14 Statistics

Lesson at a Glance

1. Mean, median and mode are the three measures of central tendency.

2. Class mark $= \dfrac{\text{upper class limit} + \text{lower class limit}}{2}$.

3. The mean $\bar{x}$ is computed by the following methods:

   (*i*) direct method: $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$.

   (*ii*) assumed mean method: $\bar{x} = a + \dfrac{\sum f_i d_i}{\sum f_i}$

   (*iii*) step deviation method: $\bar{x} = a + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) \times h$

   where symbols have their usual meanings.

4. The median for grouped data can be computed by

$$\text{median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

   where symbols have their usual meanings.

5. The x-coordinate of the point of intersection of both the ogives of grouped data gives the median.

6. The mode for grouped data can be obtained by

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h.$$

   where symbols have their usual meanings.

7. Mode = 3 median − 2 mean.

TEXTBOOK QUESTIONS SOLVED

Exercise 14.1 *(Page – 270-272)*

1. *A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.*

Number of plants	*0-2*	*2-4*	*4-6*	*6-8*	*8-10*	*10-12*	*12-14*
Number of houses	*1*	*2*	*1*	*5*	*6*	*2*	*3*

Which method did you use for finding the mean, and why?

Sol.

Number of plants (C.I.)	*Number of houses* (f_i)	*Class-mark* (x_i)	f_ix_i
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
	$\Sigma f_i = 20$		$\Sigma f_ix_i = 162$

$$\text{Mean} = \frac{\sum f_ix_i}{\sum f_i} = \frac{162}{20}$$

$$= 8.1 \text{ plants}$$

Therefore, the mean number of plants per house is 8.1. We have used direct method as numerical values of C.I. and f_i are small.

2. *Consider the following distribution of daily wages of 50 workers of a factory.*

Daily wages (in ₹)	*100-120*	*120-140*	*140-160*	*160-180*	*180-200*
Number of workers	*12*	*14*	*8*	*6*	*10*

Find the mean daily wages of the workers of the factory by using an appropriate method.

Sol.

Daily wages (C.I.)	No. of workers (f_i)	Class-mark (x_i)	$d_i = x_i - a$	$u_i = d_i$	f_iu_i
100-120	12	110	– 40	– 2	– 24
120-140	14	130	– 20	– 1	– 14
140-160	8	150	0	0	0
160-180	6	170	20	1	6
180-200	10	190	40	2	20
	$\Sigma f_i = 50$				$\Sigma f_iu_i = -12$

Here, $a = 150$, $h = 20$

Using the step-deviation method,

$$\text{Mean} = a + \frac{\sum f_iu_i}{\sum f_i} \times h$$

$$= 150 + \frac{-12}{50} \times 20$$

$$= 150 - 4.8 = 145.2$$

Therefore, the mean daily wages of the workers is ₹ 145.20.

3. *The following distribution shows the daily pocket allowance of children of a locality.*

The mean pocket allowance is ₹ 18. Find the missing frequency f.

Daily pocket allowance (in ₹)	*11-13*	*13-15*	*15-17*	*17-19*	*19-21*	*21-23*	*23-25*
Number of children	*7*	*6*	*9*	*13*	*f*	*5*	*4*

Sol.

Daily pocket allowance (in ₹) (C.I.)	*No. of children* (f_i)	*Class-mark* (x_i)	$d_i = x_i - 18$	$u_i = \frac{d_i}{2}$	$f_i u_i$
11-13	7	12	− 6	−3	−21
13-15	6	14	− 4	−2	− 12
15-17	9	16	− 2	−1	−9
17-19	13	18	0	0	0
19-21	f	20	2	1	f
21-23	5	22	4	2	10
23-25	4	24	6	3	12
	$\Sigma f_i = 44 + f_1$				$\Sigma f_i u_i = -20 + f$

Mean, *i.e.*, $\bar{x} = 18$ (given)

Here, $a = 18,\ h = 2$

Using step-deviation method.

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad 18 = 18 + \frac{-20 + f}{44 + f} \times 2$$

$$\Rightarrow \quad -20 + f = 0$$

$$\Rightarrow \quad f = 20.$$

4. *Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarised as follows. Find the mean heart beats per minute for these women, choosing a suitable method.*

Number of heart beats per minute	*65-68*	*68-71*	*71-74*	*74-77*	*77-80*	*80-83*	*83-86*
Number of women	*2*	*4*	*3*	*8*	*7*	*4*	*2*

Sol.

