10 Circles

Lesson at a Glance

1. Only one tangent is possible at a point on the circle.
2. No tangent is possible from a point in the interior of a circle, to the circle.
3. At most two tangents can be drawn from a point in the exterior of a circle, to the circle.
4. The lengths of the two tangents drawn from an external point to a circle are equal.
5. Tangent at any point on a circle is perpendicular to the radius passing through the point of contact.

TEXTBOOK QUESTIONS SOLVED

Exercise 10.1 *(Page – 209)*

1. *How many tangents can a circle have?*

Sol. A circle can have infinitely many tangents.

2. *Fill in the blanks:*

(i) A tangent to a circle intersects it in ________ point(s).

(ii) A line intersecting a circle in two points is called a ________ .

(iii) A circle can have ________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ________ .

Sol. (i) one (ii) secant

(iii) two (iv) point of contact.

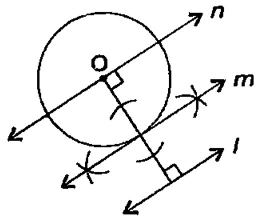
3. *A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:*

(*A*) *12 cm* (*B*) *13 cm* (*C*) *8.5 cm* (*D*) $\sqrt{119}$ *cm.*

Sol. (D).

4. *Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.*

Sol.



Here l is the given line. m and n are respectively, a tangent and a secant to a given circle with centre O and parallel to line l.

Exercise 10.2 *(Page – 213-214)*

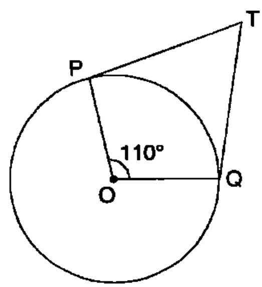
In Q. 1 to 3, choose the correct option and give justification.

1. *From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is*

   (*A*) *7 cm* (*B*) *12 cm* (*C*) *15 cm* (*D*) *24.5 cm.*

Sol. $r = \sqrt{(25)^2 - (24)^2}$ cm = 7 cm.

Option (A) is correct.



2. *In figure, if TP and TQ are the two tangents to a circle with centre O so that* $\angle POQ = 110°$, *then* $\angle PTQ$ *is equal to*

   (*A*) *60°* (*B*) *70°*

   (*C*) *80°* (*D*) *90°.*

Sol. $\because$ TQ and TP are tangents to a circle with centre O.

such that $\angle$ POQ = 110°

$\therefore$ OP $\perp$ PT and OQ $\perp$ QT

$\Rightarrow \angle OPT = 90$ and $\angle OQT = 90°$

Now, in the quadrilateral TPOQ, we get

$\therefore \angle PTQ + 90° + 110° + 90° = 360°$

$\Rightarrow \angle PTQ + 290° = 360°$

$\Rightarrow \angle PTQ = 360° - 290° = \mathbf{70°}$

Thus, the correct option is (B).

3. *If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to*

(A) 50° *(B) 60°* *(C) 70°* *(D) 80°.*

Sol. $\angle POA = \frac{1}{2}\angle AOB = \frac{1}{2} \times 100° = 50°.$

Option (A) is correct.

4. *Prove that the tangents drawn at the ends of a diameter of a circle are parallel.*

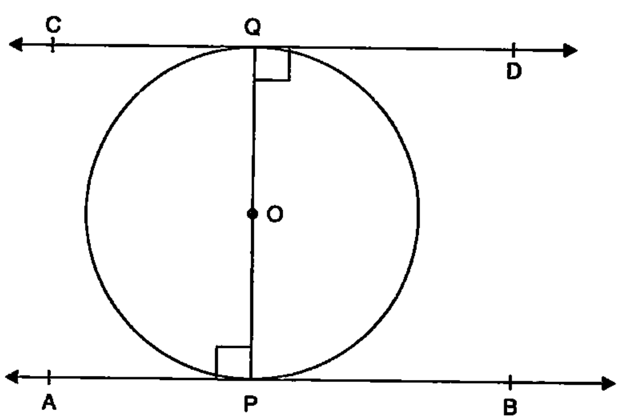
Sol. In the figure, we have:

PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.

Since the tangent at a point to a circle is perpendicular to the radius through the point.

$\therefore \quad PQ \perp AB \Rightarrow \angle APQ = 90°$



And $\quad PQ \perp CD \Rightarrow \angle PQD = 90°$

$\Rightarrow \quad \angle APQ = \angle PQD$

But they form a pair of alternate angles.

