

Lesson at a Glance

• Newton's Law of Gravitation

Newton's law of gravitation states that every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the particles.

$$F \propto m_1 m_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

$$\therefore F = G \frac{m_1 m_2}{r^2}$$

where G is a constant, called as the universal constant of gravitation.



• The value of ' G ' has been experimentally determined *i.e.*, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$. Dimensional formula of G is $[M^{-1} L^3 T^{-2}]$.

• Acceleration Due to Gravity

The acceleration produced in a body on account of the force of gravity is known as acceleration due to gravity. It is usually denoted by ' g '. It is always towards the centre of Earth.

If a body of mass ' m ' lying on the surface of the earth, the gravitational force acting on the body is given by

$$F = G \frac{M m}{R^2}$$

where, $M \rightarrow$ mass of earth, $R \rightarrow$ radius of earth

acceleration due to gravity, $g = \frac{GM}{R^2}$.

• Mass and Mean Density of Earth

Mass and Mean density of Earth is given in the following manner.

Mass of earth,
$$M = \frac{gR^2}{G}$$

Mean density of earth, $\rho = \frac{3g}{4\pi GR}$.

• Gravitational Field

The space around a body within which its gravitational force of attraction is experienced by other bodies is called gravitational field.

• Intensity of Gravitational Field

The intensity of the gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.

Intensity of gravitational field at a point p is given by

$$I = \frac{GM}{x^2}$$

where M creates a field and the point p where the field is estimated is at a distance r .

• Gravitational Potential

The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point.

Gravitational potential at a point situated at a distance r from a body or particle of mass M is given by

$$V = -\frac{GM}{r}$$

Its unit is joule/kg and it is a scalar quantity. Its dimensional formula is $[L^2 T^{-2}]$.

• Gravitational Potential Energy

The work done in carrying a mass ' m ' from infinity to a point at distance r is called gravitational potential energy.

The gravitational potential energy of the system is given by

$$U = -\frac{GMm}{r}$$

• Escape Velocity

The minimum velocity required to project a body vertically upward from the surface of earth so that it comes out of the gravitational field of earth is called escape velocity.

It is given by
$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

where $M \rightarrow$ Mass of Earth

$R \rightarrow$ radius of the Earth.

• Satellite

A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

The orbit may be either circular or elliptical. A man-made object revolving in an orbit around a planet is called an artificial satellite.

• Orbital Velocity

Orbital velocity of a satellite is the minimum velocity required to put the satellite into a given orbit around earth.

The orbital velocity is given by

$$v_o = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$

▣ TEXTBOOK QUESTIONS SOLVED ▣

8.1. Answer the following:

- You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
- An astronaut inside a small spaceship orbiting around the Earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity?
- If you compare the gravitational force on the Earth due to the Sun to that due to the Moon, you would find that the Sun's pull is greater than the Moon's pull. (You can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the Moon's pull is greater than the tidal effect of Sun. Why?

Sol. (a) No. Gravitational forces are independent of medium. A body cannot be shielded from the gravitational influence of nearby matter.

- Yes. If the size of the spaceship is extremely large, then the gravitational effect of the spaceship may become measurable. The variation in g can also be detected.

(c) Tidal effect depends inversely on the cube of the distance, unlike force which depends inversely on the square of the distance. Since the distance of moon from the ocean water is very small as compared to the distance of sun from the ocean water on earth. Therefore, the tidal effect of Moon's pull is greater than the tidal effect of the sun.

8.2. Choose the correct alternative:

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth (assume the Earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of the mass of the Earth/mass of the body.

(d) The formula $-GMm \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the Earth.

Sol. (a) decreases

(b) decreases

(c) mass of the body

(d) more.

8.3. Suppose there existed a planet that went around the Sun twice as fast as the Earth. What would be its orbital size as compared to that of the Earth?

Sol. Here, $T_e = 1$ year; $T_p = \frac{T_e}{2} = \frac{1}{2}$ year; $r_e = 1$ A.U.

Using Kepler's third law, we have $r_p = r_e \left(\frac{T_p}{T_e} \right)^{2/3}$

$$\Rightarrow r_p = 1 \left(\frac{1/2}{1} \right)^{2/3} = 0.63 \text{ AU}$$

8.4. Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the Sun.

