12 Introduction to Three Dimensional Geometry

Lesson at a Glance

1. **Coordinate planes**

   The three planes determined by the three pairs of axes are the **coordinate planes** called YZ, ZX and XY-planes.

2. **Octants.** The three coordinate planes divide the space into eight parts known as **octants.**

   These 8 octants are:

   Octant I XOYZ; Octant II X'OYZ; Octant III X'OY'Z; Octant IV XOY'Z; Octant V XOYZ'; Octant VI X'OYZ'; Octant VII X'OY'Z'; Octant VIII XOY'Z'

3. **Coordinates of a point**

   The coordinates of a point P in three dimensional geometry are always written in the form of a triplet like (x, y, z).

   Here x, y, z are the perpendicular distances of the point P from the three coordinate planes YZ, ZX and XY respectively.

4. **Equations of coordinate planes**

   (*i*) The equation of YZ-plane is $x = 0$.

   $\therefore$ Any point on YZ-plane is $(0, y, z)$.

   (*ii*) The equation of ZX-plane is $y = 0$.

   $\therefore$ Any point on ZX-plane is $(x, 0, z)$

   (*iii*) The equation of XY-plane is $z = 0$.

   $\therefore$ Any point on XY-plane is $(x, y, 0)$

5. **Equation of coordinates axes**

   (*i*) The equations of x-axis are $y = 0, z = 0$

   $\therefore$ Any point on x-axis is $(x, 0, 0)$.

   (*ii*) The equations of y-axis are $z = 0, x = 0$

   $\therefore$ Any point on y-axis is $(0, y, 0)$.

   (*iii*) The equations of z-axis are $x = 0, y = 0$.

   $\therefore$ Any point on z-axis is $(0, 0, z)$.

6. Distances of a point from coordinate axes

The perpendicular distance of the point $P(x, y, z)$ from x-axis is $\sqrt{y^2+z^2}$, from y-axis is $\sqrt{x^2+z^2}$ and from z-axis is $\sqrt{x^2+y^2}$.

7. Distance formula

The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

8. Section formula

The coordinates of the point which divides the join of the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio:

(*i*) $m_1 : m_2$ **internally** are

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right).$$

(ii) $m_1 : m_2$ **externally** are

$$\left(\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}, \frac{m_1z_2-m_2z_1}{m_1-m_2}\right).$$

9. Mid-point formula

The coordinates of the mid-point of the line segment joining the point (x_1, y_1, z_1) and (x_2, y_2, z_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

10. Centroid of a triangle

If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are the *vertices of a triangle, then coordinates of the centroid of the triangle* (i.e., *point of intersection of the medians) of a triangle ABC are*

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right).$$

TEXTBOOK QUESTIONS SOLVED

EXERCISE 12.1 (*Page No.: 271*)

1. A point is on the x-axis. What are its y-coordinate and z-coordinate?

Sol. y and z-coordinates of the point are zero.

[$\because$ Coordinates of any point on x-axis are $(x, 0, 0)$]

2. A point is in the XZ-plane. What can you say about its y-coordinate?

Sol. Any point on XZ-plane is $(x, 0, z)$. Therefore, y-coordinate of the point is zero.

3. Name the octants in which the following points lie: (1, 2, 3), (4, – 2, 3), (4, – 2, – 5), (4, 2, – 5), (– 4, 2, – 5), (– 4, 2, 5), (– 3, – 1, 6), (2, – 4, – 7).

Sol. (*i*) For the point (1, 2, 3)

$x = 1 > 0$ is measured along OX

$y = 2 > 0$ is measured along OY

$z = 3 > 0$ is measured along OZ

Hence the point (1, 2, 3) lies in the octant XOYZ, *i.e.*, in octant I.

(*ii*) For the point (4, – 2, 3)

$x = 4 > 0$ is measured along OX

$y = -2 < 0$ is measured along OY′

$z = 3 > 0$ is measured along OZ

Hence the point (4, – 2, 3) lies in the octant XOY′Z, *i.e.*, in octant IV.

(*iii*) For the point (4, – 2, – 5)

$x = 4 > 0$ is measured along OX

$y = -2 < 0$ is measured along OY′

$z = -5 < 0$ is measured along OZ′

Hence the point (4, – 2, – 5) lies in the octant XOY′Z′, *i.e.*, in octant VIII.

