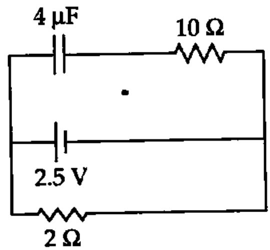
2 Electrostatic Potential and Capacitance

MULTIPLE CHOICE QUESTIONS—I



Q2.1. A capacitor of 4 μF is connected as shown in circuit here. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be

(*a*) 0 (*b*) 4 μC
(*c*) 16 μC (*d*) 8 μC

Main concepts used: (*i*) DC current does not flow across capacitor. (*ii*) P.D. across parallel branches are equal.

Ans. (*d*): As capacitor offer infinite resistance so current from cell will not flow across capacitor branch.

So current will flow across 2 Ω branch.

$$\boxed{I = \frac{V}{R + r}} = \frac{2.5}{2 + 0.5} = \frac{2.5}{2.5} = 1 \text{ Amp.}$$

So P.D. across 2 Ω resistance V = RI = 2 × 1 = 2 Volt.

As battery, capacitor and 2 Ω branches are in parallel. So P.D. will remain same across all three branches.

As current does not flow through capacitor branch so no potential drop will be across 10 Ω resistance.

So P.D. across 4 μF capacitor = 2 Volt

$$\therefore \quad \boxed{q = CV} = 4\,\mu\text{F} \times 2 = 8\,\mu\text{C}.$$

Q2.2. A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge

(*a*) remains a constant because the electric field is uniform.
(*b*) increases because the charge moves along the electric field.
(*c*) decreases because the charge moves along the electric field.
(*d*) decreases because the charge moves opposite to the electric field.

Main concepts used: (*i*) Uniform electric field. (*ii*) As K.E. increases, P.E. decreases. (*iii*) The direction of electric field is from higher to lower potential.

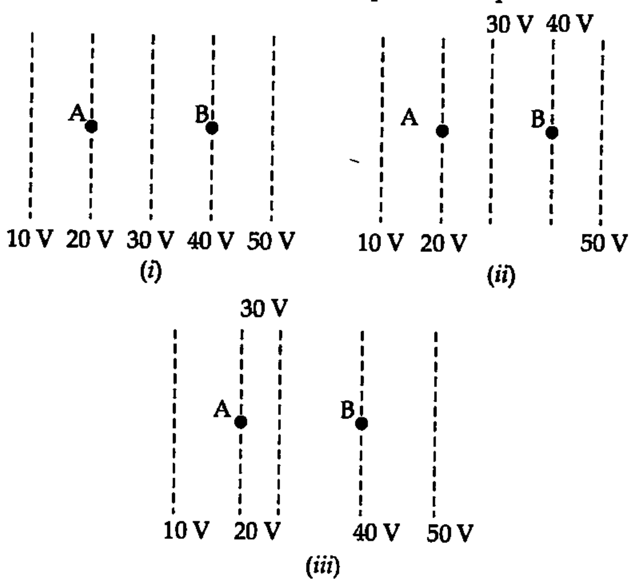
Ans. (*c*): Equipotential surface is always perpendicular to the direction of electric field.

Positive charge experiences the force in the direction of electric field.

When a positive charge is released from rest in uniform electric field, its velocity increases in the direction of electric field. So K.E. increases, and the P.E. decreases due to law of conservation of energy.

So P.E. of positively charged particle decreases because speed of charged particle moves in the direction of field due to force $q\vec{E}$.

Q2.3. Figures below show some equipotential lines distributed in space. A charged object is moved from point A to point B.



(*a*) The work done in figure (*i*) is the greatest.
(*b*) The work done in figure (*ii*) is least.
(*c*) The work done is the same in all figures (*i*), (*ii*) and (*iii*).
(*d*) The work done in figure (*iii*) is greater than figure (*ii*) but equal to that in figure (*i*).

Main concepts used: Work done in electric field by charge q.

$$\boxed{W = (V_2 - V_1)q}$$

Ans. (*c*): As the potential difference between A and B in all three figures are equal (20 V) so work done ($\Delta V.q$) by any charge in moving from A to B surface will be equal.

Q2.4. The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard:

S_1: At any point inside the sphere, electric intensity is zero.

S_2: At any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statement?

(*a*) S_1 is true but S_2 is false.

(*b*) Both S_1 and S_2 are false.

(*c*) S_1 is true, S_2 is also true and S_1 is the cause of S_2.

(*d*) S_1 is true, S_2 is also true, but the statements are independent.

Main concepts used: (*i*) Potential gradient, (*ii*) Electric field, (*iii*) Shielding (effect).

Ans. (*c*): The relation between electric field intensity E and potential (V) is $\boxed{E = -\frac{dV}{dr}}$.

Here, E = 0 inside the sphere then $\frac{dV}{dr} = 0$

i.e., V = constant.

E = 0 inside charged sphere, the potential is constant or V = 100 everywhere inside the sphere and it verifies the shielding effect also. Hence verifies the option (*c*).

Q2.5. Equipotentials at a great distance from a collection of charges, whose total sum is not zero, are approximately

(*a*) spheres (*b*) planes (*c*) paraboloids (*d*) ellipsoids

Main concepts used: (*i*) Equipotential surface, (*ii*) Properties of field lines, (*iii*) Properties of charges (*iv*) Potential, (*v*) Point charge.

Ans. (*a*): Here we have to find out the shape of equipotential surface. These surfaces are perpendicular to the field lines. So there must be electric field which cannot be without charge.

So the algebraic sum of all charges must not be zero. Equipotential surface at a great distance means that space of charge is negligible as compared to distance.

So the collection of charges is considered as a point charge.

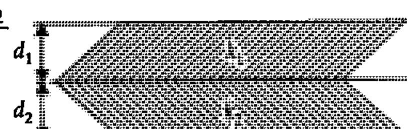
The lines of field from point charges are radial. So the equipotential surface (perpendicular to the field lines) form a sphere.

It verifies that (*a*) is the correct answer.