Sol. For a satellite of Jupiter, orbital period, $T_1 = 1.769$ days
 $= 1.769 \times 24 \times 60 \times 60$ s

Radius of the orbit of satellite, $r_1 = 4.22 \times 10^8$ m

Mass of Jupiter, M_1 is given by

$$M_1 = \frac{4\pi^2 r_1^3}{GT_1^2} = \frac{4\pi^2 \times (4.22 \times 10^8)^3}{G \times (1.769 \times 24 \times 60 \times 60)^2} \quad \dots(i)$$

We know that the orbital period of Earth around the sun,

$$T = 1 \text{ year} = 365.25 \times 24 \times 60 \times 60 \text{ s};$$

orbital radius, $r = 1 \text{ A.U.} = 1.496 \times 10^{11} \text{ m}$.

Mass of sun is given by

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times (1.496 \times 10^{11})^3}{G \times (365.25 \times 24 \times 60 \times 60)^2} \quad \dots(ii)$$

Dividing eqn. (ii) by (i), we get

$$\frac{M}{M_1} = \frac{4\pi^2 \times (1.496 \times 10^{11})^3}{G \times (365.25 \times 24 \times 60 \times 60)^2} \times \frac{G \times (1.769 \times 24 \times 60 \times 60)^2}{4\pi^2 \times (4.22 \times 10^8)^3}$$

$$= 1046$$

$$\text{or } \frac{M_1}{M} = \frac{1}{1046} \approx \frac{1}{1000} \Rightarrow M_1 = \frac{1}{1000} M. \quad \text{Proved}$$

- 8.5. Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky way to be 10^5 ly.

Sol. Here, $r = 50000 \text{ ly} = 50000 \times 9.46 \times 10^{15} \text{ m}$
 $= 4.73 \times 10^{20} \text{ m}$
 $M = 2.5 \times 10^{11} \text{ solar mass}$
 $= 2.5 \times 10^{11} \times (2 \times 10^{30}) \text{ kg} = 5.0 \times 10^{41} \text{ kg}$

We know that

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$\text{or } T = \left(\frac{4\pi^2 r^3}{GM} \right)^{1/2} = \left[\frac{4 \times (22/7)^2 \times (4.73 \times 10^{20})^3}{(6.67 \times 10^{-11}) \times (5.0 \times 10^{41})} \right]^{1/2}$$

$$= 1.12 \times 10^{16} \text{ s.}$$

- 8.6. Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.

Sol. (a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic energy.

(b) The energy required to launch an orbiting satellite out of Earth's gravitational influence is less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence.

8.7. Does the escape speed of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?

Sol. The escape speed $v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$. Hence,

(a) The escape speed of a body from the Earth does not depend on the mass of the body.

(b) The escape speed does not depend on the location from where a body is projected.

(c) The escape speed does not depend on the direction of projection of a body.

(d) The escape speed of a body depends upon the height of the location from where the body is projected, because the escape velocity depends upon the gravitational potential at the point from which it is projected and this potential depends upon height also.

8.8. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed (b) angular speed (c) angular momentum (d) kinetic energy (e) potential energy (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Sol. (a) The linear speed of the comet is variable in accordance with Kepler's second law. When comet is near the sun, its speed is higher. When the comet is far away from the sun, its speed is very less.

(b) Angular speed also varies slightly.

(c) Comet has constant angular momentum.

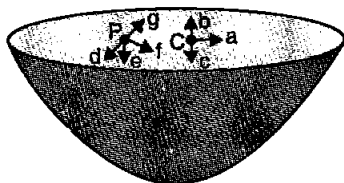
(d) Kinetic energy does not remain constant.

- (e) Potential energy varies along the path.
 (f) Total energy throughout the orbit remains constant.

8.9. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

- Sol. (a) The blood flow in feet would be lesser in zero gravity. So, the astronaut will not get swollen feet.
 (b) In the conditions of weightlessness, the face of the astronaut is expected to get more supply. Due to it, the astronaut may develop swollen face.
 (c) Due to more blood supply to face, the astronaut may get headache.
 (d) Space also has orientation. We also have the frames of reference in space. Hence, orientational problem will affect the astronaut in space.

