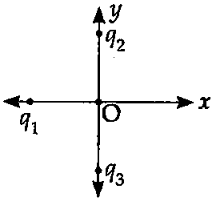
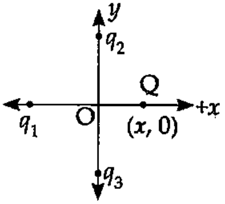
1 Electric Charges and Fields

MULTIPLE CHOICE QUESTIONS—I

Q1.1. In given figure, two positive charges q_2 and q_3 fixed along y-axis, exert a net electric force in the positive x direction on a charge q_1 fixed along the x-axis.



(*a*)



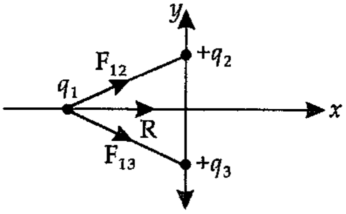
(*b*)

If a positive charge Q is added at $(x, 0)$, the force on q_1

- (*a*) shall increase along the positive x-axis.
- (*b*) shall decrease along the positive x-axis.
- (*c*) shall point along the negative x-axis.
- (*d*) shall increase but the direction changes because of the intersection of Q with q_2 and q_3.

Main concepts used: Like forces repel and unlike forces attract and vector addition of forces.

Ans. (*a*): (*i*)As the resultant force on q_1 due to q_2 and q_3 is along positive x direction. [Given]



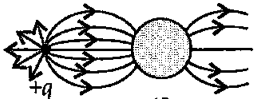
(*ii*) The vector sum of forces F_{12} and F_{13} is R, along positive x direction. So, F_{12} and F_{13} are attractive forces and $|F_{12}| = |F_{13}|$ also.

(*iii*) So q_1 is negative charge (as q_2, q_3 are positive charges.)

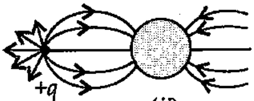
(*iv*) Charge Q $(x, 0)$ is positive, so the force due to Q will be along positive x direction.

(*v*) As the direction of forces R (due to F_{12} and F_{21} and due to Q) are along positive x-axis, so the net force on q_1 shall increase along positive x-axis.

Q1.2. A point positive charge is brought near an isolated conducting sphere in given figure. The electric field is best given by



(*i*)



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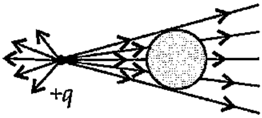

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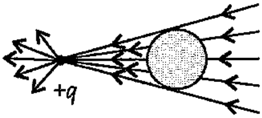


(*iv*)

(*a*) Fig. (*i*)
(*b*) Fig. (*ii*)
(*c*) Fig. (*iii*)
(*d*) Fig. (*iv*)

Main concepts used: Properties of lines of forces, induction.

Ans. (*a*): As given charge is +*q* and lines of forces in positive charge must be outward from positive charge *q*.

Now, as the positive charge is kept near an isolated conducting sphere, due to induction left part of sphere gets accumulated negative charge and right part positive, and lines of force from right part of sphere must emerge outward normally.

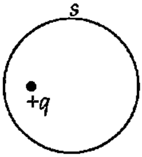
So verifies the answer (*a*).

As lines of forces are not perpendicular to the surface of sphere, so (*iii*) and (*iv*) is also rejected again.

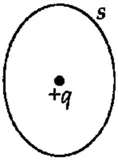
Q1.3. The electric flux through the surface

(*a*) in fig. (*iv*) is the largest.
(*b*) in fig. (*iii*) is the least.
(*c*) in fig. (*ii*) is same as fig. (*iii*) but is smaller than fig. (*iv*).
(*d*) is the same for all the figures.

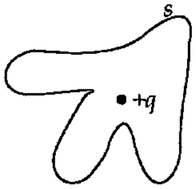
(*i*)



(*ii*)



(*iii*)



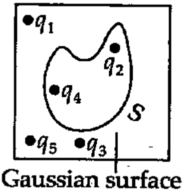
(*iv*)

Main concept used: Gauss's law of Electrostatics.

Ans. (*d*): Electric flux (ϕ) through the closed surface (or space) depends only on the charge enclosed inside the surface.

Here, charge inside all figures are same. So electric flux will remain same. $\left[\phi = \frac{q}{\varepsilon_0}\right]$

Q1.4. Five charges q_1, q_2, q_3, q_4 and q_5 are fixed at their positions as in the figure. S is a Gaussian surface. The Gauss's law is given by



Gaussian surface

$$\oint_s \mathrm{E}.ds = \frac{q}{\varepsilon_0}.$$

Which of the following statements is correct?

(*a*) E on the L.H.S. of the above equation will have a contribution from q_1, q_5 and q_3, while q on the R.H.S. will have a contribution from q_2 and q_4 only.

(*b*) E on the L.H.S. of the above equation will have a contribution from all charges while q on the R.H.S. will have a contribution from q_2 and q_4 only.

(*c*) E on the L.H.S. of the above equation will have a contribution from all charges while q on the R.H.S. will have a contribution from q_1, q_3 and q_5 only.

(*d*) Both E on the L.H.S. and q on the R.H.S. will have contributions from q_2 and q_4 only.

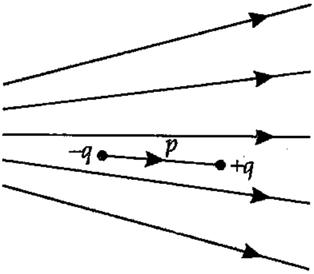
Main concepts used: Gauss's theorem, superposition of electric field.

Ans. (*b*): As all charges are positive (or of same signs) so electric field lines on R.H.S. of Gaussian surface will be due to q_2, q_3 and q_4 only.

On L.H.S. of Gaussian surface, the electric field lines on 'E' will be due to $q_1 q_2, q_3 q_4$ and q_5. So **answer** (*b*) is verified.

Q1.5. Figure here shows electric field lines in which an electric dipole $\vec{p}$ is placed as shown.

Which of the following statements is correct?



(*a*) The dipole will not experience any force.

(*b*) The dipole will experience a force towards right.

(*c*) The dipole will experience a force towards left.

(*d*) The dipole will experience a force upwards.

Main concepts used: (*i*) Magnitude of force on a charge in electric field is $\vec{F} = q\vec{E}$

(*ii*) Electric field is directly proportional to the concentration of field lines.

Ans. (*c*): Electric field lines density decreases from left to right, so electric field intensity on +*q* will be smaller than −*q* charge of dipole. As $\vec{F} = q\vec{E}$, so the force on (+*q*) will be smaller than (−*q*).

The direction of force on +*q* is along the direction of electric field, so force on −*q* will be in left direction. So net force on dipole will be towards left **verifies the answer '*c*'.**

Q1.6. A point charge +*q* is placed at a distance '*d*' from an isolated conducting plane. The field at a point P on the other side of the plane is

(*a*) directed perpendicular to the plane and away from the plane.
(*b*) directed perpendicular to the plane but towards the plane.
(*c*) directed radially away from the point charge.
(*d*) directed radially towards the point charge.

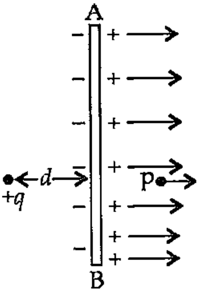
Main concepts used: (*i*) Induction; (*ii*) Electric field lines are perpendicular to the surface; (*iii*) lines of forces emerge outwards from positive charge.

Ans. (*a*): Let charge +*q* is placed to the left of isolated conducting plane AB vertical to plane of paper.