Q2.6. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant k_1 and the other has thickness d_2 and dielectric constant k_2 as shown in the figure. This arrangement can be thought as a dielectric slab of thickness $d(d_1 + d_2)$ and effective dielectric constant k. The k is

(*a*) $\frac{k_1d_1 + k_2d_2}{d_1 + d_2}$ (*b*) $\frac{k_1d_1 + k_2d_2}{k_1 + k_2}$

(*c*) $\frac{k_1k_2(d_1 + d_2)}{(k_1d_1 + k_2d_2)}$ (*d*) $\frac{2k_1k_2}{k_1 + k_2}$



Main concepts used: (*i*) Capacitance of a capacitor, (*ii*) Combination of capacitor.

Ans. (*c*): Capacitance of a parallel plate capacitor filled with dielectric of constant k_1 and thickness d_1 is $\boxed{C_1 = \frac{k_1\varepsilon_0 A}{d_1}}$.

Similarly for other, $\boxed{C_2 = \frac{k_2\varepsilon_0 A}{d_2}}$.

Both capacitors are in series so equivalent capacitance C is related as:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{k_1\varepsilon_0 A} + \frac{d_2}{k_2\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\left[\frac{k_2 d_1 + k_1 d_2}{k_1 k_2}\right]$$

So $$C = \frac{k_1 k_2 \varepsilon_0 A}{(k_1 d_2 + k_2 d_1)} \qquad \text{...I}$$

$$\boxed{C' = \frac{k\varepsilon_0 A}{d}} \qquad \text{...II}$$

where $d = (d_1 + d_2)$

So, multiply the numerator and denominator of eqn. I with $(d_1 + d_2)$,

$$C = \frac{k_1 k_2 \varepsilon_0 A}{(k_1 d_2 + k_2 d_1)} \cdot \frac{(d_1 + d_2)}{(d_1 + d_2)} = \frac{k_1 k_2 (d_1 + d_2)}{(k_1 d_2 + k_2 d_1)} \cdot \frac{\varepsilon_0 A}{(d_1 + d_2)} \qquad \text{...III}$$

Comparing eqns. II and III, the dielectric constant of new capacitor is:

$$k = \frac{k_1 k_2 (d_1 + d_2)}{(k_1 d_2 + k_2 d_1)}$$

It verifies that the correct answer is (*c*).

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q2.7. Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant

(*a*) in all space.
(*b*) for any *x* for a given z.
(*c*) for any *y* for a given z.
(*d*) on the *x*-*y* plane for a given z.

Main concepts used: (*i*) Equipotential surface, (*ii*) Electric field lines.

Ans. (*b*), (*c*) and (*d*): As we know that equipotential surfaces are perpendicular to the direction of electric field lines. Here electric field is in $+\hat{z}$ direction.

So, equipotential surfaces will be the plane perpendicular to z axis, *i.e.*, along *x*-*y*, plane, which includes any *x* or *y* axes. So answers (*b*), (*c*) and (*d*) are verified respectively.

Q2.8. Equipotential surfaces:

(*a*) are closer in regions of large electric fields compared to regions of lower electric fields.

(*b*) will be more crowded near sharp edges of a conductor.

(*c*) will be more crowded near the regions of large charge densities.

(*d*) will always be equally spaced.

Main concepts used: (*i*) Relation between electric field E and potential gradient, (*ii*) E.F. $\propto \sigma$, (*iii*) $\sigma = \frac{q}{A}$.

Ans. (*a*), (*b*) and (*c*): We know that on any two points of equipotential surface, potential difference is zero or of equal potential.

$\because$ $$E = \frac{-dV}{dr}$$

So the electric field intensity is inversely proportional to the separation between equipotential surfaces.

So equipotential surfaces are closer in regions of large electric fields. Thus, it verifies answer (*a*).

The electric field is larger near the sharp edge, due to larger charge density as A is very small.

$\because$ $$\sigma = \frac{q}{A}$$

So equipotential surfaces are closer or crowded. It verifies answer (*b*).

As the electric field $E = \frac{kq}{r^2}$ and potential or field decreases as size of body increases or vice-versa (case of earth), so the equipotential surfaces will be more crowded if the charge density $\sigma = \frac{q}{A}$ increases. It verifies the answer (*c*).

As the equipotential surface depends on distance *r* by $E = \frac{-dV}{r}$ and $V = \frac{kq}{r}$. Equipotential surface depends on charge density at that place which is different at different place, so equipotential surfaces are not equispaced all over.

Q2.9. The work done to move a charge along an equipotential surface from A to B

(*a*) cannot be defined as $-\int_A^B E.dl$ (*b*) must be defined as $-\int_A^B E.dl$

(*c*) is zero. (*d*) can have a non-zero value.

Main concepts used: (*i*) $W_{12} = (V_2 - V_1)q$, (*ii*) Equipotential surface.

Ans. (*c*): As the potential on equipotential surface does not change

so $(V_2 - V_1) = 0$

and $W = (V_2 - V_1)q.$

So, work done on moving a charge is zero, verifies answer (*c*).

We know the work done by charge *q* in moving in electric field

$$dW = F.dl$$

$$\int \quad = \int qE.dl \qquad [\because F = qE]$$

$$W = q.\int E.dl$$

So, $W \neq \int E.dl$ or answer (*b*) is wrong.

Answer (*a*) and (*b*) can be true only when $q = +1C$ which is not given in question.

Q2.10. In a region of constant potential

(*a*) the electric field is uniform.

(*b*) the electric field is zero.

(*c*) there can be no charge inside the region.

(*d*) the electric field shall necessarily change if a charge is placed outside the region.

Main concept used: Relation between E and V, *i.e.*, $E = \frac{-dV}{dr}$.

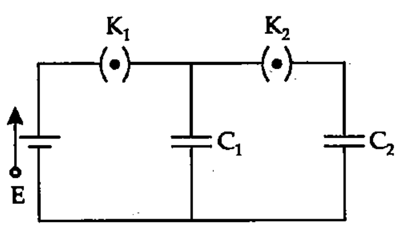
Ans. (*b*) (*c*): Constant potential $\Rightarrow dV = 0$ so by relation $E = \frac{-dV}{dr}$, $E = 0$

i.e., the E.F. is not uniform discards answer (*a*) and agree with answer (*b*).

As potential may be outside the charge also so there can be no charge inside the region of constant potential. It verifies answer (*c*).

If a charge is placed in outside region, potential difference in region will not be changed or electric field will not be changed. It makes answer (*d*) false.

Q2.11. In the circuit shown in figure, initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively].



