9 Some Applications of Trigonometry

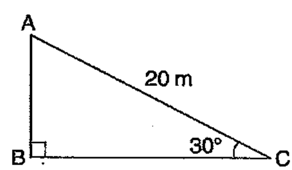
Lesson at a Glance

1. The height of an object and the distance between two distinct objects can be determined by using the trigonometric ratios.
2. The *line of sight* is the line drawn from the eye of an observer to the point in the object.
3. The *angle of elevation* is the angle formed by the line of sight with the horizontal when the object is above the horizontal level.
4. The *angle of depression* is the angle formed by the line of sight with the horizontal when the object is below the horizontal level.

TEXTBOOK QUESTIONS SOLVED

Exercise 9.1 *(Page – 203-205)*

1. *A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).*



Sol. Let height of the pole AB = x m

Length of the rope AC = 20 m

In ΔABC, $\angle ACB = 30°$

$$\therefore \quad \sin 30° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{x}{20} \Rightarrow x = 10 \text{ m}$$

$\therefore$ Height of the pole = 10 m.

2. *A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the*

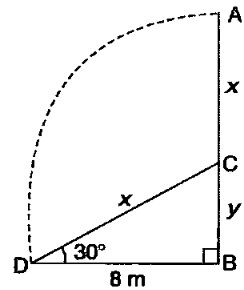
point where the top touches the ground is 8 m. Find the height of the tree.

Sol. Let the height of tree before storm be AB. Due to storm it breaks from C such that its top A touches the ground at D and makes an angle of 30°.

Let AC, *i.e.*, DC = x and BC = y; BD = 8 m [Given]

$\therefore$ Height of tree = $x + y$ ...(*i*)

In ΔBCD,



$$\frac{CD}{BD} = \sec 30°; \quad \frac{BC}{BD} = \tan 30°$$

$$\Rightarrow \frac{x}{8} = \frac{2}{\sqrt{3}}; \frac{y}{8} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \frac{16}{\sqrt{3}}; y = \frac{8}{\sqrt{3}}.$$

Substitute in (*i*) for x and y, we get

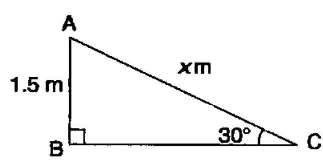
$$\text{Height of tree} = \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m.}$$

3. *A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?*

Sol. For children below 5 years:

Let x m be the length of the slide AC and height AB = 1.5 m [Given].

In ΔABC, $\angle$C = 30°



$$\therefore \frac{AB}{AC} = \sin 30° \Rightarrow \frac{1.5}{x} = \frac{1}{2}$$

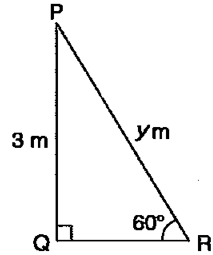
$\Rightarrow x = 3$ m

$\therefore$ Length of the slide for children below 5 years = 3 m.

For elder children:

Let length of the slide PR = y m and height PQ = 3 m (given).

In ΔPQR, $\angle R = 60°$



$$\therefore \quad \frac{PQ}{PR} = \sin 60° \quad \Rightarrow \quad \frac{3}{y} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad y = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m} = 2 \times 1.732 = 3.464 \text{ m}$$

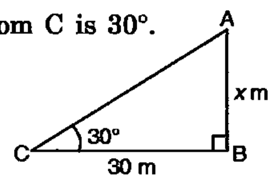
$\therefore$ Length of the slide for elder children = 3.46 m.

4. *The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.*

Sol. Let AB be the tower and C be a point on the ground such that BC = 30 m. Also let AB = x m.

The angle of elevation of the top A from C is 30°.

So, in right-angled ΔABC,



$$\frac{AB}{BC} = \tan 30° \quad \Rightarrow \quad \frac{x}{30} = \frac{1}{\sqrt{3}}$$

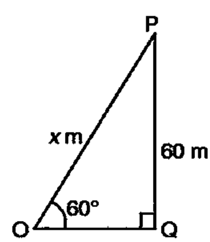
$$\Rightarrow \quad x = \frac{30}{\sqrt{3}} = 10\sqrt{3} = 10 \times 1.732 = 17.32 \text{ m.}$$

Hence, height of the tower is 17.32 m.

5. *A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.*

Sol. Let P be the temporarily position of the kite of which string is tied to a point O on the ground. Such that $\angle POQ$ is 60°, where Q is the point on the ground just below P. Therefore, PQ = 60 m.

In ΔPOQ,



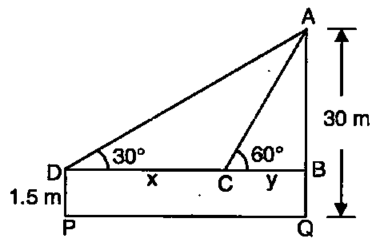
$$\frac{OP}{PQ} = \text{cosec } 60°$$

$$\Rightarrow \frac{x}{60} = \frac{2}{\sqrt{3}} \Rightarrow x = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, length of the string is $40\sqrt{3}$ m.