Due to induction by +*q* charge, R.H.S. plane will acquire positive charge.

So lines of forces will emerge perpendicularly, outward and parallel to each other.

It **verifies the answer (*a*).**



Conducting Plane

Q1.7. A hemisphere is uniformly charged positively. The electric field at a point to the diameter away from the centre is directed

(*a*) perpendicular to the diameter.
(*b*) parallel to the diameter.
(*c*) at an angle tilted towards the diameter.
(*d*) at an angle tilted away from the diameter.

Main concept used: Lines of electric field are perpendicular to the surface.

Ans. (*a*): As the side or diameter of hemisphere is plane surface, and whole hemisphere is charged with positive charge so, the electric field lines of forces emerging outward will be perpendicular to the plane surface or diameter.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q1.8. If $\oint_s E.ds = 0$ over a surface, then

(*a*) the electric field inside the surface and on it is zero.
(*b*) the electric field inside the surface is necessarily uniform.

(*c*) the number of flux lines entering the surface must be equal to the number of flux lines leaving it.
(*d*) all charges must necessarily be outside the surface.

Main concept used: Gauss's theorem of electric flux.

Ans.(*c*) and (*d*): Flux in Gaussian surface is zero. So the net charge inside the closed surface either is zero or charges are outside the surface. If charge or charges are outside the Gaussian surface, then entering leaving lines of electric field will be equal so net flux (lines of electric field) is zero **verifies answers (*c*) and (*d*).**

Q1.9. The electric field at a point is
(*a*) always continuous.
(*b*) continuous if there is no charge at that point.
(*c*) discontinuous only if there is a negative charge at that point.
(*d*) discontinuous if there is a charge at that point.

Main concept used: Properties of lines of forces.

Ans. (*b*) and (*d*): Either positive or negative charges will interact the lines of electric field so makes the electric field discontinuous.

If there is no any charge inside the electric field then the lines will not be affected. So electric field becomes continuous. So, **answers (*b*) and (*d*) are verified.**

Q1.10. If there were only one type of charge in the universe, then
(*a*) $\oint_s E.ds \neq 0$ on any surface.
(*b*) $\oint_s E.ds = 0$ if the charge is outside the surface.
(*c*) $\oint_s E.ds$ could not be defined.
(*d*) $\oint_s E.ds = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface.

Main concepts used: Gauss's theorem in Electrostatics. $\oint_s E.ds = \frac{q}{\varepsilon_0}$

Ans. (*b*) and (*d*): If a charge q is enclosed inside Gaussian surface then (*d*) is true.

If Gaussian surface (or space) is outside the charge $\oint_s E.ds = 0$ and (*c*) and (*a*) **are not true.** So, **answers (*b*) and (*d*) are verified.**

Q1.11. Consider a region inside which there are various types of charges, but the total charge is zero. At points outside the region,
(*a*) the electric field is necessarily zero.
(*b*) the electric field is due to the dipole moment of the charge distribution only.

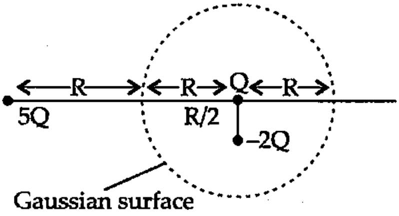
(*c*) the dominant electric field is $\propto \frac{1}{r^3}$, for large '*r*', where '*r*' is the distance of point (outside) from an origin in this region.
(*d*) the work done to move a charged particle along a closed path, away from the region, will be zero.

Main concepts used: (*i*) The Electric field due to dipole is always proportional to $\frac{1}{r^3}$.

(*ii*) A matter have so many dipoles and net charge zero.
(*iii*) Electrostatic work is conservative.
Ans. (*c*) and (*d*):

Although net charge in a dipole is zero but its electric field is proportional to $\frac{1}{r^3}$. Work done against electric field is conservative, so net work done in a closed loop is always zero. So **answers (*c*) and (*d*) are verified.**

Q1.12. Refer to the arrangement of charges in given figure and a Gaussian surface of radius R with charge Q at the centre of surface. Then



(*a*) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$.
(*b*) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$.
(*c*) flux through the surface of sphere due to 5Q is zero.
(*d*) field on the surface of sphere due to –2Q is same everywhere.

Main concepts used: (*i*) Gauss's law of electrostatic, (*ii*) Electric field on a metallic sphere.

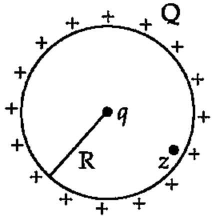
Ans. (*a*) and (*c*): (*a*) **is true** by Gauss's law. Here net charge = –2Q + Q.

(*b*) is the electric field on a conducting sphere. There is no any conducting sphere but only surface or spherical space is there.

5Q charge outside the Gaussian surface will not contribute to electric flux in Gaussian surface (or space).

For (*d*), the distance of Gaussian surface from –2Q is different, so field will not be same on the surface.

Q1.13. A positive charge +Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of ring, (Fig.). Then:



(*a*) If $q > 0$ and is displaced away from the centre in the plane of ring, it will be pushed back towards the centre.

(*b*) If $q < 0$ and is displaced away from the centre in the plane of ring, it will never return to the centre and will continue moving till it hits the ring.

(*c*) If $q < 0$, it will perform S.H.M. for small displacement along the axis.

(*d*) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$.

Main concepts used: (*i*) Interaction between charges.
(*ii*) stable and unstable equilibrium.

Ans. (*a*), (*b*), (*c*) and (*d*):

For *d*, charge is uniformly distributed along the ring. It is not sphere in which charge is only outside. So positive charge of ring will interact equally a charge placed at centre of ring but will be in unstable equilibrium.

For *c*, if q is displaced slightly (or small), it will perform S.H.M. and stops at centre.

(*a*) and **(*b*)** are verified in similar way.

VERY SHORT ANSWER TYPE QUESTIONS

Q1.14. An arbitrary surface encloses a dipole. What is the electric flux through this surface?

Main concept used: Gauss's law of electrostatics.

Ans. From Gauss's theorem, Electric flux,

$$\phi = \oint_s E.ds = \frac{q}{\varepsilon_0}$$

Net charge inside the Gaussian surface due to a dipole

$$= +q - q = 0$$

$$\therefore \qquad \phi = \frac{0}{\varepsilon_0} = 0$$

Q1.15. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (*i*) the inner surface, (*ii*) the outer surface?

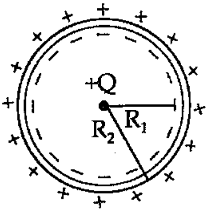
Main concepts used: (*i*) Spreading the charge on a conductor, (*ii*) Induction, (*iii*) Electric field.

Ans. Due to induction by charge +Q at centre, the inner surface acquires –Q charge and outer surface of shell will acquire +Q charge. So, Surface charge density,

$$\sigma = \frac{Q}{A}$$

$$\sigma \text{ on outside surface} = \frac{+Q}{4\pi R_2^2}$$

$$\sigma \text{ on inside surface} = \frac{-Q}{4\pi R_1^2}.$$



Q1.16. The dimensions of an atom are of the order of an Angstrom. Thus, there must be large electric fields between protons and electrons. Why, then is the electrostatic field inside a conductor zero?

Main concepts used: (*i*) Electric field due to dipole, (*ii*) Dipole moment, $p = 2aq$ (*iii*) Coulomb's law.

Ans. As we know, net charge in atom is zero and size of atom is of the order of Angstrom. So force between electron and proton is very large by Coulomb's law.

