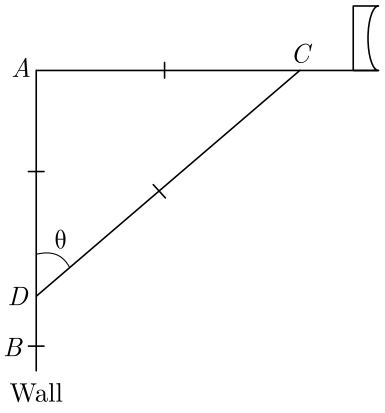
$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}$$

$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}$$

h286

$$= \frac{1}{1} = 1 = \text{RHS}$$

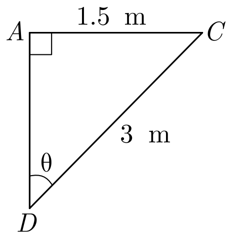
61. The rod of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Figure. If $AC = 1.5$ m long and $CD = 3$ m, find (i) $\tan\theta$ (ii) $\sec\theta + \operatorname{cosec}\theta$.



h287

Ans : [Board 2020 Delhi Standard]

From the given information we draw the figure as below



In right angle triangle $\Delta\,CAD$, applying Pythagoras theorem,

$$AD^2 + AC^2 = DC^2$$

$$AD^2 + (1.5)^2 = (3)^2$$

$$AD^2 = 9 - 2.25 = 6.75$$

$$AD = \sqrt{6.75} = 2.6\text{ m (Approx)}$$

(i) $$\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$$

(ii) $$\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$$

62. Prove that : $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{1+\cot\theta}{\sin\theta}$

Ans : [Board 2020 Delhi Standard]

h288

$$\text{LHS} = \frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1}$$

$$= \frac{\sin\theta(\cos\theta + 1 - \sin\theta)}{\sin\theta(\cos\theta - 1 + \sin\theta)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - \sin^2\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \text{RHS}$$

63. If $\sin\theta + \cos\theta = \sqrt{2}$ prove that $\tan\theta + \cot\theta = 2$

Ans : [Board 2020 OD Standard]

We have $$\sin\theta + \cos\theta = \sqrt{2}$$

Squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

h289

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta = 1$$

$$\sin\theta\cos\theta = \frac{1}{2} \quad \text{...(1)}$$

Now $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{2}} = 2 = \text{RHS}$$

64. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans : [Board 2020 SQP Standard]

Given, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring above equation, we have

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$2\sin\theta\cos\theta = 3 - 1 = 2$$

$$\sin\theta\cos\theta = 1$$

h290

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

Substituting value of $\sin\theta\cos\theta$ we have

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

65. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or ½.

Ans : [Board 2020 OD Standard]

We have, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$

h291

$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$

$$\text{cosec}^2\theta + 1 = 3\cot\theta$$

$$1 + \cot^2\theta + 1 = 3\cot\theta$$

$$\cot^2\theta - 3\cot\theta + 2 = 0$$

$$\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$

$$\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$

$$(\cot\theta - 2)(\cot\theta - 1) = 0$$

$$\cot\theta = 1 \text{ or } 2$$

$$\tan\theta = 1 \text{ or } ½.$$

66. Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Ans : [Board 2019 Delhi Standard]

$$\text{LHS} = (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= (\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta) +$$

h292

$$+(\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$

$$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta)$$

$$+2\sin\theta \times \frac{1}{\sin\theta} + 2\cos\theta \times \frac{1}{\cos\theta}$$

$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$$

$$= 7 + \tan^2\theta + \cot^2\theta$$

$$= \text{RHS}$$

67. Prove that $(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$

Ans : [Board 2019 Delhi]

$$\text{LHS} = (1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A)$$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

h293

$$= \frac{(\sin A + \cos A - 1)(\cos A + \sin A + 1)}{\sin A \cos A}$$

$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A\cos A}$$

$$= \frac{1 + 2\sin A\cos A - 1}{\sin A\cos A}$$

$$= 2 = \text{RHS}$$

68. Prove that $\frac{\sin A - \cos A - 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{1 + \sin A}{1 + \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\sin A + \cos A - 1 + \sin^2 A + \cos A\sin A - \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A\cos A}$$

h294

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A(1 - \cos A + \sin A)}$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \frac{(\sec A + \tan A)}{(\sec A - \tan A)} \times (\sec A - \tan A)$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} = \text{RHS}$$

69. Prove that : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

Ans : [Board 2020 Delhi Standard]

$$\text{LHS} = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta] + -3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$$

$$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$$

$$= -(\sin^2\theta + \cos^2\theta)^2 + 1$$

$$= -1 + 1 = 0 = \text{RHS}$$

h295

70. Prove that $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

$$= \frac{1}{1 - 2\cos^2 A}$$

$$= \text{RHS}$$

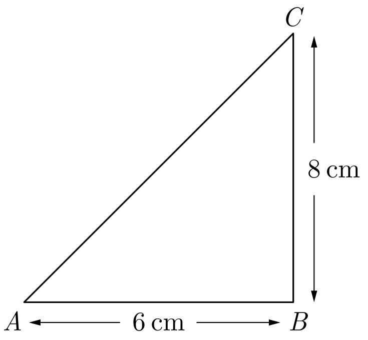
h296

71. If in a triangle ABC right angled at B, $AB = 6$ units and $BC = 8$ units, then find the value of $\sin A \cos C + \cos A \sin C$.