Number of heart beats per minute (C.I.)	*No. of women* (f_i)	*Class-mark* (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - 75.5}{3}$	$f_i u_i$
65-68	2	66.5	– 9	– 3	– 6
68-71	4	69.5	– 6	– 2	– 8
71-74	3	72.5	– 3	– 1	– 3
74-77	8	(75.5)	0	0	0
77-80	7	78.5	3	1	7
80-83	4	81.5	6	2	8
83-86	2	84.5	9	3	6
	$\Sigma f_i = 30$				$\Sigma f_i u_i = 4$

Here, $a = 75.5$, $h = 3$

Using the step-deviation method, we have

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 75.5 + \frac{4}{30} \times 3$$

$$= 75.5 + 0.4$$

$$= 75.9.$$

Hence, the mean heart beats/minute of given women is 75.9.

5. *In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.*

Number of mangoes	*50-52*	*53-55*	*56-58*	*59-61*	*62-64*
Number of boxes	*15*	*110*	*135*	*115*	*25*

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Sol.

Number of mangoes (*C.I.*)	*No. of boxes* (f_i)	*Class-mark* (x_i)	$d_i = x_i - a$	$u_i = \frac{d_i}{3}$	$f_i u_i$
50-52	15	51	– 6	– 2	– 30
53-55	110	54	– 3	– 1	– 110
56-58	135	57	0	0	0
59-61	115	60	3	1	115
62-64	25	63	6	2	50
	$\Sigma f_i = 400$				$\Sigma f_i u_i = 25$

Here, $a = 57$, $h = 3$

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 57 + \frac{25}{400} \times 3$$

$$= 57 + 0.19 = 57.19 \text{ mangoes.}$$

We choose step deviation method as f_i is large and difference between class marks is same.

6. *The table below shows the daily expenditure on food of 25 households in a locality.*

Daily expenditure (in ₹)	*100-150*	*150-200*	*200-250*	*250-300*	*300-350*
Number of households	*4*	*5*	*12*	*2*	*2*

Find the mean daily expenditure on food by a suitable method.

Sol.

Daily expenditure (in ₹) (C.I.)	No. of house-holds (f_i)	Class-mark (x_i)	$d_i = x_i - 225$	$f_i d_i$
100-150	4	125	– 100	– 400
150-200	5	175	– 50	– 250
200-250	12	225	0	0
250-300	2	275	50	100
300-350	2	325	100	200
	$\Sigma f_i = 25$			$\Sigma f_i d_i = -350$

Here, $a = 225$

Using the assumed mean method,

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 225 + \frac{-350}{25}$$

$$= 225 - 14 = 211$$

Hence, the mean daily expenditure on food is ₹ 211.

7. *To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:*

Concentration of SO_2 (in ppm)	*Frequency*
0.00-0.04	*4*
0.04-0.08	*9*
0.08-0.12	*9*

0.12-0.16	*2*
0.16-0.20	*4*
0.20-0.24	*2*

Find the mean concentration of SO_2 in the air.

Sol.

(C.I.)	(f_i)	(x_i)	$d_i = x_i - 0.10$	$u_i = \frac{x_1 - 0.10}{0.04}$	$f_i u_i$
0.00-0.04	4	0.02	– 0.08	– 2	– 8
0.04-0.08	9	0.06	– 0.04	– 1	– 9
0.08-0.12	9	(0.10)	0	0	0
0.12-0.16	2	0.14	0.04	1	2
0.16-0.20	4	0.18	0.08	2	8
0.20-0.24	2	0.22	0.12	3	6
	$\Sigma f_i = 30$				$\Sigma f_i u_i = -1$

Here, $a = 0.10$, $h = 0.04$

Using step-deviation method, we have

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 0.10 + \frac{-1}{30} \times 0.04$$

$$= 0.10 - 0.001$$

$$= 0.099 \text{ ppm. (approx.).}$$

8. *A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.*

Number of days	*0-6*	*6-10*	*10-14*	*14-20*	*20-28*	*28-38*	*38-40*
Number of students	*11*	*10*	*7*	*4*	*4*	*3*	*1*

Sol.