Then

(*a*) charge on C_1 gets redistributed such that $V_1 = V_2$

(*b*) charge on C_1 gets redistributed such that $Q_1' = Q_2'$

(*c*) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$

(*d*) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

Main concepts used: (*i*) Law of conservation of charges, (*ii*) Potential in parallel combination is equal.

Ans. (*a*) (*d*): When K_1 is closed keeping K_2 open, the capacitor C_1 gets charged by battery of emf E. Now when K_1 opens, C_1 remains charged. When K_2 closes keeping K_1 open, C_2 gets charged by redistribution of charge of C_1 between C_1 and C_2.

Let charge on C_1, which is charged by battery, was Q then after redistribution of charge $Q = Q_1' + Q_2'$ by law of conservation of charge. So answer (*d*) is verified.

As C_1 and C_2 both are in parallel combination, so their potential will be equal, *i.e.*, $V_1 = V_2$. It verifies the answer (*a*).

Q2.12. If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then

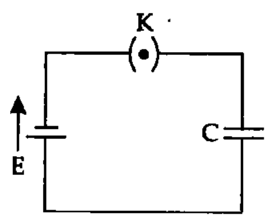
(*a*) there must be charges on the surface or inside itself.
(*b*) there cannot be any charges in the body of conductor.
(*c*) there must be charges only on the surface.
(*d*) there must be charges inside the surface.

Main concepts used: (*i*) The charge reside only on the surface of a conductor. (*ii*) Net charge inside the conductor is zero. (*iii*) Charged spherical shell.

Ans. (*a*) (*b*): As the excess charge can reside only on the surface of conductor and inside net positive and negative charge is zero. Any charge can reside inside the hollow shell or body. So verifies answer (*a*) and discards answer (*c*).

Inside the solid material of conducting body there is no charge, it comes to outer surface. So verifies answer (*b*) and discards answer (*d*).

Q2.13. A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations:



A: Key *K* is kept closed and plates of capacitors are moved apart using insulating handle.

B: Key *K* is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s):

(*a*) In A: Q remains same but C changes.
(*b*) In B: V remains same but C changes.
(*c*) In A: V remains same and hence Q changes.
(*d*) In B: Q remains same and hence V changes.

Main concepts used: (*i*) $C = \dfrac{k\varepsilon_0 A}{d}$, (*ii*) $Q = CV$

Ans. (*c*) (*d*): **(A) In situation A:** When the space between the plates of capacitor increases, the *capacitance decreases* by relation $C = \frac{k\varepsilon_0 A}{d}$, but battery remains same, *i.e.*, potential difference across plate remains 'V' same. So by Q = CV relation, Q also decreases verifies answer (*c*) and discards answer (*a*).

(B) Now for situation B: K is open and capacitance decreases by moving apart plates of capacitor, so by relation Q = CV, here K is open so charge Q remains same in turn V will increase on decreasing C hence answer (*d*) is verified.

VERY SHORT ANSWER TYPE QUESTIONS

Q2.14. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

Main concepts used: (*i*) $\sigma = \frac{q}{A}$, (*ii*) $V = \frac{kq}{r}$.

Ans. We know that $V_1 = \frac{kq_1}{R_1}$ and $V_2 = \frac{kq_2}{R_2}$.

As $V_1 = V_2$, so:

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \qquad \text{(Multiply by } \frac{1}{4\pi} \text{ on both sides)}$$

$$\frac{R_1}{R_1} \times \frac{kq_1}{4\pi R_1} = \frac{kq_2}{4\pi R_2} \times \frac{R_2}{R_2}$$

$$\frac{q_1 R_1}{4\pi R_1^2} = \frac{q_2 R_2}{4\pi R_2^2}$$

$$\frac{q_1}{A_1} R_1 = \frac{q_2}{A_2} R_2$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

$$\sigma_1 < \sigma_2 \qquad (\because R_1 > R_2)$$

So charge density of smaller sphere (R_2) will be larger than larger sphere (R_1).

Q2.15. Do free electrons travel to a region of higher potential or lower potential?

Main concepts used: Current (or positive charge) flows from higher to lower potential.

Ans. As free electrons has negative charge so the direction of flow will be opposite to positive charge, *i.e.*, free electrons will move from lower potential to higher potential.

Q2.16. Can there be a potential difference between two adjacent conductors carrying the same charge?

Main concepts used: (*i*) V = IR, (*ii*) $R = \rho\frac{l}{A}$.

Ans. If in two conductors flowing current is same then both may be considered in series. So Ohm's law becomes $V \propto R$. *i.e.,* if the resistances (which depends on ρ, *l* and A) are different then potential difference will be different.

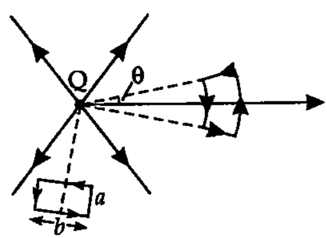
So there can be potential difference between two adjacent conductors carrying the same charge or current if either their length or area of cross-section (A) and resistivity are different.

Q2.17. Can the potential function have a maximum or minimum in free space?

Main concepts used: Electric field, potential difference.

Ans. In the absence of free space or atmosphere, the phenomenon of electric field or potential leakage cannot be prevented. Hence, the potential function do not have maximum or minimum in free space.

Q2.18. A test charge *q* is made to move in the electric field of a point charge Q along two different closed paths as in given figure. First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



Main concepts used: Work done in electrostatic force is conservative.

Ans. We know that electrostatic work done is conservative. So work done in closed loop is always zero, it does not depend on the nature of closed path.

SHORT ANSWER TYPE QUESTIONS

Q2.19. Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

Main concepts used: (*i*) Electric field lines are perpendicular to equipotential surface, (*ii*) $E = \frac{-dV}{dr}$.

Ans. Let us consider that inside the enclosed equipotential surface, potential is not same. Let the potential just inside the equipotential surface is different to that on the equipotential surface, causing in a potential gradient $\frac{dV}{dr} = E$. So electric field will exist inside surface which is equal to $E = -\frac{dV}{dr}$.

The field lines pointing inward or outward from the surface are perpendicular to equipotential surfaces or the field lines cannot be on the equipotential surface. The field lines can be on the equipotential surface if field lines can originate from the charge inside, which contradicts the original assumption. Hence, the entire volume inside equipotential surface has no charge.