Ans : [Board Term-1 2016]

As per question statement figure is shown below.

h132



We have $AC^2 = 8^2 + 6^2 = 100$

$AC = 10$ cm

Now $\sin A = \frac{BC}{AC} = \frac{8}{10}$;

$\cos A = \frac{AB}{AC} = \frac{6}{10}$

and $\sin C = \frac{AB}{AC} = \frac{6}{10}$;

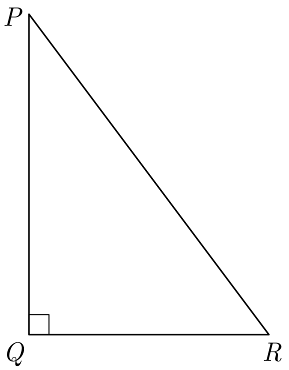
$\cos C = \frac{BC}{AC} = \frac{8}{10}$

Thus $\sin A \cos C + \cos A \sin C = \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$

$$= \frac{64}{100} + \frac{36}{100}$$

$$= \frac{100}{100} = 1$$

72. In the given $\angle PQR$, right-angled at Q, $QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Ans : [Board Term-1 2015]

Using Pythagoras theorem we have

h133

$$PQ^2 + QR^2 = PR^2$$

$$PQ^2 + 9^2 = (PQ+1)^2$$

$$PQ^2 + 81 = (PQ+1)^2$$

$$PQ^2 + 81 = PQ^2 + 1 + 2PQ$$

$$PQ = 40$$

Since $PR - PQ = 1$, thus,

$$PR = 1 + 40 = 41$$

$$\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$$

73. If $\cos(40^{o} + x) = \sin 30^{o}$, find the value of x.

Ans : [Board Term-1 2015]

We have

h135

$$\cos(40^{o} - x) = \sin 30^{o}$$

$$\cos(40^{o} + x) = \sin(90^{o} - 60^{o})$$

$$\cos(40^{o} + x) = \cos 60^{o}$$

$$40^{o} + x = 60^{o}$$

$$x = 60^{o} - 40^{o} = 20^{o}$$

Thus $x = 20^{o}$.

74. Evaluate : $\dfrac{5\cos^2 60^{o} + 4\cos^2 30^{o} - \tan^2 45^{o}}{\sin^2 30^{o} + \cos^2 60^{o}}$

Ans : [Board Term-1 2013]

h136

$$\frac{5\cos^2 60^{o} + 4\cos^2 30^{o} - \tan^2 45^{o}}{\sin^2 30^{o} + \cos^2 60^{o}}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

75. Verify : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$, for $\theta = 60^{o}$

Ans :

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^{o}}{1+\cos 60^{o}}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \qquad \left(\cos 60^{o} = \frac{1}{2}\right)$$

h139

$$\text{RHS} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^{o}}{1+\cos 60^{o}}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^{o}$.

76. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Ans : [Board Term-1 2015]

We have $\tan A + \cot A = 2$

Squaring both sides, we have

h140

$$(\tan A + \cot A)^2 = (2)^2$$

$$\tan^2 A + \cot^2 A + 2\tan A \cot A = 4$$

$$\tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} = 4$$

$$\tan^2 A + \cot^2 A + 2 = 4$$

$$\tan^2 A + \cot^2 A = 4 - 2$$

$$\tan^2 A + \cot^2 A = 2$$

77. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$.

Ans : [Board Term-1 2011]

h141

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

We have $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$$= (\sqrt{2} - 1)\cos\theta$$

$$= \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}\cos\theta$$

Thus $\sin\theta = \dfrac{1}{\sqrt{2}+1}\cos\theta$

$$(\sqrt{2}+1)\sin\theta = \cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Hence proved.

78. Prove that : $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$.

Ans : [Board Term-1 2013, 2011]

h142

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

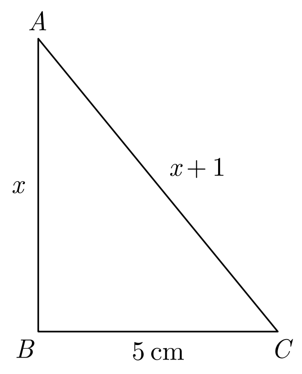
$$= \sin A + \cos A$$

$= \text{RHS}$ Hence proved.