Electric field outside the atom (or substance) will be $\propto \frac{2aq}{r^3}$.

$2a$ = Average distance between positive protons and electrons in atom.
r = Very large distance between point and atomic dipole.
$\therefore r >>> a$. So. E.F. $\to 0$.

The electric field inside surface of isolated conductor is zero.

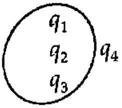
Q1.17. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero?

Conversely, if the electric field everywhere on the surface is zero, does it imply that net charge inside is zero?

Main concept used: Gauss's law.

Ans. By Gauss's law of electrostatics,

$$\oint_s E.ds = \frac{q}{\varepsilon_0}.$$



Consider a Gaussian surface enclosing charges q_1, q_2, q_3 and $q_1 + q_2 + q_3 = 0$.

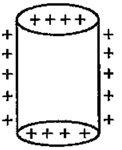
But electric field on R.H.S. due to q_4 will not be zero. But, on L.H.S., it will be zero. So first statement is **not true.**

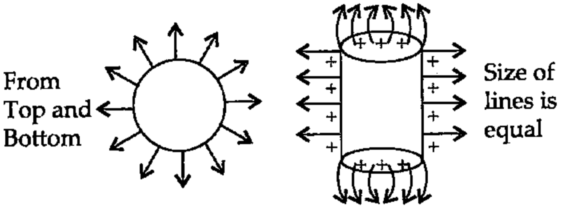
If electric field on Gaussian surface is zero, then, in above case, it is possible only when $q_4 = 0$, *i.e.*, if everywhere, on Gaussian surface, electric field is zero then net charge will be zero.

Q1.18. Sketch the electric field lines for a uniformly charged hollow cylinder shown in the figure.

Main concept used: Electric lines of forces are perpendicular to the surface.

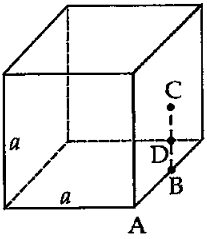
Ans. As there is no any charge inside the hollow cylinder, so not any negative charge due to induction only positive charge is spreading uniformly and lines of force emerge away perpendicularly from the surface to infinity.





Q1.19. What will be the total flux through the faces of the cube (Fig.) with side of length 'a' if the charge q is placed at

(*a*) A; a corner of the cube.
(*b*) B; mid-point of an edge of the cube.
(*c*) C; centre of a face of cube.
(*d*) D; the mid-point of B and C.



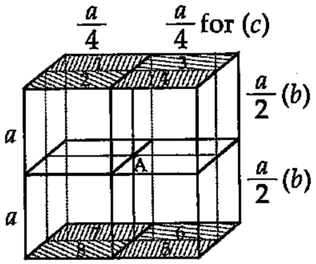
$DC = DB = \frac{a}{4}$

Main concepts used: (*i*) Gauss's law.(*ii*) Imagine the number of **cubes** so that position of charge (q) becomes symmetric to whole figure. (*iii*) Get the charge by dividing the given charge by number of cubes (or given figure).

Ans. (*a*) To make charge q at A symmetric to identical cube, there will be 8 identical cubes as shown in figure. So charge distribution for 1 cube is:

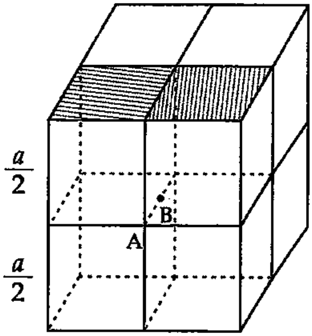
$$Q_1 = \frac{q}{8 \times 1} = \frac{q}{8}$$

$$\therefore \quad \text{Total flux by Gauss's law} = \frac{q}{8\,\varepsilon_0}$$



(*b*) To make charge q at B symmetric, we can consider the 8 identical cubes of side $\frac{a}{2}$ as shown in figure given below. So charge distribution for one cube is:

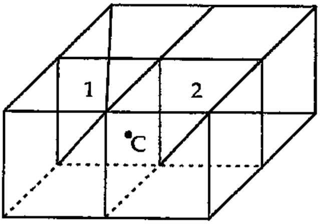
$$Q_2 = \frac{q}{8\times\frac{1}{2}} = \frac{q}{4}$$



Total flux by Gauss's law = $\frac{q}{4\,\varepsilon_0}$

(*c*) Point 'C' is symmetric to two cubes as shown in figure, so charge distribution for one cube $Q_3 = \frac{q}{8\times\frac{1}{4}} = \frac{q}{2}$

Total flux across cube by Gauss's law = $\frac{q}{2\,\varepsilon_0}$



(*d*) As the charge q is placed at D, the mid-point of BC, the charge is shared by 2 identical cubes, so the charge for each cube = $\frac{q}{2}$

Total electric flux by Gauss's law = $\frac{q}{2\,\varepsilon_0}$.

SHORT ANSWER TYPE QUESTIONS

Q1.20. A paisa coin is made up of Al-Mg alloy and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges.

Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude?

Main concepts used: (*i*) Avogadro's number, (*ii*) Mole concept, (*iii*) Atomic structure.

Ans. A 0.75 g paisa coin now is made with Al only. So,

$$\text{the mass of a paisa coin} = 0.75 \text{ g}$$

$$\text{Atomic mass of Al} \cong 27 \text{ amu}$$

So, $$\text{number of moles in } 0.75 \text{ g} = \frac{0.75}{27} \text{ mole}$$

$$\text{Number of Al atoms in coin} = \text{N}$$

$$= \frac{0.75}{27} \times 6.022 \times 10^{23} \text{ atoms}$$

$$\text{Atomic number of Al} = 13$$

∴ Number of electron (negative charge) and proton (positive charge) = 13.

So, number of either proton or electron in a coin

$$= \frac{13 \times 0.75}{27} \times 6.022 \times 10^{23}$$

Magnitude of charge on a proton or electron = 1.6×10^{-19} C

So, charge either positive or negative

$$= \frac{13 \times \overset{25}{\cancel{75}} \times 6.022 \times 10^{23} \times 1.6 \times 10^{-19} \text{C}}{\underset{9}{\cancel{27}} \times 100}$$

$$= \frac{13 \times 25 \times 6.022 \times 1.6 \times 10^{23-19-2}}{9}$$

$$= \frac{3131.44 \times 10^{21-19}}{9} = 347.9 \times 10^{2} \text{ C}$$

$$= 3.48 \times 10^{4} \text{ C}$$

Either positive or negative charge on a coin = 34.8 kC

It concludes that even a 0.75 g Al contains anormous amount of positive and negative charges and equal in magnitude.

Q1.21. Consider a coin of question 1.20. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two point charges separated by:

(*i*) 1 cm (~ 1/2 × diagonal of one paisa coin), (*ii*) 100 m (~ length of a long building), (*iii*) 10^{6} m (radius of earth). Find the force on each such point charge in each case. What do you conclude from these results?

Main concept used: Coulombian force.

Ans. Here charges are equal and opposite. So, by Coulomb's law, force of attraction between charges,

$$\text{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$q_1 = q_2 = 34.8 \text{ kC}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

$$F = \frac{34.8\times10^3 \times 34.8\times10^3 \times 9\times10^9}{r^2}$$

$$= \frac{34.8\times 34.8\times 9\times 10^{9+3+3}}{r^2}$$

$$= \frac{10899.36\times10^{15}}{r^2}\text{ N} \cong \frac{1.1\times10^{19}}{r^2}\text{ N}$$

$$\boxed{F = \frac{1.1\times10^{19}}{r^2}\text{ N}}$$

(*i*) $r_1 = 1 \text{ cm} = 0.01 \text{ m}$

$\therefore$ $F_1 = \dfrac{1.1\times10^{19}}{0.01\times0.01} = 1.1 \times 10^{19+4}$

$= \mathbf{1.1 \times 10^{23}}$ **N** towards the charges.