6. *A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.*

Sol. Let a 1.5 m tall boy DP finds the angle of elevation to the top A of the building A.



Let DCB be the horizontal eye sight. When that boy walks towards the building finds increased angle of 60° at C.

Given: AQ = 30 m.

$$AB = AB - BQ = (30 - 1.5) \text{ m} = 28.5 \text{ m}.$$

In right-angled ΔABC,

$$\frac{BC}{AB} = \cot 60° \Rightarrow \frac{y}{28.5} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y = \frac{28.5}{\sqrt{3}} = 16.45 \text{ m} \quad ...(i)$$

In right-angled Δ ABD,

$$\frac{BD}{AB} = \cot 30° \Rightarrow \sqrt{3} = \frac{x+y}{28.5}$$

$$\Rightarrow x + y = 28.5 \times \sqrt{3} = 49.36 \text{ m}$$

Putting from (*i*), we get

$$\Rightarrow x = (49.36 - 16.45) \text{ m} = 32.91 \text{ m}.$$

Hence, the distance he walked towards the building is 32.91 m.

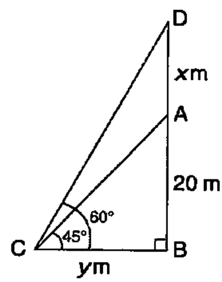
7. *From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top* *of a 20 m high building are 45° and 60° respectively. Find the height of the tower.*

Sol. Let AD be the transmission tower fixed at the top A of the building BA of 20 m high. Angles of elevation of A and D from a point C on the ground are 45° and 60° respectively, *i.e.*, $\angle ACB = 45°$ and $\angle DCB = 60°$

Let $AD = x$ m and $BC = y$ m

In right-angled ΔABC,

$$\frac{BC}{AB} = \cot 45° \Rightarrow \frac{y}{20} = 1 \Rightarrow y = 20 \qquad ...(i)$$



In right ΔDBC, $\frac{DB}{BC} = \tan 60°$

$$\Rightarrow \frac{x+20}{y} = \sqrt{3} \Rightarrow x + 20$$

$$= \sqrt{3}\,y \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$$x + 20 = 20\sqrt{3}$$

$$\Rightarrow \quad x = 20\sqrt{3} - 20 = 20(1.732 - 1) = 14.64 \text{ m}$$

Hence, the height of the tower is 14.64 m.

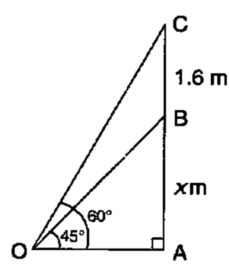
8. *A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.*

Sol. Let height of pedestal $AB = x$ m.

Height of statue $BC = 1.6$ m

$$\angle AOB = 45°, \angle AOC = 60°$$

In right-angled ΔOAB,



$$\frac{OA}{AB} = \cot 45°$$

$$\frac{OA}{x} = 1 \Rightarrow OA = x \qquad ...(i)$$

Also in right-angled triangle OAC,

$$\frac{OA}{AC} = \cot 60° \Rightarrow \frac{x}{x+1.6} = \frac{1}{\sqrt{3}} \quad \text{[From (i)]}$$

$$\Rightarrow \quad \sqrt{3}x = x + 1.6 \Rightarrow (\sqrt{3} - 1)x = 1.6$$

$$x = \frac{1.6}{\sqrt{3}-1} = \frac{1.6(\sqrt{3}+1)}{2} = 0.8(\sqrt{3} + 1) \text{ m}$$

$\therefore$ Height of pedestal = $0.8(\sqrt{3} + 1)$ m or 2.19 m.

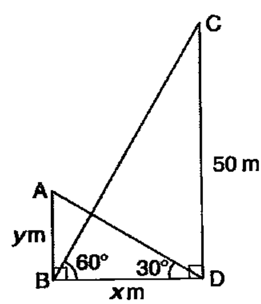
9. *The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.*

Sol. In the figure, CD is a tower of 50 m high and AB is a building.

Let height of the building = y m.

Let BD = x m.

$\angle CBD = 60°$, $\angle ADB = 30°$.



In right-angled triangle CDB,

$$\frac{BD}{CD} = \cot 60° \Rightarrow \frac{x}{50} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{50}{\sqrt{3}} \quad ...(i)$$

In right-angled triangle ABD,

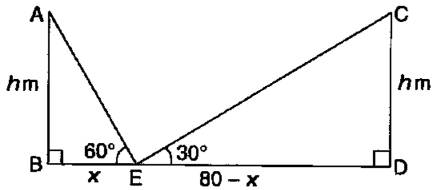
$$\frac{AB}{BD} = \tan 30° \Rightarrow \frac{y}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad y = \frac{x}{\sqrt{3}} = \frac{50}{3} \text{ m} \quad \text{[From (i)]}$$

$\therefore$ Height of the building = $16\frac{2}{3}$ m.