Q2.20. A capacitor has some dielectric between its plates, and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

Main concepts used: (*i*) $C = \frac{k\varepsilon_0 A}{d}$, (*ii*) Q = CV, (*iii*) $E = \frac{1}{2}\frac{q^2}{C}$, (*iv*) $E = \frac{V}{d}$.

Ans. $\because \quad C = \frac{k\varepsilon_0 A}{d}$

As k is positive and more than one, so by removing dielectric slab, and keeping A and d constant, capacitance of *capacitor* will decrease.

When battery and dielectric slab from capacitor is removed, the charge remains same as it was when battery connected earlier.

As the energy stored in capacitor is $\frac{q^2}{2C}$. When capacitance C is decreased by removing dielectric slab but q remains same, so the **energy stored in capacitor will increase.**

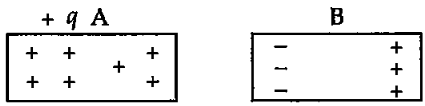
We know that $V = \frac{q}{C}$, where q is same and C is decreased so **potential will increase.**

As $E = \frac{V}{d}$, distance between plates of capacitor is same and potential is increased as discussed above, so ***electric field* between the plates of capacitor will increase.**

Q2.21. Prove that, if an insulated uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

Main concept used: $V = \frac{kq}{r}$.

Ans. Consider a charged body (A) (say with positive charge) and an insulated uncharged conductor (B) is placed near the charged conductor (A) as shown in the figure:



As $V = \frac{kq}{r}$ where k and q are constants, so

$$V \propto \frac{1}{r} \text{ or at infinity, } V \to 0$$

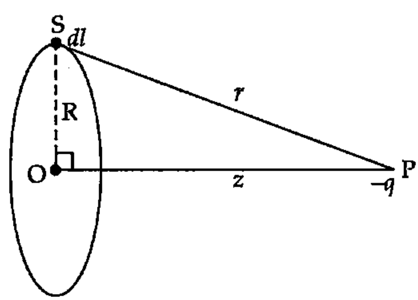
Uncharged conductor is between charged conductor and infinity, so potential decreases from body A to infinity.

So the potential of uncharged body varies between potential of A and infinity.

Q2.22. Calculate the potential energy of a point charge $-q$ placed along the axis due to a charge +Q uniformly distributed along a ring of radius R. Sketch P.E. as a function of axial distance Z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis)?

Main concepts used: (*i*) $V = \frac{kq}{r}$, (*ii*) $PE = V \cdot q$.

Ans. Let us consider a ring of radius R having charge +Q distributed uniformly over the ring. Also a point P at distance z on its axis passing through centre O and perpendicular to plane of ring.



Again consider an element of ring at S of length dl having charge dq and SP is equal to r. Then potential energy due to element dl at P. If dq is charge on element dl of ring

$dV = \frac{-kdq}{r}$, where $k = \frac{1}{4\pi\varepsilon_0}$ and as Q is positive charge so potential due to dq charge will be negative.

Charge on $2\pi R$ length of ring = Q

Charge on dl length of ring $dq = \frac{Q}{2\pi R} dl$

So potential due to element dl at P

$$dV = \frac{-k \cdot Q \cdot dl}{2\pi R r}$$

$$\because dW = dV.q \quad \text{and} \quad r = \sqrt{R^2 + z^2}$$

So $$dW = \frac{-kQq dl}{2\pi R\sqrt{R^2 + z^2}}$$

Integrating both sides, over a ring, we have

$$\int_0^W dW = -\int_0^{2\pi R} \frac{kqQdl}{2\pi R\sqrt{R^2 + z^2}}$$

$$W = -\frac{kqQ\, 2\pi R}{2\pi R\, R\sqrt{1 + \frac{z^2}{R^2}}}$$

This work done converts into P.E. at P, so

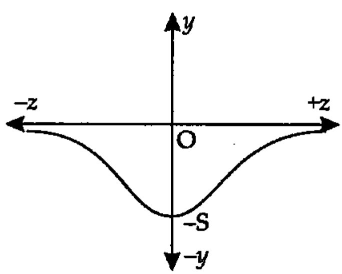
$$\text{P.E., } V = \frac{-Qq}{4\pi\varepsilon_0 R\sqrt{1 + \frac{z^2}{R^2}}}$$

Let $\boxed{\frac{Qq}{4\pi\varepsilon_0 R} = S}$ (a new constant)

$$V = \frac{-S}{\left[1 + \frac{z^2}{R^2}\right]^{1/2}}$$

at $z = -\infty$

$$V_z = \frac{-S}{\left(1 + \frac{z^2}{R^2}\right)^{1/2}}$$



$\because$ $z >>> R$

$\therefore$ $z^2 >>> R^2$

$$\left(1 + \frac{z^2}{R^2}\right)^{1/2} = \infty$$

$\therefore$ $V_{-z} = \frac{-S}{\infty} \to 0$

$V_{-z} \to 0$

$V_{+z} \to 0$

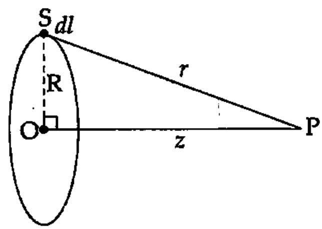
at $z = 0$

$$V = -S$$

Q2.23. Calculate the potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Main concept used: $V = \frac{kq}{r}$.

Ans. Let us consider a ring of radius R having charge +Q distributed uniformly. Also a point P at distance z on its axis passing through centre O and perpendicular to plane of ring.



Again consider an element of ring at S of length dl having charge dq and SP is equal to r. Then potential energy due to element dl at P, $dV = \frac{-kdq}{r}$ where $k = \frac{1}{4\pi\varepsilon_0}$

Charge on $2\pi R$ length of ring = Q

Charge on dl length of ring $= \frac{Q}{2\pi R} dl$

So potential due to element dl at P,

$$dV = \frac{-k.Q.dl}{2\pi R r}$$

Integrating over a ring the potential at P, V_P

$$\int_0^V dV_p = \int_0^{2\pi R} \frac{kQdl}{2\pi Rr} \quad \text{where } r = \sqrt{R^2 + z^2}$$

$$V_P = \frac{kQ2\pi R}{2\pi R\sqrt{R^2 + z^2}} = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2 + z^2}}.$$

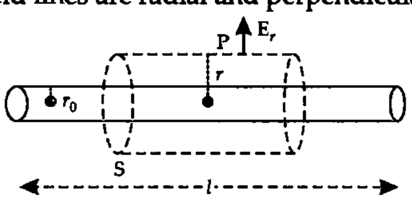
LONG ANSWER TYPE QUESTIONS

Q2.24. Find the equation of the equipotentials for an infinite cylinder of radius r_0, carrying charge of linear density λ.