(*iv*) For the point (4, 2, – 5)

$x = 4 > 0$ is measured along OX

$y = 2 > 0$ is measured along OY

$z = -5 < 0$ is measured along OZ′

Hence the point (4, 2, – 5) lies in the octant XOYZ′, *i.e.*, in octant V.

(*v*) For the point (– 4, 2, – 5)

$x = -4 < 0$ is measured along OX′

$y = 2 > 0$ is measured along OY

$z = -5 < 0$ is measured along OZ′

Hence the point (– 4, 2, – 5) lies in the octant X′OYZ′, *i.e.*, in octant VI.

(*vi*) For the point (– 4, 2, 5)

$x = -4 < 0$ is measured along OX′

$y = 2 > 0$ is measured along OY

$z = 5 > 0$ is measured along OZ

Hence the point (– 4, 2, 5) lies in the octant X′OYZ, *i.e.*, in octant II.

(*vii*) For the point (– 3, – 1, 6)

$x = -3 < 0$ is measured along OX′

$y = -1 < 0$ is measured along OY′

$z = 6 > 0$ is measured along OZ

Hence the point (– 3, – 1, 6) lies in the octant X′OY′Z, *i.e.*, in octant III.

(*viii*) For the point (2, – 4, – 7)

$x = 2 > 0$ is measured along OX

$y = -4 < 0$ is measured along OY′

$z = -7 < 0$ is measured along OZ′

Hence the point (2, – 4, – 7) lies in the octant XOY′Z′, *i.e.*, in octant VIII.

4. Fill in the blanks:

(*i*) The *x*-axis and *y*-axis taken together determine a plane known as ....

(*ii*) The coordinates of points in the XY-plane are of the form ....

(*iii*) Coordinate planes divide the space into .... octants.

Sol. (*i*) XY-plane (*ii*) $(x, y, 0)$ (*iii*) 8.

EXERCISE 12.2 (*Page No.: 273*)

1. Find the distance between the following pairs of points:

(*i*) (2, 3, 5) and (4, 3, 1) (*ii*) (–3, 7, 2) and (2, 4, –1)

(*iii*) (–1, 3, –4) and (1, –3, 4) (*iv*) (2, –1, 3) and (–2, 1, 3).

Sol. (*i*) The distance between the points P(2, 3, 5) and Q(4, 3, 1) by distance formula is,

$$PQ = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$
$$= \sqrt{4+0+16} = \sqrt{20} = \sqrt{4\times 5} = 2\sqrt{5}.$$

(*ii*) The distance between the points P(– 3, 7, 2) and Q(2, 4, – 1) is

$$PQ = \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{25+9+9} = \sqrt{43}.$$

(*iii*) The distance between the points P(–1, 3, –4) and Q(1, – 3, 4) is

$$PQ = \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = \sqrt{4\times 26} = 2\sqrt{26}.$$

(*iv*) The distance between the points P(2, – 1, 3) and Q(– 2, 1, 3) is

$$PQ = \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$$
$$= \sqrt{16+4+0} = \sqrt{20} = \sqrt{4\times 5} = 2\sqrt{5}.$$

2. Show that the points (– 2, 3, 5), (1, 2, 3) and (7, 0, – 1) are collinear.

Sol. Let P(– 2, 3, 5), Q(1, 2, 3), R(7, 0, – 1) be the given points.

$$\text{Distance } PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$

(By distance formula)

$$= \sqrt{36+4+16}$$
$$= \sqrt{56} = \sqrt{4\times 14} = 2\sqrt{14}$$

$$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{81+9+36}$$

$$= \sqrt{126} = \sqrt{9 \times 14} = 3\sqrt{14}$$

Thus $PQ + QR = \sqrt{14} + 2\sqrt{14} = (1 + 2)\sqrt{14} = 3\sqrt{14} = PR.$

Hence points P, Q, R are collinear.

3. Verify the following:

(i) (0, 7, – 10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (– 1, 6, 6) and (– 4, 9, 6) are the vertices of a right angled triangle.

(iii) (– 1, 2, 1), (1, – 2, 5), (4, – 7, 8) and (2, – 3, 4) are the vertices of a parallelogram.