(*ii*) $r_2 = 100 \text{ m}$

$\therefore$ $F_2 = \dfrac{1.1\times10^{19}}{100\times100} = 1.1 \times 10^{19-4}$

$= \mathbf{1.1 \times 10^{15}}$ **N** towards the charges.

(*iii*) $r_3 = 10^6 \text{ m}$

$\therefore$ $F_3 = \dfrac{1.1\times10^{19}}{10^6\times10^6} = 1.1 \times 10^{19-6-6}$

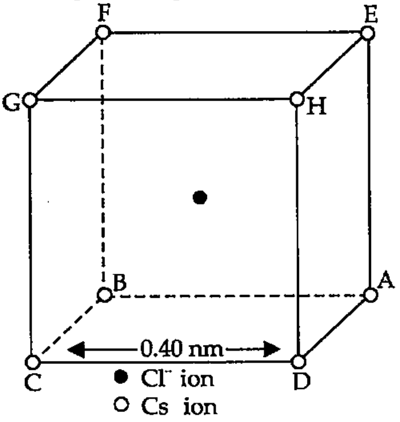
$= \mathbf{1.1 \times 10^{7}}$ **N** towards the charges.

This electrostatic force varies from order 10^7 N to 10^{23} N. The minimum force 10^7 is equivalent to the force of attraction between earth and 1 million kg body which is too much high.

So electrostatic force is so many times larger than gravitational force.

Q1.22. Figure shows a crystal unit of cesium chloride, CsCl. The cesium atoms represented by open circles are situated at the corners of a cube of side 0.40 nm, whereas a chlorine atom is situated at the centre of the cube. The cesium atoms are deficient by one electron while the Cl atom carries an excess electron.

(*i*) What is the net electric field (force) on Cl atom due to eight Cs atoms?



(*ii*) Suppose that the Cs atom at the corner A is missing. What is the net force on the Cl atom due to seven remaining Cs atoms?

Main concept used: Electric force is vector quantity.

Ans. The charge on Cs atom = $+e$

The charge on Cl atom = $-e$

The distance of Cl^- ion from any Cs^+ ion

$$= \frac{1}{2} \text{ diagonal of cube of side } l$$

$$r = \frac{1}{2}\sqrt{3l^2}$$

$$= \frac{1}{2}\sqrt{3 \times 0.4 \times 10^{-9} \times 0.4 \times 10^{-9}} \text{ m}$$

$$= \frac{1}{2} \times 0.4 \times 10^{-9} \times \sqrt{3}$$

$$\boxed{r = 0.2\sqrt{3} \times 10^{-9} \text{ m}}$$

(*i*) As the distance of Cl^- ion from Cs^+ is equal and charge on each Cs^+ ion is same, so electrostatic force due to Cs^+ ion at A and D will be equal in magnitude and opposite in direction. So by $F = \frac{kq_1q_2}{r^2}$ net force on Cl^- due to Cs^+ ion will be zero, as atoms of Cs attracts the Cl^- equally in opp. direction with pairs diagonally *i.e.*, (B, H), (C, E), (D, F).

(*ii*) As the Cs^+ ion at A is missing so net force on Cl^- ion will be only due to its' opposite Cs^+ ion and other forces will be cancelled out.

Net force on Cl^- ion when a Cs^+ ion from A is removed,

$$= \frac{kq_1q_2}{r^2} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$r = 0.2\sqrt{3} \times 10^{-9} \text{ m and } |q_1| = |q_2| = |e|$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\therefore \qquad F = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{0.2\sqrt{3} \times 10^{-9} \times 0.2\sqrt{3} \times 10^{-9}}$$

$$= \frac{9 \times 16 \times 16 \times 10^{-38+9}}{2 \times 2 \times 3 \times 10^{-18}}$$

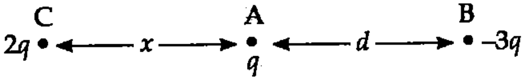
$$= 3 \times 16 \times 4 \times 10^{-29+18} = 192 \times 10^{-11}$$

Force on Cl^- ion $F = 1.92 \times 10^{-9}$ N towards Cl^- ion

Q1.23. Two charges q and $-3\,q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge '$2q$' be placed such that it will not experience any force?

Main concepts used: (*i*) Coulomb's law (*ii*) Vector properties of addition.

Ans.



(*i*) If we place the third charge $2q$ between A and B the direction of the force on $2q$ due to A and B on C will be same.

So the net force cannot be zero, so the charge q cannot be placed between A and B.

(*ii*) If $2q$ is placed right side of A then $r_{AC} > r_{BC}$ as $q_A < q_B$. So $F_{CA} < F_{CB}$ always as the direction of F_{CA} is towards right and F_{CB} is left so $F_{CA} + F_{CB} \neq 0$ we cannot obtain required condition.

(*iii*) Now consider $2q$ at C to the left of q at distance x from q.

Force on $2q$ at C (left of q) is in opposite direction so net force will be zero if magnitude is equal so,

$$F_{CA} + F_{CB} = 0 \text{ or } F_{CA} = -F_{CB}$$

$$\frac{\cancel{K}q_C q_A}{r^2_{CA}} = \frac{-\cancel{K}q_C q_B}{r^2_{CB}}$$

$$\Rightarrow \qquad \frac{2q.q}{x^2} = \frac{-2q(-3q)}{(x+d)^2}$$

$$\Rightarrow \qquad \frac{2q^2}{x^2} = \frac{6q^2}{(x+d)^2} \Rightarrow \frac{1}{x^2} = \frac{3}{(x+d)^2}$$

$$\Rightarrow \qquad 3x^2 = x^2 + d^2 + 2xd$$

$$3x^2 - x^2 - 2xd - d^2 = 0$$

$$2x^2 - 2xd - d^2 = 0$$

$$x = \frac{+2d \pm \sqrt{(-2d)^2 - 4.2.(-d^2)}}{2.2}$$

$$\Rightarrow \qquad x = \frac{+2d \pm \sqrt{4d^2 + 8d^2}}{4} = \frac{2d \pm 2d\sqrt{3}}{4}$$

$$\Rightarrow \qquad x = \frac{\cancel{2}\left(d \pm d\left(\sqrt{3}\right)\right)}{\cancel{4}\,2} = \frac{d\left(1 \pm \sqrt{3}\right)}{2}$$

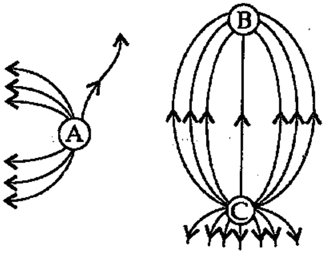
$$x = \frac{d}{2}\left(1 + \sqrt{3}\right) \text{ m to the left of } q.$$

Q1.24. Figure shows the electric field lines around three point charges A, B and C.

(*a*) Which charges are positive?

(*b*) Which charge has the largest magnitude? Why?

(*c*) In which region or regions of the picture could the electric field be zero? Justify your answer. (*i*) near A (*ii*) near B (*iii*) near C (*iv*) nowhere.



Main concepts used: (*i*) Properties of electric lines of force (*ii*) neutral point.

Ans. (*a*) Electric lines of forces emerge out from positive charge. So, A and C are positive charges.