Number of days (C.I.)	*No. of students* (f_i)	*Class-mark* (x_i)	f_ix_i
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	(17)	68
20-28	4	24	96
28-38	3	33	99
38-40	1	39	39
	$\Sigma f_i = 40$		$\Sigma f_ix_i = 499$

Using direct method, we have

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{499}{40} = 12.48$$

Hence, the mean number of days a student was absent = 12.48 days.

9. *The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.*

Literacy rate (in %)	*45-55*	*55-65*	*65-75*	*75-85*	*85-95*
Number of cities	*3*	*10*	*11*	*8*	*3*

Sol.

Literacy rate (in %) (C.I.)	*Number of cities* (f_i)	*Class-marks* (x_i)	$d_i = x_i - 70$	f_id_i
45-55	3	50	– 20	– 60
55-65	10	60	– 10	– 100
65-75	11	(70)	0	0

75-85	8	80	10	80
85-95	3	90	20	60
	$\Sigma f_i = 35$			$\Sigma f_i d_i = -20$

Here, $a = 70$

Using the assumed mean method, we have

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 70 + \frac{-20}{35}$$

$$= 70 - 0.57 = 69.43\%.$$

Hence, the mean literacy rate of the cities is 69.43%.

Exercise 14.2 *(Page – 275-276)*

1. *The following table shows the ages of the patients admitted in a hospital during a year:*

Age (in years)	*5-15*	*15-25*	*25-35*	*35-45*	*45-55*	*55-65*
Number of patients	*6*	*11*	*21*	*23*	*14*	*5*

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Sol. For Mode:

Age (in years) C.I.	*Number of patients (f)*	
5-15	6	
15-25	11	
25-35	21	
35-45	23	← Modal class
45-55	14	
55-65	5	

Maximum frequency = 23

Modal class is 35-45.

$f_0 = 21, f_1 = 23, f_2 = 14, h = 10, l = 35$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 35 + \frac{23 - 21}{46 - 21 - 14} \times 10$$

$$= 35 + \frac{2}{11} \times 10$$

$$= 35 + 1.8 = 36.8 \text{ years}$$

For Arithmetic Mean:

(C.I.)	(f_i)	(x_i)	$u_i = \frac{x_i - 40}{10}$	$f_i u_i$
5-15	6	10	– 3	– 18
15-25	11	20	– 2	– 22
25-35	21	30	– 1	– 21
35-45	23	40	0	0
45-55	14	50	1	14
55-65	5	60	2	10
	$\Sigma f_i = 80$			$\Sigma f_i u_i = -37$

Here, $a = 40, h = 10$

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 40 + \frac{-37}{80} \times 10$$

$$= 40 - 4.63 = 35.37 \text{ years.}$$

Maximum number of patients admitted at the age of 36.8 years. Average age of the patient is 35.37 years.

2. *The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:*

Lifetimes (in hours)	*0-20*	*20-40*	*40-60*	*60-80*	*80-100*	*100-120*
Frequency	*10*	*35*	*52*	*61*	*38*	*29*

Determine the modal lifetimes of the components.

Sol.

C.I.	f
0-20	10
20-40	35
40-60	52
60-80	61 ← Modal class
80-100	38
100-120	29

Maximum frequency $(f_1) = 61$

$\therefore$ Modal class is 60-80

Here, $l = 60, f_1 = 61, f_0 = 52, f_2 = 38, h = 20$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{61 - 52}{122 - 52 - 38}\right) \times 20$$

$$= 60 + 5.625 = 65.625 \text{ hours.}$$

3. *The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:*

Expenditure (in ₹)	*Number of families*
1000-1500	*24*
1500-2000	*40*
2000-2500	*33*

2500-3000	*28*
3000-3500	*30*
3500-4000	*22*
4000-4500	*16*
4500-5000	*7*

Sol. For Mode:

Expenditure (in ₹) C.I.	*Number of families (f)*	
1000-1500	24	
1500-2000	40	← Modal class
2000-2500	33	
2500-3000	28	
3000-3500	30	
3500-4000	22	
4000-4500	16	
4500-5000	7	

Maximum frequency = 40

Modal class is 1500-2000

$l = 1500, f_0 = 24, f_1 = 40, f_2 = 33, h = 500$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 1500 + \left(\frac{40 - 24}{80 - 24 - 33}\right) \times 500$$

$$= 1500 + \frac{16 \times 500}{23} = 1500 + 347.826$$

$$= 1847.826 \sim 1847.83$$

Hence, modal monthly expenditure is ₹ 1847.83.