8.10. In the following two exercises, choose the correct answer from among the given ones: The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig.) (i) a, (ii) b, (iii) c, (iv) 0.



- Sol. At all points inside a hollow spherical shell, potential is same. So, gravitational intensity, which is negative of gravitational potential gradient, is zero. Due to zero gravitational intensity, the gravitational forces acting on any particle at any point inside a spherical shell will be symmetrically placed. It follows from here that if we remove the upper hemispherical shell, the net gravitational force acting on a particle at P will be downwards. Since gravitational intensity is gravitational force per unit mass therefore, the direction of gravitational intensity will be along c. So, option (iii) is correct.
- 8.11. For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Sol. Using the explanation given in the solution of the previous problem, the direction of the gravitational field intensity at P will be along e. So, option (ii) is correct.

8.12. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of

the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

Sol. Mass of Sun, $M = 2 \times 10^{30}$ kg; Mass of Earth, $m = 6 \times 10^{24}$ kg
Distance between Sun and Earth, $r = 1.5 \times 10^{11}$ m

Let at the point P , the gravitational force on the rocket due to Earth

= gravitational force on the rocket due to Sun

Let x = distance of the point P from the Earth

$$\text{Then } \frac{Gm}{x^2} = \frac{GM}{(r-x)^2}$$

$$\Rightarrow \frac{(r-x)^2}{x^2} = \frac{M}{m} = \frac{2 \times 10^{30}}{6 \times 10^{24}} = \frac{10^6}{3}$$

$$\text{or } \frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \Rightarrow \frac{r}{x} = \frac{10^3}{\sqrt{3}} + 1 \approx \frac{10^3}{\sqrt{3}}$$

$$\text{or } x = \frac{\sqrt{3}r}{10^3} = \frac{1.732 \times 1.5 \times 10^{11}}{10^3} = 2.6 \times 10^8 \text{ m.}$$

8.13. How will you 'weigh the sun', that is, estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

Sol. The mean orbital radius of the Earth around the Sun

$$R = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^{11} \text{ m}$$

Time period, $T = 365.25 \times 24 \times 60 \times 60$ s

Let the mass of the Sun be M and that of Earth be m .

According to law of gravitation

$$F = G \frac{Mm}{R^2} \quad \dots(i)$$

Centripetal force,

$$F = \frac{mv^2}{R} = m.R.\omega^2 \quad \dots(ii)$$

From eqn. (i) and (ii), we have

$$\begin{aligned} \frac{GMm}{R^2} &= m.R.\omega^2 \\ &= \frac{mR.4\pi^2}{T^2} \quad \left[\because \omega = \frac{2\pi}{T} \right] \end{aligned}$$

$$\begin{aligned} \therefore M &= \frac{4\pi^2 R^3}{G.T^2} \\ &= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (365.25 \times 24 \times 60 \times 60)^2} \\ &= 2.009 \times 10^{30} \text{ kg} = 2.0 \times 10^{30} \text{ kg.} \end{aligned}$$

8.14. A Saturn year is 29.5 times the Earth year. How far is the Saturn from the Sun if the Earth is 1.50×10^8 km away from the Sun?

Sol. We know that $T^2 \propto R^3$

$$\therefore \frac{T_s^2}{T_e^2} = \frac{R_s^3}{R_e^3}$$

where subscripts s and e refer to the Saturn and Earth respectively.

$$\text{Now } \frac{T_s}{T_e} = 29.5 \text{ [given]; } R_e = 1.50 \times 10^8 \text{ km}$$

$$\left(\frac{R_s}{R_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$$

$$\begin{aligned} R_s &= R_e \times [(29.5)^2]^{1/3} = 1.50 \times 10^8 \times (870.25)^{1/3} \text{ km} \\ &= 1.43 \times 10^9 \text{ km} = 1.43 \times 10^{12} \text{ m} \end{aligned}$$

\therefore Distance of Saturn from Sun = 1.43×10^{12} m.

