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Chapter Eight

ELECTROMAGNETIC WAVES

8.1 Introduction

In Chapter 4, we learnt that an electric current produces magnetic field and that two current-carrying wires exert a magnetic force on each other. Further, in Chapter 6, we have seen that a magnetic field changing with time gives rise to an electric field. Is the converse also true? Does an electric field changing with time give rise to a magnetic field? James Clerk Maxwell (1831-1879), argued that this was indeed the case – not only an electric current but also a time-varying electric field generates magnetic field. While applying the Ampere's circuital law to find magnetic field at a point outside a capacitor connected to a time-varying current, Maxwell noticed an inconsistency in the Ampere's circuital law. He suggested the existence of an additional current, called by him, the displacement current to remove this inconsistency.

Maxwell formulated a set of equations involving electric and magnetic fields, and their sources, the charge and current densities. These equations are known as Maxwell's equations. Together with the Lorentz force formula (Chapter 4), they mathematically express all the basic laws of electromagnetism.

The most important prediction to emerge from Maxwell's equations is the existence of electromagnetic waves, which are (coupled) time-varying electric and magnetic fields that propagate in space. The speed of the waves, according to these equations, turned out to be very close to

JAMES CLERK MAXWELL (1831–1879)

James Clerk Maxwell (1831 – 1879) Born in Edinburgh, Scotland, was among the greatest physicists of the nineteenth century. He derived the thermal velocity distribution of molecules in a gas and was among the first to obtain reliable estimates of molecular parameters from measurable quantities like viscosity, etc. Maxwell's greatest acheivement was the unification of the laws of electricity and magnetism (discovered by Coulomb, Oersted, Ampere and Faraday) into a consistent set of equations now called Maxwell's equations. From these he arrived at the most important conclusion that light is an electromagnetic wave. Interestingly, Maxwell did not agree with the idea (strongly suggested by the Faraday's laws of electrolysis) that electricity was particulate in nature.

the speed of light(3×10^8 m/s), obtained from optical measurements. This led to the remarkable conclusion that light is an electromagnetic wave. Maxwell's work thus unified the domain of electricity, magnetism and light. Hertz, in 1885, experimentally demonstrated the existence of electromagnetic waves. Its technological use by Marconi and others led in due course to the revolution in communication that we are witnessing today.

In this chapter, we first discuss the need for displacement current and its consequences. Then we present a descriptive account of electromagnetic waves. The broad spectrum of electromagnetic waves, stretching from γ rays (wavelength $\sim 10^{-12}$ m) to long radio waves (wavelength $\sim 10^6$ m) is described. How the electromagnetic waves are sent and received for communication is discussed in Chapter 15.

8.2 Displacement Current

We have seen in Chapter 4 that an electrical current produces a magnetic field around it. Maxwell showed that for logical consistency, a changing electric field *must also* produce a magnetic field. This effect is of great importance because it explains the existence of radio waves, gamma rays and visible light, as well as all other forms of electromagnetic waves.

To see how a changing electric field gives rise to a magnetic field, let us consider the process of charging of a capacitor and apply Ampere's circuital law given by (Chapter 4)

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0\, i\,(t) \qquad (8.1)$$

to find magnetic field at a point outside the capacitor. Figure 8.1(a) shows a parallel plate capacitor C which is a part of circuit through which a time-dependent current $i\,(t)$ flows . Let us find the magnetic field at a point such as P, in a region outside the parallel plate capacitor. For this, we consider a plane circular loop of radius r whose plane is perpendicular to the direction of the current-carrying wire, and which is centred symmetrically with respect to the wire [Fig. 8.1(a)]. From symmetry, the magnetic field is directed along the circumference of the circular loop and is the same in magnitude at all points on the loop so that if B is the magnitude of the field, the left side of Eq. (8.1) is $B\,(2\pi r)$. So we have

$$B\,(2\pi r) = \mu_0 i\,(t) \qquad (8.2)$$

Now, consider a different surface, which has the same boundary. This is a pot like surface [Fig. 8.1(b)] which nowhere touches the current, but has its bottom between the capacitor plates; its mouth is the circular loop mentioned above. Another such surface is shaped like a tiffin box (without the lid) [Fig. 8.1(c)]. On applying Ampere's circuital law to such surfaces with the *same* perimeter, we find that the left hand side of Eq. (8.1) has not changed but the right hand side is *zero* and *not* $\mu_0 i$, since *no* current passes through the surface of Fig. 8.1(b) and (c). So we have a *contradiction*; calculated one way, there is a magnetic field at a point P; calculated another way, the magnetic field at P is zero.

Since the contradiction arises from our use of Ampere's circuital law, this law must be missing something. The missing term must be such that one gets the same magnetic field at point P, no matter what surface is used.

We can actually guess the missing term by looking carefully at Fig. 8.1(c). Is there anything passing through the surface S *between* the plates of the capacitor? Yes, of course, the electric field! If the plates of the capacitor have an area A, and a total charge Q, the magnitude of the electric field **E** between the plates is $(Q/A)/\varepsilon_0$ (see Eq. 2.41). The field is perpendicular to the surface S of Fig. 8.1(c). It has the same magnitude over the area A of the capacitor plates, and vanishes outside it. So what is the *electric* flux Φ_E through the surface S? Using Gauss's law, it is

$$\Phi_E = |\mathbf{E}|A = \frac{1}{\varepsilon_0}\frac{Q}{A}A = \frac{Q}{\varepsilon_0} \qquad (8.3)$$

Now if the charge Q on the capacitor plates changes with time, there is a current $i = (\mathrm{d}Q/\mathrm{d}t)$, so that using Eq. (8.3), we have

$$\frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{\mathrm{d}Q}{\mathrm{d}t}$$

This implies that for consistency,

$$\varepsilon_0\left(\frac{\mathrm{d}\Phi_E}{\mathrm{d}t}\right) = i \qquad (8.4)$$

This is the missing term in Ampere's circuital law. If we generalise this law by adding to the total current carried by conductors through the surface, another term which is ε_0 times the rate of change of electric flux through the same surface, the *total* has the same value of current i for all surfaces. If this is done, there is no contradiction in the value of B obtained anywhere using the generalised Ampere's law. B at the point P is non-zero no matter which surface is used for calculating it. B at a point P outside the plates [Fig. 8.1(a)] is the same as at a point M just inside, as it should be. The current carried by conductors due to flow of charges is called *conduction current*. The current, given by Eq. (8.4), is a new term, and is due to changing electric field (or electric *displacement*, an old term still used sometimes). It is, therefore, called *displacement current* or Maxwell's displacement current. Figure 8.2 shows the electric and magnetic fields inside the parallel plate capacitor discussed above.

The generalisation made by Maxwell then is the following. The source of a magnetic field is not *just* the conduction electric current due to flowing

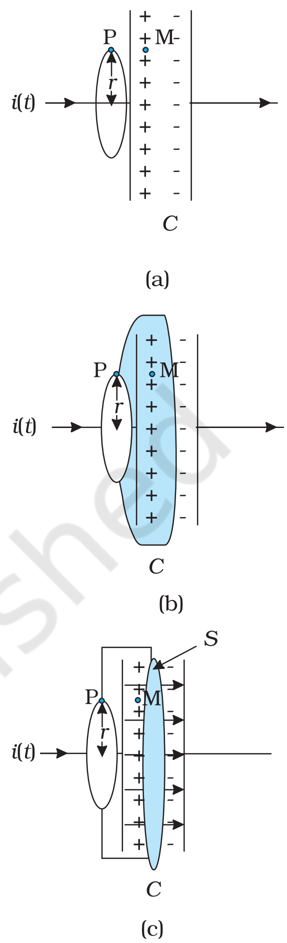

FIGURE 8.1 A parallel plate capacitor C, as part of a circuit through which a time dependent current $i\,(t)$ flows, (a) a loop of radius r, to determine magnetic field at a point P on the loop; (b) a pot-shaped surface passing through the interior between the capacitor plates with the loop shown in (a) as its rim; (c) a tiffin-shaped surface with the circular loop as its rim and a flat circular bottom S between the capacitor plates. The arrows show uniform electric field between the capacitor plates.

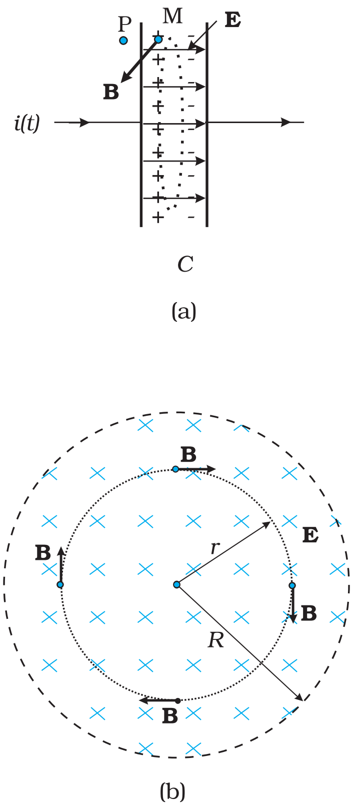

FIGURE 8.2 (a) The electric and magnetic fields **E** and **B** between the capacitor plates, at the point M. (b) A cross sectional view of Fig. (a).

charges, but also the time rate of change of electric field. More precisely, the total current i is the sum of the conduction current denoted by i_c, and the displacement current denoted by i_d (= ε_0 ($d\Phi_E/dt$)). So we have

$$i = i_c + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \tag{8.5}$$

In explicit terms, this means that outside the capacitor plates, we have only conduction current $i_c = i$, and no displacement current, i.e., $i_d = 0$. On the other hand, inside the capacitor, there is no conduction current, i.e., $i_c = 0$, and there is only displacement current, so that $i_d = i$.

The generalised (and correct) Ampere's circuital law has the same form as Eq. (8.1), with one difference: "the *total current* passing through any surface of which the closed loop is the perimeter" is the sum of the conduction current and the displacement current. The generalised law is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \tag{8.6}$$

and is known as Ampere-Maxwell law.

In all respects, the displacement current has the same physical effects as the conduction current. In some cases, for example, steady electric fields in a conducting wire, the displacement current may be zero since the electric field **E** does not change with time. In other cases, for example, the charging capacitor above, both conduction and displacement currents may be present in different regions of space. In most of the cases, they both may be present in the same region of space, as there exist no perfectly conducting or perfectly insulating medium. Most interestingly, there may be large regions of space where there is *no* conduction current, but there is only a displacement current due to time-varying electric fields. In such a region, we expect a magnetic field, though there is no (conduction) current source nearby! The prediction of such a displacement current can be verified experimentally. For example, a *magnetic* field (say at point M) between the plates of the capacitor in Fig. 8.2(a) can be measured and is seen to be the same as that just outside (at P).

The displacement current has (literally) far reaching consequences. One thing we immediately notice is that the laws of electricity and magnetism are now more symmetrical*. Faraday's law of induction states that there is an induced emf *equal to the rate of* change of magnetic flux. Now, since the emf between two points 1 and 2 is the work done per unit charge in taking it from 1 to 2, the existence of an emf implies the existence of an electric field. So, we can rephrase Faraday's law of electromagnetic induction by saying that a *magnetic field*, changing with time, gives rise to an *electric field*. Then, the fact that an *electric field* changing with time gives rise to a *magnetic field*, is the symmetrical counterpart, and is

* They are still not perfectly symmetrical; there are no known sources of magnetic field (magnetic monopoles) analogous to electric charges which are sources of electric field.

a consequence of the displacement current being a source of a magnetic field. Thus, time- dependent electric and magnetic fields give rise to each other! Faraday's law of electromagnetic induction and Ampere-Maxwell law give a quantitative expression of this statement, with the current being the total current, as in Eq. (8.5). One very important consequence of this symmetry is the existence of electromagnetic waves, which we discuss qualitatively in the next section.