79. In ΔABC, $\angle B = 90°$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1 + \sin C}{1 + \cos C}$.

Ans : [Board Term-1 2011]

As per question we have drawn the figure given below.



h143

We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC = x + 1$$

Now $$AC^2 = AB^2 + BC^2$$

$$(x + 1)^2 = x^2 + 5^2$$

$$x^2 + 2x + 1 = x^2 + 25$$

$$2x = 24$$

$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now $$\sin C = \frac{AB}{AC} = \frac{12}{13}$$

$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now $$\frac{1 + \sin C}{1 + \cos C} = \frac{1 + \frac{12}{13}}{1 + \frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$$

80. Prove that : $\frac{\cos A}{1 + \tan A} - \frac{\sin A}{1 + \cot A} = \cos A - \sin A$

Ans : [Board Term-1 2016]

$$\frac{\cos A}{1 + \tan A} - \frac{\sin A}{1 + \cot A}$$

h165

$$= \frac{\cos A}{1 + \frac{\sin A}{\cos A}} - \frac{\sin A}{1 + \frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$

$= \cos A - \sin A$ Hence Proved.

81. If $b \cos \theta = a$, then prove that $\text{cosec}\,\theta + \cot \theta = \sqrt{\frac{b + a}{b - a}}$.

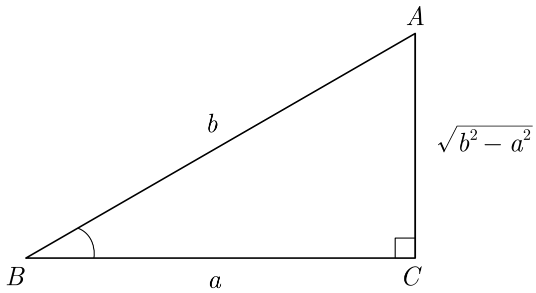
Ans : [Board Term-1 2015]

We have $$b \cos \theta = a$$

or, $$\cos \theta = \frac{a}{b}$$

h166

Now consider the triangle shown below.



$$AC^2 = AB^2 - BC^2$$

or, $$\cos\theta = \frac{a}{b}$$

$$AC = \sqrt{b^2-a^2}$$

Now $$\operatorname{cosec}\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{a}{\sqrt{b^2-a^2}}$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

82. Prove that : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

Ans : [Bard Term-1 2015]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$

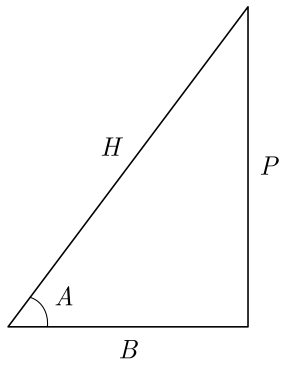
$$= \tan\theta$$

h167

83. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Ans : [Board Term-1 2015, NCERT]

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement. Consider the triangle shown below.



Let $\tan A = \dfrac{P}{B}$ and $\sec A = \dfrac{H}{B}$

h168

$$H^2 = P^2 + B^2$$

Now $$1 + \tan^2 A = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$$

$$= \frac{B^2+P^2}{B^2} = \frac{H^2}{B^2}$$

$$= \left(\frac{H}{B}\right)^2$$

$$= \sec^2 A$$ Hence Proved.

84. Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Ans : [Board Term-1 2015]

$$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$(\cot\theta - \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

h169

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad [[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$ Hence Proved.

85. Prove that :

$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Ans : [Board Term-1 2015]

LHS $= (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta\cos\theta}\right) \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \cos\theta\sin\theta \times \frac{1}{\sin\theta\cos\theta} = 1$$

h170

86. Show that :

$\operatorname{cosec}^2\theta - \tan^2(90^o - \theta) = \sin^2\theta + \sin(90^o - \theta)$

Ans : [Board Term-1 2013]

$$\operatorname{cosec}^2\theta - \tan^2(90^o - \theta)$$

$$= \operatorname{cosec}^2\theta - \cot^2\theta$$

h171

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}$$

$$= 1$$

$$= \sin^2\theta + \cos^2\theta$$

$$= \sin^2\theta + \sin^2(90^o - \theta)$$

Hence Proved

87. Prove that : $\dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Ans : [Board Term-1 2013]

We have

$$\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = \operatorname{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$$

$$= \operatorname{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}\right]$$

$$= \frac{1}{\sin^2\theta}\sin\theta\left[\frac{(1+\sin\theta)-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

h172

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

Hence Proved

88. Prove that :

$$\frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}.$$

Ans : [Board Term-1 2011]

$$\frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}$$

$$\frac{1}{\operatorname{cosec}A - \cot A} + \frac{1}{\operatorname{cosec}A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\frac{1}{\operatorname{cosec}A - \cot A} + \frac{1}{\operatorname{cosec}A + \cot A} = \frac{2}{\sin A}$$

$$\frac{\operatorname{cosec}A + \cot A + \operatorname{cosec}A - \cot A}{(\operatorname{cosec}A - \cot A)(\operatorname{cosec}A + \cot a)} = \frac{2}{\sin A}$$

h173

$$\frac{2\operatorname{cosec}A}{\operatorname{cosec}^2A - \cot^2A} = \frac{2}{\sin A}$$

$$\frac{2\frac{1}{\sin A}}{1} = \frac{2}{\sin A}$$

$$\frac{2}{\sin A} = \frac{2}{\sin A}$$

Hence Proved.