For Arithmetic Mean:

(C.I.)	(x_i)	(f_i)	$u_i = \frac{x_i - 2750}{500}$	f_iu_i
1000-1500	1250	24	– 3	– 72
1500-2000	1750	40	– 2	– 80
2000-2500	2250	33	– 1	– 33
2500-3000	2750	28	0	0
3000-3500	3250	30	1	30
3500-4000	3750	22	2	44
4000-4500	4250	16	3	48
4500-5000	4750	7	4	28
		$\Sigma f_i = 200$		$\Sigma f_iu_i = -35$

Here, $a = 2750$, $h = 500$

$$\text{Mean} = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h = 2750 + \frac{-35}{200} \times 500$$

$$= 2750 - 87.5 = 2662.5.$$

Hence, mean monthly expenditure = ₹ 2662.50.

4. *The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.*

Number of students per teacher	*Number of states / U.T.*
15-20	*3*
20-25	*8*
25-30	*9*
30-35	*10*

35-40	*3*
40-45	*0*
45-50	*0*
50-55	*2*

Sol.

C.I.	x_i	f_i	$u_i = \frac{x_i - 32.5}{5}$	$f_i u_i$
15-20	17.5	3	– 3	– 9
20-25	22.5	8	– 2	– 16
25-30	27.5	9	– 1	– 9
30-35	(32.5)	10	0	0 ← Modal Class
35-40	37.5	3	1	3
40-45	42.5	0	2	0
45-50	47.5	0	3	0
50-55	52.5	2	4	8
		$\Sigma f_i = 35$		$\Sigma f_i u_i = -23$

Here, $a = 32.5$, ∴ $= 5$

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 32.5 + \frac{-23}{35} \times 5$$

$$= 32.5 - 3.28 = 29.22$$

Maximum frequency $(f_1) = 10$

So, modal class is 30-35

We have $l = 30$, $f_1 = 10$, $f_0 = 9$, $f_2 = 3$, $h = 5$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{10 - 9}{20 - 9 - 3}\right) \times 5$$

$$= 30 + \frac{5}{8} = 30 + 0.62 = 30.62$$

Most states/U.T. have 30.62 students per teacher and average number of students per teacher is 29.22.

5. *The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.*

Runs scored	*Number of batsmen*
3000-4000	*4*
4000-5000	*18*
5000-6000	*9*
6000-7000	*7*
7000-8000	*6*
8000-9000	*3*
9000-10000	*1*
10000-11000	*1*

Find the mode of the data.

Sol.

C.I.	f	
3000-4000	4	
4000-5000	18	← Modal class
5000-6000	9	
6000-7000	7	
7000-8000	6	
8000-9000	3	
9000-10000	1	
10000-11000	1	

Maximum frequency $(f_1) = 18$

So, modal class is 4000-5000.

Here, $l = 4000, f_1 = 18, f_0 = 4, f_2 = 9, h = 1000$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 4000 + \left(\frac{18-4}{36-4-9}\right) \times 1000$$

$$= 4000 + 608.7 = 4608.7.$$

Hence, 4608.7 runs scored by maximum number of batsmen.

6. *A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:*

Number of cars	*0-10*	*10-20*	*20-30*	*30-40*	*40-50*	*50-60*	*60-70*	*70-80*
Frequency	*7*	*14*	*13*	*12*	*20*	*11*	*15*	*8*

Sol.

C.I	f	
0-10	7	
10-20	14	
20-30	13	
30-40	12	
40-50	20	← Modal class
50-60	11	
60-70	15	
70-80	8	

Maximum frequency = 20

So, modal class is 40-50.

Here, $l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10.$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{20 - 12}{40 - 12 - 11}\right) \times 10$$

$$= 40 + \frac{8}{17} \times 10 = 40 + 4.7 = 44.7$$

44.7 cars passed maximum number of times in each of 3 minutes period.