MAXWELL'S EQUATIONS

1. $\oint \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$ (Gauss's Law for electricity)
2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ (Gauss's Law for magnetism)
3. $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\Phi_B}{dt}$ (Faraday's Law)
4. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ (Ampere – Maxwell Law)

Example 8.1 A parallel plate capacitor with circular plates of radius 1 m has a capacitance of 1 nF. At $t = 0$, it is connected for charging in series with a resistor $R = 1\ \mathrm{M\Omega}$ across a 2V battery (Fig. 8.3). Calculate the magnetic field at a point P, halfway between the centre and the periphery of the plates, after $t = 10^{-3}$ s. (The charge on the capacitor at time t is $q(t) = CV[1 - \exp(-t/\tau)]$, where the time constant τ is equal to CR.)

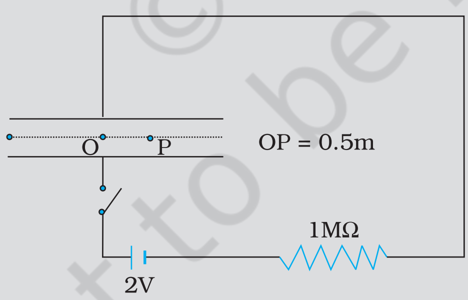

FIGURE 8.3

Solution The time constant of the CR circuit is $\tau = CR = 10^{-3}$ s. Then, we have

$q(t) = CV[1 - \exp(-t/\tau)]$
$\quad = 2 \times 10^{-9} [1 - \exp(-t/10^{-3})]$

The electric field in between the plates at time t is

$E = \frac{q(t)}{\varepsilon_0 A} = \frac{q}{\pi \varepsilon_0}$; $A = \pi (1)^2\ \mathrm{m}^2$ = area of the plates.

Consider now a circular loop of radius (1/2) m parallel to the plates passing through P. The magnetic field **B** at all points on the loop is

EXAMPLE 8.1

EXAMPLE 8.1

along the loop and of the same value.
The flux Φ_E through this loop is
Φ_E = $E \times$ area of the loop

$$= E \times \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\varepsilon_0}$$

The displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4}\frac{dq}{dt} = 0.5 \times 10^{-6} \exp(-1)$$

at $t = 10^{-3}$s. Now, applying Ampere-Maxwell law to the loop, we get

$$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0 (i_c + i_d) = \mu_0 (0 + i_d) = 0.5\times10^{-6} \mu_0 \exp(-1)$$

or, $B = 0.74 \times 10^{-13}$ T

8.3 Electromagnetic Waves

8.3.1 Sources of electromagnetic waves

How are electromagnetic waves produced? Neither stationary charges nor charges in uniform motion (steady currents) can be sources of electromagnetic waves. The former produces only electrostatic fields, while the latter produces magnetic fields that, however, do not vary with time. It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves. The proof of this basic result is beyond the scope of this book, but we can accept it on the basis of rough, qualitative reasoning. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other, so to speak, as the wave propagates through the space. The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source – the accelerated charge.

From the preceding discussion, it might appear easy to test the prediction that light is an electromagnetic wave. We might think that all we needed to do was to set up an ac circuit in which the current oscillate at the frequency of visible light, say, yellow light. But, alas, that is not possible. The frequency of yellow light is about 6×10^{14} Hz, while the frequency that we get even with modern electronic circuits is hardly about 10^{11} Hz. This is why the experimental demonstration of electromagnetic wave had to come in the low frequency region (the radio wave region), as in the Hertz's experiment (1887).

Hertz's successful experimental test of Maxwell's theory created a sensation and sparked off other important works in this field. Two important achievements in this connection deserve mention. Seven years after Hertz, Jagdish Chandra Bose, working at Calcutta (now Kolkata), succeeded in producing and observing electromagnetic waves of much shorter wavelength (25 mm to 5 mm). His experiment, like that of Hertz's, was confined to the laboratory.

At around the same time, Guglielmo Marconi in Italy followed Hertz's work and succeeded in transmitting electromagnetic waves over distances of many kilometres. Marconi's experiment marks the beginning of the field of communication using electromagnetic waves.

8.3.2 Nature of electromagnetic waves

It can be shown from Maxwell's equations that electric and magnetic fields in an electromagnetic wave are perpendicular to each other, *and* to the direction of propagation. It appears reasonable, say from our discussion of the displacement current. Consider Fig. 8.2. The electric field inside the plates of the capacitor is directed perpendicular to the plates. The magnetic field this gives rise to via the displacement current is along the perimeter of a circle parallel to the capacitor plates. So **B** and **E** are perpendicular in this case. This is a general feature.

In Fig. 8.4, we show a typical example of a plane electromagnetic wave propagating along the *z* direction (the fields are shown as a function of the *z* coordinate, at a given time *t*). The electric field E_x is along the *x*-axis, and varies sinusoidally with *z*, at a given time. The magnetic field B_y is along the *y*-axis, and again varies sinusoidally with *z*. The electric and magnetic fields E_x and B_y are perpendicular to each other, and to the direction *z* of propagation. We can write E_x and B_y as follows:

$$E_x = E_0 \sin(kz - \omega t) \quad [8.7(a)]$$

$$B_y = B_0 \sin(kz - \omega t) \quad [8.7(b)]$$

Here *k* is related to the wave length λ of the wave by the usual equation

$$k = \frac{2\pi}{\lambda} \quad (8.8)$$

and ω is the angular frequency. *k* is the magnitude of the wave vector (or propagation vector) **k** and its direction describes the direction of propagation of the wave. The speed of propagation of the wave is (ω/k). Using Eqs. [8.7(a) and (b)] for E_x and B_y and Maxwell's equations, one finds that

Heinrich Rudolf Hertz (1857 – 1894) German physicist who was the first to broadcast and receive radio waves. He produced electro-magnetic waves, sent them through space, and measured their wave-length and speed. He showed that the nature of their vibration, reflection and refraction was the same as that of light and heat waves, establishing their identity for the first time. He also pioneered research on discharge of electricity through gases, and discovered the photoelectric effect.

HEINRICH RUDOLF HERTZ (1857–1894)

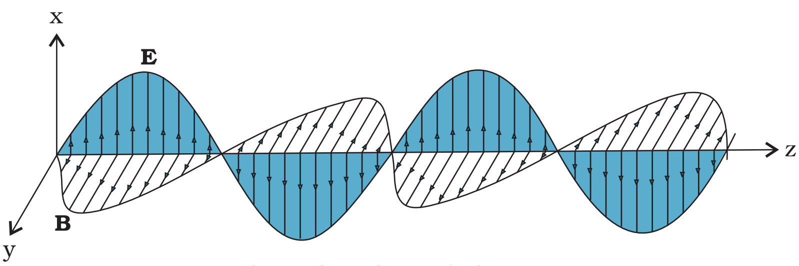

FIGURE 8.4 A linearly polarised electromagnetic wave, propagating in the *z*-direction with the oscillating electric field **E** along the *x*-direction and the oscillating magnetic field **B** along the *y*-direction.

$\omega = ck$, where, $c = 1/\sqrt{\mu_0 \varepsilon_0}$ [8.9(a)]

The relation $\omega = ck$ is the standard one for waves (see for example, Section 15.4 of class XI Physics textbook). This relation is often written in terms of frequency, ν $(=\omega/2\pi)$ and wavelength, λ $(=2\pi/k)$ as

$$2\pi\nu = c\left(\frac{2\pi}{\lambda}\right) \quad \text{or}$$

$$\nu\lambda = c \qquad [8.9(b)]$$

It is also seen from Maxwell's equations that the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as

$$B_0 = (E_0/c) \qquad (8.10)$$

We here make remarks on some features of electromagnetic waves. They are self-sustaining oscillations of electric and magnetic fields in free space, or vacuum. They differ from all the other waves we have studied so far, in respect that *no material medium* is involved in the vibrations of the electric and magnetic fields. Sound waves in air are longitudinal waves of compression and rarefaction. Transverse elastic (sound) waves can also propagate in a solid, which is rigid and that resists shear. Scientists in the nineteenth century were so much used to this mechanical picture that they thought that there must be some medium pervading all space and all matter, which responds to electric and magnetic fields just as any elastic medium does. They called this medium *ether*. They were so convinced of the reality of this medium, that there is even a novel called *The Poison Belt* by *Sir Arthur Conan Doyle* (the creator of the famous detective *Sherlock Holmes*) where the solar system is supposed to pass through a poisonous region of ether! We now accept that no such physical medium is needed. The famous experiment of Michelson and Morley in 1887 demolished conclusively the hypothesis of ether. Electric and magnetic fields, oscillating in space and time, can sustain each other in vacuum.

But what if a material medium is actually there? We know that light, an electromagnetic wave, does propagate through glass, for example. We have seen earlier that the total electric and magnetic fields inside a medium are described in terms of a permittivity ε and a magnetic permeability μ (these describe the factors by which the total fields differ from the external fields). These replace ε_0 and μ_0 in the description to electric and magnetic fields in Maxwell's equations with the result that in a material medium of permittivity ε and magnetic permeability μ, the velocity of light becomes,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (8.11)$$

Thus, the velocity of light depends on electric and magnetic properties of the medium. We shall see in the next chapter that the *refractive index* of one medium with respect to the other is equal to the ratio of velocities of light in the two media.

The velocity of electromagnetic waves in free space or vacuum is an important fundamental constant. It has been shown by experiments on electromagnetic waves of different wavelengths that this velocity is the

PHYSICS

Simulate propagation of electromagnetic waves
(i) http://www.amanogawa.com/waves.html
(ii) http://www.phys.hawaii.edu/~teb/java/ntnujava/emWave/emWave.html

same (independent of wavelength) to within a few metres per second, out of a value of 3×10^8 m/s. The constancy of the velocity of em waves in vacuum is so strongly supported by experiments and the actual value is so well known now that this is used to define a standard of *length*. Namely, the metre is now *defined* as the distance travelled by light in vacuum in a time $(1/c)$ seconds = $(2.99792458 \times 10^8)^{-1}$ seconds. This has come about for the following reason. The basic unit of time can be defined very accurately in terms of some atomic frequency, i.e., frequency of light emitted by an atom in a particular process. The basic unit of length is harder to define as accurately in a direct way. Earlier measurement of c using earlier units of length (metre rods, etc.) converged to a value of about 2.9979246×10^8 m/s. Since c is such a strongly fixed number, unit of length can be defined in terms of c and the unit of time!

Hertz not only showed the existence of electromagnetic waves, but also demonstrated that the waves, which had wavelength ten million times that of the light waves, could be diffracted, refracted and polarised. Thus, he conclusively established the wave nature of the radiation. Further, he produced stationary electromagnetic waves and determined their wavelength by measuring the distance between two successive nodes. Since the frequency of the wave was known (being equal to the frequency of the oscillator), he obtained the speed of the wave using the formula $\upsilon = \nu\lambda$ and found that the waves travelled with the same speed as the speed of light.

The fact that electromagnetic waves are polarised can be easily seen in the response of a portable AM radio to a broadcasting station. If an AM radio has a telescopic antenna, it responds to the electric part of the signal. When the antenna is turned horizontal, the signal will be greatly diminished. Some portable radios have horizontal antenna (usually inside the case of radio), which are sensitive to the magnetic component of the electromagnetic wave. Such a radio must remain horizontal in order to receive the signal. In such cases, response also depends on the orientation of the radio with respect to the station.