89. If $\sec\theta = x + \frac{1}{4x}$ prove that $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$

Ans : [Board Term-1 2011]

We have $\sec\theta = x + \dfrac{1}{4x}$ (1)

Squaring both side we have

h174

$$\sec^2\theta = x^2 + 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$1 + \tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$\tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2} - 1$$

$$= x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

$$= x^2 - 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

Now $\tan\theta = x - \dfrac{1}{4x}$ (2)

or $\tan\theta = -\left(x - \dfrac{1}{4x}\right) = -x + \dfrac{1}{4x}$ (3)

Adding (1) and (2) we have

$$\tan\theta + \sec\theta = 2x$$

Adding (1) and (3) we have

$$\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x} \quad \text{Hence proved.}$$

90. Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$

Ans : [Board Term-1 2011]

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1} = \text{RHS}$$

h175

Hence Proved.

91. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove that $x^2 + y^2 = 1$.

Ans : [Board Term-1 2011]

We have $\quad x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \quad (1)$

and $\quad x\sin\theta = y\cos\theta$

or, $\quad x = \frac{y\cos\theta}{\sin\theta} \quad (2)$

Eliminating x from equation (1) and (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y(\sin^2\theta + \cos^2\theta) = \sin\theta$$

$$y = \sin\theta \quad ...(3)$$

h176

Substituting this value of y in equation (2) we have,

$$x = \cos\theta \quad (4)$$

Squaring and adding equation (3) and (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1 \quad \text{Hence Proved.}$$

92. Prove that $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Ans : [Board Term-1 2011]

$$X = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$= (1 - \sin\theta\cos\theta)$$

$$Y = \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

h177

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= (1 + \sin\theta\cos\theta)$$

Now given expression

$$X + Y = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$

$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 2 = \text{RHS} \quad \text{Hence Proved.}$$

93. Express : $\sin A, \tan A$ and $\operatorname{cosec} A$ in terms of $\sec A$.

Ans : [Board Term-1 2011]

(1) $\quad \sin^2 A + \cos^2 A = 1$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

h181

(2) $\quad \tan A = \frac{\sin A}{\cos A} = \sin A \sec A$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$

$$= \sqrt{\sec^2 A - 1}$$

(3) $\quad \operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

94. If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate $\tan\theta + \cot\theta$.

Ans : [Board SQP 2018]

We have $\quad \sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

h184

$$2\sin\theta\cos\theta \quad 2-1 = 1$$

$$\frac{1}{\sin\theta\cos\theta} = 2$$

Now,
$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} = 2$$

FOUR MARKS QUESTIONS

95. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans : [Board 2020 Delhi Standard]

h297

We have
$$\sin\theta + \cos\theta = \sqrt{3}$$

Squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$2\sin\theta\cos\theta = 3 - 1 = 2$$

$$\sin\theta\cos\theta = 1 \quad ...(1)$$

Now
$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

or
$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

Substituting the value of $\sin\theta\cos\theta$ from equation (1) we have

$$\tan\theta + \cot\theta = \frac{1}{1} = 1$$

Hence,
$$\tan\theta + \cot\theta = 1$$

96. If $\sec\theta = x + \frac{1}{4x}$, $x \neq 0$ find $(\sec\theta + \tan\theta)$.

Ans : [Board 2019 Delhi]

h298

We have
$$\sec\theta = x + \frac{1}{4x} \quad ...(1)$$

Since,
$$\tan^2\theta = \sec^2\theta - 1$$

Substituting value of $\sec\theta$ we have

$$\tan^2\theta$$

$$= \left(x + \frac{1}{4x}\right)^2 - 1$$

$$= x^2 + \frac{2x}{4x} + \frac{1}{16x^2} - 1$$

$$= x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

When $\sec\theta = x + \frac{1}{4x}$ and $\tan\theta = x - \frac{1}{4x}$ we have

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right) = 2x$$

When $\sec\theta = x + \frac{1}{4x}$ and $\tan\theta = -\left(x - \frac{1}{4x}\right)$ we have

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left\{-\left(x - \frac{1}{4x}\right)\right\}$$

$$= x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$= \frac{2}{4x} = \frac{1}{2x}$$

97. If $\sin A = \frac{3}{4}$ calculate $\sec A$.

Ans : [Board 2019 OD]

h299

We have
$$\sin A = \frac{3}{4}$$

Now
$$\cos^2 A = 1 - \sin^2 A$$

$$\cos^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos A = \frac{\sqrt{7}}{4}$$

Thus
$$\sec A = \frac{1}{\cos A} = \frac{4}{\sqrt{7}}$$

98. Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

Ans : [Board 2019 OD]