$\therefore$ AB $\parallel$ CD.

5. *Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.*

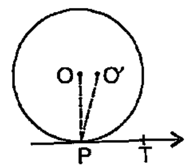
Sol. Let perpendicular at the point of contact to the tangent does not pass through at centre.

O′P $\perp$ PT ...(*i*) [Given]

Join OP. As OP is radius.

$\therefore$ OP $\perp$ PT ...(*ii*)

[Radius is perpendicular to tangent at the point of contact]



From (*i*) and (*ii*), we get

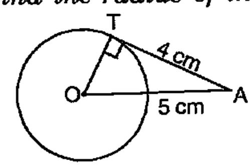
OP and O′P are perpendicular to PT

$\Rightarrow$ OP and O′P must coincide.

As only one perpendicular can be drawn from a point on a line.

Hence perpendicular from the point of contact, passes through the centre.

6. *The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.*



Sol. OT = $\sqrt{5^2 - 4^2}$

cm = $\sqrt{9}$ cm = 3 cm.

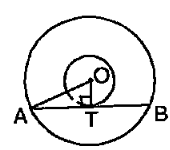
7. *Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.*

Sol. OA = 5 cm, OT = 3 cm

Also OT $\perp$ AB,

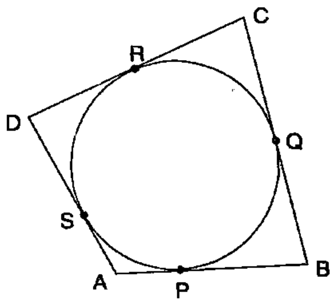
Therefore, AT = $\sqrt{25-9}$ cm = 4 cm

$\therefore$ AB = 2AT = 8 cm.



8 *A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that*

$$AB + CD = AD + BC$$



Sol. We have AS = AP; BP = BQ; CQ = CR and DR = DS.

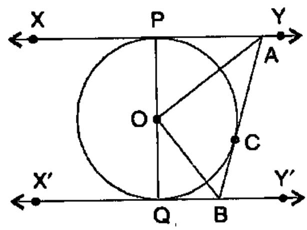
Consider AB + CD = (AP + PB) + (CR + RD)

= AS + BQ + CQ + DS

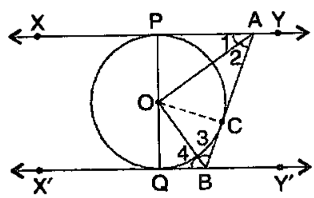
= (AS + DS) + (BQ + CQ)

= AD + BC. **Hence proved.**

9. *In figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X′Y′ at B. Prove that ∠AOB = 90°.*



Sol. Given: A circle with centre O has three tangents XY, X′Y′ and AB at the points P, Q and C respectively. Also XY ∥ X′Y′.



To prove: ∠AOB = 90°

Construction: Join OC, OP, OQ.

Proof: In ΔAOP and ΔAOC,

PA = PC [Two tangents drawn from a point outside the circle]

PO = CO [Radii of same circle]

AO = AO [Common]

Therefore, ΔAOP ≅ ΔAOC [SSS criterion)]

∴ ∠1 = ∠2 ...(*i*)

Similarly, we can prove that

∠3 = ∠4 ...(*ii*)

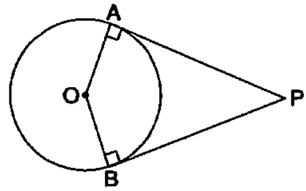
Now, $\angle PAB + \angle QBA = 180°$ [Sum of interior angles on the same side of transversal]

$\Rightarrow \quad 2\angle 2 + 2\angle 3 = 180°$ [Using (*i*) and (*ii*)]

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$ [Sum of angles of a triangle is 180°]

$\Rightarrow \quad \angle AOB = 90°$. **Hence proved.**

10. *Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.*



Sol. $\angle OAP = \angle OBP = 90°$ ...(*i*)

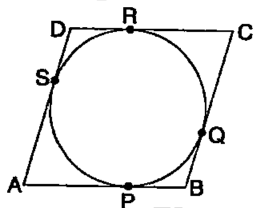
Also $\angle AOB + \angle OBP + \angle BPA + \angle OAP = 360°$

[Sum of angles of a quadrilateral is 360°]

$\Rightarrow \quad \angle AOB + 90° + \angle BPA + 90° = 360°$

$\Rightarrow \quad \angle AOB + \angle BPA = 180°$.