8.15. A body weighs 63 N on the surface of the Earth. What is the gravitational force on it due to the Earth at a height equal to half the radius of the Earth?

Sol. Let g_h be the acceleration due to gravity at a height equal to half the radius of the Earth ($h = R/2$) and g its value on Earth's surface. Let the body have mass m .

We know that

$$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2$$

$$\text{or } \frac{g_h}{g} = \left(\frac{R}{R+\frac{R}{2}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let W be the weight of body on the surface of Earth and W_h the weight of the body at height h .

$$\text{Then, } \frac{W_h}{W} = \frac{mg_h}{mg} = \frac{g_h}{g} = \frac{4}{9}$$

$$\text{or } W_h = \frac{4}{9}W = \frac{4}{9} \times 63 \text{ N} = 28 \text{ N}.$$

8.16. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

$$\text{Sol. As } g_d = g \left(1 - \frac{d}{R}\right) \Rightarrow mg_d = mg \left(1 - \frac{d}{R}\right)$$

$$\text{Here } d = \frac{R}{2}$$

$$\therefore mg_d = (250) \times \left(1 - \frac{R/2}{R}\right) = 250 \times \frac{1}{2} = 125 \text{ N}.$$

8.17. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = $6.0 \times 10^{24} \text{ kg}$; mean radius of the earth = $6.4 \times 10^6 \text{ m}$; $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

$$\begin{aligned} \text{Sol. Initial kinetic energy of rocket} &= \frac{1}{2}mv^2 = \frac{1}{2} \times m \times (5000)^2 \\ &= 1.25 \times 10^7 \text{ m J} \end{aligned}$$

At distance r from centre of earth, kinetic energy becomes zero

$$\therefore \text{Change in kinetic energy} = 1.25 \times 10^7 - 0 = 1.25 \times 10^7 \text{ m J}$$

This energy changes into potential energy.

$$\begin{aligned} \text{Initial potential energy at the surface of earth} &= \frac{GM_e m}{r} \\ &= \frac{-(6.67 \times 10^{-11}) \times (6 \times 10^{24}) m}{6.4 \times 10^6} = -6.25 \text{ m} \times 10^7 \text{ J} \end{aligned}$$

Final potential energy at distance,

$$\begin{aligned} r &= -\frac{GM_e m}{r} \\ &= \frac{-(6.67 \times 10^{-11}) \times (6 \times 10^{24}) m}{r} = -4 \times 10^{14} \frac{m}{r} \end{aligned}$$

$$\therefore \text{Change in potential energy} = 6.25 \times 10^7 \text{ m} - 4 \times 10^{14} \frac{m}{r}$$

Using law of conservation of energy,

$$6.25 \times 10^7 \text{ m} - \frac{4 \times 10^{14} \text{ m}}{r} = 1.25 \times 10^7 \text{ m}$$

$$\text{i.e.,} \quad r = \frac{4 \times 10^{14}}{5 \times 10^7} \text{ m} = 8 \times 10^{16} \text{ m}.$$

8.18. The escape speed of a projectile on the Earth's surface is 11.2 km s^{-1} . A body is projected out with thrice this speed. What is the speed of the body far away from the Earth? Ignore the presence of the Sun and other planets.

Sol. Let v_{es} be the escape speed from surface of Earth having a value

$$v_{\text{es}} = 11.2 \text{ km s}^{-1} = 11.2 \times 10^3 \text{ m s}^{-1}. \text{ By definition}$$

$$\frac{1}{2} m v_e^2 = \frac{GMm}{R^2} \quad \dots(i)$$

When a body is projected with a speed $v_i = 3v_{\text{es}} = 3 \times 11.2 \times 10^3 \text{ m/s}$, then it will have a final speed v_f such that

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - \frac{GMm}{R^2} = \frac{1}{2} m v_i^2 - \frac{1}{2} m v_e^2$$

$$\begin{aligned} \Rightarrow \quad v_f &= \sqrt{v_i^2 - v_e^2} \\ &= \sqrt{(3 \times 11.2 \times 10^3)^2 - (11.2 \times 10^3)^2} \\ &= 11.2 \times 10^3 \times \sqrt{8} = 31.7 \times 10^3 \text{ ms}^{-1} \text{ or } 31.7 \text{ km s}^{-1}. \end{aligned}$$