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^2\theta+1+\tan\theta}{\tan\theta}$$

h301

$$= \tan\theta+\cot\theta+1$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1$$

$$= \frac{1}{\sin\theta\cos\theta}+1$$

$$= \operatorname{cosec}\theta\sec\theta+1$$

$$= 1+\sec\theta\operatorname{cosec}\theta$$ Hence Proved

99. Prove that: $\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

Ans : [Board 2019 OD]

$$\text{LHS} = \frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}$$

$$= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta+1}$$

$$= \frac{1-\cos^2\theta}{\cos\theta+1} = \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta+1}$$

$$= 1-\cos\theta \qquad ...(1)$$

Now, $\text{RHS} = 2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

$$= 2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}} = 2+\frac{\sin^2\theta}{\cos\theta-1}$$

h302

$$2+\frac{1-\cos^2\theta}{\cos\theta-1} = 2-\frac{(\cos^2\theta-1)}{(\cos\theta-1)}$$

$$= 2-\frac{(\cos\theta-1)(\cos\theta+1)}{\cos\theta-1}$$

$$= 2-(\cos\theta+1) = 1-\cos\theta$$

$= \text{LHS}$ Hence Proved

100. Find A and B if $\sin(A+2B)=\frac{\sqrt{3}}{2}$ and $\cos(A+4B)=0$, where A and B are acute angles.

Ans : [Board 2019 OD]

We have $\sin(A+2B) = \frac{\sqrt{3}}{2}$

$$\sin(A+2B) = \sin 60^\circ \quad \left(\sin 60^\circ = \tfrac{\sqrt{3}}{2}\right)$$

$$A+2B = 60^\circ \qquad ...(1)$$

Also, given $\cos(A+4B) = 0$

$$\cos(A+4B) = \cos 90^\circ \quad (\cos 90^\circ = 0)$$

$$A+4B = 90^\circ \qquad ...(2)$$

Subtracting equation (2) from equation (1) we get

$$-2B = -30^\circ \Rightarrow B = 15^\circ$$

From equation (1) we have

h303

$$A+2(15^\circ) = 60^\circ$$

$$A = 60^\circ-30^\circ$$

$= 30^\circ$

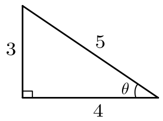
Hence angle $A=30^\circ$ and angle $B=15^\circ$.

101. If $4\tan\theta=3$, evaluate $\left(\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$

Ans : [Board 2018]

We have $4\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{4}$

h304



We know very well that if $\tan\theta = \frac{3}{4}$, then

$$\sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

Substituting above values in given expression,

$$\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1} = \frac{4\times\frac{3}{5}-\frac{4}{5}+1}{4\times\frac{3}{5}+\frac{4}{5}-1} = \frac{13}{11}$$

102. Evaluate :

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

Ans : [Board Term-1 2015]

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2\times 1\times 1^2\times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 3 - 2$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

h144

103. Given that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

find the values of $\tan 75^\circ$ and $\tan 90^\circ$ by taking suitable values of A and B.

Ans : [Board Term-1 2012, NCERT]

We have $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

h145

(i)
$$\tan 75^\circ = \tan(45^\circ + 30^\circ)$$
$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$
$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2}$$
$$= 2 + \sqrt{3}$$

Hence $\tan 75^\circ = 2 + \sqrt{3}$

(ii)
$$\tan 90^\circ = \tan(60^\circ + 30^\circ)$$
$$= \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ}$$
$$= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^\circ = \infty$

104. Evaluate :

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$$

Ans : [Board Term-1 2013]

h146

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3 + 32 + 12 + 1}{24} = \frac{48}{24} = 2$$

105. Evaluate : $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45 - \sin^2 90^\circ)$

Ans : [Board Term-1 2013]

$$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45 - \sin^2 90^\circ)$$
$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$
$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$
$$= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

h147

106. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$.

Ans : [Board Term-1 2012]

We have
$$15\tan^2\theta + 4\sec^2\theta = 23$$
$$15\tan^2\theta + 4(\tan^2\theta + 1) = 23$$
$$15\tan^2\theta + 4\tan^2\theta + 4 = 23$$
$$19\tan^2\theta = 19$$
$$\tan\theta = 1 = \tan 45^\circ$$

h148

Thus $\theta = 45^\circ$

Now, $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$
$$= (\sec 45^\circ + \operatorname{cosec} 45^\circ)^2 - \sin^2 45^\circ$$
$$= (\sqrt{2} + \sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$

107. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have
$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$
$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$
$$(x - \sqrt{3})(\sqrt{3}x - 1) = 0$$
$$x = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

h149

Thus $\cot\theta = \sqrt{3}$ or $\cot\theta = \dfrac{1}{\sqrt{3}}$

Therefore $\theta = 30^\circ$ or $\theta = 60^\circ$

If $\theta = 30^\circ$, then

$$\cot^2 30^o + \tan^2 30^o = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^o$, then

$$\cot^2 60^o + \tan^2 60^o = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

108. Evaluate the following :

$$\frac{2\cos^2 60^o + 3\sec^2 30^o - 2\tan^2 45^o}{\sin^2 30^o + \cos^2 45^o}$$

Ans : [Board Term-1 2012]

$$\frac{2\cos^2 60^o + 3\sec^2 30^o - 2\tan^2 45^o}{\sin^2 30^o + \cos^2 45^o} = \frac{2(\frac{1}{2})^2 + 3(\frac{2}{\sqrt{3}})^2 - 2(1)^2}{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$= \frac{2(\frac{1}{2})^2 + 3(\frac{2}{\sqrt{3}})^2 - 2(1)^2}{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}$$

h150

109. Prove that : $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta.$

Ans : [Board Term-1 2012]