11. *Prove that the parallelogram circumscribing a circle is a rhombus.*



Sol. AB = CD and BC = DA ...(*i*)

AB + CD = AP + BP + CR + DR

= AS + BQ + CQ + DS

= AD + BC [Using (*i*)]

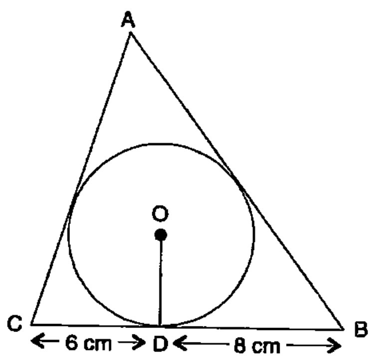
$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB = AD

As adjacent sides of a parallelogram are equal.

$\therefore$ Parallelogram is a rhombus.

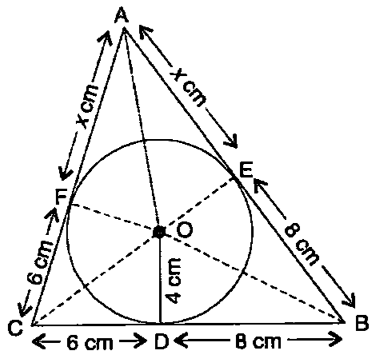
12. *A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of*



lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.

Sol. Let the circumcircle touches AB and AC at E and F respectively.

Join OA, OB, OC, OE and OF.



We have

$$OD = 4 \text{ cm} = OE = OF \quad \text{[Radii]}$$

Also $\quad CD = 6$ cm $= CF$, $BD = 8$ cm $= BE$

and $\quad AE = AF = x$ cm $\quad$ [Say]

[$\because$ The lengths of the two tangents from an external point to a circle are equal]

From figure,

$$ar(\Delta ABC) = ar(\Delta OAB) + ar(\Delta OBC) + ar(\Delta OCA) \quad ...(i)$$

But $\quad ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{(14+x)(14+x-8-x)(14+x-14)(14+x-6-x)}$$

$$\left[\because \; s = \frac{AB+BC+CA}{2} = \frac{8+x+8+6+6+x}{2} = 14+x\right]$$

$$= \sqrt{(14+x)\,6 \times x \times 8} = \sqrt{48x\,(14+x)}.$$

$$ar(\Delta OAB) = \frac{1}{2} \times (8 + x) \times 4 = 16 + 2x$$

$$ar(\Delta OBC) = \frac{1}{2}\,(6 + 8) \times 4 = 28$$

and $\quad ar(\Delta OCA) = \frac{1}{2}\,(6 + x) \times 4 = 12 + 2x$

Now, putting these values in eqn. (*i*), we have

$$\sqrt{48x\,(14+x)} = (16 + 2x) + (28) + (12 + 2x)$$

$\Rightarrow \quad \sqrt{48x\,(14+x)} = 56 + 4x = 4(14 + x)$

$\Rightarrow \quad 48x(14 + x) = 16(14 + x)^2$ [Squaring both sides]

$\Rightarrow \quad 48x(14 + x) - 16(14 + x)^2 = 0$

$\Rightarrow \quad 16(14 + x)(3x - 14 - x) = 0$

$\Rightarrow \quad 16(14 + x)(2x - 14) = 0$

$\Rightarrow \quad x = -14, 7$

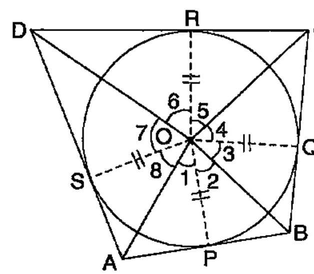
Ignoring $x = -14$ because length cannot be negative.

$\therefore \quad x = 7$ cm

Hence, AB = BE + AE = 8 + x = 8 + 7 = 15 cm

and AC = CF + AF = 6 + x = 6 + 7 = 13 cm.

13. *Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.*



Sol. $\Delta AOP \cong \Delta AOS$

[As AP = AS, OP = OS, AO is common]

$\angle 1 = \angle 8$

Similarly $\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \quad 2\angle 1 + 2\angle 2 + 2\angle 6 + 2\angle 5 = 360°$

$\Rightarrow \quad (\angle 1 + \angle 2) + (\angle 6 + \angle 5) = 180°$

$\Rightarrow \quad \angle AOB + \angle COD = 180°$

Similarly, we can show

$\angle AOD + \angle BOC = 180°$