Sol. (i) Let A(0, 7, – 10), B(1, 6, – 6) and C(4, 9, – 6) be the given points.

$$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6.$$

Since AB = BC $(= 3\sqrt{2})$, triangle ABC is isosceles.

(ii) Let A(0, 7, 10), B(– 1, 6, 6) and C(– 4, 9, 6) be the given points.

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

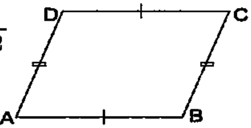
Since $AB^2 + BC^2 = 18 + 18 = 36 = AC^2$

$\therefore$ ΔABC is right angled at B.

Note: Also, AB = BC $\Rightarrow$ ΔABC is isosceles.

Hence, ABC is a right-angled isosceles triangle.

(*iii*) Let A(– 1, 2, 1), B(1, – 2, 5), C(4, – 7, 8) and D(2, – 3, 4) be the given points.



$$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$$

$$= \sqrt{4+16+16} = \sqrt{36} = 6$$

$$DC = \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2}$$

$$= \sqrt{4+16+16} = \sqrt{36} = 6$$

$$AD = \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2}$$

$$= \sqrt{9+25+9} = \sqrt{43}$$

$$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$$

$$= \sqrt{9+25+9} = \sqrt{43}$$

Since AB = DC (=6) and AD = BC ($=\sqrt{43}$)

$\Rightarrow$ The opposite sides of quadrilateral ABCD are equal.

$\Rightarrow$ Quadrilateral ABCD is a parallelogram.

4. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, – 1).

Sol. Let P(x, y, z) be equidistant from the points A(1, 2, 3) and B(3, 2, – 1).

$\therefore$ AP = BP

$\Rightarrow$ $AP^2 = BP^2$

$\Rightarrow$ $(x-1)^2 + (y-2)^2 + (z-3)^2$

$= (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow$ $(x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 - 6z + 9)$

$= (x^2 - 6x + 9) + (y^2 - 4y + 4) + (z^2 + 2z + 1)$

$\Rightarrow$ $-2x + 6x - 6z - 2z + 14 - 14 = 0$

$\Rightarrow$ $4x - 8z = 0$

Dividing every term by 4, $x - 2z = 0$

which is the required equation of the set of points P(x, y, z).

5. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(– 4, 0, 0) is equal to 10.

Sol. Given points are A(4,0,0) and B(–4,0,0). Let the coordinates of P be (x, y, z).

Given: $\quad AP + BP = 10$

$$\Rightarrow \sqrt{(x-4)^2+(y-0)^2+(z-0)^2} + \sqrt{(x+4)^2+(y-0)^2+(z-0)^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2} + \sqrt{(x+4)^2+y^2+z^2} = 10$$

Shifting one square root term to R.H.S;

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2} = 10 - \sqrt{(x+4)^2+y^2+z^2}$$

Squaring both sides, we have

$$(x-4)^2 + y^2 + z^2 = 100 + (x+4)^2 + y^2 + z^2 - 20\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow \quad x^2 + 16 - 8x = 100 + x^2 + 16 + 8x - 20\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow \quad -16x - 100 = -20\sqrt{(x+4)^2+y^2+z^2}$$

Dividing both sides by (– 4), we get

$$4x + 25 = 5\sqrt{(x+4)^2+y^2+z^2}$$

Squaring both sides again, we have

$$16x^2 + 200x + 625 = 25[x^2 + 8x + 16 + y^2 + z^2]$$

$\Rightarrow 16x^2 + 200x + 625 = 25x^2 + 200x + 400 + 25y^2 + 25z^2$

$\Rightarrow -9x^2 - 25y^2 - 25z^2 = -225$

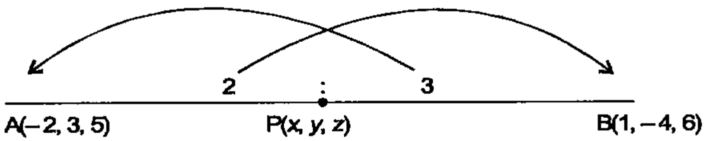
multiplying by –1, $9x^2 + 25y^2 + 25z^2 = 225$

Which is required equation (*i.e.*, locus) of set of points P.