Do electromagnetic waves carry energy and momentum like other waves? Yes, they do. We have seen in chapter 2 that in a region of free space with electric field E, there is an energy density $(\varepsilon_0 E^2/2)$. Similarly, as seen in Chapter 6, associated with a magnetic field B is a magnetic energy density $(B^2/2\mu_0)$. As electromagnetic wave contains both electric and magnetic fields, there is a non-zero energy density associated with it. Now consider a plane perpendicular to the direction of propagation of the electromagnetic wave (Fig. 8.4). If there are, on this plane, electric charges, they will be set and sustained in motion by the electric and magnetic fields of the electromagnetic wave. The charges thus acquire energy and momentum from the waves. This just illustrates the fact that an electromagnetic wave (like other waves) carries energy and momentum. Since it carries momentum, an electromagnetic wave also exerts pressure, called *radiation pressure*.

If the total energy transferred to a surface in time t is U, it can be shown that the magnitude of the total momentum delivered to this surface (*for complete absorption*) is,

$$p = \frac{U}{c} \qquad (8.12)$$

When the sun shines on your hand, you feel the energy being absorbed from the electromagnetic waves (your hands get warm). Electromagnetic waves also transfer momentum to your hand but because *c* is very large, the amount of momentum transferred is extremely small and you do not feel the pressure. In 1903, the American scientists Nicols and Hull succeeded in measuring radiation pressure of visible light and verified Eq. (8.12). It was found to be of the order of 7×10^{-6} N/m^2. Thus, on a surface of area 10 cm^2, the force due to radiation is only about 7×10^{-9} N.

The great technological importance of electromagnetic waves stems from their capability to carry energy from one place to another. The radio and TV signals from broadcasting stations carry energy. Light carries energy from the sun to the earth, thus making life possible on the earth.

Example 8.2 A plane electromagnetic wave of frequency 25 MHz travels in free space along the *x*-direction. At a particular point in space and time, $\mathbf{E} = 6.3\ \hat{\mathbf{j}}$ V/m. What is **B** at this point?

Solution Using Eq. (8.10), the magnitude of **B** is

$$B = \frac{E}{c}$$

$$= \frac{6.3\ \text{V/m}}{3 \times 10^8\ \text{m/s}} = 2.1 \times 10^{-8}\ \text{T}$$

To find the direction, we note that **E** is along *y*-direction and the wave propagates along *x*-axis. Therefore, **B** should be in a direction perpendicular to both *x*- and *y*-axes. Using vector algebra, **E × B** should be along *x*-direction. Since, $(+\hat{\mathbf{j}}) \times (+\hat{\mathbf{k}}) = \hat{\mathbf{i}}$, **B** is along the *z*-direction.

Thus, $\mathbf{B} = 2.1 \times 10^{-8}\ \hat{\mathbf{k}}$ T

EXAMPLE 8.2

Example 8.3 The magnetic field in a plane electromagnetic wave is given by $B_y = (2 \times 10^{-7})$ T sin $(0.5\times10^3 x + 1.5\times10^{11} t)$.

(a) What is the wavelength and frequency of the wave?

(b) Write an expression for the electric field.

Solution

(a) Comparing the given equation with

$$B_y = B_0 \sin\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right]$$

We get, $\lambda = \dfrac{2\pi}{0.5 \times 10^3}$ m = 1.26 cm,

and $\dfrac{1}{T} = \nu = \left(1.5 \times 10^{11}\right)/2\pi = 23.9$ GHz

(b) $E_0 = B_0 c = 2\times10^{-7}$ T × 3 × 10^8 m/s = 6 × 10^1 V/m

The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along the z-axis is obtained as

$E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11}\ t)$ V/m

EXAMPLE 8.3

Example 8.4 Light with an energy flux of 18 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm^2, find the average force exerted on the surface during a 30 minute time span.

Solution
The total energy falling on the surface is

$$U = (18 \text{ W/cm}^2) \times (20 \text{ cm}^2) \times (30 \times 60)$$
$$= 6.48 \times 10^5 \text{ J}$$

Therefore, the total momentum delivered (for complete absorption) is

$$p = \frac{U}{c} = \frac{6.48 \times 10^5 \text{ J}}{3 \times 10^8 \text{ m/s}} = 2.16 \times 10^{-3} \text{ kg m/s}$$

The average force exerted on the surface is

$$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{0.18 \times 10^4} = 1.2 \times 10^{-6} \text{N}$$

How will your result be modified if the surface is a perfect reflector?

EXAMPLE 8.4

Example 8.5 Calculate the electric and magnetic fields produced by the radiation coming from a 100 W bulb at a distance of 3 m. Assume that the efficiency of the bulb is 2.5% and it is a point source.

Solution The bulb, as a point source, radiates light in all directions uniformly. At a distance of 3 m, the surface area of the surrounding sphere is

$$A = 4\pi r^2 = 4\pi (3)^2 = 113 \text{m}^2$$

The intensity I at this distance is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100 \text{ W} \times 2.5 \%}{113 \text{ m}^2}$$
$$= 0.022 \text{ W/m}^2$$

Half of this intensity is provided by the electric field and half by the magnetic field.

$$\frac{1}{2} I = \frac{1}{2}\left(\varepsilon_0 E_{rms}^2 c\right)$$
$$= \frac{1}{2}\left(0.022 \text{ W/m}^2\right)$$

$$E_{rms} = \sqrt{\frac{0.022}{\left(8.85 \times 10^{-12}\right)\left(3 \times 10^8\right)}} \text{ V/m}$$
$$= 2.9 \text{ V/m}$$

The value of E found above is the root mean square value of the electric field. Since the electric field in a light beam is sinusoidal, the peak electric field, E_0 is

$$E_0 = \sqrt{2} E_{\text{rms}} = \sqrt{2} \times 2.9 \text{ V/m}$$
$$= 4.07 \text{ V/m}$$

Thus, you see that the electric field strength of the light that you use for reading is fairly large. Compare it with electric field strength of TV or FM waves, which is of the order of a few microvolts per metre.

EXAMPLE 8.5

EXAMPLE 8.5

Now, let us calculate the strength of the magnetic field. It is

$$B_{rms} = \frac{E_{rms}}{c} = \frac{2.9 \quad \text{V m}^{-1}}{3\times10^{8}\,\text{m s}^{-1}} = 9.6 \times 10^{-9}\ \text{T}$$

Again, since the field in the light beam is sinusoidal, the peak magnetic field is $B_0 = \sqrt{2}\ B_{rms} = 1.4 \times 10^{-8}$ T. Note that although the energy in the magnetic field is equal to the energy in the electric field, the magnetic field strength is evidently very weak.

8.4 Electromagnetic Spectrum

At the time Maxwell predicted the existence of electromagnetic waves, the only familiar electromagnetic waves were the visible light waves. The existence of ultraviolet and infrared waves was barely established. By the end of the nineteenth century, X-rays and gamma rays had also been discovered. We now know that, electromagnetic waves include visible light waves, X-rays, gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of em waves according to frequency is the electromagnetic spectrum (Fig. 8.5). *There is no sharp division between one kind of wave and the next.* The classification is based roughly on how the waves are produced and/or detected.

Electromagnetic spectrum
http://www.fnal.gov/pub/inquiring/more/light
http://imagine.gsfc.nasa.gov/docs/science/

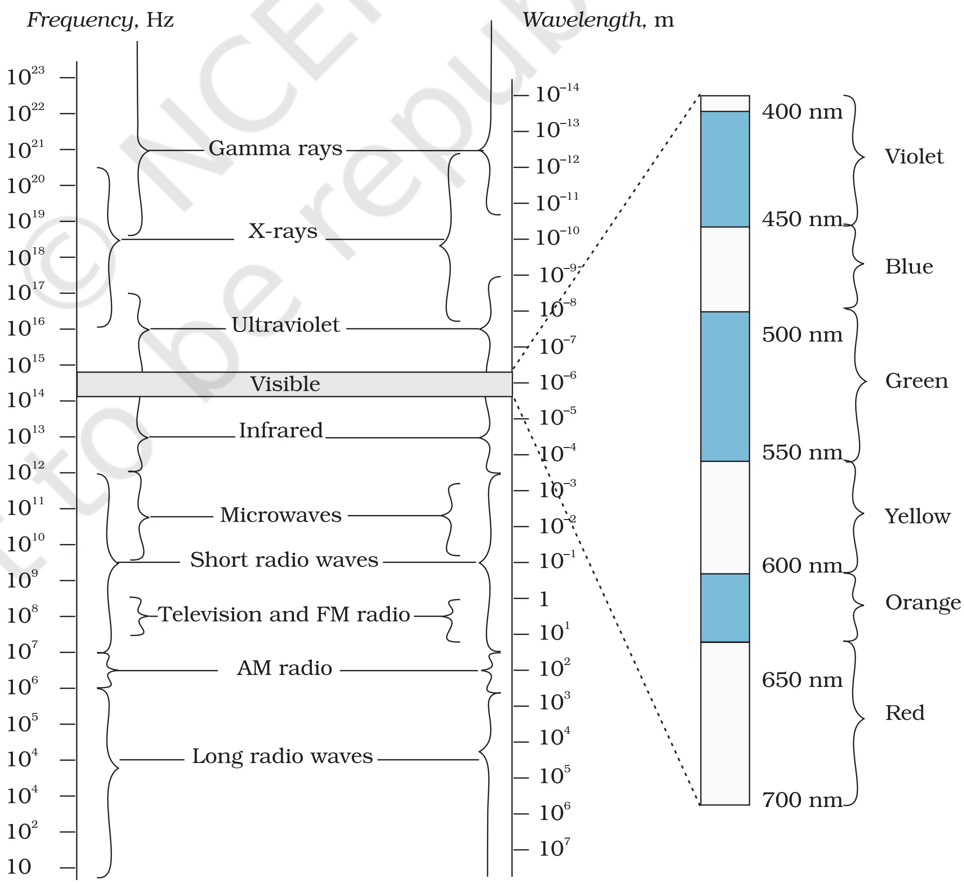

FIGURE 8.5 The electromagnetic spectrum, with common names for various part of it. The various regions do not have sharply defined boundaries.

We briefly describe these different types of electromagnetic waves, in order of decreasing wavelengths.

8.4.1 Radio waves

Radio waves are produced by the accelerated motion of charges in conducting wires. They are used in radio and television communication systems. They are generally in the frequency range from 500 kHz to about 1000 MHz. The AM (amplitude modulated) band is from 530 kHz to 1710 kHz. Higher frequencies upto 54 MHz are used for *short wave* bands. TV waves range from 54 MHz to 890 MHz. The FM (frequency modulated) radio band extends from 88 MHz to 108 MHz. Cellular phones use radio waves to transmit voice communication in the ultrahigh frequency (UHF) band. How these waves are transmitted and received is described in Chapter 15.

8.4.2 Microwaves

Microwaves (short-wavelength radio waves), with frequencies in the gigahertz (GHz) range, are produced by special vacuum tubes (called klystrons, magnetrons and Gunn diodes). Due to their short wavelengths, they are suitable for the radar systems used in aircraft navigation. Radar also provides the basis for the speed guns used to time fast balls, tennis-serves, and automobiles. Microwave ovens are an interesting domestic application of these waves. In such ovens, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water.