Main concepts used: (*i*) Gauss's law of electrostatics, (*ii*) Properties of electric field lines, (*iii*) Drawing of proper Gaussian surface, (*iv*) $E = \frac{-dV}{dr}$.

Ans. Consider a Gaussian cylindrical dotted surface, S at a distance r from the centre of the cylinder of radius r_0 of infinite length.

The electric field lines are radial and perpendicular to the surface.



Let electric field intensity on Gaussian surface at P is E_r and total charge q on cylinder will be $q = \lambda l$.

So, by Gauss's law,

$$\oint_S E_r ds = \frac{\lambda l}{\varepsilon_0} \Rightarrow \left[E_r s \cos\theta\right]_0^{2\pi l} = \frac{\lambda l}{\varepsilon_0}$$

$$E_r 2\pi r l \cos 90^\circ = \frac{\lambda l}{\varepsilon_0}$$ [$\angle\theta$ is between E_r and curved surface of dotted cylinder is 90°]

$$E_r = \frac{\lambda}{2\pi r\varepsilon_0}$$

We know that electric field E_r at distance r from centre of cylinder $E_r = \frac{-dV}{dr}$.

So potential difference d at distance r_0 and r from the centre of cylinder,

$$dV = -E_r . dr \qquad \left[\because E = \frac{-dV}{dr}\right]$$

$$V(r) - V(r_0) = -\int E_r . dr$$

$$= -\int_{r_0}^{r} \frac{\lambda}{2\pi\varepsilon_0 r} \cdot dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{dr}{r} = \frac{-\lambda}{2\pi\varepsilon_0}[\log_e r]_{r_0}^{r}$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}[\log_e r - \log_e r_0] = \frac{-\lambda}{2\pi\varepsilon_0}\log_e\frac{r}{r_0}$$

$$\log_e\frac{r}{r_0} = \frac{-2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

$$\frac{r}{r_0} = e^{\frac{-2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]}$$

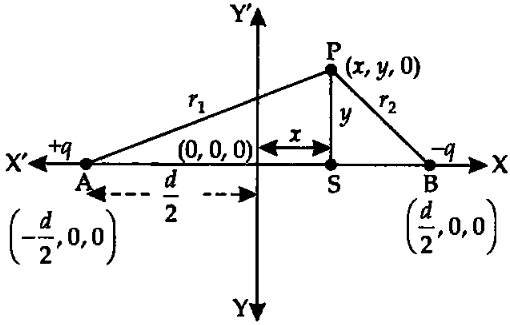
$$r = r_0\, e^{\frac{-2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]}$$

So equipotential surfaces are the coaxial curved surfaces of cylinders with given cylinder of radius r related as above.

Q2.25. Two point charges of magnitudes $+q$ and $-q$ are placed at $\left(-\frac{d}{2}, 0, 0\right)$ and $\left(\frac{d}{2}, 0, 0\right)$ respectively. Find the equation of the equipotential surface where the potential is zero.

Main concepts used: Net potential at a point is equal to the vector sum of all potentials due to different charges in the system and $V = \frac{kq}{r}$.

Ans. The potential due to charges $+q$ and $-q$ will be zero in between the line joining the two charges $+q$ and $-q$. Let zero potential is at S.



Then equipotential surface will pass through S and perpendicular to line joining two chargers or AB.

So
$$r_1^2 = AS^2 + SP^2$$
$$= \left(x + \frac{d}{2}\right)^2 + y^2$$
$$r_1 = \sqrt{\left(x + \frac{d}{2}\right)^2 + y^2}$$

Similarly,
$$r_2 = \sqrt{\left(x - \frac{d}{2}\right)^2 + y^2}$$

So net potential at P = 0

$$\frac{kq}{r_1} + \frac{k(-q)}{r_2} = 0 \qquad \text{where, } k = \frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow \quad kq\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 0 \qquad [\because kq \neq 0]$$

$$\Rightarrow \quad \frac{1}{r_1} - \frac{1}{r_2} = 0$$

$$\Rightarrow \quad \frac{1}{r_1} = \frac{1}{r_2} \quad \Rightarrow \quad r_1 = r_2$$

$$\Rightarrow \quad \left(x + \frac{d}{2}\right)^2 + y^2 = \left(x - \frac{d}{2}\right)^2 + y^2$$

$$\Rightarrow \quad \left(x + \frac{d}{2}\right)^2 = \left(x + \frac{d}{2}\right)^2$$

$$\Rightarrow \quad x^2 + \frac{d^2}{4} + dx = x^2 + \frac{d^2}{4} - dx$$
$$2dx = 0$$

$$2d \neq 0$$

$$\therefore \quad x = 0$$

So equipotential surface will be perpendicular to X-axis passing through $x = 0$ *i.e.*, origin in Y-Z plane.

Q2.26. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $\alpha = 2\ V^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78$ V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.9650852605

Main concepts used: (*i*) $Q = CV$, (*ii*) $C = \varepsilon C_0$.

Ans. Let C be the capacitance of capacitor C_1 without dielectric then charge $q_1 = CU$ where U is the final potential of C_1 when connected to C_2 the capacitor filled with dielectric ε_0

$$C_2 = \varepsilon C$$

$$q_2 = \varepsilon CU = \alpha UCU = \alpha CU^2$$

Initial charge q_0 of C_1 when charged at potential of $U_0 = 78$ V is,

$$q_0 = CU_0 = 78\ C$$

By the law of conservation of charge

$$q_0 = q_1 + q_2$$

$$78\ C = CU + \alpha.CU^2$$

$$78 = U + \alpha U^2 \qquad [\alpha = 2 \text{ per volt}]$$

$$\therefore \quad = U + 2U^2$$

$$\text{or} \quad 2U^2 + U - 78 = 0$$

$$U = \frac{-1 \pm \sqrt{1 - 4.2(-78)}}{2.2} = \frac{-1 \pm \sqrt{1 + 624}}{4}$$

$$= \frac{-1 \pm \sqrt{625}}{4} = \frac{-1 \pm 25}{4}$$

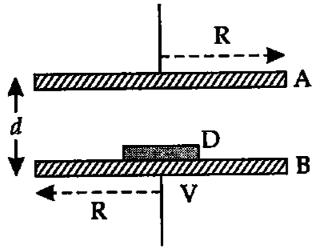
$$= \frac{-1 + 25}{4} \quad \text{as U is positive}$$

$$= \frac{24}{4} = 6 \text{ Volts.}$$

Final potential on both the capacitors becomes 6 Volts.

