3 Current Electricity

MULTIPLE CHOICE QUESTIONS—I

Q3.1. Consider a current carrying wire (current I), in the shape of a circle. Note that as the current progresses along the wire, the direction of J (current density) changes in an exact manner, while the current I remains unaffected. The agent that is essentially responsible for it is:

(*a*) source of emf.

(*b*) electric field produced by the charges accumulated on the surfaces of wire.

(*c*) the charges just behind a given segment of the wire which push them just the right way by repulsion.

(*d*) the charges ahead.

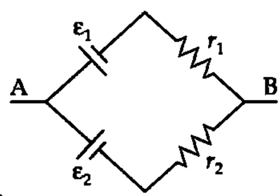
Main concepts used: $\vec{J} = \sigma\vec{E}, J = \frac{I}{A}, \sigma = \frac{1}{\rho} = \frac{l}{RA}$.

Ans. (*b*): Current density (J) depends on conductivity $\sigma = \frac{1}{\rho} = \frac{l}{R.A}$,

Electric field (J = σE), current and length and area of cross-section.

In our options only E *i.e.*, electric field can be varied by the charges accumulated on the surface of wire.

Q3.2. Two batteries of emf ε_1 and ε_2 ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.



(*a*) The equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$.

(*b*) The equivalent emf ε_{eq} is smaller than ε_1.

(*c*) The equivalent emf is given by

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 \text{ always.}$$

(*d*) ε_{eq} is independent of internal resistances r_1 and r_2.

Main concept used: Combination of cells in parallel

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Ans. (*a*): We know that equivalent emf ε_{eq} in parallel combination of cells is:

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Clearly, part '*c*' and '*d*' are discarded by formula. This formula suggests that $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$. So verifies answer (*a*).

Q3.3. The resistance R is to be measured using a meter bridge. Student chooses the standard resistance S to be 100 Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to improve the accuracy. Which of the following is a useful way?

(*a*) He should measure l_1 more accurately.
(*b*) He should change S to 1000 Ω and repeat the experiment.
(*c*) He should change S to 3 Ω and repeat the experiment.
(*d*) He should give up hope of a more accurate measurement with a meter bridge.

Main concept used: $\frac{R}{S} = \frac{l_1}{100 - l_1}$

Ans. (*c*): R is the unknown resistance

$$\Rightarrow \qquad R = S\left(\frac{l_1}{100 - l_1}\right) = 100\left[\frac{2.9}{97.1}\right] = 2.98\ \Omega.$$

So to get balance point near to 50 cm (middle) we should take S = 3 Ω, as here R : S = 2.9 : 97.1 implies that S is nearly 33 times to R. In order to make ratio R and S = 1 : 1, we must take the resistance S = 3 Ω, verifies answer (*c*).

Q3.4. Two cells of emf approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.

(*a*) The battery that runs the potentiometer should have voltage of 8 V.
(*b*) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V.
(*c*) The first portion of 50 cm of wire itself should have a potential drop of 10 V.
(*d*) Potentiometer is usually used for comparing resistances and not voltage.

Main concept used: Potential drop across the potentiometer wire should be slightly more than the emf of primary cell which is to be measured.

Ans. (*b*): Here, emf of primary cells are 5 V and 10 V. So the potential drop across potentiometer wire must be slightly more than that larger emf 10 V. So the battery should be of 15 V and about 4 V potential is dropped by using rheostat or resistances. So verifies answer (*b*).

Q3.5. A metal rod of length 10 cm and a rectangular cross-section of $1\text{ cm} \times \frac{1}{2}$ cm is connected to a battery across opposite faces.

The resistance will be

(*a*) maximum when the battery is connected across $1\text{ cm} \times \frac{1}{2}$ cm faces.
(*b*) maximum when the battery is connected across 10 cm × 1 cm faces.

(*c*) maximum when the battery is connected across 10 cm × $\frac{1}{2}$ cm faces.
(*d*) same irrespective of the three faces.

Main concept used: $R = \rho\frac{l}{A}$.

Ans. (*a*): As we know $R = \rho.\frac{l}{A}$. The maximum resistance will be when the value of $\frac{l}{A}$ is maximum, i.e., 'A' must be minimum, it is minimum when area of cross section is 1 cm × $\frac{1}{2}$ cm. Verifies option (*a*).

