11 Constructions

Lesson at a Glance

1. The bisector of an angle passes between the two arms of the angles.
2. Each point on the bisector of an angle is equidistant from the arms of the angle.
3. In the process of constructing the perpendicular bisector of a line segment, the radius of each arc must be greater than the half of the line segment.
4. Each point on the perpendicular bisector of a line segment is equidistant from the end points of the line segment.
5. To construct a triangle when the sum of the three sides and the two base angles are given, first we draw the line segment consisting the sum of the three sides and the two base angles.

TEXTBOOK QUESTIONS SOLVED

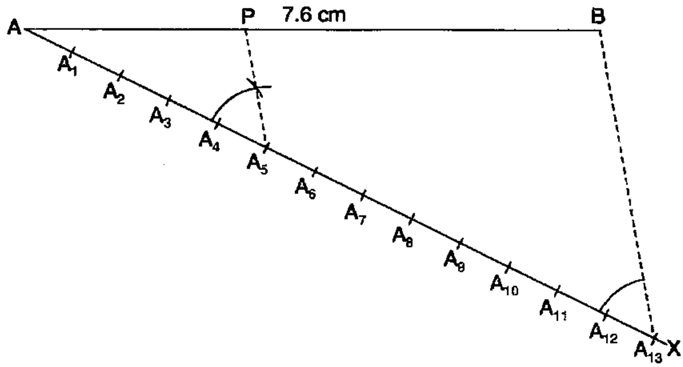
Exercise 11.1 *(Page – 219-220)*

In each of the following, give the justification of the construction also:

1. *Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.*

Sol. Steps of construction:

1. A line AB of length 7.6 cm is drawn.
2. Any angle BAX is drawn.



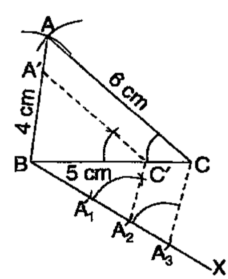
3. On AX, points A_1, A_2, ..., A_{13} are taken such that $AA_1 = A_1A_2 = A_2A_3 = ... = A_{12}A_{13}$.
4. B and A_{13} are joined.
5. PA_5 is drawn parallel to BA_{13}, meeting AB at P.
6. Then AP : PB = 5 : 8. AP = 2.9 cm, PB = 4.7 cm.

Justification: PA_5 is parallel to BA_{13}

$$\therefore \quad \frac{AP}{PB} = \frac{AA_5}{A_5A_{13}} = \frac{5}{8} \qquad \Rightarrow \quad AP : PB = 5 : 8.$$

2. *Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.*

Sol. Steps of construction:

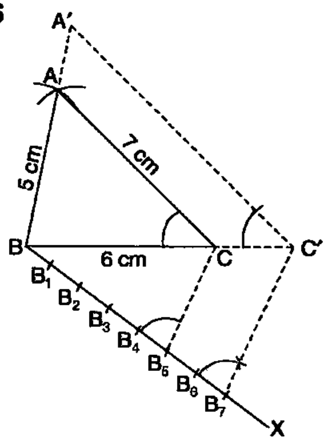


1. A line segment BC of length 5 cm is drawn.
2. With centre B, an arc of radius 4 cm and with centre C, another arc of radius 6 cm are drawn cutting at point A of arcs. A is joined to B and C to form ΔABC.
3. Any angle CBX is drawn.
4. On BX, points A_1, A_2 and A_3 are taken, such that $BA_1 = A_1A_2 = A_2A_3$.
5. C and A_3 are joined.

6. $C'A_2$ is drawn parallel to CA_3, meeting BC at C′.
7. A′C′ is drawn parallel to AC, meeting BA at A′.
8. Then, BA′C′ is the required triangle whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

3. *Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.*

Sol. Steps of construction:

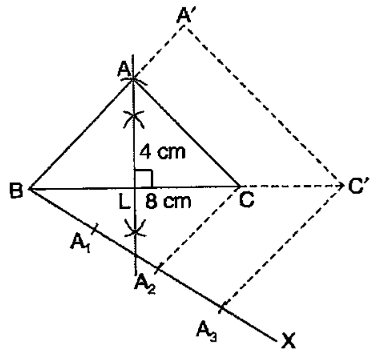


1. A line segment BC of length 6 cm is drawn.
2. Two arcs taking B and C as centres and respectively of length 5 cm and 7 cm as radius are drawn.
3. Point A, *i.e.*, meeting point of arcs is joined to the points B and C, so ΔABC is formed.
4. Any angle CBX is drawn.
5. On BX, points B_1, B_2, ..., B_7 are taken such that $BB_1 = B_1B_2 = B_2B_3 = \ldots\ldots B_6B_7$.
6. B_5 and C are joined.
7. B_7C' is drawn parallel to B_5C where C′ lies to BC produced.
8. Now C′A′ parallel to CA is drawn, A′ lies on BA produced. Therefore, the required $\Delta A'BC'$ is formed whose sides are $\frac{7}{5}$ times the corresponding sides of the given triangle.