(*b*) As the density of electric lines of forces from a charge increases, the intensity or of electric field or magnitude of charge increases. So the magnitude of charge C is maximum.

(*c*) The neutral point lies between two like charges. At neutral point force acting is zero, or no electric lines of force. As the lines of force repel each other sideways and the lines of force emerging from A and C are of same kind so repel each other and there will be no any electric lines of force at a point between A and C.

As the magnitude of charge C is larger than A so lines of force of C will be stronger than of A.

So, the Neutral point lies near A

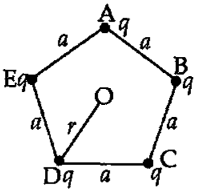
So, option (*i*) is the correct answer.

Q1.25. Five charges, q each are placed at the corners of a regular pentagon of side 'a' as shown in figure.

(*a*) (*i*) What will be the electric field at O, the centre of the regular pentagon?

(*ii*) What will be the electric field at O, if the charge from one of the corners (say A) is removed?

(*iii*) What will be the electric field at O if the charge q at A is replaced by $-q$?



(*b*) How would your answer to (*a*) be affected if pentagon is replaced by $n-$ sided regular polygon with charge q at each of its corners?

Main concepts used: (*i*) Properties of electric field (*ii*) concept of Geometry of regular polygon.

Ans. (*a*) (*i*) Point O is symmetric to all the five equal charges of $+q$ placed at vertices of regular polygon. So net electric field will be zero at O.

(*ii*)When a charge q from A is removed from symmetric charge distribution about 'O', the net electric field due to rest of the charges will be equal in magnitude as there is $-q$ charge at 'A'. So Electric field at 'O' in this case

$$E = \frac{-1q}{4\pi\varepsilon_0 r^2}$$

The direction of E is from O to A.

(*iii*) When charge q from A is removed and $-q$ charge is placed then electric field at O

$$E_2 = \frac{-q}{4\pi\varepsilon_0 r^2}\,.$$

When a new charge $-q$ is placed the E.F. will increase by $\frac{-q}{4\pi\varepsilon_0 r^2}$.

Now resultant becomes $= E_3 = E_2 + \frac{-q}{4\pi\varepsilon_0 r^2} = \frac{-q}{4\pi\varepsilon_0 r^2} + \frac{-q}{4\pi\varepsilon_0 r^2}$

$$\boxed{E_3 = \frac{-2q}{4\pi\varepsilon_0 r^2}}$$

(*b*) Now when pentagon is replaced by n – sided polygon

(*i*) electric field at 'O' will be again zero as all charge distribution about O is symmetric as in case of electric field at the centre of conducting ring or shell.

(*ii*) electric field at O will remain same as in *a*(*ii*) if a charge q is removed. The resultant will be equal to the electric field due to charge $-q$ at A.

So electric field at 'O' at the centre of regular polygon of n side if a charge from one vertex is removed is equal to $E_2 = \frac{-q}{4\pi\varepsilon_0 r^2}$. The direction from O is opposite to OA.

(*iii*) If charge $-q$ is placed at one vertex after removing $+q$ from there then resultant electric field at 'O' will be due to charge $(-2q)$ at A.

So the net electric field at O after removing q from A by placing $-q$ at A.

$$E = \frac{-q}{4\pi\varepsilon_0 r^2} - \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{-2q}{4\pi\varepsilon_0 r^2} \text{ from O to A.}$$

LONG ANSWER TYPE QUESTIONS

Q1.26. In 1959 Lyttleton and Bondi suggested that the expansion of the Universe could be explained if matter carried a net charge. Suppose that the Universe is made up of hydrogen atoms with a number density N which is maintained a constant. Let the charge on the proton be: $e_p = -(1+y)\,e$, where e is the electronic charge.

(*a*) Find the critical value of y such that expansion may start.

(*b*) Show that the velocity of expansion is proportional to the distance from the centre.

Main concepts used: (*i*) Expansion will start if gravitational force (F_G) between H atoms is smaller than Coulombian repulsive force (F_C), (*ii*) Atom has equal no. of proton and electron, (*iii*) for critical value of $y \rightarrow F_C = F_G$.

Ans. (*a*) Consider that the Universe is spherical with radius R made up of H atoms.

$$\text{Charge on proton} = -(1+y)\,e$$

$$\text{So the total charge on a H atom} = e_p + e$$
$$= -(1+y)\,e + e$$
$$= [-1-y+1]\,e$$

$$\text{Charge on 1 H atom} = -ye$$

$$\text{Number of H atoms in spherical Universe} = \text{N.V.}$$
$$= \text{N}.\frac{4}{3}\pi\text{R}^3$$

$$\therefore \text{ The net charge in Universe} = \frac{4}{3}\pi\text{NR}^3.ye$$

Consider the boundaries of Universe as Gaussian surface then by Gauss's law of electrostatics,

$$\oint_s \text{E}.ds = \frac{q}{\varepsilon_0}$$

$$\text{E}.\,4\pi\text{R}^2 = \frac{-4\pi\text{NR}^3 ye}{3\varepsilon_0}$$

$$\Rightarrow \qquad \text{E} = \frac{-4\pi\text{NR}^3 ye}{3\varepsilon_0.4\pi\text{R}^2} = \frac{-\text{NR}ye}{3\varepsilon_0}$$

Electrostatic force acting on one H atom $F_C = q\text{E}$

$$\text{F}_\text{C} = \frac{(-ye)(-\text{NR}ye)}{3\varepsilon_0} = \frac{+y^2e^2\text{N.R.}}{3\varepsilon_0}$$

Positive sign of F_C shows repulsive force.

$$\text{Gravitational potential at boundary of Universe} = \frac{\text{GM}}{\text{R}^2}$$

M = mass of Universe (or all H atoms)

So, the gravitational force acting on a H atom at boundary of Universe = Gravitational potential × m

[m = mass of H-atom]

$$F_G = \frac{GM}{R^2}\, m_H$$

Mass of 1 H atom = mass of a proton

$= m_p$

$\therefore$ Mass of Universe = No. of H atoms in Universe × m_p

$$= N.\frac{4}{3}\pi R^3 . m_p$$

$$\therefore \qquad F_G = \frac{G\left(N\frac{4}{3}\pi R^3\right)m_p^2}{R^2}$$

$$F_G = \frac{4\pi GNR\, m_p^2}{3}$$

If $F_C > F_G$, then universe will start to expand. So for critical value of expansion

$$F_C = F_G$$

$$\frac{y^2e^2NR}{3\varepsilon_0} = \frac{4\pi\, GNR\, m_p^2}{3}$$

$$\Rightarrow \qquad y^2 = \frac{\cancel{3}\varepsilon_0 4\pi\, GNR\, m_p^2}{\cancel{3}e^2\, NR} = 4\pi\varepsilon_0 G\left(\frac{m_p}{e}\right)^2$$

$$\Rightarrow \qquad y^2 = \frac{6.67\times10^{-11}}{9\times10^9}\left(\frac{1.66\times10^{-27}}{1.6\times10^{-19}}\right)^2$$

$$\Rightarrow \qquad y^2 \cong \frac{6.67\times10^{-20}}{9}\,[10^{-27+19}]^2$$

$$= 0.741 \times 10^{-20} \times (10^{-8})^2$$

$$\Rightarrow \qquad y^2 = 74.1 \times 10^{-22} \times 10^{-16} \Rightarrow y = \sqrt{74.1\times10^{-38}}$$

$$y = 8.6 \times 10^{-19} \cong 10^{-18}$$

So critical value of y is of the order of 10^{-18} so that Universe start to expand.