EXERCISE 12.3 (*Page No.: 277*)

1. Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio (*i*) 2 : 3 internally, (*ii*) 2 : 3 externally.

Sol. (*i*) Let P(x, y, z) be the point which divides the line segment joining A(– 2, 3, 5) and B(1, – 4, 6) internally in the ratio 2 : 3.



Then, by section formula;

$$x = \frac{2(1)+3(-2)}{2+3} = -\frac{4}{5},\ y = \frac{2(-4)+3(3)}{2+3} = \frac{1}{5}$$

$$z = \frac{2(6)+3(5)}{2+3} = \frac{27}{5}$$

Therefore, the required point is $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$.

(*ii*) Let P(x, y, z) be the point which divides the line segment joining A(– 2, 3, 5) and B(1, – 4, 6) externally in the ratio 2 : 3. Then, by section formula

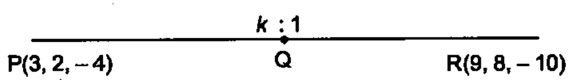
$$x = \frac{2(1)-3(-2)}{2-3} = -8,\ y = \frac{2(-4)-3(3)}{2-3} = 17$$

$$z = \frac{2(6)-3(5)}{2-3} = 3$$

Therefore, the required point is (– 8, 17, 3).

2. Given that P(3, 2, – 4), Q(5, 4, – 6) and R(9, 8, – 10) are collinear. Find the ratio in which Q divides PR.

Sol. Let Q divide PR in the ratio k : 1, then the coordinates of Q are $\left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1}\right)$.



But the coordinates of Q are given to be (5, 4, – 6)

Equating coordinates, we have

$$\therefore \quad \frac{9k+3}{k+1} = 5 \text{ and } \frac{8k+2}{k+1} = 4 \text{ and } \frac{-10k-4}{k+1} = -6$$

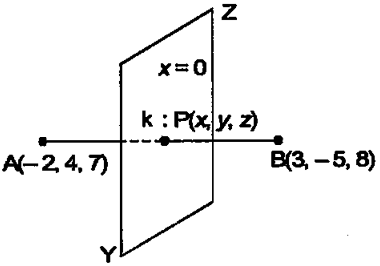
Solving any one of these equations for k, say first, we have

on cross-multiplication, $9k + 3 = 5k + 5$ or $4k = 2$ $\therefore$ $k = \frac{1}{2}$.

Hence, Q divides PR in the ratio $\frac{1}{2} : 1 = 1 : 2$.

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (– 2, 4, 7) and (3, – 5, 8).

Sol. Let the YZ-plane divide the line segment joining A(– 2, 4, 7) and B(3, –5, 8) at P(x, y, z) in the ratio $k : 1$.



Then the coordinates of P are

$$\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$$

Since P lies on the YZ-plane, its x-coordinate is zero, *i.e.*,

$$\frac{3k-2}{k+1} = 0$$

cross-multiplying, $3k - 2 = 0 \Rightarrow 3k = 2$

or $k = \frac{2}{3}$ (+ve)

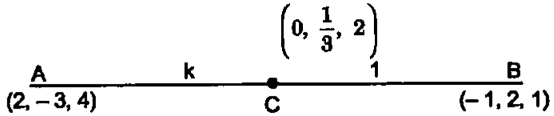
Therefore, YZ-plane divides AB internally in the ratio $\frac{2}{3} : 1$ *i.e.*, $2 : 3$.

4. Using section formula, show that the points A(2, – 3, 4), B(– 1, 2, 1) and C$\left(0, \frac{1}{3}, 2\right)$ are collinear.

Sol. Let the point C divide the line joining the points A and B in the ratio $k : 1$.

Then the coordinates of C are

$$\left(\frac{-k+2}{k+1}, \frac{2k-3}{k+1}, \frac{k+4}{k+1}\right)$$



But coordinates of C are given $\left(0, \frac{1}{3}, 2\right)$

Equating coordinates of C given and obtained, we have

	$\frac{-k+2}{k+1} = 0,$	$\frac{2k-3}{k+1} = \frac{1}{3}$	$\frac{k+4}{k+1} = 2$
cross-multiplying	$-k+2=0$	or $6k-9=k+1$	or $k+4=2k+2$
or	$k=2$	or $5k=10$	or $2=k$
		or $k=2$	

Because the value of k from all the three equations is the same.