4. *Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.*

Sol. Steps of Construction:

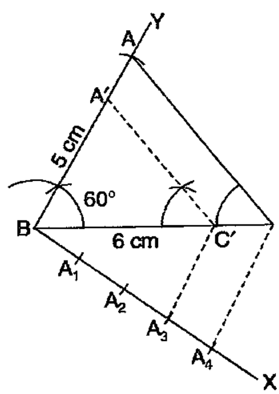
1. Base BC = 8 cm is drawn.
2. Perpendicular bisector l of BC is drawn.
3. On line l, point A is taken such that AL = 4 cm where L lies on BC.
4. AB and AC are joined. Then ABC is an isosceles triangle formed.
5. An angle CBX is drawn.
6. Points A_1, A_2 and A_3 are taken on BX, such that $BA_1 = A_1A_2 = A_2A_3$.
7. C and A_2 are joined.
8. $C'A_3$ is drawn parallel to A_2C, meeting BC produced at C′.
9. C′A′ is drawn parallel to CA, meeting BA produced at A′.
10. Then A′BC′ is the required triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of an isosceles triangle.



5. *Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the isosceles triangle.*

Sol. Steps of construction:

1. A line segment BC = 6 cm is drawn.
2. $\angle YBC = 60°$ is drawn.
3. On BY, AB = 5 cm is cut.
4. AC is joined to form triangle ABC.
5. Any angle CBX is drawn.
6. On BX, points A_1, A_2, A_3 and A_4 are taken such that $BA_1 = A_1A_2 = A_2A_3 = A_3A_4$.
7. A_4 and C are joined.
8. A_3C' is drawn parallel to the A_4C meeting BC at C′.



9. C′A′ is drawn parallel to AC meeting AB at A′.
10. Then triangle A′BC′ is the required triangle whose each side is $\frac{3}{4}$ times the side of given triangle.

6. *Draw a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of Δ ABC.*

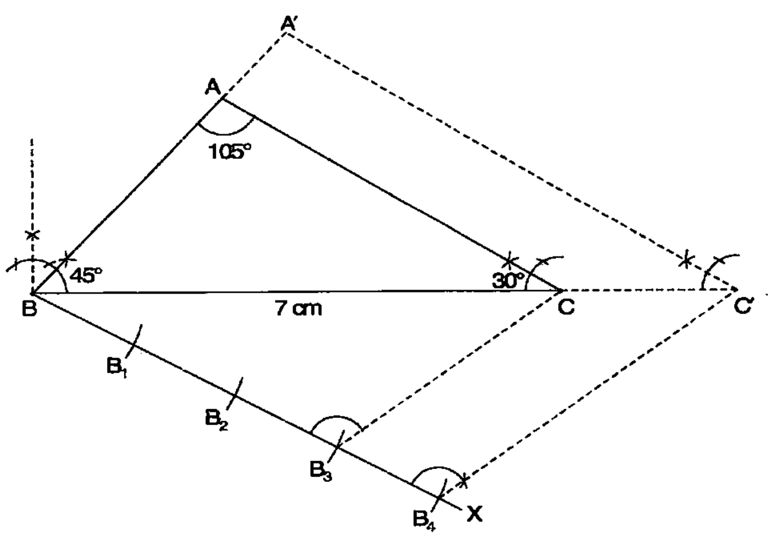
Sol. Analysis: In ΔABC, ∠A = 105°, ∠B = 45° and base BC = 7 cm are given.

$$\angle C = 180° - (\angle A + \angle B)$$
$$= 180° - (105° + 45°)$$
$$= 180° - 150° = 30°.$$

Steps of constructions:

1. A line segment BC = 7 cm is drawn.
2. An angle of measure 45° is constructed at B, also at C another angle of measure 30° is formed with common side BC.
3. Thus, uncommon sides of these angles are met in A, so ΔABC is formed.