MICROWAVE OVEN

The spectrum of *electromagnetic radiation* contains a part known as *microwaves*. These waves have frequency and energy smaller than visible light and wavelength larger than it. What is the principle of a microwave oven and how does it work?

Our objective is to cook food or warm it up. All food items such as fruit, vegetables, meat, cereals, etc., contain water as a constituent. Now, what does it mean when we say that a certain object has become warmer? When the temperature of a body rises, the energy of the random motion of atoms and molecules increases and the molecules travel or vibrate or rotate with higher energies. The frequency of rotation of water molecules is about 2.45 gigahertz (GHz). If water receives microwaves of this frequency, its molecules absorb this radiation, which is equivalent to heating up water. These molecules share this energy with neighbouring food molecules, heating up the food.

One should use porcelain vessels and not metal containers in a microwave oven because of the danger of getting a shock from accumulated electric charges. Metals may also melt from heating. The porcelain container remains unaffected and cool, because its large molecules vibrate and rotate with much smaller frequencies, and thus cannot absorb microwaves. Hence, they do not get heated up.

Thus, the basic principle of a microwave oven is to generate microwave radiation of appropriate frequency in the working space of the oven where we keep food. This way energy is not wasted in heating up the vessel. In the conventional heating method, the vessel on the burner gets heated first, and then the food inside gets heated because of transfer of energy from the vessel. In the microwave oven, on the other hand, energy is directly delivered to water molecules which is shared by the entire food.

8.4.3 Infrared waves

Infrared waves are produced by hot bodies and molecules. This band lies adjacent to the low-frequency or long-wave length end of the visible spectrum. Infrared waves are sometimes referred to as *heat waves*. This is because water molecules present in most materials readily absorb infrared waves (many other molecules, for example, CO_2, NH_3, also absorb infrared waves). After absorption, their thermal motion increases, that is, they heat up and heat their surroundings. Infrared lamps are used in physical therapy. Infrared radiation also plays an important role in maintaining the earth's warmth or average temperature through the greenhouse effect. Incoming visible light (which passes relatively easily through the atmosphere) is absorbed by the earth's surface and re-radiated as infrared (longer wavelength) radiations. This radiation is trapped by greenhouse gases such as carbon dioxide and water vapour. Infrared detectors are used in Earth satellites, both for military purposes and to observe growth of crops. Electronic devices (for example semiconductor light emitting diodes) also emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders and hi-fi systems.

8.4.4 Visible rays

It is the most familiar form of electromagnetic waves. It is the part of the spectrum that is detected by the human eye. It runs from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range of about 700 – 400 nm. Visible light emitted or reflected from objects around us provides us information about the world. Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths. For example, snakes can detect infrared waves, and the 'visible' range of many insects extends well into the utraviolet.

8.4.5 Ultraviolet rays

It covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10}m (0.6 nm). Ultraviolet (UV) radiation is produced by special lamps and very hot bodies. The sun is an important source of ultraviolet light. But fortunately, most of it is absorbed in the ozone layer in the atmosphere at an altitude of about 40 – 50 km. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV radiation is absorbed by ordinary glass. Hence, one cannot get tanned or sunburn through glass windows.

Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs. Due to its shorter wavelengths, UV radiations can be focussed into very narrow beams for high precision applications such as LASIK (*Laser-assisted in situ keratomileusis*) eye surgery. UV lamps are used to kill germs in water purifiers.

Ozone layer in the atmosphere plays a protective role, and hence its depletion by chlorofluorocarbons (CFCs) gas (such as freon) is a matter of international concern.

8.4.6 X-rays

Beyond the UV region of the electromagnetic spectrum lies the X-ray region. We are familiar with X-rays because of its medical applications. It covers wavelengths from about 10^{-8} m (10 nm) down to 10^{-13} m (10^{-4} nm). One common way to generate X-rays is to bombard a metal target by high energy electrons. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because X-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary or over exposure.

8.4.7 Gamma rays

They lie in the upper frequency range of the electromagnetic spectrum and have wavelengths of from about 10^{-10}m to less than 10^{-14}m. This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nuclei. They are used in medicine to destroy cancer cells.

Table 8.1 summarises different types of electromagnetic waves, their production and detections. As mentioned earlier, the demarcation between different regions is not sharp and there are overlaps.

TABLE 8.1 DIFFERENT TYPES OF ELECTROMAGNETIC WAVES

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	$<10^{-3}$ nm	Radioactive decay of the nucleus	-do-

SUMMARY

1. Maxwell found an inconsistency in the Ampere's law and suggested the existence of an additional current, called displacement current, to remove this inconsistency. This displacement current is due to time-varying electric field and is given by

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

and acts as a source of magnetic field in exactly the same way as conduction current.

2. An accelerating charge produces electromagnetic waves. An electric charge oscillating harmonically with frequency ν, produces electromagnetic waves of the same frequency ν. An electric dipole is a basic source of electromagnetic waves.

3. Electromagnetic waves with wavelength of the order of a few metres were first produced and detected in the laboratory by Hertz in 1887. He thus verified a basic prediction of Maxwell's equations.

4. Electric and magnetic fields oscillate sinusoidally in space and time in an electromagnetic wave. The oscillating electric and magnetic fields, **E** and **B** are perpendicular to each other, and to the direction of propagation of the electromagnetic wave. For a wave of frequency ν, wavelength λ, propagating along *z*-direction, we have

$$E = E_x(t) = E_0 \sin (kz - \omega t)$$

$$= E_0 \sin \left[2\pi\left(\frac{z}{\lambda} - \nu t\right)\right] = E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

$$B = B_y(t) = B_0 \sin (kz - \omega t)$$

$$= B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \nu t\right)\right] = B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

They are related by $E_0/B_0 = c$.

5. The speed *c* of electromagnetic wave in vacuum is related to μ_0 and ε_0 (the free space permeability and permittivity constants) as follows: $c = 1/\sqrt{\mu_0 \varepsilon_0}$. The value of *c* equals the speed of light obtained from optical measurements.

   Light is an electromagnetic wave; c is, therefore, also the speed of light. Electromagnetic waves other than light also have the same velocity c in free space.

   The speed of light, or of electromagnetic waves in a material medium is given by $v = 1/\sqrt{\mu \varepsilon}$

   where μ is the permeability of the medium and ε its permittivity.

6. Electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields.

   Electromagnetic waves transport momentum as well. When these waves strike a surface, a pressure is exerted on the surface. If total energy transferred to a surface in time *t* is *U*, total momentum delivered to this surface is $p = U/c$.

7. The spectrum of electromagnetic waves stretches, in principle, over an infinite range of wavelengths. Different regions are known by different

names; γ-rays, X-rays, ultraviolet rays, visible rays, infrared rays, microwaves and radio waves in order of increasing wavelength from 10^{-2} Å or 10^{-12} m to 10^{6} m.

They interact with matter via their electric and magnetic fields which set in oscillation charges present in all matter. The detailed interaction and so the mechanism of absorption, scattering, etc., depend on the wavelength of the electromagnetic wave, and the nature of the atoms and molecules in the medium.

POINTS TO PONDER

1. The basic difference between various types of electromagnetic waves lies in their wavelengths or frequencies since all of them travel through vacuum with the same speed. Consequently, the waves differ considerably in their mode of interaction with matter.
2. Accelerated charged particles radiate electromagnetic waves. The wavelength of the electromagnetic wave is often correlated with the characteristic size of the system that radiates. Thus, gamma radiation, having wavelength of 10^{-14} m to 10^{-15} m, typically originate from an atomic nucleus. X-rays are emitted from heavy atoms. Radio waves are produced by accelerating electrons in a circuit. A transmitting antenna can most efficiently radiate waves having a wavelength of about the same size as the antenna. Visible radiation emitted by atoms is, however, much longer in wavelength than atomic size.
3. The oscillating fields of an electromagnetic wave can accelerate charges and can produce oscillating currents. Therefore, an apparatus designed to detect electromagnetic waves is based on this fact. Hertz original 'receiver' worked in exactly this way. The same basic principle is utilised in practically all modern receiving devices. High frequency electromagnetic waves are detected by other means based on the physical effects they produce on interacting with matter.
4. Infrared waves, with frequencies lower than those of visible light, vibrate not only the electrons, but entire atoms or molecules of a substance. This vibration increases the internal energy and consequently, the temperature of the substance. This is why infrared waves are often called *heat waves*.
5. The centre of sensitivity of our eyes coincides with the centre of the wavelength distribution of the sun. It is because humans have evolved with visions most sensitive to the strongest wavelengths from the sun.

EXERCISES

8.1 Figure 8.6 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.

(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

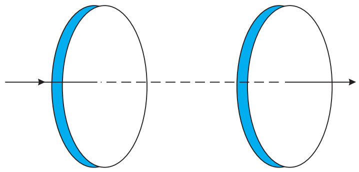

FIGURE 8.6

8.2 A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of **B** at a point 3.0 cm from the axis between the plates.

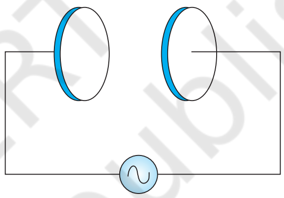

FIGURE 8.7

8.3 What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å and radiowaves of wavelength 500m?

8.4 A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

8.5 A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

8.6 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

8.7 The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0 = 510 nT. What is the amplitude of the electric field part of the wave?

8.8 Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is ν = 50.0 MHz. (a) Determine, B_0,ω, k, and λ. (b) Find expressions for **E** and **B**.

8.9 The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

8.10 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.

(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the **E** field equals the average energy density of the **B** field. [$c = 3 \times 10^8$ m s^{-1}.]

ADDITIONAL EXERCISES

8.11 Suppose that the electric field part of an electromagnetic wave in vacuum is $\mathbf{E} = \{(3.1\ \text{N/C}) \cos [(1.8\ \text{rad/m})\ y + (5.4 \times 10^6\ \text{rad/s})t]\}\hat{\mathbf{i}}$.

(a) What is the direction of propagation?
(b) What is the wavelength λ ?
(c) What is the frequency v ?
(d) What is the amplitude of the magnetic field part of the wave?
(e) Write an expression for the magnetic field part of the wave.

8.12 About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(a) at a distance of 1m from the bulb?
(b) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

8.13 Use the formula $\lambda_m T = 0.29$ cmK to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

8.14 Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.

(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).
(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift).
(c) 2.7 K [temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe].
(d) 5890 Å - 5896 Å [double lines of sodium]
(e) 14.4 keV [energy of a particular transition in ^{57}Fe nucleus associated with a famous high resolution spectroscopic method (Mössbauer spectroscopy)].

8.15 Answer the following questions:

(a) Long distance radio broadcasts use short-wave bands. Why?
(b) It is necessary to use satellites for long distance TV transmission. Why?
(c) Optical and radiotelescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?
(d) The small ozone layer on top of the stratosphere is crucial for human survival. Why?
(e) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?
(f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

ANSWERS

CHAPTER 1

1.1 6×10^{-3} N (repulsive)

1.2 (a) 12 cm
(b) 0.2 N (attractive)

1.3 2.4×10^{39}. This is the ratio of electric force to the gravitational force (at the same distance) between an electron and a proton.

1.5 Charge is not created or destroyed. It is merely transferred from one body to another.

1.6 Zero N

1.8 (a) 5.4×10^{6} N C^{-1} along OB
(b) 8.1×10^{-3} N along OA

1.9 Total charge is zero. Dipole moment $= 7.5 \times 10^{-8}$ C m along *z*-axis.

1.10 10^{-4} N m

1.11 (a) 2×10^{12}, from wool to polythene.
(b) Yes, but of a negligible amount ($= 2 \times 10^{-18}$ kg in the example).