8.19. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = $6.0 \times 10^{24} \text{ kg}$; radius of the earth = $6.4 \times 10^6 \text{ m}$; $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Sol. Total energy of orbiting satellite at a height h

$$= -\frac{GMm}{(R+h)} + \frac{1}{2} m v^2 = -\frac{GMm}{(R+h)} + \frac{1}{2} m \frac{GM}{(R+h)} = -\frac{GMm}{2(R+h)}$$

Energy expended to rocket the satellite out of the earth's gravitational field

$$\begin{aligned} &= -(\text{total energy of the orbiting satellite}) \\ &= \frac{GMm}{2(R+h)} = \frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24}) \times 200}{2 \times (6.4 \times 10^6 + 4 \times 10^5)} \\ &= 5.9 \times 10^9 \text{ J}. \end{aligned}$$

8.20. Two stars each of one solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are at a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Sol. Here, mass of each star, $M = 2 \times 10^{30}$ kg

Initial potential between two stars, $r = 10^9$ km $= 10^{12}$ m.

Initial potential energy of the system $= -\frac{GMm}{r}$

Total K.E. of the stars $= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

where v is the speed of stars with which they collide. When the stars are about to collide, the distance between their centres,

$$r' = 2R.$$

\therefore Final potential energy of two stars $= -\frac{GMM}{2R}$

Since gain in K.E. is at the cost of loss in P.E.

$$\therefore Mv^2 = -\frac{GMM}{r} - \left(-\frac{GMM}{2R}\right) = -\frac{GMM}{r} + \frac{GMM}{2R}$$

or $2 \times 10^{30} v^2$

$$\begin{aligned} &= -\frac{6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{10^{12}} + \frac{6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{2 \times 10^7} \\ &= -2.668 \times 10^{38} + 1.334 \times 10^{43} \\ &= 1.334 \times 10^{43} \text{ J} \end{aligned}$$

$$\therefore v = \sqrt{\frac{1.334 \times 10^{43}}{2 \times 10^{30}}} = 2.583 \times 10^6 \text{ ms}^{-1}.$$

8.21. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Sol. Here $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$; $M = 100$ kg; $R = 0.1$ m, distance between the two spheres, $d = 1.0$ m

Suppose that the distance of either sphere from the mid-point of the line joining their centre is r . Then $r = \frac{d}{2} = 0.5$ m. The

gravitational field at the mid-point due to two spheres will be equal and opposite.

Hence, the resultant gravitational field at the mid point = 0

$$\begin{aligned} \text{The gravitational potential at the mid point} &= \left(-\frac{GM}{r} \right) \times 2 \\ &= -\frac{6.67 \times 10^{-11} \times 100 \times 2}{0.5} = -2.668 \times 10^{-8} \text{ J kg}^{-1}. \end{aligned}$$

The object placed at that point would be in stable equilibrium.

- 8.22.** As you have learnt in the text, a geostationary satellite orbits the Earth at a height of nearly 36,000 km from the surface of the Earth. What is the potential due to Earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the Earth = 6.0×10^{24} kg, radius = 6400 km.

Sol. Distance of satellite from the centre of earth = $R = r + x$
 $= 6400 + 36000 = 42400 \text{ km} = 4.24 \times 10^7 \text{ m}$

Using potential, $V = -\frac{GM}{R}$, we get

$$V = -\frac{(6.67 \times 10^{-11}) \times (6 \times 10^{24})}{(4.24 \times 10^7)} = -9.44 \times 10^6 \text{ J kg}^{-1}$$

- 8.23.** A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = 2×10^{30} kg).

Sol. Acceleration due to gravity of the star, $g = \frac{GM}{R^2}$... (i)

Here M is the mass and R is the radius of the star.