Exercise 14.3 *(Page – 287-289)*

1. *The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.*

Monthly consumption (in units)	*Number of consumers*
65-85	*4*
85-105	*5*
105-125	*13*
125-145	*20*
145-165	*14*
165-185	*8*
185-205	*4*

Sol. Table for Median:

Monthly consumption of electricity (C.I.)	*No. of consumers (f)*	*cf*
65-85	4	4
85-105	5	9

105-125	13	22	
125-145	20	42	← Median class
145-165	14	56	
165-185	8	64	
185-205	4	68	
	N = 68		

$\frac{N}{2} = \frac{68}{2} = 34$ $\quad\therefore$ Median class is 125-145.

$l = 125$, $c.f. = 22$, $f = 20$, $h = 20$.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$$= 125 + 12 = 137 \text{ units.}$$

Table for Mean and Mode:

C.I.	x_i	f_i	$u_i = \frac{x_i - 135}{20}$	$f_i u_i$
65-85	75	4	– 3	– 12
85-105	95	5	– 2	– 10
105-125	115	13	– 1	– 13
125-145	(135)	20	0	0
145-165	155	14	1	14
165-185	175	8	2	16
185-205	195	4	3	12
		$\Sigma f_i = 68$		$\Sigma f_i u_i = 7$

Here, $a = 135$, $h = 20$

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 135 + \frac{7}{68} \times 20$$

$$= 135 + 2.05$$

$$= 137.05 \text{ units}$$

Mode: Maximum frequency $(f_1) = 20$

So, modal class is 125-145

Here, $l = 125, f_1 = 20, f_0 = 13, f_2 = 14, h = 20$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 125 + \left(\frac{20 - 13}{40 - 13 - 14}\right) \times 20$$

$$= 125 + 10.76 = 135.76 \text{ units.}$$

Comparison: The three measures of central tendency are approximately the same.

2. *If the median of the distribution given below is 28.5, find the values of x and y.*

Class interval	*Frequency*
0-10	*5*
10-20	*x*
20-30	*20*
30-40	*15*
40-50	*y*
50-60	*5*
Total	*60*

Sol.

C.I.	f	cf	
0-10	5	5	
10-20	x	$5 + x$	
20-30	20	$25 + x$	← Median class
30-40	15	$40 + x$	
40-50	y	$40 + x + y$	
50-60	5	$45 + x + y$	
	60		

We have $45 + x + y = 60 \Rightarrow x + y = 15$ ...(*i*)

Median class is 20-30 as median is 28.5 (given)

$\frac{N}{2} = \frac{60}{2} = 30,\ l = 20,\ cf = 5 + x,\ f = 20,\ h = 10$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 28.5 = 20 + \frac{30 - (5 + x)}{20} \times 10$$

$$\Rightarrow \quad 17 = 25 - x \Rightarrow x = 8$$

Substituting this value for x in (*i*), we get $y = 7$

Hence, $x = 8$, $y = 7$.

3. *A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.*

Age (in years)	*Number of policy holders*
Below 20	*2*
Below 25	*6*
Below 30	*24*

Below 35	*45*
Below 40	*78*
Below 45	*89*
Below 50	*92*
Below 55	*98*
Below 60	*100*

Sol. Here, we are given less than cumulative frequency table, first we make frequency table.

C.I.	f	cf	
15-20	2	2	
20-25	4	6	
25-30	18	24	
30-35	21	45	
35-40	33	78	← Median class
40-45	11	89	
45-50	3	92	
50-55	6	98	
55-60	2	100	
	N = 100		

$\frac{N}{2} = 50$, we locate 50 in *cf* column.

Median class is 35-40.

$\therefore \quad l = 35, f = 33, cf = 45, h = 5$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 35 + \frac{50 - 45}{33} \times 5$$

$$= 35 + 0.76 = 35.76 \text{ years.}$$

4. *The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table:*

Length (in mm)	*Number of leaves*
118-126	*3*
127-135	*5*
136-144	*9*
145-153	*12*
154-162	*5*
163-171	*4*
172-180	*2*

Find the median length of the leaves.

(**Hint:** *The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5-126.5, 126.5-135.5, ..., 171.5-180.5.*)

Sol. The data is not continuous, so we make the data continuous.

C.I.	*C.I.*	*f*	*cf*	
118-126	117.5-126.5	3	3	
127-135	126.5-135.5	5	8	
136-144	135.5-144.5	9	17	
145-153	144.5-153.5	12	29	← Median class
154-162	153.5-162.5	5	34	
163-171	162.5-171.5	4	38	
172-180	171.5-180.5	2	40	
		N = 40		

$\frac{N}{2} = \frac{40}{2} = 20$, we locate 20 in *cf* column.