10. *Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the* *heights of the poles and the distances of the point from the poles.*

Sol. Let height of the each pole = h m.



Let BE = x m, ED = $(80 - x)$ m.

$\angle AEB = 60°$, $\angle CED = 30°$

In right-angled triangle ABE,

$$\frac{AB}{BE} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}\,x \quad ...(i)$$

In right-angled triangle CDE,

$$\frac{CD}{DE} = \tan 30°$$

$$\frac{h}{80-x} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{80-x}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we get

$$\sqrt{3}\,x = \frac{80-x}{\sqrt{3}} \Rightarrow 3x = 80 - x$$

$$\Rightarrow \quad 4x = 80 \quad \Rightarrow \quad x = 20 \text{ m}$$

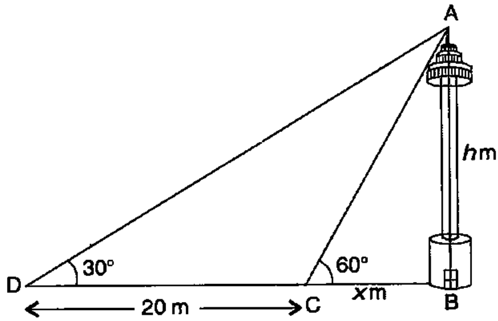
$\therefore$ Distance of the point from one pole = 20 m.

Substituting in (i), we get

$$h = 20\sqrt{3} \text{ m}$$

$\therefore$ Height of each pole = $20\sqrt{3}$ m.

11. *A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the canal.*



Sol. Let width (BC) of the canal is x m and height (AB) of the tower is h m.

In right-angled triangle ABC,

$$\frac{AB}{BC} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}\,x \quad ...(i)$$

In right-angled triangle ABD,

$$\frac{AB}{BD} = \tan 30° \Rightarrow \frac{h}{20+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{20+x}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we get

$$\sqrt{3}\,x = \frac{20+x}{\sqrt{3}} \Rightarrow 3x = 20 + x$$

$$\Rightarrow \quad 2x = 20 \Rightarrow x = 10 \text{ m}$$

$\therefore$ Width of the canal = 10 m.

Substituting in (i), we get $\quad h = 10\sqrt{3}$ m

$\therefore$ Height of the TV tower = $10\sqrt{3}$ m.

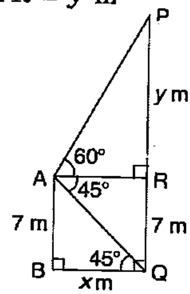
12. *From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.*

Sol. In the figure, AB represents a 7 m high building and PQ represents a cable tower. Angle of elevation to the top P from A is 60° and angle of depression to the bottom Q from same point A is 45°.

So, $\angle PAR = 60°$ and $\angle RAQ = 45° = \angle AQB$.

Let AB = 7 m = QR, AR = BQ = x m and PR = y m

In rt ΔABQ,



$$\frac{AB}{BQ} = \tan 45° \Rightarrow \frac{7}{x} = 1$$

$\Rightarrow \quad x = 7$ m ...(i)

In rt. ΔPRA,

$$\frac{PR}{AR} = \tan 60° \Rightarrow \frac{y}{x} = \sqrt{3}$$

$\Rightarrow \quad y = \sqrt{3}x = 7\sqrt{3}$ m [From (i)]

$\therefore$ Height of the tower = $7 + y = (7 + 7\sqrt{3})$ m

$= 7(1 + \sqrt{3})$ m $= 7 \times 2.732$ m

$= 19.12$ m.

13. *As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.*

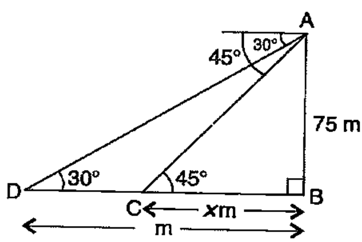
Sol. In the figure, AB represents a 75 m high lighthouse, C and D are the positions of two ships where angles of depression from top of the tower are 45° and 30° respectively.

Let BC = x m and BD = y m.