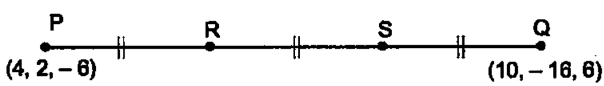
∴ Points A, B and C are collinear (and the point C divides AB in the ratio 2 : 1).

5. Find the coordinates of the points which trisect the line segment joining the points P(4, 2, – 6) and Q(10, – 16, 6).

Sol. Given: points P(4, 2, – 6) and Q(10, – 16, 6).

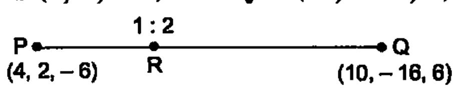
Let us take two points R and S within the segment PQ such that PR = RS = SQ.

Then points R and S are called **points of trisection** of the segment PQ.



∴ One point of trisection R divides the join of P and Q internally in the ratio 1 : 2(= 1 + 1).

Point P is (4, 2, – 6) and Q is (10, – 16, 6).

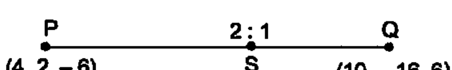


∴ Coordinates of point R are

$$R\left[\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2}\right]$$

i.e., $$R\left(\frac{18}{3}, \frac{-12}{3}, \frac{-6}{3}\right) = (6, -4, -2)$$

Again the second point of trisection S divides the join of P and Q internally in the ratio of 2(= 1 + 1) : 1.



$\therefore$ Coordinates of point S are

$$S\left[\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)+1(-6)}{2+1}\right]$$

i.e., $$S\left(\frac{24}{3}, \frac{-30}{3}, \frac{6}{3}\right) = (8, -10, 2)$$

Note. We can find coordinates of point S by a second method also.

Point S is the mid-point of RQ

$$\therefore \quad S\left(\frac{6+10}{2}, \frac{-4-16}{2}, \frac{-2+6}{2}\right) \Bigg| \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$$= (8, -10, 2).$$

MISCELLANEOUS EXERCISE ON CHAPTER 12

(Page No.: 278-279)

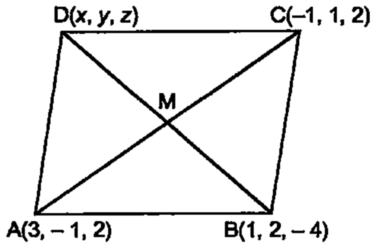
1. Three vertices of a parallelogram ABCD are A(3, – 1, 2), B(1, 2, – 4) and C(– 1, 1, 2). Find the coordinates of the fourth vertex.

Sol. Let the coordinates of D be (x, y, z).

Let M be the point of intersection of the diagonals.

We know that diagonals of a $||^{gm}$ bisect each other.

(Here at point M)



Since M is the mid-point AC,

$\therefore$ Coordinates of M are

$$\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = (1, 0, 2) \qquad ...(i)$$

Also M is the mid-point of BD

$\therefore$ Coordinates of M are $\left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$ ...(*ii*)

From (*i*) and (*ii*) equating corresponding entries, we have

$$1 = \frac{x+1}{2}, \quad 0 = \frac{y+2}{2} \text{ and } 2 = \frac{z-4}{2}$$

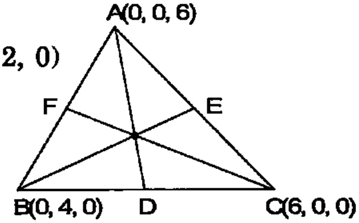
cross-multiplying $x + 1 = 2$, $y + 2 = 0$ and $z - 4 = 4$

$\Rightarrow$ $x = 1$, $y = -2$ and $z = 8$

Hence, vertex D is (1, – 2, 8).