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1 - \tan\theta)\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

h152

$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

110. In an acute angled triangle ABC if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Ans : [Board Term-1 2012]

We have $\sin(A + B - C) = \frac{1}{2} = \sin 30^o$

$$A + B - C = 30^o \quad ...(1)$$

and $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^o$

$$B + C - A = 45^o \quad ...(2)$$

Adding equation (1) and (2), we get

$$2B = 75^o \Rightarrow B = 37.5^o$$

Subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^o$$

$$A - C = -7.5^o \quad ...(3)$$

Now $A + B + C = 180^o$

$$A + C = 180^\circ - 37.5^\circ = 142.5^o \quad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^o \Rightarrow A = 67.5^o$$

h153

and, $C = 75^o$

Hence, $\angle A = 67.5^o, \angle B = 37.5^o, \angle C = 75^o$

111. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :

(1) $x = a\sec\theta, y = b\tan\theta$, or

(2) $x = a\,\text{cosec}\,\theta, y = b\cot\theta$

Ans : [Board Term-1 2015]

(1) We have $x = a\sec\theta, y = b\tan\theta$,

$$\frac{x^2}{a^2} = \sec^2\theta, \ \frac{y^2}{b^2} = \tan^2\theta$$

h185

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

(ii) We have $x = a\,\text{cosec}\,\theta, y = b\cot\theta$

$$\frac{x^2}{a^2} = \text{cosec}^2\theta, \ \frac{y^2}{b^2} = \cot^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \text{cosec}^2\theta - \cot\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

112. If $\text{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\text{cosec}\,\theta + \cot\theta = \sqrt{2}\,\text{cosec}\,\theta$.

Ans : [Board Term-1 2015]

We have $\text{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$

h186

Squaring both sides we have

$$\text{cosec}^2\theta + \cot^2\theta - 2\,\text{cosec}\,\theta\cot\theta = 2\cot^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta)$$

$$(\operatorname{cosec}\theta + \cot\theta)\sqrt{2}\cot\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$$

Hence Proved.

113. Prove that :

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2} = \frac{\sec\theta\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Ans : [Board Term-1 2015]

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

h187

$$= \frac{\frac{\cos^3\theta}{\sin^3\theta}\times\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta}\times\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{\cos^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{1-\sin\theta\cos\theta}{\cos\theta+\sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\operatorname{cosec}\theta\sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Hence Proved

114. Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta$.

Ans : [Board Terim-1, 2012, Set-9]

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = \frac{(\sec\theta-1)+(\sec\theta+1)}{\sqrt{(\sec\theta+1)(\sec\theta-1)}}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta-1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$= 2\times\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}$$

h188

$$= 2\times\frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

Hence Proved

115. Prove that : $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\sec\theta+1}{\sec\theta-1}$.

Ans : [Board Term-1 2012]

We have
$$\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}$$

h189

$$= \frac{\sin\theta(\frac{1}{\cos\theta}+1)}{\sin\theta(\frac{1}{\cos\theta}-1)}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

Hence Proved.

116. Prove that : $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

Ans : [Board Term-1 2012]

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$$

$$= \frac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A - 1} = \frac{2\operatorname{cosec}^2 A}{\cot^2 A}$$

h190

$$= \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\sin^2 A}\times\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{2}{\cos^2 A} = 2\sec^2 A$$

Hence Proved.

117. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$.

Ans : [Board Term-1 2016

$$\frac{p^2-1}{p^2+1} = \frac{(\operatorname{cosec}\theta+\cot\theta)^2-1}{(\operatorname{cosec}\theta+\cot\theta)^2+1}$$

$$= \frac{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta-1+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{2\cot\theta(\cot\theta+\operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+\cot\theta)}$$

h191

$$= \frac{\cos\theta}{\sin\theta}\times\sin\theta = \cos\theta$$

118. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $m^2+n^2 = a^2+b^2$

Ans : [Board Term-1 2012]

We have

$$m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta \quad ...(1)$$

and, $n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta$ ...(2)

Adding equations (1) and (2) we get

$$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$
$$= a^2(1) + b^2(1)$$
$$= a^2 + b^2$$

h192

119. Prove that : $\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta$.

Ans : [Board Term-1 2012]

$$\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$
$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$
$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$
$$= 1+\sin\theta\cos\theta$$

Hence Proved

h193

120. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \mathrm{cosec}\,\theta = q$, prove that $q(p^2-1) = 2p$

Ans : [Board Term-1 2012]