Q3.6. Which of the following characteristics of electrons determines the current in a conductor?
(*a*) Drift velocity alone.
(*b*) Thermal velocity alone.
(*c*) Both the drift and thermal velocity.
(*d*) Neither drift nor thermal velocity.

Main concept used: $I = Anev_d$

Ans. (*a*): As $I = Anev_d$, so current $I \propto v_d$ *i.e.*, verifies answer (*a*).

Although I also depends on *n*, the number of free electrons which increases on increasing temperature which makes more collision between electrons increase resistance or decrease current. So Ans. '*a*' verified.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

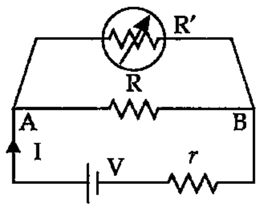
Q3.7. Kirchhoff's junction rule is a reflection of
(*a*) conservation of current density vector.
(*b*) conservation of charge.
(*c*) the fact that the momentum with which a charged particle approaches a junction is uncharged (as vector), as the charged particle leaves the junction.
(*d*) the fact that there is no accumulation of charges at a junction.

Main concept used: Kirchhoff's junction rule.

Ans. (*b*) (*d*): According to junction rule, algebraic sum of current or charge flowing per unit time towards a junction in an electric network is zero, *i.e.*, law of conservation of charges verifies answer (*b*) and no any charges accumulate at junction as the sum of entering and out going charge are equal, at any time interval. It verifies answer (*d*).

Q3.8. Consider a simple circuit shown in figure.  stands for a variable resistance R′. R′ can vary from R_0 to infinity, *r* is internal resistance of the battery ($r << R << R_0$).

(*a*) Potential drop across AB is nearly constant as R′ is varied.
(*b*) Current through R′ is nearly a constant as R′ is varied.
(*c*) Current I depends sensitively on R′.
(*d*) $I \geq \frac{V}{r+R}$ always.



Main concept used: (*i*) Property of resistances in series and parallel, (*ii*) V = IR.

Ans. (*a*) (*d*): As $r \ll R \ll R_0 < R'$ from question, R′ > R and R′ and R are in parallel combination, so the equivalent resistance will be always less than R. So $I \geq \frac{V}{r+R}$ and potential across AB will remain nearly constant. It verifies answers (*a*) and (*d*).

Q3.9. The temperature dependence of resistivity ρ(T) of semiconductors, insulators and metals is significantly based on the following factors:
(*a*) Number of charge carriers can change with temperature T.
(*b*) Time-interval between two successive collisions can depend on T.
(*c*) Length of material can be a function of T.
(*d*) Mass of carriers is a function of T.

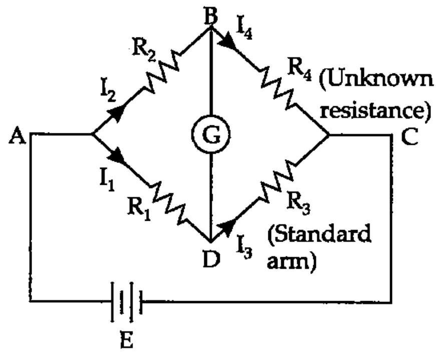
Main concept used: Resistivity $(\rho) = \frac{m}{ne^2\tau}$

Ans. (*a*) and (*b*): We know that resistivity (ρ) depends on mass of charge-carrier (*m*), relaxation time (τ). Length and mass cannot be function of T as the mass of a body is constant everywhere. So discards answer (*d*) and length of body changes negligibly with temperature discards answer (*c*).

As τ decreases on increasing T due to rise in speed of charge-carriers and *n* increases on increasing temperature. So will affect the ρ or ρ is function of T verifies answers (*a*) and (*b*).

Q3.10. The measurement of an unknown resistance R is to be carried out using Wheatstone Bridge. Two students perform an experiment in two ways. The first student takes $R_2 = 10\ \Omega$ and $R_1 = 5\ \Omega$. The other student takes $R_2 = 1000\ \Omega$ and $R_1 = 500\ \Omega$. In the standard arm, both students take $R_3 = 5\ \Omega$. Both find $R = \frac{R_2}{R_1}R_3 = 10\ \Omega$ within errors.
(*a*) The errors of measurement of the two students are the same.
(*b*) Errors of the measurement do depend on the accuracy with which R_2 and R_1 can be measured.
(*c*) If the student uses large values of R_2 and R_1, the current through the arms will be feeble. This will make determination of null point accurately more difficult.