2. Find the lengths of the medians of the triangle with vertices A(0, 0, 6), B(0, 4, 0) and C(6, 0, 0).

Sol. Let D, E, F be the mid-points of sides BC, CA, AB respectively. Then



$$D = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3, 2, 0)$$

$\therefore$ Length of median AD

$$= \sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}$$

$$= \sqrt{9+4+36} = \sqrt{49} = 7$$

$$E = \left(\frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right) = (3, 0, 3)$$

$\therefore$ Length of median BE

$$= \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2}$$

$$= \sqrt{9+16+9} = \sqrt{34}$$

$$F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$$

$\therefore$ Length of median CF

$$= \sqrt{(0-6)^2 + (2-0)^2 + (3-0)^2}$$

$$= \sqrt{36+4+9} = \sqrt{49} = 7.$$

3. If the origin is the centroid of the triangle PQR with vertices P(2*a*, 2, 6), Q(– 4, 3*b*, – 10) and R(8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Sol. By formula; centroid of the triangle PQR with vertices P(2*a*, 2, 6), Q(– 4, 3*b*, – 10) and R(8, 14, 2*c*) is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

i.e., $\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$

i.e., $\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$ = origin (given)

= (0, 0, 0)

Equating coordinates, we have

$\frac{2a+4}{3} = 0,$	$\frac{3b+16}{3} = 0,$	$\frac{2c-4}{3} = 0$
$\therefore\ 2a + 4 = 0,$	$3b + 16 = 0,$	$2c - 4 = 0$
or $2a = -4,$	$3b = -16,$	$2c = 4$
$\therefore\ a = -2,$	$b = -\frac{16}{3},$	$c = 2.$

4. Find the coordinates of points on *y*-axis which are at a distance of $5\sqrt{2}$ from the point P(3, – 2, 5).

Sol. Let the required point on *y*-axis be A(0, y, 0).

Given: Point P (3,–2,5) and distance AP = $5\sqrt{2}$

$\Rightarrow \sqrt{(0-3)^2+(y+2)^2+(0-5)^2} = 5\sqrt{2}$

Squaring, $9 + (y + 2)^2 + 25 = 50$

$\Rightarrow (y + 2)^2 = 50 - 34 \quad \Rightarrow (y + 2)^2 = 16$

$\Rightarrow y + 2 = \pm 4 \Rightarrow y = -2 \pm 4 \quad \Rightarrow y = 2$ or -6

Therefore, the required points are (0, *y*, 0) = (0, 2, 0) and (0, – 6, 0).

5. A point R with *x*-coordinate 4 lies on the line segment joining the points P(2, – 3, 4) and Q(8, 0, 10). Find the coordinates of the point R.

Sol. Since R lies on PQ, therefore, R divides PQ in some ratio, say *k* : 1.

$\therefore$ By section formula, R has coordinates $\left(\frac{8k+2}{k+1}, \frac{0-3}{k+1}, \frac{10k+4}{k+1}\right)$ ...(*i*)

x-coordinate of R is given to be 4

$$\therefore \quad \frac{8k+2}{k+1} = 4 \qquad \Rightarrow \quad 8k + 2 = 4(k + 1)$$

$$\Rightarrow \quad 8k + 2 = 4k + 4 \quad \Rightarrow 4k = 2 \quad \therefore \quad k = \frac{1}{2}$$

Putting this value of k in (i), the required coordinates of R are

$$\left(4, \frac{-3}{1/2+1}, \frac{10\times 1/2+4}{1/2+1}\right) = \left(4, \frac{-3}{\left(\frac{3}{2}\right)}, \frac{9}{\left(\frac{3}{2}\right)}\right) = (4, -2, 6).$$

6. If A and B be the points (3, 4, 5) and (– 1, 3, – 7) respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Sol. Let the coordinates of P be (x, y, z).

Given: Points A(3, 4, 5) and B(–1, 3, –7) and $PA^2 + PB^2 = k^2$

$$\Rightarrow \quad [(x-3)^2 + (y-4)^2 + (z-5)^2] + [(x+1)^2 + (y-3)^2 + (z+7)^2] = k^2$$

$$\Rightarrow \quad (x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25) + (x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49) = k^2$$

$$\Rightarrow \quad 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$\Rightarrow \quad 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$$

Dividing by 2, $x^2 + y^2 + z^2 - 2x - 7y + 2z = \dfrac{k^2 - 109}{2}$.

Which is the required equation of set of points P.