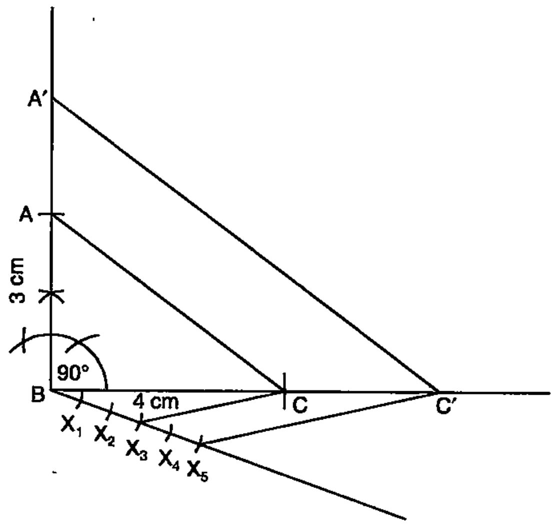
Further follow the steps as of solution 3.



7. *Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then, construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.*

Sol. Steps of construction:

(*i*) Construct the right triangle ABC such that $\angle B = 90°$, BC = 4 cm and BA = 3 cm.



(*ii*) Draw a ray BX such that an acute angle $\angle CBX$ is formed.

(*iii*) Mark 5 points X_1, X_2, X_3, X_4 and X_5 on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_4X_5$.

(*iv*) Join X_3 to C.

(*v*) Draw a line through X_5 parallel to X_3 C, intersecting the extended line segment BC at C′.

(*vi*) Draw another line through C′ parallel to CA intersecting the extended line segment BA at A′.

Thus, Δ A′ BC′ is the required triangle.

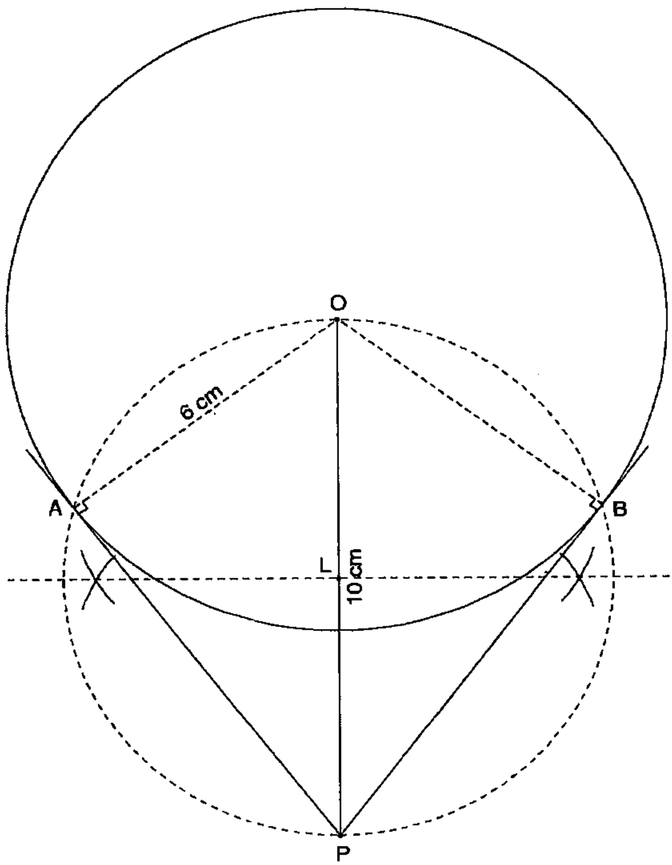
Exercise 11.2 *(Page – 221-222)*

In each of the following, give also the justification of the construction:

1. *Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.*

Sol. Steps of construction:

1. A circle of radius 6 cm is drawn.



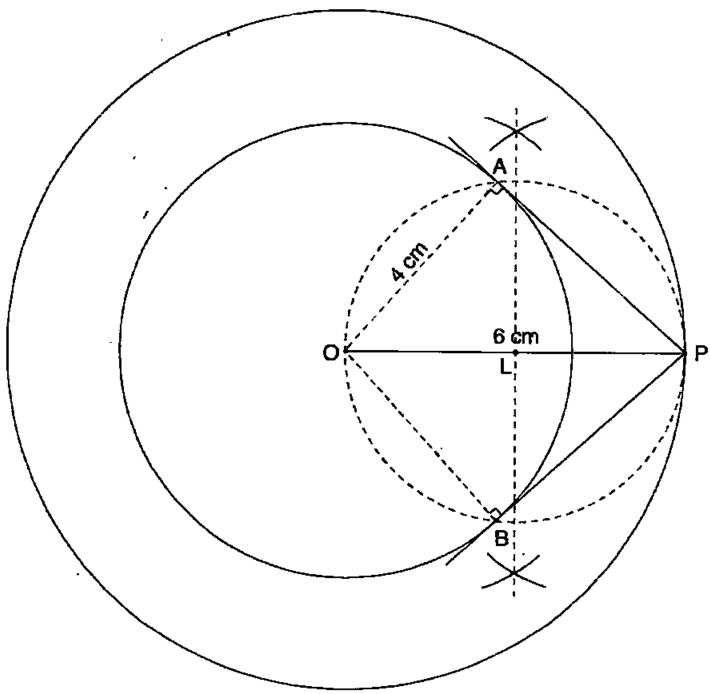
2. A point P is taken outside the circle such that OP = 10 cm.
3. Perpendicular bisector of OP is drawn, meeting OP at L.
4. With L as centre and OL as radius a circle is drawn, meeting the given circle at A and B.

5. PA and PB are joined.
6. PA and PB are the required tangents PA = 8 cm = PB.

2. *Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.*

Sol. Steps of construction:

1. Two concentric circles of radii 4 cm and 6 cm are drawn.
2. P is a point on the circle with radius 6 cm.
3. Perpendicular bisector of OP is drawn, meeting OP at L.
4. With L as centre and OL as radius a circle is drawn, meeting the circle with radius 4 cm at A and B.



PA = PB = 4.5 cm

5. PA and PB are joined.
6. PA and PB are the required tangents, such that PA = 4.5 cm = PB.

Calculation:

In ΔAOP, OA = 4 cm and OP = 6 cm

Then $AP = \sqrt{OP^2 - OA^2} = \sqrt{(6)^2 - (4)^2}$

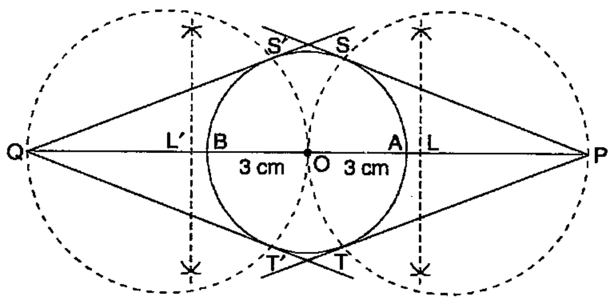
$= \sqrt{36 - 16}$

$= \sqrt{20} = 4.47 \approx 4.5$ cm.

3. *Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.*

Sol. Steps of construction:

1. A circle of 3 cm radius is drawn.

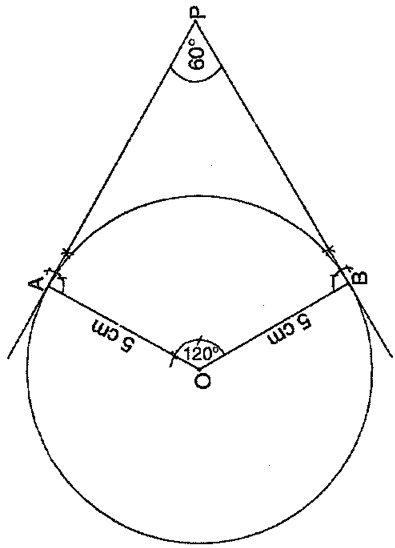


2. Points P and Q are taken on extended diameter AB such that OP = OQ = 7 cm.
3. Perpendicular bisectors of OP and OQ are drawn meeting OP at L and OQ at L′.
4. With L and L′ as centres and OL and OL′ as radii circles are drawn meeting the circle at S, T and S′, T′ respectively.
5. PS, PT and QS′, QT′ are drawn.
6. Then PS, PT, QS′, QT′ are the required tangents.

4. *Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.*

Sol. Steps of construction:

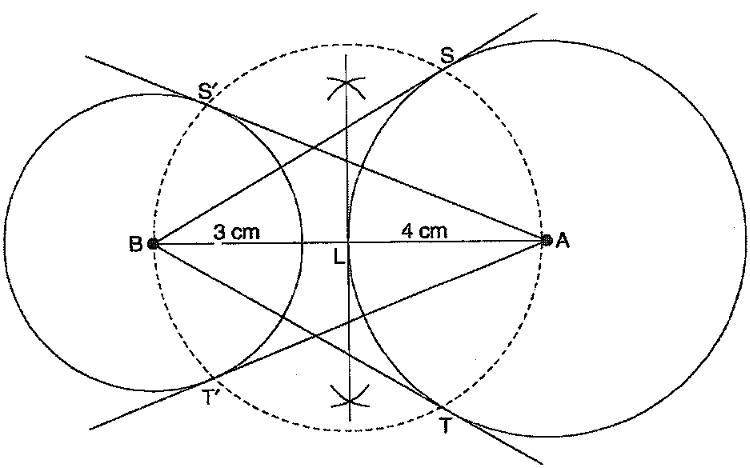
1. A circle with centre O of radius 5 cm is drawn.
2. Taking a radius OB as base and O as centre, an angle of measure 120° is constructed. So $\angle AOB = 120°$.
3. Tangents AP and BP are drawn to the circle at A and B, meeting each other at P.
4. Then PA and PB are the required tangents, such that $\angle APB = 60°$.



5. *Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.*

Sol. Steps of Construction:

1. A line segment AB = 8 cm is drawn.



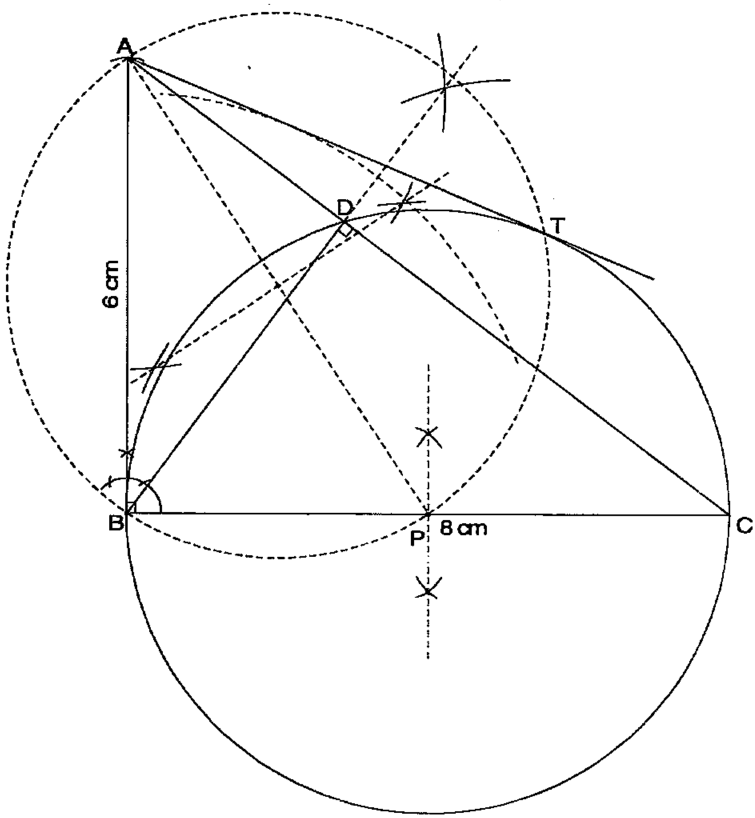
2. With A as centre a circle of radius 4 cm is drawn and B as centre a circle of radius 3 cm is drawn.

4. With L as centre and AL as radius a circle is drawn meeting smaller circle at S′ and T′ and bigger circle at S and T.
5. AS′, AT′ and BS, BT are joined. Then AS′, AT′, BS, BT are the required tangents.

6. *Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and ∠B = 90°, BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.*

Sol. Steps of construction:

1. A right-angled triangle ABC is drawn such that AB = 6 cm, BC = 8 cm, ∠ABC = 90°.
2. BD is drawn perpendicular to AC, meeting AC at D.



3. ∠BDC = 90°, therefore, BC is hypotenuse. Hence, perpendicular bisector of BC is drawn to find mid-point P.

4. A circle through B, C, D is drawn, such that BC is diameter and P is its centre.
5. AP is joined.
6. A circle is drawn with AP as diameter meeting the circle at B and T.
7. AT is joined.
8. Then AB and AT are the required tangents.

7. *Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.*

Sol. Steps of construction:

(*i*) Draw the given circle using a bangle.

(*ii*) Take two non parallel chords PQ and RS of this circle.

(*iii*) Draw the perpendicular bisectors of PQ and RS such that they intersect at O. Therefore, O is the centre of the given circle.

(*iv*) Take a point P outside this circle.

(*v*) Join OP and bisect it. Let M be the mid point of OP.

(*vi*) Taking M as centre and OM as radius, draw a circle. Let it intersect the given circle at A and B.

(*vii*) Join PA and PB. Thus, PA and PB are the required two tangents.