Q2.27. A capacitor is made of two circular plates of radius R each separated by a distance $d \ll R$. The capacitor is connected to a constant voltage. A thin conducting disc of radius $r \ll R$ and thickness 't' $\ll r$ is placed at the centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.

Main concepts used: (*i*) $E = \frac{-dV}{dr}$, (*ii*) Electrostatic force is balanced by weight, (*iii*) Electrostatic force $F = qE$.



Ans. Let A and B are circular plates of radius R separated by distance $d \ll R$ kept horizontally.

A thin conducting disc D of radius $r \ll R$ of thickness 't' is placed concentrically on lower plate B as shown in figure.

Let plates A and B charged with potential V.

The magnitude of electric field E between plates of capacitor

$$E = \frac{V}{d} \qquad \left[\because E = \frac{-dV}{dr}\right]$$

Consider Gaussian surface along circular disc D.

By Gauss's law, $\oint E.ds = \frac{q'}{\varepsilon_0}$

$$\frac{V}{d} \cdot \pi r^2 = \frac{q'}{\varepsilon_0}$$

q' is the charge conducted by plate B to disc D during charging. Nature of charge on plate B and disc will be same so repulsive force acts between B and D.

So, the charge on disc $D = q' = \frac{V}{d}\pi r^2 \varepsilon_0$

Electrostatic repulsive force acting on disc in upward direction

$$F = q'E = \frac{V}{d} \cdot \pi r^2 \varepsilon_0 \cdot \frac{V}{d} = \frac{V^2}{d^2} \pi r^2 \varepsilon_0$$

This repulsive force will be balanced by weight mg of disc D.

$$mg = \frac{V^2}{d^2} \pi r^2 \varepsilon_0$$

$$V^2 = \frac{mg\, d^2}{\pi r^2 \varepsilon_0}$$

So minimum voltage V to lift the disc

$$V = \sqrt{\frac{mg\, d^2}{\pi r^2 \varepsilon_0}}$$

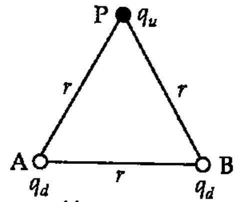
Q2.28. (*a*) In a quark model of elementary particles, a neutron is made of **one up quark** $\left[\text{charge}\left(\frac{2}{3}\right)e\right]$ and **two down quarks** $\left[\text{charges}\left(-\frac{1}{3}\right)e\right]$. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate the electrostatic potential energy of a neutron and compare it with its mass 939 MeV.

(*b*) Repeat the above exercise for a proton which is made of two up and one down quark.

Main concepts used: The potential energy is equal to the sum of potential energy or energies required to form the configuration, if charge particles are carried with zero acceleration, from infinity to that point.

Ans. (*a*) $q_d = -\frac{1}{3}e$ [charge on down quark]

$q_u = +\frac{2}{3}e$ [charge on up quark]



(A neutron)

Potential energy $U = \frac{kq_1q_2}{r}$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$U = \frac{kq_1q_2}{r} + \frac{kq_1q_3}{r} + \frac{kq_2q_3}{r}$$

$$\therefore \quad U_n = \frac{1}{4\pi\varepsilon_0}\frac{(-q_d)(-q_d)}{r} + \frac{(-q_d)q_u}{4\pi\varepsilon_0 r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$$

$$= \frac{q_d}{4\pi\varepsilon_0 r}[+q_d - q_u - q_u] \quad \text{(Taking sign of charge)}$$

$$= \frac{q_d}{4\pi\varepsilon_0 r}[q_d - 2q_u] = \frac{9\times10^9\times\frac{1}{3}e}{10^{-15}}\left[\frac{1}{3}e - 2\cdot\frac{2}{3}e\right]$$

[nature sign of charges taken already]

$$= \frac{9\times10^9\times e}{3\times10^{-15}}\cdot\frac{e}{3}[1-4] \text{ Joule}$$

$$= \frac{-3\times9\times10^9\times1.6\times10^{-19}}{9\times10^{-15}}e \text{ Joule}$$

$$= -4.8\times10^{9-19+15}\,eV = -4.8\times10^5\,eV = -0.48\times10^6\,eV$$

$$U = -0.48 \text{ MeV}$$

So charges inside neutron [$1q_u$ and $2q_d$] are attracted by energy of 0.48 MeV.

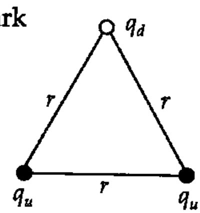
Energy released by a neutron when converted into energy is 939 MeV.

$$\therefore \text{Required ratio} = \frac{1-0.481 \text{ MeV}}{939 \text{ MeV}} = 0.0005111 = \mathbf{5.11\times10^{-4}}$$

(*b*) P.E. of proton consists of 2 up and 1 down quark

$r = 10^{-15}$ m

$q_d = -\frac{1}{3}e,\quad q_u = \frac{2}{3}e$



$$U_p = \frac{1}{4\pi\varepsilon_0}\frac{q_u\times q_u}{r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$$

$$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - q_d - q_d]$$

$$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - 2q_d] = \frac{9\times10^9}{10^{-15}}\frac{2}{3}e\left[\frac{2}{3}e - 2\cdot\frac{1}{3}e\right] = 0.$$

Q2.29. Two metal spheres, one of radius R and the other of radius 2R, both have same surface charge density '*s*'. They are brought in contact and separated. What will be new surface charge densities of them?

Main concepts used: (*i*) $Q = \sigma.A$, (*ii*) $V = \frac{kq}{R}$.

Ans. Let surface charge density of both the spheres are σ and their charges are q_1 and q_2.