(*d*) Wheatstone Bridge is a very accurate instrument and has no errors of measurement.



Main concept used:

$$\frac{R_2}{R_3}=\frac{R_1}{R_4}$$

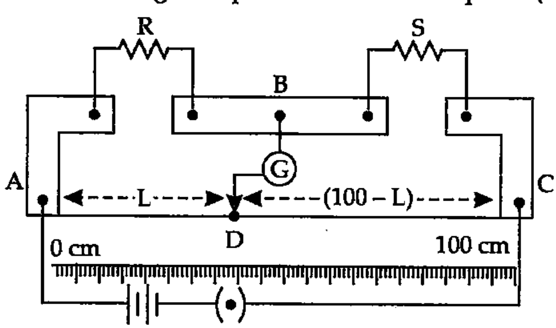
$$R=\frac{R_2}{R_1}\times R_3$$

Ans. (*b*) and (*c*): As the ratio of $\frac{R_2}{R_1}$ and standard resistance are same, so value of unknown resistance for both students are 10 Ω. So we can say the Wheatstone Bridge is most sensitive.

The results of both students depend on the accuracy of resistances used. **So answer (*b*) is verified.**

When R_1 and R_2 is larger, the current through galvanometer becomes weak. It will make difficult to find out null point more accurately. So answer (*c*) is verified.

Q3.11. In the meter bridge the point D is a neutral point (Fig.).



(*a*) The meter bridge can have no other neutral point for this set of resistances.

(*b*) When the jockey contacts a point on meter wire left of D, current flows to B from the wire.

(*c*) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer.

(*d*) When R is increased, the neutral point shifts to left.

Main concepts used: (*i*) Principal of meter bridge $\frac{R}{S}=\frac{L}{100-L}$, (*ii*) When potential difference between two points is zero the current does not flow, (*iii*) Potential decreases from A to B.

Ans. (*a*) and (*c*): When jockey is at D, the current does not flow through galvanometer. So the potentials at B and D are equal. Potentials at different points on the wire are different.

The point D is unique to get null point verifies the answer (*a*). When jockey is shifted to right of D on wire the potential in wire towards right side becomes smaller or $V_B > V_D$ becomes smaller so current flows from B to D in wire verify the answer (*c*) and discards answer (*b*).

When R is increased potential drop across R increases. So potential at B increases to get null point. Jockey must move to right.

VERY SHORT ANSWER TYPE QUESTIONS

Q3.12. Is the motion of a charge across junction momentum conserving? Why or why not?

Main concept used: $p_C = mv_d$, $v_d = \dfrac{eE\tau}{m}$.

Ans. As we know that drift velocity depends on e, E, τ and m as for a junction point, if the temperature is constant e, τ, m is constant so drift velocity (v_d) of electron depends on electric field only i.e., ($\theta \propto$ E).

When a free electron approaches a junction in addition of the uniform electric field (E) facing it normally as E is constant by cell or battery so v_d is constant.

There is a accumulation of charges on the junction which will affect the drift velocity or the momentum, so the momentum is not conserved at junction.

Q3.13. The relaxation time τ is nearly independent of applied electric field E whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's law whereas the second fact leads to variation of ρ with temperature. Elaborate (why)?

Main concept used: (*i*) Higher v_d makes higher collision, v_d increases on increasing temperature, (*ii*) Increase in v_d decreases relaxation time.

Ans. As the drift velocity increases, the relaxation time (τ) (average time between successive collision) decreases which increases the ρ by formula: $\rho = \dfrac{m}{ne^2\tau}$.

The drift velocity (v_d) changes of the order of one mm on increasing electric field, whereas the drift velocity increases of the order of 10^2 m/s when the number of free electrons (n) increases on increasing temperature (T). So, due to increase in v_d the relaxation time (τ) considerably decreases in metal or conductor.

Q3.14. What are the advantages of the null point method in Wheatstone Bridge? What additional measurements would be required to calculate R (unknown) by any other method?

Main concept used: At null point in Wheatstone Bridge experiment no current flows in the arm of galvanometer.

Ans. The main advantage in Wheatstone Bridge is that at null point current does not flow in arm of galvanometer, so no effect of resistance of galvanometer or no consumption of electric energy or potential across galvanometer. It is convenient and easy method.