We have to find the distance between the two ships,

i.e., CD = BD – BC = $y - x$

In right-angled ΔABC,



$$\frac{BC}{AB} = \cot 45°$$

$$\Rightarrow \quad \frac{x}{75} = 1$$

$\Rightarrow \quad x = 75$ m

In right-angled ΔABD,

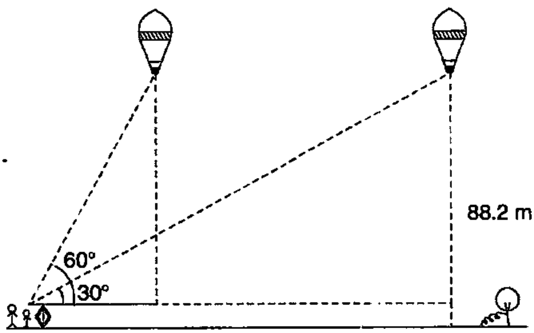
$$\frac{BD}{AB} = \cot 30° \Rightarrow \frac{y}{75} = \sqrt{3}$$

$\Rightarrow \quad y = 75\sqrt{3}$ m

$\therefore$ The distance between two ships

$$= y - x = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1) \text{ m.}$$

14. *A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig.). Find the distance travelled by the balloon during the interval.*

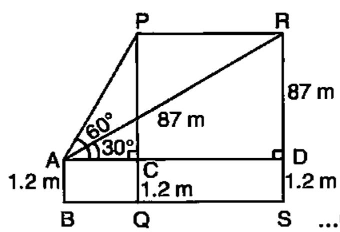


Sol. When the horizontally moving balloon is at P, a girl AB (say) finds ∠PAC is 60° and at the position R, ∠RAD is 30°.

We have to find the distance travelled in horizontal line, *i.e.*, PR or CD.

Consider PC = PQ – CQ = (88.2 – 1.2) m = 87 m = RD

In rt ΔACP,



$$\tan 60° = \frac{PC}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{AC}$$

$$\Rightarrow \quad AC = \frac{87}{\sqrt{3}} \qquad ...(i)$$

In ΔADR,

$$\tan 30° = \frac{RD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{AD} \Rightarrow AD = 87\sqrt{3} \qquad ...(ii)$$

Now CD = AD – AC

$$= 87\sqrt{3} - \frac{87}{\sqrt{3}} = 87\frac{(3-1)}{\sqrt{3}} = \frac{87\times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

[From (*i*) and (*ii*)]

$$= 29 \times 2 \times \sqrt{3} = 58\sqrt{3}$$

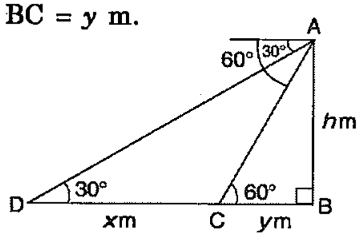
∴ The distance travelled by the balloon during the interval is $58\sqrt{3}$ m.

15. *A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.*

Sol. Let AB be the tower on the top of which a man is standing and finds angle of depression to the position D of the running car as 30°.

After 6 seconds, angle is found 60° at C.

Let AB = h m, DC = x m and BC = y m.



When car is at D:

$$\frac{x+y}{h} = \cot 30°$$

$$\Rightarrow \quad x + y = \sqrt{3}\,h \qquad ...(i)$$

When car is at C:

$$\frac{y}{h} = \cot 60° \quad \Rightarrow \quad \frac{y}{h} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad h = \sqrt{3}\,y \qquad ...(ii)$$

From (*i*) and (*ii*), we have

$$x + y = \sqrt{3}\,.\sqrt{3}\,y = 3y \quad \Rightarrow \quad x = 2y \quad \Rightarrow \quad y = \frac{x}{2}$$

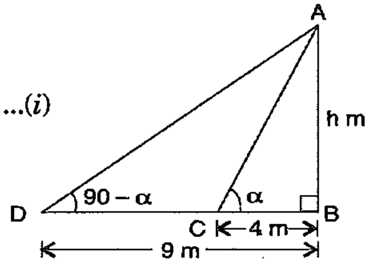
∵ Distance x is covered in 6 seconds

∴ Distance y, *i.e.*, $\frac{x}{2}$ is covered in 3 seconds.

16. *The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower* *and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.*

Sol. In the drawn figure, AB represents a tower, C and D are two points distant 4 m and 9 m away from the base B.

Let angle of elevation to the top A from C is α and from D is $90° - \alpha$.



In right-angled ΔABC,

$$\frac{h}{4} = \tan\alpha \quad \text{...}(i)$$

In right-angled ΔABD,

$$\frac{h}{9} = \tan(90° - \alpha)$$

$$\Rightarrow \quad \frac{h}{9} = \cot\alpha \quad \text{...}(ii)$$

Multiplying the corresponding sides of (*i*) and (*ii*), we get

$$\frac{h}{4} \cdot \frac{h}{9} = \tan\alpha \cdot \cot\alpha \quad \Rightarrow \quad \frac{h^2}{36} = 1$$

$$\Rightarrow \quad h^2 = 36 \qquad \Rightarrow \quad h = 6$$

$\therefore$ Height of the tower = 6 m. **Hence proved.**