$\therefore$
$$q_1 = \sigma.A_1 = \sigma.4\pi R^2$$
$$q_2 = \sigma A_2 = \sigma.4\pi(2R)^2 = \sigma.4\pi R^2.4 = 4q_1$$

Both charged spheres are kept in contact, so charge flows between them and their potential becomes equal, let the charges on them now become q_1' and q_2'.

So, $V_1 = V_2$ $\quad\left(\because V = \frac{kq}{r}\right)$

So $\frac{kq_1'}{R} = \frac{kq_2'}{(2R)}$ $\quad\left[\because k = \frac{1}{4\pi\varepsilon_0}\right]$

Where q_1' and q_2' are the charges on spheres after redistribution of charges

$$\frac{q_1'}{R} = \frac{q_2'}{2R}$$

$\therefore$ $q_2' = 2q_1'$ ...I

By law of conservation of charges

$$q_1 + q_2 = q_1' + q_2'$$
$$q_1 + 4q_1 = q_1' + 2q_1' \quad \text{(from I)}$$
$$5q_1 = 3q_1'$$
$$3q_1' = 5.4.\sigma.\pi R^2$$
$$q_1' = \frac{20}{3}\pi R^2\sigma$$

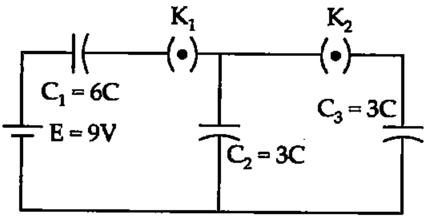
$$\sigma_1 = \frac{q_1'}{A_1} = \frac{\frac{20}{3}\pi R^2\sigma}{4\pi R^2} = \frac{5}{3}\sigma$$

$$\sigma_2 = \frac{q_2'}{A_2} = \frac{2.q_1'}{4.\pi(2R)^2} = \frac{2.\frac{20}{3}\pi R^2\sigma}{4\pi.4R^2} = \frac{5}{6}\sigma$$

Hence, $\sigma_1 = \frac{5}{3}\sigma$ and $\sigma_2 = \frac{5}{6}\sigma$

Q2.30. In the circuit in given figure initially K_1 is closed and K_2 is open. What are the charges on each capacitor?

Then K_1 was opened and K_2 was closed (order is important). What will be the charge on each capacitor now?



[C = 1 μF]

Main concepts used: (*i*) $V = \frac{q}{C}$, (*ii*) Law of conservation of charge, (*iii*) In series combination charges on capacitors are equal.

Ans. When K_2 is open and K_1 is closed the capacitors C_1 and C_2 will charge and potential develops across them *i.e.*, V_1 and V_2 respectively which will be equal to the potential of battery 9 V.

$$\therefore \quad V_1 + V_2 = 9 \quad \text{...I}$$

$$\because \quad V = \frac{q}{C} = \quad \text{or} \quad V \propto \frac{1}{C}$$

$$\text{or} \quad \frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$\frac{V_1}{V_2} = \frac{3C}{6C}$$

$$3V_2 = 6V_1$$

$$V_2 = 2V_1 \quad \text{...II}$$

From Eqns. I and II,

$$V_1 + 2\,V_1 = 9$$

$$3\,V_1 = 9$$

$$V_1 = 3 \text{ Volt}$$

$$V_2 = 2 \times 3 \text{ Volt} = 6 \text{ Volt} \quad \begin{bmatrix}\text{(from II)} \\ C = 1\,\mu F\end{bmatrix}$$

$$\therefore \quad q_1 = C_1V_1 = 6C \times 3 = 18C$$

$$= 18 \times 1\,\mu F = 18\,\mu C$$

$$q_2 = C_2V_2 = 3C \times 6$$

$$= 3 \times 1\,\mu F \times 6 = 18\,\mu C$$

So, charges on each capacitor i.e., $q_1 = q_2 = 18\,\mu C$

When K_1 is open and K_2 is closed then charge q_2 will be distributed among C_2 and C_3. Let it be q_2' and q_3.

$$\therefore \quad q_2 = q_2' + q_3$$

As C_2 and C_3 are now in parallel combination so their potentials remain same (V)

$$\therefore \quad q_2 = C_2V + C_3V$$

$$18\ \mu C = 3 \times 1\ \mu F \times V + 3 \times 1\ \mu F \times V$$
$$18 = 6\ V$$
$$V = 3\ \text{Volt.}$$

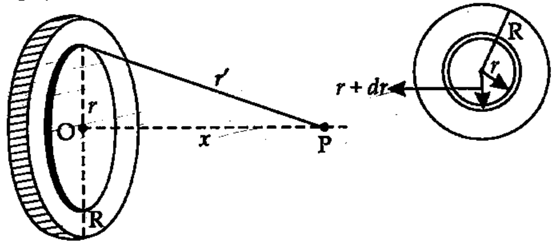
So potential on C_2 and C_3 capacitors are 3 Volt each

$$\left.\begin{aligned} q_2' &= C_2 V = 3 \times 1\ \mu F \times 3\ \text{Volt} = 9\ \mu C \\ q_3 &= C_3 V = 3 \times 1\ \mu F \times 3\ \text{Volt} = 9\ \mu C \end{aligned}\right] \textbf{Ans.}$$

Q2.31. Calculate the potential on the axis of a disc of radius R due to a charge Q, uniformly distributed on its surface.

Main concepts used: A disc can be considered as the combination of rings of different radii and V for ring $= \frac{kq}{r}$ where, r is the distance of axial point from ring.

Ans. Consider a point P on the axis perpendicular to the plane of disc and at distance x from the centre O of disc as shown in the figure.