Median class is 144.5-153.5

$$l = 144.5,\ f = 12,\ cf = 17,\ h = 9$$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 144.5 + \frac{20-17}{12} \times 9$$

$$= 144.5 + 2.25 = 146.75 \text{ mm.}$$

5. *The following table gives the distribution of the lifetime of 400 neon lamps:*

Lifetime (in hours)	*Number of lamps*
1500-2000	*14*
2000-2500	*56*
2500-3000	*60*
3000-3500	*86*
3500-4000	*74*
4000-4500	*62*
4500-5000	*48*

Find the median lifetime of a lamp.

Sol.

C.I.	*f*	*cf*	
1500-2000	14	14	
2000-2500	56	70	
2500-3000	60	130	
3000-3500	86	216	← Median class
3500-4000	74	290	
4000-4500	62	352	
4500-5000	48	400	
	N = 400		

$\frac{N}{2} = 200$, we locate 200 in *cf* column.

Median class is 3000-3500.

Here $l = 3000$, $f = 86$, $c.f. = 130$, $h = 500$.

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 3000 + \frac{200 - 130}{86} \times 500$$

$$= 3000 + 406.98 = 3406.98 \text{ hours.}$$

6. *100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:*

Number of letters	*1-4*	*4-7*	*7-10*	*10-13*	*13-16*	*16-19*
Number of surnames	*6*	*30*	*40*	*16*	*4*	*4*

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Sol. Table for Mean, Mode and Median:

C.I.	x_i	f_i	*cf*	$u_i = \frac{x_i - 8.5}{3}$	f_iu_i	
1-4	2.5	6	6	– 2	– 12	
4-7	5.5	30	36	– 1	– 30	
7-10	8.5	40	76	0	0	← Modal class ← Median class
10-13	11.5	16	92	1	16	
13-16	14.5	4	96	2	8	

16-19	17.5	4	100	3	12
		$\Sigma f_i =$ 100			$\Sigma f_i u_i$ $= -6$

Mean: $a = 8.5$, $h = 3$

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 8.5 + \frac{-6}{100} \times 3$$

$$= 8.5 - 0.18 = 8.32$$

Mode: Maximum frequency $(f_1) = 40$

$\therefore$ Modal class is 7-10.

Here, $l = 7$, $f_1 = 40$, $f_0 = 30$, $f_2 = 16$, $h = 3$

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 7 + \left(\frac{40-30}{80-30-16}\right) \times 3$$

$$= 7 + 0.88 = 7.88$$

Median: $\frac{N}{2} = \frac{100}{2} = 50$, we locate 50 in *cf* column.

$\therefore$ Median class is 7-10.

Here, $l = 7$, $f = 40$, $cf = 36$, $h = 3$

$$\therefore \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 7 + \frac{50-36}{40} \times 3$$

$$= 7 + 1.05 = 8.05.$$

7. *The distribution below gives the weights of 30 students of a class. Find the median weight of the students.*

Weight (in kg)	*40-45*	*45-50*	*50-55*	*55-60*	*60-65*	*65-70*	*70-75*
Number of students	*2*	*3*	*8*	*6*	*6*	*3*	*2*

Sol.

C.I.	f	cf	
40-45	2	2	
45-50	3	5	
50-55	8	13	
55-60	6	19	← Median class
60-65	6	25	
65-70	3	28	
70-75	2	30	
	30		

$\frac{N}{2} = \frac{30}{2} = 15$, we locate 15 in *cf* column.

Median class is 55-60.