1.12 (a) 1.5×10^{-2} N
(b) 0.24 N

1.13 5.7×10^{-3} N

1.14 Charges 1 and 2 are negative, charge 3 is positive. Particle 3 has the highest charge to mass ratio.

1.15 (a) 30Nm^2/C, (b) 15 Nm^2/C

1.16 Zero. The number of lines entering the cube is the same as the number of lines leaving the cube.

1.17 (a) 0.07 μC
(b) No, only that the net charge inside is zero.

1.18 2.2×10^{5} N m^2/C

1.19 1.9×10^{5} N m^2/C

1.20 (a) -10^{3} N m^2/C; because the charge enclosed is the same in the two cases.
(b) –8.8 nC

1.21 –6.67 nC

1.22 (a) 1.45×10^{-3} C
(b) 1.6×10^{8} Nm^2/C

1.23 10 μC/m

1.24 (a) Zero, (b) Zero, (c) 1.9 N/C

1.25 9.81×10^{-4} mm.

1.26 Only (c) is right; the rest cannot represent electrostatic field lines, (a) is wrong because field lines must be normal to a conductor, (b) is wrong because field lines cannot start from a negative charge, (d) is wrong because field lines cannot intersect each other, (e) is wrong because electrostatic field lines cannot form closed loops.

1.27 The force is 10^{-2} N in the negative z-direction, that is, in the direction of decreasing electric field. You can check that this is also the direction of decreasing potential energy of the dipole; torque is zero.

1.28 (a) *Hint:* Choose a Gaussian surface lying wholly within the conductor and enclosing the cavity.

(b) Gauss's law on the same surface as in (a) shows that q must induce $-q$ on the inner surface of the conductor.

(c) Enclose the instrument fully by a metallic surface.

1.29 *Hint:* Consider the conductor with the hole filled up. Then the field just outside is $(\sigma/\varepsilon_0)\ \hat{\mathbf{n}}$ and is zero inside. View this field as a superposition of the field due to the filled up hole plus the field due to the rest of the charged conductor. Inside the conductor, these fields are equal and opposite. Outside they are equal both in magnitude and direction. Hence, the field due to the rest of the conductor is $\left(\dfrac{\sigma}{2\,\varepsilon_0}\right)\hat{\mathbf{n}}$.

1.31 p;uud; n;udd.

1.32 (a) *Hint:* Prove it by contradiction. Suppose the equilibrium is stable; then the test charge displaced slightly in any direction will experience a restoring force towards the null-point. That is, all field lines near the null point should be directed inwards towards the null-point. That is, there is a net inward flux of electric field through a closed surface around the null-point. But by Gauss's law, the flux of electric field through a surface, not enclosing any charge, must be zero. Hence, the equilibrium cannot be stable.

(b) The mid-point of the line joining the two charges is a null-point. Displace a test charge from the null-point slightly along the line. There is a restoring force. But displace it, say, normal to the line. You will see that the net force takes it away from the null-point. Remember, stability of equilibrium needs restoring force in all directions.

1.34 1.6 cm

CHAPTER 2

2.1 10 cm, 40 cm away from the positive charge on the side of the negative charge.

2.2 2.7×10^{6} V

2.3 (a) The plane normal to AB and passing through its mid-point has zero potential everywhere.

(b) Normal to the plane in the direction AB.

2.4 (a) Zero

(b) 10^5 N C^{-1}

(c) 4.4×10^4 N C^{-1}

2.5 96 pF

2.6 (a) 3 pF

(b) 40 V

2.7 (a) 9 pF

(b) 2×10^{-10} C, 3×10^{-10} C, 4×10^{-10} C

2.8 18 pF, 1.8×10^{-9} C

2.9 (a) $V = 100$ V, $C = 108$ pF, $Q = 1.08 \times 10^{-8}$ C

(b) $Q = 1.8 \times 10^{-9}$ C, $C = 108$ pF, $V = 16.6$ V

2.10 1.5×10^{-8} J

2.11 6×10^{-6} J

2.12 1.2 J; the point R is irrelevant to the answer.

2.13 Potential = $4q/(\sqrt{3}\,\pi\,\varepsilon_0\,b)$; field is zero, as expected by symmetry.

2.14 (a) 2.4×10^5 V; 4.0×10^5 Vm^{-1} from charge 2.5 μC to 1.5 μC.

(b) 2.0×10^5 V; 6.6×10^5 Vm^{-1} in the direction that makes an angle of about 69° to the line joining charge 2.5 μC to 1.5 μC.

2.15 (a) $-q/(4\pi r_1^2)$, $(Q+q)/(4\pi r_2^2)$

(b) By Gauss's law, the net charge on the inner surface enclosing the cavity (not having any charge) must be zero. For a cavity of arbitrary shape, this is not enough to claim that the electric field inside must be zero. The cavity may have positive and negative charges with total charge zero. To dispose of this possibility, take a closed loop, part of which is inside the cavity along a field line and the rest inside the conductor. Since field inside the conductor is zero, this gives a net work done by the field in carrying a test charge over a closed loop. We know this is impossible for an electrostatic field. Hence, there are no field lines inside the cavity (i.e., no field), and no charge on the inner surface of the conductor, whatever be its shape.

2.17 $\lambda/(2\,\pi\,\varepsilon_0\,r)$, where r is the distance of the point from the common axis of the cylinders. The field is radial, perpendicular to the axis.

2.18 (a) –27.2 eV

(b) 13.6 eV

(c) –13.6 eV, 13.6 eV. Note in the latter choice the total energy of the hydrogen atom is zero.

2.19 –19.2 eV; the zero of potential energy is taken to be at infinity.

2.20 The ratio of electric field of the first to the second is (b/a). A flat portion may be equated to a spherical surface of large radius, and a pointed portion to one of small radius.

2.21 (a) On the axis of the dipole, potential is $(\pm\, 1/4\,\pi\,\varepsilon_0)\ \ p/(x^2 - a^2)$ where $p = 2qa$ is the magnitude of the dipole moment; the + sign when the point is closer to q and the – sign when it is closer to $-q$. Normal to the axis, at points $(x, y, 0)$, potential is zero.

(b) The dependence on r is $1/r^2$ type.

(c) Zero. No, because work done by electrostatic field between two points is independent of the path connecting the two points.

2.22 For large r, quadrupole potential goes like $1/r^3$, dipole potential goes like $1/r^2$, monopole potential goes like 1/r.

2.23 Eighteen 1 μF capacitors arranged in 6 parallel rows, each row consisting of 3 capacitors in series.

2.24 1130 km^2

2.25 Equivalent capacitance = (200/3) pF.
$Q_1 = 10^{-8}$ C, V_1 = 100 V ; $Q_2 = Q_3 = 10^{-8}$ C
$V_2 = V_3$ = 50 V
$Q_4 = 2.55 \times 10^{-8}$ C, V_4 = 200 V

2.26 (a) 2.55×10^{-6} J
(b) u = 0.113 J m^{-3}, $u = (½)\ \varepsilon_0 E^2$

2.27 2.67×10^{-2} J

2.28 *Hint:* Suppose we increase the separation of the plates by Δx. Work done (by external agency) = $F\,\Delta x$. This goes to increase the potential energy of the capacitor by $u\,a\,\Delta x$ where u is energy density. Therefore, $F = u\,a$ which is easily seen to be (1/2) QE, using $u = (1/2)\ \varepsilon_0 E^2$. The physical origin of the factor 1/2 in the force formula lies in the fact that just outside the conductor, field is E, and inside it is zero. So, the average value $E/2$ contributes to the force.

2.30 (a) 5.5×10^{-9} F
(b) 4.5×10^2 V
(c) 1.3×10^{-11} F

2.31 (a) No, because charge distributions on the spheres will not be uniform.
(b) No.
(c) Not necessarily. (True only if the field line is a straight line.) The field line gives the direction of acceleration, not that of velocity, in general.
(d) Zero, no matter what the shape of the complete orbit.
(e) No, potential is continuous.
(f) A single conductor is a capacitor with one of the 'plates' at infinity.
(g) A water molecule has permanent dipole moment. However, detailed explanation of the value of dielectric constant requires microscopic theory and is beyond the scope of the book.

2.32 1.2×10^{-10} F, 2.9×10^4 V

2.33 19 cm^2

2.34 (a) Planes parallel to x-y plane.
(b) Same as in (a), except that planes differing by a fixed potential get closer as field increases.
(c) Concentric spheres centred at the origin.
(d) A periodically varying shape near the grid which gradually reaches the shape of planes parallel to the grid at far distances.

2.35 *Hint:* By Gauss's law, field between the sphere and the shell is determined by q_1 alone. Hence, potential difference between the sphere and the shell is independent of q_2. If q_1 is positive, this potential difference is always positive.

2.36 (a) Our body and the ground form an equipotential surface. As we step out into the open, the original equipotential surfaces of open air change, keeping our head and the ground at the same potential.

(b) Yes. The steady discharging current in the atmosphere charges up the aluminium sheet gradually and raises its voltage to an extent depending on the capacitance of the capacitor (formed by the sheet, slab and the ground).

(c) The atmosphere is continually being charged by thunderstorms and lightning all over the globe and discharged through regions of ordinary weather. The two opposing currents are, on an average, in equilibrium.

(d) Light energy involved in lightning; heat and sound energy in the accompanying thunder.

CHAPTER 3

3.1 30 A

3.2 17 Ω, 8.5 V

3.3 (a) 6 Ω
(b) 2 V, 4 V, 6 V

3.4 (a) (20/19) Ω
(b) 10A, 5 A, 4A; 19A

3.5 1027 °C

3.6 2.0×10^{-7} Ωm

3.7 0.0039 $°C^{-1}$

3.8 867 °C

3.9 Current in branch AB = (4/17) A,
in BC = (6/17) A, in CD = (–4/17) A,
in AD = (6/17) A, in BD. = (–2/17) A, total current = (10/17) A.

3.10 (a) X = 8.2 Ω; to minimise resistance of the connection which are not accounted for in the bridge formula.
(b) 60.5 cm from A.
(c) The galvanometer will show no current.

3.11 11.5 V; the series resistor limits the current drawn from the external source. In its absence, the current will be dangerously high.

3.12 2.25 V

3.13 2.7×10^{4} s (7.5 h)

3.14 Take the radius of the earth = 6.37×10^{6} m and obtain total charge of the globe. Divide it by current to obtain time = 283 s. Still this method gives you only an estimate; it is not strictly correct. Why?

3.15 (a) 1.4 A, 11.9 V
(b) 0.005 A; impossible because a starter motor requires large current (~ 100 A) for a few seconds.

3.16 The mass (or weight) ratio of copper to aluminium wire is (1.72/2.63) × (8.9/2.7) ≅ 2.2. Since aluminium is lighter, it is preferred for long suspensions of cables.

3.17 Ohm's law is valid to a high accuracy; the resistivity of the alloy manganin is nearly independent of temperature.

3.18 (a) Only current (because it is given to be steady!). The rest depends on the area of cross-section inversely.

(b) No, examples of non-ohmic elements: vacuum diode, semiconductor diode.

(c) Because the maximum current drawn from a source = ε/r.

(d) Because, if the circuit is shorted (accidentally), the current drawn will exceed safety limits, if internal resistance is not large.

3.19 (a) greater, (b) lower, (c) nearly independent of, (d) 10^{22}.

3.20 (a) (i) in series, (ii) all in parallel; n^2.