Now consider a ring of radius r of thickness dr on disc of radius R, as shown in figure. Again let the charge on the ring is dq then potential dV due to ring at P, will be,

$$dV = \frac{k\,dq}{r'} \qquad [\because r' = \sqrt{r^2 + x^2}]$$

dq is the charge on the ring = $\sigma \cdot$area of ring

$$= \sigma \cdot [\pi(r + dr)^2 - \pi r^2]$$
$$dq = \sigma \cdot \pi[r^2 + dr^2 + 2rdr - r^2]$$

Because dr is small therefore, dr^2 is negligible.

$$\therefore \qquad dq = \sigma\pi(2rdr) = 2\pi r\sigma \cdot dr$$

$$\therefore \qquad dV = \frac{k.2\pi r\sigma dr}{\sqrt{(r^2 + x^2)}}$$

So the potential due to charged disc

$$\int_0^V dV = \int_0^R \frac{k2\pi r\sigma dr}{\sqrt{r^2 + x^2}}$$

$$V = k.2\pi\sigma \cdot \int_0^R \frac{rdr}{(r^2 + x^2)^{1/2}} = 2\pi k\sigma \int_0^R r \cdot (r^2 + x^2)^{1/2} dr$$

$$= 2\pi k\sigma[(R^2 + x^2)^{1/2} - x] = \frac{2\pi\sigma}{4\pi\varepsilon_0}[(R^2 + x^2)^{1/2} - x]$$

[$\because \pi R^2\sigma = Q$ (charge on disc)

$\sigma = \frac{Q}{\pi R^2}$]

$$= \frac{2\pi R^2\sigma}{4\pi\varepsilon_0 R^2}\left[\sqrt{R^2 + x^2} - x\right]$$

$$V = \frac{2Q}{4\pi\varepsilon_0 R^2}\left[\sqrt{R^2 + x^2} - x\right]$$

Q2.32. Two charges q_1 and q_2 are placed at (0, 0, d) and (0, 0, $-d$) respectively. Find the locus of points where the potential is zero.

Main concepts used: Where the net potential due to different charges are zero. $V = \frac{kq}{r}$.

Ans. Let the potential at any point P(x, y, z) is zero then—

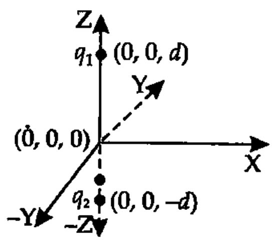
$$V_1 + V_2 = 0$$

$$\frac{kq_1}{\sqrt{x^2 + y^2 + (z-d)^2}} + \frac{kq_2}{\sqrt{x^2 + y^2 + (z+d)^2}} = 0$$

$$\frac{q_1}{\sqrt{x^2 + y^2 + (z-d)^2}} + \frac{q_2}{\sqrt{x^2 + y^2 + (z+d)^2}} = 0$$

$$\frac{q_1}{\sqrt{x^2 + y^2 + (z-d)^2}} = \frac{-q_2}{\sqrt{x^2 + y^2 + (z+d)^2}}$$

$$\frac{q_1}{q_2} = \frac{-\sqrt{x^2 + y^2 + (z-d)^2}}{\sqrt{x^2 + y^2 + (z+d)^2}}$$



$$\frac{q_1^2}{q_2^2} = \frac{x^2 + y^2 + z^2 + d^2 - \ \ zd}{x^2 + y^2 + z^2 + d^2 + \ \ zd}$$

Componendo and dividendo

of $\frac{x}{a} = \frac{y}{b}$ is $\frac{x+a}{x-a} = \frac{y+b}{y-b}$

Then componendo and dividendo of

$$\frac{\left(\frac{q_1}{q_2}\right)^2}{1} = \frac{x^2 + y^2 + z^2 + d^2 - 2zd}{x^2 + y^2 + z^2 + d^2 + 2dz}$$

$$\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}=\frac{x^2+y^2+z^2+d^2-2dz+(x^2+y^2+z^2+d^2+2dz)}{x^2+y^2+z^2+d^2-2dz-(x^2+y^2+z^2+d^2+2dz)}$$

$$\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]=\frac{2(x^2+y^2+z^2+d^2)}{-4dz}$$

$$x^2+y^2+z^2+d^2=-2dz\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]$$

$$x^2+y^2+z^2+2d\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]z+d^2=0$$

$$x^2+y^2+z^2+2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]z+d^2=0$$

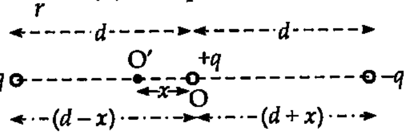
This is the equation of sphere with centre (a, b, c) as required point is on z axis so $a = 0$, $b = 0$ and $z = 2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]$

$$\left(0, 0, -\ 2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]\right).$$

Q2.33. Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid-point 'O'. Find the potential energy of $+q$ as a function of small distance x from 'O' due to $-q$ charges. Sketch P.E. v/s x and convince yourself that the charge at O is in an unstable equilibrium.

Main concepts used: (*i*) P.E. $U=\frac{kq_1q_2}{r}$, (*ii*) At equilibrium, $F = 0$ or

$$\frac{dF}{dx}\cdot dx=0 \Rightarrow \frac{dU}{dx}=0$$



Ans. $V=\frac{kq}{r}$

Let equilibrium of $+q$ is at P at a distance x from mid-point of line joining two charges.

Force F_A on $+q$ is towards left side and force F_B is towards right side, so for equilibrium of $+q$ at P,

$$F_A = F_B$$

$$\frac{-kq^2}{(d-x)^2} = \frac{-kq^2}{(d+x)^2}$$

$$\therefore \quad (d-x)^2 = (d+x)^2$$

$$d - x = d + x \qquad \text{(Taking square root)}$$

$$-2x = 0$$

$$x = 0$$

So, equilibrium position of charge $+q$ between two $-q$ charges is at mid-point (O) of line joining the two charges $(-q)$ and $(-q)$.

Now we have to find out potential energy of $+q$ as a function of small distance x from balance condition (O) towards any of $(-q)$ charge.

Let new position of charge $(+q)$ from a small distance x from (O)

$$U = \frac{k.(q)(-q)}{(d-x)} + \frac{k(q)(-q)}{(d+x)} \qquad \left(\because U = \frac{kq_1q_2}{r_1}\right)$$

$$= -kq^2\left[\frac{1}{(d-x)} + \frac{1}{(d+x)}\right]$$

$$= -kq^2\left[\frac{d+x+d-x}{(d-x)(d+x)}\right] = -kq^2\left[\frac{2d}{d^2-x^2}\right]$$

$$\boxed{U = \frac{-q^2}{4\pi\varepsilon_0} \cdot \frac{2d}{(d^2-x^2)}}$$

So, U is the P.E. as a function of x.

x	U
0	$\frac{-2k}{d}q^2$
$\frac{d}{2}$	$\frac{4}{3}\left(\frac{-2kq^2}{d}\right)$
$-\frac{d}{2}$	$\frac{4}{3}\left(\frac{-2kq^2}{d}\right)$
$+d$	$-\alpha$
d	$-\alpha$