The outward centrifugal force acting on a body of mass m at the equator of the star

$$= \frac{mv^2}{R} = mR\omega^2 \quad \dots(ii)$$

From equation (i), the acceleration due to the gravity of the star

$$= \frac{6.67 \times 10^{-11} \times 2.5 \times 2 \times 10^{30}}{(12 \times 10^3)^2} = 2.316 \times 10^{12} \text{ m/s}^2$$

$$\therefore \text{Inward force due to gravity on a body of mass } m \\ = m \times 2.316 \times 10^{12} \text{ N}$$

From equation (ii), the outward centrifugal force = $mR\omega^2$

$$= m \times (12 \times 10^3) \times \left(\frac{2\pi \times 1.5}{-1} \right)^2 = m \times 1.06 \times 10^6 \text{ N}$$

Since the inward force due to gravity on a body at the equator of the star is about 2.2 million times more than the outward centrifugal force, the body will remain stuck to the surface of the star.

8.24. A space-ship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system? Mass of the spaceship = 1000 kg, Mass of the Sun = 2×10^{30} kg. Mass of the Mars = 6.4×10^{23} kg, Radius of Mars = 3395 km. Radius of the orbit of Mars = 2.28×10^{11} m, $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Sol. Let R be the radius of orbit of Mars and R' be the radius of the Mars. M be the mass of the Sun and M' be the mass of Mars. If m is the mass of the space-ship, then

Potential energy of space-ship due to gravitational attraction of the Sun = $-GMm/R$

Potential energy of space-ship due to gravitational attraction of Mars = $-GM'm/R'$

Since the K.E. of space ship is zero, therefore, total energy of spaceship

$$= -\frac{GMm}{R} - \frac{GM'm}{R'} = -Gm \left(\frac{M}{R} + \frac{M'}{R'} \right)$$

\therefore Energy required to rocket out the spaceship from the solar system = - (total energy of spaceship)

$$= - \left[-Gm \left(\frac{M}{R} + \frac{M'}{R'} \right) \right] = Gm \left[\frac{M}{R} + \frac{M'}{R'} \right]$$

$$= 6.67 \times 10^{-11} \times 1000 \times \left[\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3395 \times 10^3} \right]$$

$$= 6.67 \times 10^{-8} \left[\frac{20}{2.28} + \frac{6.4}{33.95} \right] \times 10^{18} \text{ J} = 5.98 \times 10^{11} \text{ J.}$$

8.25. A rocket is fired 'vertically' from the surface of Mars with a speed of 2 km s^{-1} . If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of Mars = 6.4×10^{23} kg; radius of Mars = 3395 km; $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Sol. Initial K.E. = $\frac{1}{2} mv^2$; Initial P.E. = $-\frac{GMm}{R}$

where m = Mass of rocket, M = Mass of Mars, R = Radius of Mars

$$\therefore \text{Total initial energy} = \frac{1}{2} mv^2 - \frac{GMm}{R}$$

Since 20% of K.E. is lost, only 80% remains to reach the height.

\therefore Total initial energy available

$$= \frac{4}{5} \times \frac{1}{2} mv^2 - \frac{GMm}{R} = 0.4 mv^2 - \frac{GMm}{R}$$

When the rocket reaches the highest point, at a height h above

the surface, its K.E. is zero and P.E. = $-\frac{GMm}{R+h}$.

Using principle of conservation of energy.

$$0.4 mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\text{or } \frac{GMm}{R+h} = \frac{GMm}{R} - 0.4 mv^2$$

$$\Rightarrow \frac{GM}{R+h} = \frac{GM}{R} - 0.4 v^2$$

$$\text{or } \frac{GM}{R+h} = \frac{1}{R} [GM - 0.4 Rv^2]$$

$$\Rightarrow \frac{R+h}{R} = \frac{GM}{GM - 0.4 Rv^2}$$

$$\text{or } \frac{h}{R} = \frac{GM}{GM - 0.4 Rv^2} - 1$$

$$\text{or } \frac{h}{R} = \frac{0.4 Rv^2}{GM - 0.4 Rv^2} \Rightarrow h = \frac{0.4 R^2 v^2}{GM - 0.4 Rv^2}$$

$$\begin{aligned} \text{or } h &= \frac{0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times 3.395 \times 10^6} \text{ m} \\ &= 495 \text{ km.} \end{aligned}$$