(*b*) For expansion repulsive force F_C must be greater than, attractive gravitational force

So net force on H atom to expand

$= F_C - F_G$

$$F_H = \frac{y^2e^2NR}{3\varepsilon_0} - \frac{4\pi}{3}\, GNR\, m_p^2$$

This force F_H will produce acceleration in H atom

$$\therefore \qquad F_H = m_p \frac{d^2R}{dt^2}$$

Here R (size of Universe) changes with time as Universe expands with velocity

$$\therefore \qquad m_p \frac{d^2R}{dt^2} = \left[\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi GNm_p^2}{3}\right]R$$

$$\frac{d^2R}{dt^2} = \frac{1}{m_p}\left[\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi GNm_p^2}{3}\right]R$$

As N, y, e, ε_0 π, G, m_p are constants so taking a new constant α^2 such that

$$\alpha^2 = \frac{1}{m_p}\left[\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi GNm_p^2}{3}\right]$$

$$\therefore \qquad \frac{d^2R}{dt^2} = \alpha^2 R$$

It is differential equation of order 2. Its solution is

$$R = Ae^{\alpha t} + Be^{-\alpha t}$$

For expansion of Universe B = 0

$$\therefore \qquad R = Ae^{\alpha t} \qquad \text{...(I)}$$

Receding velocity of Universe $v = \frac{dR}{dt}$

$$v = A\alpha e^{\alpha t} \quad or \quad v = R\alpha$$

as α is constant. So the receding (expanding) velocity of Universe is directly proportional to the distance of matter (H atom) from centre of Universe.

Q1.27. Consider a sphere of radius R with charge density distributed as

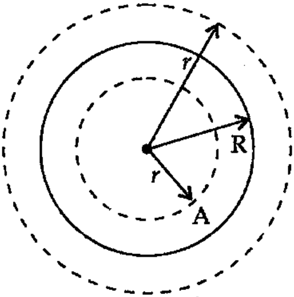
$$\rho(r) = kr \text{ (for } r \le R)$$
$$\rho(r) = 0 \text{ (for } r > R)$$

(*a*) Find the electric field at all points r.

(*b*) Suppose the total charge on the sphere is $2e$ where e is electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

Main concept used: Gauss's law of electrostatics.

Ans. (*a*) Consider a sphere (solid) of radius R with charge density $\rho(r) = kr$ (for $r \le R$) and $\rho(r) = 0$ (for $r > R$)



Case-I: Consider the Gaussian surface at radius $r < R$. Applying Gaussian law at A

$$v = \text{Volume of Gaussian surface}$$

$$\therefore \quad q = \rho(r)\cdot\frac{4}{3}\pi r^3$$

$$\oint_s E.ds = \frac{1}{\varepsilon_0}\rho(r).dv$$

$$v = \frac{4}{3}\pi r^3$$

$$dv = \frac{4}{3}\pi\, 3r^2\, dr = 4\pi r^2\, dr$$

$$\rho(r) = kr \text{ for } r < R$$

$$\oint_s E.ds = \int_0^r \frac{kr.4\pi r^2 dr}{\varepsilon_0}$$

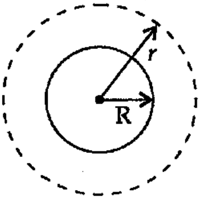
$$E4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\int_0^r r^3 dr$$

$$E = \frac{\cancel{4}\cancel{\pi}k}{\cancel{4}\cancel{\pi}r^2\varepsilon_0}\cdot\frac{r^4}{4}$$

$$\boxed{E = \frac{kr^2}{4\varepsilon_0}}$$

As field is positive so direction of E is radially outward.

Case-II: Consider a Gaussian surface of radius $r > R$



As charge inside the Gaussian surface is upto $r = 0$ to $r = R$ remains same as earlier. So net charge in this Gaussian surface

$$q = \rho(r).dv$$

$$q = \int_0^R kr\, 4\pi r^2 dr$$ as charge reside upto radius R only

By Gaussian law

$$\oint_{s = 4\pi r^2} E.ds = \int_0^R \frac{4\pi k r^3 dr}{\varepsilon_0}$$

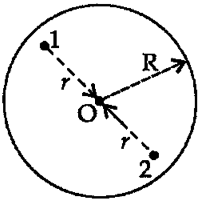
$$E.4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\left[\frac{r^4}{4}\right]_0^R$$

$$E = \frac{4\pi k}{4\varepsilon_0}\frac{(R^4 - 0)}{(4\pi r^2)}$$

$$\boxed{E(r) = \frac{k}{4\varepsilon_0}\cdot\frac{R^4}{r^2}}$$

The direction of E.F. is outward radially.

(*b*) As the total negative charge on sphere is $2e$ (e is charge on electron) is distributed in sphere of radius R symmetrically. So two protons must be symmetrical in sphere. *i.e.*, two protons must be on the opposite sides equidistant from the centre as shown in fig.



Charge on sphere $\quad q = \rho(r)dv$

From last part (*a*) $\quad dv = 4\pi r^2.dr$

$$\therefore \quad q = \int_0^R (kr)\, 4\pi r^2 dr = 4\pi k\cdot\frac{R^4}{4}$$

$$2e = \pi k R^4$$

$$\Rightarrow \quad k = \frac{2e}{\pi R^4} \qquad \text{...(I)}$$

Protons 1 and 2 shown in fig. are embedded at distance r from centre O, of sphere, thus force of attraction between a proton and negative charge distribution in sphere as E is due to (–) charge. E.F due to charge distribution inside the charge sphere at $r < R$ from part (*a*) is

$$E = \frac{kr^2}{4\varepsilon_0}$$

$$F_1 = -eE = \frac{-e\cdot kr^2}{4\varepsilon_0}$$

Repulsive force on proton 1 due to proton 2 = F_2 by Coulomb's law

$$F_2 = \frac{e^2}{4\pi\varepsilon_0 (2r)^2}$$

Net force on proton 1, $F = F_1 + F_2 = 0$

$$\Rightarrow \quad F = \frac{-ekr^2}{4\varepsilon_0} + \frac{e^2}{4\pi\varepsilon_0 4r^2} = 0$$

$$\Rightarrow \quad \frac{e^2}{4\pi\varepsilon_0 4r^2} = \frac{ekr^2}{4\varepsilon_0}$$

$$\Rightarrow \quad r^4 = \frac{4\varepsilon_0 e^2}{4\pi\varepsilon_0 4.ek}$$

$$\Rightarrow \quad r^4 = \frac{e}{4\pi k}$$

$$\Rightarrow \quad r^4 = \frac{e\,\pi R^4}{4\pi \cdot 2e} \qquad \left(\because k = \frac{2e}{\pi R^4} - \text{from I}\right)$$

$$\Rightarrow \quad r^4 = \frac{R^4}{8}$$

$$\Rightarrow \quad r = \frac{R}{(8)^{1/4}}$$

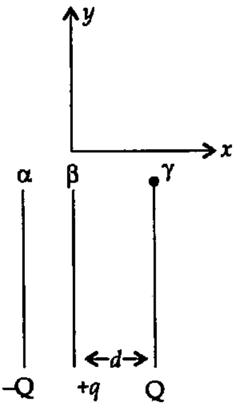
So protons must be embedded at a distance of $R/(8)^{1/4}$ from the centre of sphere of radius R.

Q1.28. Two fixed identical conducting plates (α & β), each of surface area S are charged to $-Q$ and q, respectively, where $Q > q > 0$. A third identical plate (γ), free to move is located on the other side of the plate with charge Q at a distance d as shown in fig. The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.