We have $\cos\theta+\sin\theta = p$ and $\sec\theta + \mathrm{cosec}\,\theta = q$

$$q(p^2-1) = (\sec\theta+\mathrm{cosec}\,\theta)[(\cos\theta+\sin\theta)^2-1]$$
$$= (\sec\theta+\mathrm{cosec}\,\theta)(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta-1)$$
$$= (\sec\theta+\mathrm{cosec}\,\theta)[1+2\sin\theta\cos\theta-1]$$
$$= \left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$
$$= \left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)2\sin\theta\cos\theta$$
$$= 2(\sin\theta+\cos\theta) = 2p$$

Hence Proved.

h194

121. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2+y^2+z^2 = r^2$

Ans : [Board Term-1 2012, Set-50]

Since, $x^2 = r^2\sin^2 A\cos^2 C$

$y^2 = r^2\sin^2 A\sin^2 C$

and $z^2 = r^2\cos^2 A$

h195

$$x^2+y^2+z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$$
$$= r^2\sin^2 A(\cos^2 C+\sin^2 C) + r^2\cos^2 A$$
$$= r^2\sin^2 A + r^2\cos^2 A$$
$$= r^2(\sin^2 A+\cos^2 A)$$
$$= r^2$$

Hence Proved.

122. Prove that: $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$.

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$
$$= \frac{2}{\cos\theta} = 2\sec\theta$$

Hence Proved

h196

123. Prove that

$(1-\sin\theta+\cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta)$.

Ans : [Board Term-1 2012]

$$(1-\sin\theta+\cos\theta)^2$$
$$= 1+\sin^2\theta+\cos^2\theta-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$
$$= 1+1-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$
$$= 2+2\cos\theta-2\sin\theta-2\sin\theta\cos\theta$$
$$= 2(1+\cos\theta)-2\sin\theta(1+\cos\theta)$$
$$= (1+\cos\theta)(2-2\sin\theta)$$
$$= 2(1+\cos\theta)(1-\sin\theta)$$

Hence Proved

h197

124. Prove that : $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1} = \sec\theta+\tan\theta$

Ans : [Board Term-1 2012]

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$
$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$
$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

h198

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta \qquad \text{Hence Proved}$$

125. Prove that :

$(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta\cot^2\theta$

Ans : [Board Term-1 2012]

h199

$$(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$$

$$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + 2 + \sec^2\theta + 2$$

$$= 1 + (1 + \cot^2\theta) + 2 + (1 + \tan^2\theta) + 2$$

$$= 7 + \tan^2\theta + \cot^2\theta \qquad \text{Hence Proved}$$

126. If $\sin\theta = \frac{c}{\sqrt{c^2 + d^2}}$ and $d > 0$, find the value of $\cos\theta$ and $\tan\theta$.

Ans : [Board Term-1 2013]

h200

We have $\sin\theta = \frac{c}{\sqrt{c^2 + d^2}}$

Now
$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)^2 = 1 - \frac{c^2}{c^2 + d^2} = \frac{c^2 + d^2 - c^2}{c^2 + d^2} = \frac{d^2}{c^2 + d^2}$$

Thus $\cos\theta = \frac{d}{\sqrt{c^2 + d^2}}$

Again,
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{c}{\sqrt{c^2 + d^2}}}{\frac{d}{\sqrt{c^2 + d^2}}} = \frac{c}{d}$$

Thus $\tan\theta = \frac{c}{d}$

127. If $\tan\theta = \frac{1}{\sqrt{5}}$,

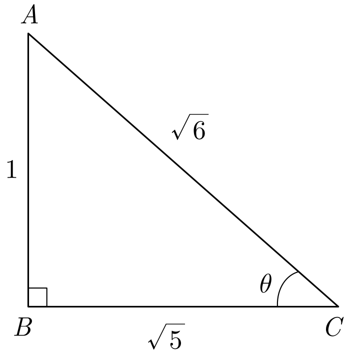
(1) Evaluate : $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

(2) Verify the identity : $\sin^2\theta + \cos^2\theta = 1$

Ans : [Board Term-1 2012]

We have $\tan\theta = \frac{1}{\sqrt{5}}$

We draw the triangle as shown below and write all dimensions.



h201

Now
$$\cot\theta = \frac{1}{\tan\theta} = \sqrt{5}$$
$$\sin\theta = \frac{1}{\sqrt{6}}$$
$$\cos\theta = \frac{\sqrt{5}}{\sqrt{6}}$$

(1)
$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(1 + \cot^2\theta) - (1 + \tan^2\theta)}{(1 + \cot^2\theta) + (1 + \tan^2\theta)} = \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta + \tan^2\theta}$$
$$= \frac{(\sqrt{5})^2 - (\frac{1}{5})^2}{2 + (\sqrt{5})^2 + (\frac{1}{\sqrt{5}})^2} = \frac{5 - \frac{1}{5}}{2 + 5 + \frac{1}{5}} = \frac{25 - 1}{35 + 1} = \frac{24}{36} = \frac{2}{3}$$

(2)
$$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2 = \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1 \qquad \text{Hence proved.}$$

128. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Ans : [Board Term-1 2015]