(b) (i) Join 1 Ω, 2 Ω in parallel and the combination in series with 3Ω, (ii) parallel combination of 2 Ω and 3 Ω in series with 1 Ω, (iii) all in series, (iv) all in parallel.

(c) (i) (16/3) Ω, (ii) 5 R.

3.21 *Hint:* Let X be the equivalent resistance of the infinite network. Clearly, $2 + X/(X+1) = X$ which gives $X = (1 + \sqrt{3})$ Ω; therefore the current is 3.7 A.

3.22 (a) ε = 1.25 V.

(b) To reduce current through the galvanometer when the movable contact is far from the balance point.

(c) No.

(d) No. If ε is greater than the emf of the driver cell of the potentiometer, there will be no balance point on the wire AB.

(e) The circuit, as it is, would be unsuitable, because the balance point (for ε of the order of a few mV) will be very close to the end A and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor R in series with the wire AB so that potential drop across AB is only slightly greater than the emf to be measured. Then, the balance point will be at larger length of the wire and the percentage error will be much smaller.

3.23 1.7 Ω

Chapter 4

4.1 $\pi \times 10^{-4}$ T $\simeq 3.1 \times 10^{-4}$ T

4.2 3.5×10^{-5} T

4.3 4×10^{-6} T, vertical up

4.4 1.2×10^{-5} T, towards south

4.5 0.6 N m^{-1}

4.6 8.1×10^{-2} N; direction of force given by Fleming's left-hand rule

4.7 2×10^{-5} N; attractive force normal to A towards B

4.8 $8\pi \times 10^{-3}$ T $\simeq 2.5 \times 10^{-2}$ T

4.9 0.96 N m

4.10 (a) 1.4, (b) 1

4.11 4.2 cm

4.12 18 MHz

4.13 (a) 3.1 Nm, (b) No, the answer is unchanged because the formula $\boldsymbol{\tau} = N\,I\,\mathbf{A} \times \mathbf{B}$ is true for a planar loop of any shape.

4.14 $5\pi \times 10^{-4}$ T = 1.6×10^{-3} T towards west.

4.15 Length about 50 cm, radius about 4 cm, number of turns about 400, current about 10 A. These particulars are not unique. Some adjustment with limits is possible.

4.16 (b) In a small region of length $2d$ about the mid-point between the coils,

$$B = \frac{\mu_0 I R^2 N}{2} \times \left[\left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-3/2} + \left\{\left(\frac{R}{2}-d\right)^2 + R^2\right\}^{-3/2}\right]$$

$$\simeq \frac{\mu_0 I R^2 N}{2} \times \left(\frac{5R^2}{4}\right)^{-3/2} \times \left[\left(1+\frac{4d}{5R}\right)^{-3/2} + \left(1-\frac{4d}{5R}\right)^{-3/2}\right]$$

$$\simeq \frac{\mu_0 I R^2 N}{2R^3} \times \left(\frac{4}{5}\right)^{3/2} \times \left[1-\frac{6d}{5R}+1+\frac{6d}{5R}\right]$$

where in the second and third steps above, terms containing d^2/R^2 and higher powers of d/R are neglected since $\frac{d}{R} << 1$. The terms linear in d/R cancel giving a uniform field B in a small region:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 IN}{R} \simeq 0.72 \frac{\mu_0 IN}{R}$$

4.17 *Hint:* B for a toroid is given by the same formula as for a solenoid: $B = \mu_0 nI$, where n in this case is given by $n = \frac{N}{2\pi r}$. The field is non-zero only inside the core surrounded by the windings. (a) Zero, (b) 3.0×10^{-2} T, (c) zero. Note, the field varies slightly across the cross-section of the toroid as r varies from the inner to outer radius. Answer (b) corresponds to the mean radius r = 25.5 cm.

4.18 (a) Initial **v** is either parallel or anti-parallel to **B**.

(b) Yes, because magnetic force can change the direction of **v**, not its magnitude.

(c) **B** should be in a vertically downward direction.

4.19 (a) Circular trajectory of radius 1.0 mm normal to **B**.

(b) Helical trajectory of radius 0.5 mm with velocity component 2.3×10^7 m s^{-1} along **B**.

4.20 Deuterium ions or deuterons; the answer is not unique because only the ratio of charge to mass is determined. Other possible answers are He^{++}, Li^{+++}, etc.

4.21 (a) A horizontal magnetic field of magnitude 0.26 T normal to the conductor in such a direction that Fleming's left-hand rule gives a magnetic force upward.

(b) 1.176 N.

4.22 1.2 N m^{-1}; repulsive. Note, obtaining total force on the wire as 1.2 × 0.7 = 0.84 N, is only approximately correct because the formula $F = \frac{\mu_0}{2\pi r} I_1 I_2$ for force per unit length is strictly valid for infinitely long conductors.

4.23 (a) 2.1 N vertically downwards

(b) 2.1 N vertically downwards (true for any angle between current and direction and **B** since $l \sin\theta$ remains fixed, equal to 20 cm)

(c) 1.68 N vertically downwards

4.24 Use $\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$ and $\mathbf{F} = I\,\mathbf{l} \times \mathbf{B}$

(a) 1.8×10^{-2} N m along $-y$ direction

(b) same as in (a)

(c) 1.8×10^{-2} N m along $-x$ direction

(d) 1.8×10^{-2} N m at an angle of 240° with the $+x$ direction

(e) zero

(f) zero

Force is zero in each case. Case (e) corresponds to stable, and case (f) corresponds to unstable equilibrium.

4.25 (a) Zero, (b) zero, (c) force on each electron is $evB = IB/(nA) = 5 \times 10^{-25}$ N. Note: Answer (c) denotes only the magnetic force.

4.26 108 A

4.27 Resistance in series = 5988 Ω

4.28 Shunt resistance = 10 mΩ

Chapter 5

5.1 (a) Magnetic declination, angle of dip, horizontal component of earth's magnetic field.

(b) Greater in Britain (it is about 70°), because Britain is closer to the magnetic north pole.

(c) Field lines of **B** due to the earth's magnetism would seem to come out of the ground.

(d) A compass is free to move in a horizontal plane, while the earth's field is exactly vertical at the magnetic poles. So the compass can point in any direction there.

(e) Use the formula for field **B** on the normal bisector of a dipole of magnetic moment **m**,

$$\mathbf{B}_A = -\frac{\mu_0}{4\pi}\frac{\mathbf{m}}{r^3}$$

Take $m = 8 \times 10^{22}$ J T^{-1}, $r = 6.4 \times 10^6$ m; one gets $B = 0.3$ G, which checks with the order of magnitude of the observed field on the earth.

(f) Why not? The earth's field is only approximately a dipole field. Local N-S poles may arise due to, for instance, magnetised mineral deposits.

5.2 (a) Yes, it does change with time. Time scale for appreciable change is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

(b) Because molten iron (which is the phase of the iron at the high temperatures of the core) is not ferromagnetic.

(c) One possibility is the radioactivity in the interior of the earth. But nobody really knows. You should consult a good modern text on geomagnetism for a proper view of the question.

(d) Earth's magnetic field gets weakly 'recorded' in certain rocks during solidification. Analysis of this rock magnetism offers clues to geomagnetic history.

(e) At large distances, the field gets modified due to the field of ions in motion (in the earth's ionosphere). The latter is sensitive to extra-terrestrial disturbances such as, the solar wind.

(f) From the relation $R = \frac{mv}{eB}$, an extremely minute field bends charged particles in a circle of very large radius. Over a small distance, the deflection due to the circular orbit of such large R may not be noticeable, but over the gigantic interstellar distances, the deflection can significantly affect the passage of charged particles, for example, cosmic rays.

5.3 0.36 JT^{-1}

5.4 (a) **m** parallel to **B**; $U = -mB = -4.8 \times 10^{-2}$ J: stable.

(b) **m** anti-parallel to **B**; $U = +mB = +4.8 \times 10^{-2}$ J; unstable.

5.5 0.60 JT^{-1} along the axis of the solenoid determined by the sense of flow of the current.

5.6 7.5×10^{-2} J

5.7 (a) (i) 0.33 J (ii) 0.66 J

(b) (i) Torque of magnitude 0.33 J in a direction that tends to align the magnitude moment vector along **B**. (ii) Zero.

5.8 (a) 1.28 A m^2 along the axis in the direction related to the sense of current via the right-handed screw rule.

(b) Force is zero in uniform field; torque = 0.048 Nm in a direction that tends to align the axis of the solenoid (i.e., its magnitude moment vector) along **B**.

5.9 Use $\mathcal{I} = mB/(4\pi^2 \nu^2)$; $m = NIA$ to get $\mathcal{I} = 1.2 \times 10^{-4}\ \text{kg m}^2$.

5.10 B = 0.35 sec 22° $\simeq$ 0.38 G.

5.11 The earth's lies in the vertical plane 12° west of the geographic meridian making an angle of 60° (upwards) with the horizontal (magnetic south to magnetic north) direction. Magnitude = 0.32 G.

5.12 (a) 0.96 g along S-N direction.

(b) 0.48 G along N-S direction.

5.13 0.54 G in the direction of earth's field.

5.14 At $14 \times 2^{-1/3} = 11.1$ cm on the normal bisector.

5.15 (a) $(\mu_0 m)/(4\pi r^3) = 0.42 \times 10^{-4}$ which gives r = 5.0 cm.

(b) $(2\mu_0 m)/(4\pi r_1^3) = 0.42 \times 10^{-4}$ i.e., $r_1 = 2^{1/3}\ r$ = 6.3 cm.

5.16 (a) The tendency to disrupt the alignment of dipoles (with the magnetising field) arising from random thermal motion is reduced at lower temperatures.

(b) The induced dipole moment in a diamagnetic sample is always opposite to the magnetising field, no matter what the internal motion of the atoms is.

(c) Slightly less, since bismuth is diamagnetic.

(d) *No*, as it evident from the magnetisation curve. From the slope of magnetisation curve, it is clear that *m* is greater for lower fields.

(e) Proof of this important fact (of much practical use) is based on boundary conditions of magnetic fields (**B** and **H**) at the interface of two media. (When one of the media has $\mu >> 1$, the field lines meet this medium nearly normally.) Details are beyond the scope of this book.

(f) *Yes*. Apart from minor differences in strength of the individual atomic dipoles of two different materials, a paramagnetic sample with saturated magnetisation will have the same order of magnetisation. But of course, saturation requires impractically high magnetising fields.

5.17 (b) Carbon steel piece, because heat lost per cycle is proportional to the area of hysteresis loop.

(c) Magnetisation of a ferromagnet is not a single-valued function of the magnetising field. Its value for a particular field depends both on the field and also on history of magnetisation (i.e., how many cycles of magnetisation it has gone through, etc.). In other words, the value of magnetisation is a record or *memory* of its cycles of magnetisation. If information bits can be made to correspond to these cycles, the system displaying such a hysteresis loop can act as a device for storing information.

(d) Ceramics (specially treated barium iron oxides) also called ferrites.

(e) Surround the region by soft iron rings. Magnetic field lines will be drawn into the rings, and the enclosed space will be free of magnetic field. But this shielding is only approximate, unlike the perfect electric shielding of a cavity in a conductor placed in an external electric field.