Here, $l = 55, f = 6, cf = 13, h = 5$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 55 + \frac{15-13}{6} \times 5$$

$$= 55 + 1.67 = 56.67 \text{ kg.}$$

Exercise 14.4 *(Page – 293)*

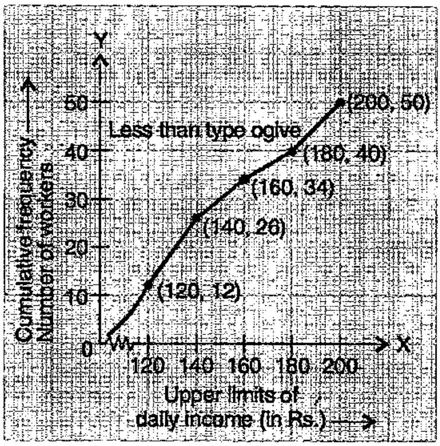
1. *The following distribution gives the daily income of 50 workers of a factory.*

Daily income (in ₹)	*100-120*	*120-140*	*140-160*	*160-180*	*180-200*
Number of workers	*12*	*14*	*8*	*6*	*10*

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Sol. Less than type cumulative frequency distribution:

C.I.	f	cf	Point
100-120	12	12	(120, 12)
120-140	14	26	(140, 26)
140-160	8	34	(160, 34)
160-180	6	40	(180, 40)
180-200	10	50	(200, 50)



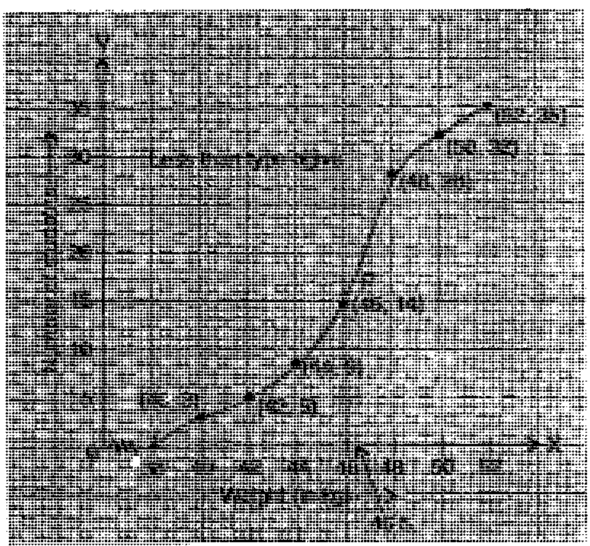
2. *During the medical check-up of 35 students of a class, their weights were recorded as follows:*

Weight (in kg)	*Number of students*
Less than 38	*0*
Less than 40	*3*
Less than 42	*5*
Less than 44	*9*
Less than 46	*14*
Less than 48	*28*
Less than 50	*32*
Less than 52	*35*

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

Sol.

Weight (in kg)	Number of students	Point
Less than 38	0	(38, 0)
Less than 40	3	(40, 3)
Less than 42	5	(42, 5)
Less than 44	9	(44, 9)
Less than 46	14	(46, 14)
Less than 48	28	(48, 28)
Less than 50	32	(50, 32)
Less than 52	35	(52, 35)



$\frac{N}{2} = 17.5$. From 17.5 on y-axis, we draw a line parallel to x-axis meeting curve at P. From P, we draw perpendicular to x-axis, which meets x-axis at L(46.5, 0).

Hence, median is 46.5 kg.

Table for median:

C.I.	f	cf	
38-40	3	3	
40-42	2	5	
42-44	4	9	
44-46	5	14	
46-48	14	28	← Median class
48-50	4	32	
50-52	3	35	
	N = 35		

$\frac{N}{2} = \frac{35}{2} = 17.5$, we locate 17.5 in *cf* column.

Median class is 46-48.

Here, $l = 46$, $f = 14$, $cf = 14$, $h = 2$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 46 + \frac{17.5 - 14}{14} \times 2$$

$$= 46 + 0.5 = 46.5 \text{ kg.}$$

Hence, from both the ways, the median is same, *i.e.*, 46.5 kg.

3. *The following table gives production yield per hectare of wheat of 100 farms of a village.*

Production yield (in kg/ha)	*50-55*	*55-60*	*60-65*	*65-70*	*70-75*	*75-80*
Number of farms	*2*	*8*	*12*	*24*	*38*	*16*

Change the distribution to a more than type distribution, and draw its ogive.

Sol. Table for more than type cumulative frequency distribution.

Production yield (in kg / ha)	*Number of farms (f)*	*cf*	*Point*
50-55 more than 50	2	100	(50, 100)
55-60 more than 55	8	98	(55, 98)
60-65 more than 60	12	90	(60, 90)
65-70 more than 65	24	78	(65, 78)
70-75 more than 70	38	54	(70, 54)
75-80 more than 75	16	16	(75, 16)