(*a*) Find the electric field acting on the plate γ before collision.

(*b*) Find the charges on β and γ plates after collision.

(*c*) Find the velocity of plate γ after collision and at a distance d from the plate β.



Main concepts used: Electric field of infinite plate of charge density $= \dfrac{\sigma}{2\varepsilon_0}$

(*i*) WD = F.d. (*ii*) KE = $\frac{1}{2}mv^2$

Ans. (*a*) As the plate γ has Q charge. So electric field on plate γ due to plate α = $\frac{\sigma}{2\varepsilon_0} = \frac{-Q}{S2\varepsilon_0}$

$$E_\alpha = \frac{-Q}{2S\varepsilon_0} \text{ towards left}$$

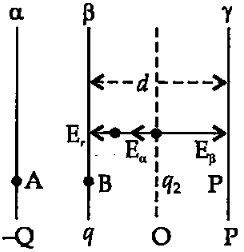
Similarly, electric field on γ due to plate β = $E_\beta = \frac{+q}{2S\varepsilon_0}$ towards right. So this net electric field on plate γ = E = $F_\alpha + E_\beta$

$$E = \frac{-Q}{2S\varepsilon_0} + \frac{q}{2S\varepsilon_0}$$

$$E = \frac{1}{2S\varepsilon_0}[q - Q] \text{ towards left as } (Q > q)$$

So, the electric field on plate γ before collision is towards left as $(|Q| > |q|) = \frac{1}{2S\varepsilon_0}[q - Q]$.

(*b*) On collision between plate β and γ their potential becomes same. Suppose during collision at any point P between plates β and γ the charges on β and γ plates are q_1 and q_2 respectively. Consider a point O to the right of plate B and left to plate γ such that net electric field at O is zero.



Electric field due to plate α at O

$$= \frac{-Q}{2S\varepsilon_0} \text{ (towards left)}$$

Electric field at O due to plate β = $\frac{+q_1}{2S\varepsilon_0}$ (towards right)

Electric field at O due to plate γ = $\frac{-q_2}{2S\varepsilon_0}$ (towards left)

$\because |-Q| > |q|$

Net electric field at O must be zero so,

$$\frac{-Q}{2S\varepsilon_0} + \frac{-q_2}{2S\varepsilon_0} + \frac{q_1}{2S\varepsilon_0} = 0$$

or $$Q = q_1 - q_2 \qquad \text{... I}$$

As there is no loss of charge on collision by law of conservation of charges so,

$$+Q + q = q_1 + q_2 \qquad \text{... II}$$

$$Q = q_1 - q_2 \qquad \text{[From I]}$$

$$q = 2q_2 \qquad \text{[Subtract I from II]}$$

or $$q_2 = \frac{q}{2}$$

So, charge on plate γ after collision $= \frac{q}{2}$ unit.

Charge on plate $\beta = Q + q - \frac{q}{2}$

Charge on plate $\beta = \left(Q - \frac{q}{2}\right)$

(*c*) After collision of plate γ with α, after charge distribution between plate γ and β, the plates will repel each other and plate γ will move towards its initial position as plate γ is free to move but plate α, β are fixed.

Let the velocity of plate γ after collision at distance *d* is *v* and mass of plate is *m* then gain in KE round the trip from P to B and B to P must be equal to the work done by electric field.

After collision the electric field on plate γ at O, due to plate at α and β

$$= \frac{-Q}{2S\varepsilon_0} + \frac{Q + q/2}{2\varepsilon_0 S}$$

$$E_2 = \frac{q/2}{2\varepsilon_0 S} \text{ towards right}$$

Electric field on plate γ just before collision due to plate α, and β

$$E_1 = \frac{-Q}{2\varepsilon_0 S} + \frac{q}{2\varepsilon_0 S}$$

$$E_1 = \frac{-Q + q}{2\varepsilon_0 S}$$

Force on plate γ just before collision $= -E_1 Q$

$$\Rightarrow \quad F_1 = \frac{(-Q+q)(-Q)}{2\varepsilon_0 S} = \frac{(Q-q)Q}{2\varepsilon_0 S}$$

$$\Rightarrow \quad F_2 = E_2 \cdot \frac{q}{2} = \frac{\frac{q}{2} \cdot \frac{q}{2}}{2S\varepsilon_0} = \frac{\left(\frac{q}{2}\right)^2}{2\varepsilon_0 S}$$

$$W = (F_1 + F_2)d$$

$$= \left[\frac{(Q-q)Q}{2\varepsilon_0 S} + \frac{\left(\frac{q}{2}\right)^2}{2\varepsilon_0 S}\right] d = \frac{\left[Q^2 - qQ + \frac{q^2}{4}\right] d}{2\varepsilon_0 S}$$

$$= \frac{\left[Q^2 + \left(\frac{q}{2}\right)^2 - 2 . Q . \frac{q}{2}\right] d}{2\varepsilon_0 S}$$

$$W = \frac{\left(Q - \frac{q}{2}\right)^2 d}{2\varepsilon_0 S} = KE$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{\left(Q - \frac{q}{2}\right)^2 .d}{2\varepsilon_0 S} \Rightarrow v^2 = \frac{2d\left(Q - \frac{q}{2}\right)^2}{2\varepsilon_0 Sm}$$

$$v = \left(Q - \frac{q}{2}\right)\sqrt{\left(\frac{d}{\varepsilon_0 S.m}\right)}$$ is the velocity of plate γ at a distance d after collision.

Q1.29. There is another useful system of units, besides the SI/mks. A system, called the cgs (centimeter-gram-second) system. In this system Coulomb's law is given by

$$F = \frac{Qq}{r^2}\hat{r}$$

where the distance r is measured in cm ($= 10^{-2}$m), F in dynes ($= 10^{-5}$ N) and the charges in electrostatic units (*es* units), where 1es unit of charge $= \frac{1}{[3]} \times 10^{-9}$C

The number [3] actually arises from the speed of light in vaccum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c then is $c = [3] \times 10^8$ m/s.

(*i*) Show that the Coulomb law in cgs units yields
1 esu of charge = 1(dyne)½ cm.
Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(*ii*) Write 1 esu of charge = χ Coulomb, where χ is a dimensionless number. Show that this gives:

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{\chi^2}\frac{Nm^2}{C^2}$$

With $\chi = \frac{1}{[3]} \times 10^{-9}$,

We have

$$\frac{1}{4\pi\varepsilon_0} = [3]^2 \times 10^9 \frac{Nm^2}{C^2}$$

or $$\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9 \frac{Nm^2}{C^2} \text{ (Exactly)}$$

Main concept used: Homogeneity of a formula in Dimensions.