5.18 Parallel to and above the cable at a distance at 1.5 cm.

5.19 Below the cable:

$R_h = 0.39 \cos 35° - 0.2$
$= 0.12$ G

$R_v = 0.36 \sin 35° = 0.22$ G

$$R = \sqrt{R_h^{\ 2} + R_v^{\ 2}} = 0.25 \text{ G}$$

$$\theta = \tan^{-1}\frac{R_v}{R_h} = 62°$$

Above the cable:

$R_h = 0.39 \cos 35° + 0.2$
$= 0.52$ G

$R_v = 0.224$ G

R = 0.57 G, $\theta \simeq 23°$

5.20 (a) $B_h = (\mu_0 IN/2r)\cos 45^o = 0.39\,\text{G}$

(b) East to west (i.e., the needle will reverse its original direction).

5.21 Magnitude of the other field

$$= \frac{1.2\times10^{-2}\times\sin 15^o}{\sin 45^o}$$

$$= 4.4\times10^{-3}\ \text{T}$$

5.22 $R = \dfrac{meV}{eB}$

$$= \frac{\sqrt{2m_e \times \text{kinetic energy}}}{eB}$$

= 11.3 m

Up or down deflection = R (1-cosθ) where sinθ = 0.3/11.3. We get deflection $\simeq$ 4 mm.

5.23 Initially, total dipole moment

= 0.15 × 1.5 × 10^{-23} × 2.0 ×10^{24}

= 4.5 J T^{-1}

Use Curie's Law $m \propto B/T$ to get the final dipole moment

= 4.5 × (0.98/0.84) × (4.2/2.8)

= 7.9 J T^{-1}

5.24 Use the formula $B = \dfrac{\mu_r \mu_o NI}{2\pi R}$ where μ_r (relative permeability) to get B = 4.48 T.

5.25 Of the two, the relation $\boldsymbol{\mu}_l = -(e/2m)\,\boldsymbol{l}$ is in accordance with classical physics. It follows easily from the definitions of $\boldsymbol{\mu}_l$ and $\boldsymbol{l}$:

$\mu_l = IA = (e/T)\pi r^2$

$l = mvr = m\dfrac{2\pi r^2}{T}$

where r is the radius of the circular orbit which the electron of mass m and charge $(-e)$ completes in time T. Clearly, $\mu_r / l = e/2m$.

Since charge of the electron is negative $(= -e)$, it is easily seen that $\boldsymbol{\mu}$ and $\boldsymbol{l}$ are antiparallel, both normal to the plane of the orbit. Therefore, $\boldsymbol{\mu}_l = -e/2m\,\boldsymbol{l}$. Note μ_s/S in contrast to μ_l/l is e/m, i.e., twice the classically expected value. This latter result (verified experimentally) is an outstanding consequence of modern quantum theory and cannot be obtained classically.

Chapter 6

6.1 (a) Along qrpq

(b) Along prq, along yzx

(c) Along yzx

(d) Along zyx

(e) Along xry

(f) No induced current since field lines lie in the plane of the loop.

6.2 (a) Along adcd (flux through the surface increases during shape change, so induced current produces opposing flux).

(b) Along a′d′c′b′ (flux decreases during the process)

6.3 7.5×10^{-6} V

6.4 (1) 2.4×10^{-4} V, lasting 2 s

(2) 0.6×10^{-4} V, lasting 8 s

6.5 100 V

6.6 Flux through each turn of the loop = $\pi r^2 B \cos(\omega t)$

$\varepsilon = -N \omega \pi r^2 B \sin(\omega t)$

$\varepsilon_{max} = -N \omega \pi r^2 B$

$= 20 \times 50 \times \pi \times 64 \times 10^{-4} \times 3.0 \times 10^{-2} = 0.603$ V

ε_{avg} is zero over a cycle

$I_{max} = 0.0603$ A

$$P_{average} = \frac{1}{2} \varepsilon_{max} I_{max} = 0.018 \, W$$

The induced current causes a torque opposing the rotation of the coil. An external agent (rotor) must supply torque (and do work) to counter this torque in order to keep the coil rotating uniformly. Thus, the source of the power dissipated as heat in the coil is the external rotor.

6.7 (a) 1.5×10^{-3} V, (b) West to East, (c) Eastern end.

6.8 4H

6.9 30 Wb

6.10 Vertical component of **B**

$= 5.0 \times 10^{-4} \sin 30°$

$= 2.5 \times 10^{-4}$ T

$\varepsilon = Blv$

$\varepsilon = 2.5 \times 10^{-4} \times 25 \times 500$

$= 3.125$ V

The emf induced is 3.1 V (using significant figures).

The direction of the wing is immaterial (as long as it is horizontal) for this answer.

6.11 Induced emf = $8 \times 2 \times 10^{-4} \times 0.02 = 3.2 \times 10^{-5}$ V

Induced current = 2×10^{-5} A

Power loss = 6.4×10^{-10} W

Source of this power is the external agent responsible for changing the magnetic field with time.

6.12 Rate of change of flux due to explicit time variation in B

$= 144 \times 10^{-4} \text{ m}^2 \times 10^{-3} \text{ T s}^{-1}$

$= 1.44 \times 10^{-5} \text{ Wb s}^{-1}$

Rate of change of flux due to motion of the loop in a non-uniform B

$= 144 \times 10^{-4} \text{ m}^2 \times 10^{-3} \text{ T cm}^{-1} \times 8 \text{ cm s}^{-1}$

$= 11.52 \times 10^{-5} \text{ Wb s}^{-1}$

The two effects add up since both cause a *decrease* in flux along the positive *z*-direction. Therefore, induced emf = 12.96 × 10^{-5} V; induced current = 2.88 × 10^{-2} A. The direction of induced current is such as to *increase* the flux through the loop along positive *z*-direction. If for the observer the loop moves to the right, the current will be seen to be anti-clockwise. A proper proof of the procedure above is as follows:

$$\Phi(t) = \int_0^a aB(x,t)\,\mathrm{d}x$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = a\int_0^a dx \frac{\mathrm{d}B(x,t)}{\mathrm{d}t}$$

using,

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\partial B}{\partial t} + \frac{\partial B}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

$$= \left[\frac{\partial B}{\partial t} + v\frac{\partial B}{\partial x}\right]$$

we get,

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = a\int_0^a \mathrm{d}x\left[\frac{\partial B(x,t)}{\partial t} + v\frac{\partial B(x,t)}{\partial x}\right]$$

$$= A\left[\frac{\partial B}{\partial t} + v\frac{\partial B}{\partial x}\right]$$

where $A = a^2$

The last step follows because $\left(\frac{\partial B}{\partial t}\right)$, $\left(\frac{\partial B}{\partial x}\right)$ and v are given to be constants in the problem. Even if you do not understand this formal proof (which requires good familiarity with calculus), you will still appreciate that flux change can occur both due to the motion of the loop as well as time variations in the magnetic field.

6.13 $$Q = \int_{t_i}^{t_f} I\mathrm{d}t$$

$$= \frac{1}{R}\int_{t_i}^{t_f} \varepsilon\,\mathrm{d}t$$

$$= -\frac{N}{R}\int_{\Phi_i}^{\Phi_f} \mathrm{d}\Phi$$

$$= \frac{N}{R}(\Phi_i - \Phi_f)$$

for $N = 25$, $R = 0.50\ \Omega$, $Q = 7.5 \times 10^{-3}$ C

$\Phi_f = 0$, $A = 2.0 \times 10^{-4}\ \mathrm{m}^2$, $\Phi_i = 1.5 \times 10^{-4}$ Wb

$B = \Phi_i/A = 0.75$ T

6.14 $|\varepsilon| = vBl = 0.12 \times 0.50 \times 0.15 = 9.0$ mV;

P positive end and Q negative end.

(b) Yes. When K is closed, the excess charge is maintained by the continuous flow of current.

(c) Magnetic force is cancelled by the electric force set-up due to the excess charge of opposite signs at the ends of the rod.

(d) Retarding force = IBl

$$= \frac{9\,\text{mV}}{9\,\text{m}\Omega} \times 0.5\ \text{T} \times 0.15\ \text{m}$$

$$= 75 \times 10^{-3}\ \text{N}$$

(e) Power expended by an external agent against the above retarding force to keep the rod moving uniformly at 12 cm s^{-1}

$$= 75 \times 10^{-3} \times 12 \times 10^{-2} = 9.0 \times 10^{-3}\ \text{W}$$

When K is open, no power is expended.

(f) $I^2R = 1 \times 1 \times 9 \times 10^{-3} = 9.0 \times 10^{-3}$ W

The source of this power is the power provided by the external agent as calculated above.

(g) Zero; motion of the rod does not cut across the field lines. [Note: length of PQ has been considered above to be equal to the spacing between the rails.]

6.15 $B = \frac{\mu_0 NI}{l}$

(Inside the solenoid away from the ends)

$$\Phi = \frac{\mu_0 NI}{l} A$$

Total flux linkage = $N\Phi$

$$= \frac{\mu_0 N^2 A}{l} I$$

(Ignoring end variations in **B**)

$$|\varepsilon| = \frac{\text{d}}{\text{d}t}(N\Phi)$$

$$|\varepsilon|_{av} = \frac{\text{total change in flux}}{\text{total time}}$$

$$|\varepsilon|_{av} = \frac{4\pi \times 10^{-7} \times 25 \times 10^{-4}}{0.3 \times 10^{-3}} \times (500)^2 \times 2.5$$

$$= 6.5\ \text{V}$$

6.16 $M = \frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{a}{x}\right)$

$\varepsilon = 1.7 \times 10^{-5}$ V

6.17 $-\frac{B\pi a^2 \lambda}{MR}\hat{\mathbf{k}}$

Chapter 7

7.1 (a) 2.20 A
(b) 484 W

7.2 (a) $\frac{300}{\sqrt{2}} = 212.1\,\text{V}$

(b) $10\sqrt{2} = 14.1\,\text{A}$

7.3 15.9 A

7.4 2.49 A

7.5 Zero in each case.

7.6 125 s^{-1}; 25

7.7 $1.1 \times 10^3\ s^{-1}$

7.8 0.6 J, same at later times.

7.9 2,000 W

7.10 $\nu = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$, i.e., $C = \frac{1}{4\pi^2\nu^2 L}$

For $L = 200\ \mu\text{H}$, $\nu = 1200$ kHz, $C = 87.9$ pF.
For $L = 200\ \mu\text{H}$, $\nu = 800$ kHz, $C = 197.8$ pF.
The variable capacitor should have a range of about 88 pF to 198 pF.

7.11 (a) 50 rad s^{-1}
(b) 40 Ω, 8.1 A
(c) $V_{Lrms} = 1437.5$ V, $V_{Crms} = 1437.5$ V, $V_{Rrms} = 230$ V

$$V_{LCrms} = I_{rms}\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

7.12 (a) 1.0 J. Yes, sum of the energies stored in L and C is conserved if $R = 0$.
(b) $\omega = 10^3\ \text{rad s}^{-1}$, $\nu = 159$ Hz
(c) $q = q_0 \cos \omega t$

(i) Energy stored is completely electrical at $t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$

(ii) Energy stored is completely magnetic (i.e., electrical energy is zero) at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, ...$, where $T = \frac{1}{\nu} = 6.3\,\text{ms}$.

(d) At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$, because $q = q_0 \cos\frac{\omega T}{8} = q_0 \cos\frac{\pi}{4} = \frac{q_0}{\sqrt{2}}$.

Therefore, electrical energy $= \frac{q^2}{2C} = \frac{1}{2}\left(\frac{q_0^2}{2C}\right)$ which is half the total energy.

(e) R damps out the LC oscillations eventually. The whole of the initial energy (= 1.0 J) is eventually dissipated as heat.