We have $\sec\theta + \tan\theta = p \qquad (1)$

h204

Now
$$\frac{1}{p} = \frac{1}{\sec\theta + \tan\theta} \times \frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)} = \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta - \tan\theta$$

or $$\frac{1}{p} = \sec\theta - \tan\theta \qquad (2)$$

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \frac{1}{p}$,

$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$$

Thus $$\cos\theta = \frac{2p}{p^2+1}$$

and $$\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right) = \frac{p^2-1}{2p}$$

and $$\sin\theta = \tan\theta\cos\theta = \frac{p^2-1}{p^2+1}$$

129. Prove that : $(\operatorname{cosec}\theta + \cot\theta)^2 = \frac{\sec\theta+1}{\sec\theta-1}$

Ans :

$$(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta.\cot\theta$$

$$= \left(\frac{1}{\sin\theta}\right)^2 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 + \frac{2\times 1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

h205

$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$ Hence Prove.

130. Prove that :

$(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A\operatorname{cosec} A)^2$

Ans : [Board Term-1 2012]

$$\text{LHS} = (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$$

$$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$$

$$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + +2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

h206

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A\cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right)$$

$$= 1 + \frac{1}{\sin^2 A\cos^2 A} + \frac{2}{\sin A\cos A}$$

$$= \left(1 + \frac{1}{\sin A\cos A}\right)^2$$

$$= (1 + \sec A\operatorname{cosec} A)^2$$ Hence Proved

131. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
Prove that each of the side is equal to ± 1.

Ans : [Board Term-1 2012]

We have

h207

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

Multiply both sides by

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

or, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)\times$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

or, $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec A - \tan B)^2(\sec C - \tan C)^2$

or, $1 = \left[(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)\right]^2$

or, $(\sec A - \tan A)(\sec B - \tan B)(\sec C + \tan C) = \pm 1$

132. If $4\sin\theta = 3$, find the value of x if
$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$

Ans : [Board Term-1 2012]

We have $$\sin\theta = \frac{3}{4}$$

h208

or, $$\sin^2\theta = \frac{9}{16}$$

Since $\sin^2\theta + \cos^2 = 1$, we have

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

and $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$$

Thus $$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$$

$$\sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

Thus $x = \frac{4}{3}$

133. Prove that $\sec^2\theta + \mathrm{cosec}^2\theta$ can never be less than 2.

Ans : [Board-Term 1 2011]

h209

Let $\sec^2\theta + \mathrm{cosec}^2\theta = x$

$$1 + \tan^2\theta + 1 + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$\tan^2\theta \geq 0$ and $\cot^2\theta \geq 0$

Thus $x > 2$

Thus $\sec^2\theta + \mathrm{cosec}^2\theta > 2$

Hence $\sec^2\theta + \mathrm{cosec}^2\theta$ can never be less than 2.

134. (a) Solve for ϕ, if $\tan 5\phi = 1$

(b) Solve for ϕ, if $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

Ans :

h210

(a) $\tan 5\phi = 1$

$$\tan 5\phi = \tan 45^\circ$$

$$5\phi = 45^\circ$$

Thus $\phi = 9^\circ$

(b) $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

$$\frac{\sin^2\phi + (1+\cos\theta)^2}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{\sin^2\phi + 1 + 2\cos\phi + \cos^2\phi}{\sin\phi + \sin\phi\cos\phi} = 4$$

$$\frac{\sin^2\phi + \cos^2\phi + 1 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2}{\sin\phi} = 4$$

$$\sin\phi = \frac{1}{2}$$

$$\sin\phi = \sin 30^\circ$$

Thus $\phi = 30^\circ$

135. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Ans : [Board-Term 1 2009

h211

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$

$$= (\tan^2 A + \sin^2 A + 2\sin A\tan A)$$

$$-(\tan^2 A + \sin^2 A - 2\sin A\tan A)$$

$$= \tan^2 A + \sin^2 A + 2\sin A\tan A$$

$$-\tan^2 A - \sin^2 A + 2\sin A\tan A$$

$$= 4\sin A\tan A$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A\cos^2 A}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$

$$= 4\frac{\sin A \times \sin A}{\cos A}$$

$$= 4\sin A \times \frac{\sin A}{\cos A}$$

$$= 4\sin A\tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

136. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that

$(m^2+n^2)\cos^2\beta = n^2$.

Ans : [Board-Term 1 2010]

h212

We have $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$(m^2+n^2)\cos^2\beta = \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta}$$

$$= n^2$$ Hence Proved.

137. If $7\operatorname{cosec}\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\operatorname{cosec}\phi = 3$.

Ans :

h213

We have $7\operatorname{cosec}\phi - 3\cot\phi = 7$

$$7\operatorname{cosec}\phi - 7 = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1) = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1)(\operatorname{cosec}\phi + 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7(\operatorname{cosec}^2\phi - 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot^2\phi = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi = 3(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi - 3\operatorname{cosec}\phi = 3$$ Hence Proved

138. Prove that : $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Ans : [Board SQP 2018]

h214

$$\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1 - \cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - [(1 - \cos\theta)(1 + \cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1 + \cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1 + \cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta + \cot\theta$$ Hence Proved