Ans. (*i*)
$$F = \frac{Qq}{r^2}$$

$$1 \text{ dyne} = \frac{[1 \text{ esu}]^2}{(1 \text{ cm})^2}$$

$$1 \text{ esu} = \text{cm}\sqrt{\text{dyne}}$$

$$1 \text{ esu} = L^1 \ F^{1/2} = L^1 \quad [MLT^{-2}]^{1/2}$$

$$1 \text{ esu} = M^{1/2} \ L^{3/2} \ T^{-1}$$

So esu of charge is represented in terms of fractional powers of $\frac{1}{2}$ of M, $\frac{3}{2}$ of L and (–1) of T

(*ii*) 1 esu = χC [Given]

Where, χ is dimensionless number. Coulombian force between two charges each of magnitude 1 esu separated by 1 cm is one dyne = 10^{-5}N. This situation is equivalent to two charges of magnitude χC separated by 10^{-2} m. By Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ or } \frac{1}{4\pi\varepsilon_0} = \frac{F \cdot r^2}{q_1q_2} \qquad \ldots \text{I}$$

$$q_1 = q_2 = \chi C$$

$$r = 1 \text{ cm} = 10^{-2} \text{ m}$$

If
$$F = 1 \text{ dyne},$$

From I,
$$\frac{1}{4\pi\varepsilon_0} = \frac{1 \text{ dyne } (1 \text{ cm})^2}{\chi \ \chi}$$

$$= \frac{10^{-5} \text{N } (10^{-2}\text{m})^2}{\chi^2 \ C^2}$$

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{\chi^2}\ \frac{\text{N-m}^2}{C^2}$$

$$\chi = \frac{1}{[3] \times 10^9}$$

$$\therefore \qquad \frac{1}{4\pi\varepsilon_0} = 10^{-9} \times [3]^2 \times (10^9)^2 \ \frac{\text{N-m}^2}{C^2}$$

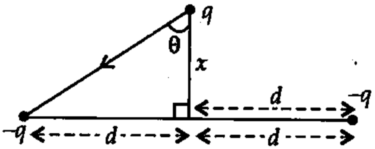
$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \frac{\text{N-m}^2}{C^2}$$

If [3] → 2.99792458 we get

$$\frac{1}{4\pi\varepsilon_0} = 8.98755 \times 10^9 \text{ N-m}^2 \ C^{-2}$$

Q1.30. Two charges $-q$ each are fixed separated by a distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by x (x <<< d) perpendicular to the line joining the two fixed charges as shown in figure. Show that q will perform simple harmonic oscillation of time period

$$T = \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{\frac{1}{2}}$$



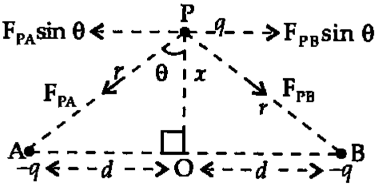
Main concepts used: (*i*) Properties of S.H.M. (*ii*) Coulombian force.

Ans. Force acting on charge q due to $-q$ at A will be F_{PA} along P to A

So, $$F_{PA} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(-q)}{r^2}$$

$$F_{PA} = \frac{-q^2}{4\pi\varepsilon_0 r^2}$$

Similarly, $$F_{PB} = \frac{-q^2}{4\pi\varepsilon_0 r^2}$$ $[\because |F_{PA}| = |F_{PB}| = F]$



So, the horizontal components of F_{PA} and F_{PB} are equal and opposite so neutralise each other.

The vertical components of F_{PA} and F_{PB} are downward and so add up.

Net force on q at P $= F_{PB}\cos\theta + F_{PA}\cos\theta = 2F\cos\theta$

$\because |F_{PA}| = |F_{PB}| = |F|$

$$= \frac{-2\cdot q^2}{4\pi\varepsilon_0 r^2}\cos\theta$$

Force on q downwards $= \dfrac{-2q^2}{4\pi\varepsilon_0 r^2}\,\dfrac{x}{r}$

$r^2 = x^2 + d^2$ [By Pythagoras theorem]

$\therefore$ Force on $q = \dfrac{-2q^2}{4\pi\varepsilon_0}\,\dfrac{x}{(x^2+d^2)^{3/2}}$

$\because x <<< d \quad \therefore x^2 <<<< d^2$

As negative sign shows force of attraction.

So net force on q at P downwards $= \left(\dfrac{-2q^2}{4\pi\varepsilon_0 d^3}\right)x$

So force on q is directly proportional to the displacement from mean position O (mid point of segment joining $-q$ and $-q$ charges). So motion of q about O will be S.H.M.

$$F = -kx \qquad \text{where, } k = \frac{2q^2}{4\pi\varepsilon_0 d^3}$$

$$\therefore \qquad \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{\frac{2q^2}{4\pi\varepsilon_0 d^3}}} = 2\pi\sqrt{\frac{\cancel{4}\pi m\varepsilon_0 d^3}{\cancel{2}q^2}}$$

$$T = \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{\frac{1}{2}}$$

Hence Proved.

Q1.31. Total charge –Q is uniformly spread along the length of a ring of radius R. A small test charge +*q* of mass *m* is kept at the centre of the ring and is given a gentle push along the axis of the ring

(*a*) Show that particle executes a simple harmonic oscillation.

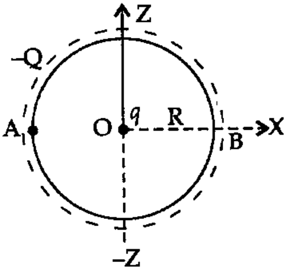
(*b*) Obtain its time period.

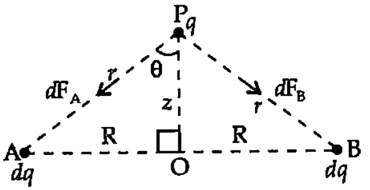
Main concepts used: (*i*) Properties of SHM, (*ii*) Centre of ring is symmetric to all charge distribution so electric field at centre is zero.

Here ring is along x - y plane and its axis is along z-axis.

Ans.(*a*) As –Q charge is equally distributed along a conducting ring, so, at the point O, the centre of ring is symmetric to charge distribution. So the electric field at O will be zero, or force on charge q placed at O will be equal to qE *i.e.,* $q \times 0 = 0$.

But when the charge q is displaced gently from 'O', or mean position the electric field on q will not be zero so force acts on q as in fig. below.





Force on q at P when z is small

$$F = F_A + F_B \qquad \boxed{\because |F_A| = |F_B| = |F_A|}$$

$$F_Y = F_A\cos\theta + F_A\cos\theta$$

$$F_Y = 2F_A\cos\theta \text{ (downward)}$$

The horizontal component of F_A and F_B will be equal and opposite, so cancelled out.

So net force acting on q due to small element dl of ring $= dF\cos\theta$

$$dF_y = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q.dq}{r^2}\cdot\frac{z}{r} \qquad \left[\because \cos\theta = \frac{z}{r}\right]$$

Charge on small element dl of ring

$$dq = \lambda dl$$

$$dq = \frac{-Q}{2\pi r}\cdot dl$$

$$\therefore \quad dF_y = \frac{qz}{4\pi\varepsilon_0 r^3}\cdot\frac{(-Q)dl}{2\pi R}$$

$$dF_y = \frac{-Qqzdl}{4\pi\varepsilon_0\cdot 2\pi R r^3}$$

Integrating both sides we get,

$$\int_0^F dF_y \downarrow = \int_0^{2\pi R} \frac{-qQzdl}{4\pi\varepsilon_0 2\pi R r^3}$$

$$\therefore \quad F = \frac{-qQz}{4\pi\varepsilon_0 2\pi R\,(R^2+z^2)^{3/2}}\int_0^{2\pi R} dl \quad (\because r = \sqrt{R^2+z^2}$$

[if $z <<< R$, $z^2 <<<< R^2$ or z^2 can be neglected]

$$\therefore \quad F = \frac{-qQz.2\pi r}{4\pi\varepsilon_0 2\pi r R^3}$$

$$F = \frac{-Qq}{4\pi\varepsilon_0 R^3}z \qquad \text{... [I]}$$

$$F \propto z$$

So motion of q is SHM

$$F = -kz \qquad \text{... [II]}$$

(*b*) $\therefore$
$$k = \frac{Qq}{4\pi\varepsilon_0 R^3} \qquad \text{[Comparing I and II]}$$

$$\therefore \quad T = 2\neq\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m R^3}{Qq}}$$

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