7.13 For an *LR* circuit, if $V = V_0 \sin \omega t$

$$I = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \phi), \text{ where } \tan\phi = (\omega L / R).$$

(a) I_0 = 1.82 A

(b) V is maximum at t = 0, I is maximum at $t = (\phi / \omega)$.

Now, $\tan\phi = \dfrac{2\pi \nu L}{R} = 1.571$ or $\phi \approx 57.5°$

Therefore, time lag $= \dfrac{57.5\pi}{180} \times \dfrac{1}{2\pi \times 50} = 3.2$ ms

7.14 (a) $I_0 = 1.1 \times 10^{-2}$ A

(b) tan ϕ = 100 π, ϕ is close to π/2.

I_0 is much smaller than the low frequency case (Exercise 7.13) showing thereby that at high frequencies, L nearly amounts to an open circuit. In a dc circuit (after steady state) ω = 0, so here L acts like a pure conductor.

7.15 For a *RC* circuit, if $V = V_0 \sin \omega t$

$$I = \frac{V_0}{\sqrt{R^2 + (1/\omega C)^2}} \sin(\omega t + \phi) \quad \text{where } \tan\phi = \frac{1}{\omega C R}$$

(a) I_0 = 3.23 A

(b) $\phi = 33.5°$

Time lag $= \dfrac{\phi}{\omega} = 1.55$ ms

7.16 (a) I_0 = 3.88 A

(b) $\phi \approx 0.2$ and is nearly zero at high frequency. Thus, at high frequency, C acts like a conductor. For a dc circuit, after steady state, ω = 0 and C amounts to an open circuit.

7.17 Effective impedance of the parallel *LCR* circuit is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

which is minimum at $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$

Therefore, $|Z|$ is maximum at $\omega = \omega_0$, and the total current amplitude is minimum.

In R branch, $I_{Rrms} = 5.75$ A

In L branch, $I_{Lrms} = 0.92$ A

In C branch, $I_{Crms} = 0.92$ A

Note: total current $I_{rms} = 5.75$ A, since the currents in L and C branch are 180° out of phase and add to zero at every instant of the cycle.

7.18 (a) For $V = V_0 \sin \omega t$

$$I = \frac{V_0}{\left|\omega L - \frac{1}{\omega C}\right|}\sin\left(\omega t + \frac{\pi}{2}\right); \quad \text{if } R = 0$$

where – sign appears if $\omega L > 1/\omega C$, and + sign appears if $\omega L < 1/\omega C$.
I_0 = 11.6 A, I_{rms} = 8.24 A

(b) $V_{L\,rms} = 207\,\text{V}, V_{C\,rms} = 437\,\text{V}$

(Note: 437 V – 207 V = 230 V is equal to the applied rms voltage as should be the case. The voltage across L and C gets subtracted because they are 180° out of phase.)

(c) Whatever be the current I in L, actual voltage leads current by $\pi/2$. Therefore, average power consumed by L is zero.

(d) For C, voltage lags by $\pi/2$. Again, average power consumed by C is zero.

(e) Total average power absorbed is zero.

7.19 I_{rms} = 7.26 A

Average power to $R = I_{rms}^2 R = 791\,\text{W}$

Average power to L = Average power to C = 0

Total power absorbed = 791 W

7.20 (a) ω_0= 4167 rad s^{-1}; ν_0 = 663 Hz

$I_0^{max} = 14.1$ A

(b) $\overline{P} = (1/2)I_0^2 R$ which is maximum at the same frequency (663 Hz) for which I_0 is maximum $\overline{P}_{max} = (1/2)(I_{\max})^2 R$ = 2300 W.

(c) At $\omega = \omega_0 \pm \Delta\omega$ [Approximation good if $(R/2L) << \omega_0$].
$\Delta\omega = R/2L$ = 95.8 rad s^{-1}; $\Delta\nu = \Delta\omega/2\pi$ = 15.2 Hz.
Power absorbed is half the peak power at ν = 648 Hz and 678 Hz.

At these frequencies, current amplitude is $(1/\sqrt{2})$ times I_0^{max}, i.e., current amplitude(at half the peak power points) is 10 A.

(d) Q = 21.7

7.21 ω_0= 111 rad s^{-1}; Q = 45

To double Q without changing ω_0, reduce R to 3.7 Ω.

7.22 (a) Yes. The same is *not* true for rms voltage, because voltages across different elements may not be in phase. See, for example, answer to Exercise 7.18.

(b) The high induced voltage, when the circuit is broken, is used to charge the capacitor, thus avoiding sparks, etc.

(c) For dc, impedance of L is negligible and of C very high (infinite), so the dc signal appears across C. For high frequency ac, impedance of L is high and that of C is low. So, the ac signal appears across L.

(d) For a steady state dc, L has no effect, even if it is increased by an iron core. For ac, the lamp will shine dimly because of additional impedance of the choke. It will dim further when the iron core is inserted which increases the choke's impedance.

(e) A choke coil reduces voltage across the tube without wasting power. A resistor would waste power as heat.

7.23 400

7.24 Hydroelectric power = $h \rho g \times A \times v = h \rho g \beta$

where $\beta = Av$ is the flow (volume of water flowing per second across a cross-section).

Electric power available = $0.6 \times 300 \times 10^3 \times 9.8 \times 100$ W

= 176 MW

7.25 Line resistance = 30 × 0.5 = 15 Ω.

rms current in the line $= \dfrac{800 \times 1000\ \text{W}}{4000\ \text{V}} = 200\ \text{A}$

(a) Line power loss = $(200\ \text{A})^2 \times 15\ \Omega$ = 600 kW.

(b) Power supply by the plant = 800 kW + 600 kW = 1400 kW.

(c) Voltage drop on the line = 200 A × 15 Ω = 3000 V.

The step-up transformer at the plant is 440 V – 7000 V.

7.26 Current $= \dfrac{800 \times 1000\ \text{W}}{40{,}000\ \text{V}} = 20\ \text{A}$

(a) Line power loss = $(20\ \text{A})^2 \times (15\ \Omega)$ = 6 kW.

(b) Power supply by the plant = 800 kW + 6 kW = 806 kW.

(c) Voltage drop on the line = 20 A × 15 Ω = 300 V.

The step-up transformer is 440 V – 40, 300 V. It is clear that percentage power loss is greatly reduced by high voltage transmission. In Exercise 7.25, this power loss is (600/1400) × 100 = 43%. In this exercise, it is only (6/806) × 100 = 0.74%.

CHAPTER 8

8.1 (a) $C = \varepsilon_0 A/d = 80.1$ pF

$$\frac{dQ}{dt} = C\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{0.15}{80.1 \times 10^{-12}} = 1.87 \times 10^9\ \text{V s}^{-1}$$

(b) $i_d = \varepsilon_0 \dfrac{d}{dt}\Phi_E$. Now across the capacitor $\Phi_E = EA$, ignoring end corrections.

Therefore, $i_d = \varepsilon_0 A \dfrac{d\Phi_E}{dt}$

Now, $E = \dfrac{Q}{\varepsilon_0 A}$. Therefore, $\dfrac{dE}{dt} = \dfrac{i}{\varepsilon_0 A}$, which implies $i_d = i = 0.15$ A.

(c) Yes, provided by 'current' we mean the sum of conduction and displacement currents.

8.2 (a) $I_{rms} = V_{rms}\,\omega C = 6.9\mu\text{A}$

(b) Yes. The derivation in Exercise 8.1(b) is true even if i is oscillating in time.

(c) The formula $B = \frac{\mu_0}{2\pi}\frac{r}{R^2} i_d$

goes through even if i_d (and therefore B) oscillates in time. The formula shows they oscillate in phase. Since $i_d = i$, we have

$B_0 = \frac{\mu_0}{2\pi}\frac{r}{R^2} i_0$, where B_0 and i_0 are the amplitudes of the oscillating magnetic field and current, respectively. $i_0 = \sqrt{2} I_{rms}$ = 9.76 μA. For r = 3 cm, R = 6 cm, B_0 = 1.63 × 10^{-11} T.

8.3 The speed in vacuum is the same for all: $c = 3 \times 10^8$ m s^{-1}.

8.4 **E** and **B** in x-y plane and are mutually perpendicular, 10 m.

8.5 Wavelength band: 40 m – 25 m.

8.6 10^9 Hz

8.7 153 N/C

8.8 (a) 400 nT, 3.14 × 10^8 rad/s, 1.05 rad/m, 6.00 m.

(b) $\mathbf{E}$ = { (120 N/C) sin[(1.05 rad/m)]x – (3.14 × 10^8 rad/s)t]} $\hat{\mathbf{j}}$

$\mathbf{B}$ = { (400 nT) sin[(1.05 rad/m)]x – (3.14 × 10^8 rad/s)t]} $\hat{\mathbf{k}}$

8.9 Photon energy (for λ = 1 m)

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}}\text{eV} = 1.24\times10^{-6}\text{ eV}$$

Photon energy for other wavelengths in the figure for electromagnetic spectrum can be obtained by multiplying approximate powers of ten. Energy of a photon that a source produces indicates the spacings of the relevant energy levels of the source. For example, $\lambda = 10^{-12}$ m corresponds to photon energy = 1.24 × 10^6 eV = 1.24 MeV. This indicates that nuclear energy levels (transition between which causes γ-ray emission) are typically spaced by 1 MeV or so. Similarly, a visible wavelength λ = 5 × 10^{-7} m, corresponds to photon energy = 2.5 eV. This implies that energy levels (transition between which gives visible radiation) are typically spaced by a few eV.

8.10 (a) $\lambda = (c/\nu) = 1.5 \times 10^{-2}$ m

(b) $B_0 = (E_0/c) = 1.6 \times 10^{-7}$ T

(c) Energy density in **E** field: $u_E = (1/2)\varepsilon_0 E^2$

Energy density in **B** field: $u_B = (1/2\mu_0)B^2$

Using $E = cB$, and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $u_E = u_B$

8.11 (a) – $\hat{\mathbf{j}}$, (b) 3.5 m, (c) 86 MHz, (d) 100 nT,

(e) {(100 nT) cos[(1.8 rad/m)y + (5.4 × 10^6 rad/s)t]} $\hat{\mathbf{k}}$

8.12 (a) 0.4 W/m^2, (b) 0.004 W/m^2

8.13 A body at temperature T produces a continuous spectrum of wavelengths. For a black body, the wavelength corresponding to maximum intensity of radiation is given according to Wein's law by the relation: λ_m=0.29 cm K/T. For λ_m=10^{-6} m, T = 2900 K. Temperatures for other wavelengths can be found. These numbers tell us the temperature ranges required for obtaining radiations in different parts of the electromagnetic spectrum. Thus, to obtain visible radiation, say $\lambda = 5 \times 10^{-7}$ m, the source should have a temperature of about 6000 K.
Note: a lower temperature will also produce this wavelength but not the maximum intensity.

8.14 (a) Radio (short wavelength end)
(b) Radio (short wavelength end)
(c) Microwave
(d) Visible (Yellow)
(e) X-rays (or soft γ-rays) region

8.15 (a) Ionosphere reflects waves in these bands.
(b) Television signals are not properly reflected by the ionosphere (see text). Therefore, reflection is effected by satellites.
(c) Atmosphere absorbs X-rays, while visible and radiowaves can penetrate it.
(d) It absorbs ultraviolet radiations from the sun and prevents it from reaching the earth's surface and causing damage to life.
(e) The temperature of the earth would be lower because the Greenhouse effect of the atmosphere would be absent.
(f) The clouds produced by global nuclear war would perhaps cover substantial parts of the sky preventing solar light from reaching many parts of the globe. This would cause a 'winter'.

Notes