We can calculate the unknown resistance by Ohm's law in which we need to calculate the least counts and readings of ammeter and voltmeter.

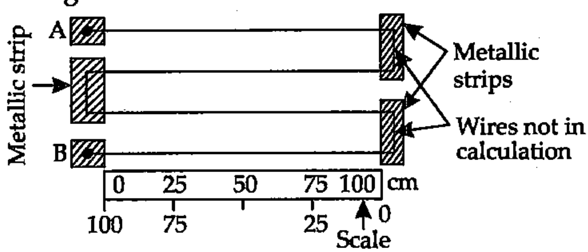
Unknown resistance can also be calculated by applying Kirchhoff's laws to the circuit in which unknown resistance is connected. Here we have to measure the currents and potential differences across all components of circuit which makes the method difficult.

Q3.15. What is the advantage of using thick metallic strips to join wires in a potentiometer?

Main concept used: $R = \rho \frac{l}{A}$.

Ans. As the area of cross-section of metallic strips in potentiometer (and meter bridge) is larger than a single wire. So the resistance of strip is much smaller by formula: $R = \rho \frac{l}{A}$.

A long resistance wire is used in potentiometer by winding it as shown in the figure.



The resistance below the metallic strips are out of circuit and calculation which makes easy to take readings and calculations.

Q3.16. For wiring in the home one uses Cu wires or Al wires. What considerations are involved in this?

Main concept used: Resistance and resistivity of metal, cost availability.

Ans. As the metals have low resistivities so metals have low resistance. The cost of metals used in electric circuits decreases from Ag, Cu, Al, Fe (steel). But Ag is costly so Cu or Al wires are used in wiring.

Q3.17. Why are alloys used for making standard resistance coils?

Main concept used: Dependence of resistance on temperature, atmospheric condition must be minimum and cost also.

Ans. Dependence of resistance on change of temperature, humidity, pressure, etc. must be negligible. Alloys has small value of temperature coefficient and are not affected by moisture, etc.

Alloys has higher resistivity in turn the higher resistance so need smaller length to make coils which decrease the effect of inductance. Due to these reasons alloys are used to make standard resistance coils.

Q3.18. Power P is to be delivered to a device via transmission cables having resistance R_c. If V is the voltage across R and I the current flowing through it, find the power wasted, and how can it be reduced.

Main concept used: $P = VI$, $P = I^2R$.

Ans. As we know that P = VI so, to transmit a constant power P through transmission cable there are two ways:

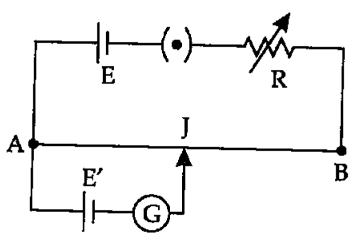
(*i*) If a constant power P is transmitted at low voltage (V) and high current (I). In this method high current will produce higher heat by $H = I^2R$ and power loss through cable is higher.

(*ii*) If a constant or same power be transmitted at high voltage (V) and low current. It gives lower loss of power as heat. But need thicker insulation during transmission.

So to transmit high power at long distance, we use low current and high (132 kV) voltage to minimize heat losses through towers and thicker (long) insulator.

To transmit power supply at short distance, we can transmit power at low 440 V, 220 V, 11 kV with higher current.

Q3.19. In given figure AB is potentiometer wire. If the value of R is increased, in which direction will the balance point J shift?



Main concept used: At null point current in galvanometer circuit is zero so potentials at A and J are equal and $K = \frac{V}{AB}$.

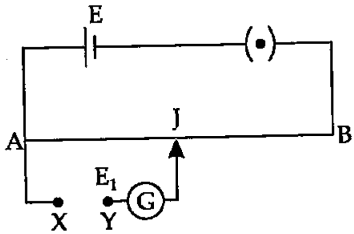
Ans. If R is increased current in main circuit will decrease (by V = IR) as the potential (E) is constant. So in turn potential difference across AB will decrease (by V = IR). As R of AB is constant so potential gradient $K = \frac{V}{AB}$ will decrease. So to balance potential across AB equal to potential of secondary circuit (E′) the length AJ′ must be larger than earlier AJ. So the point J shifts towards B.

Q3.20. While doing an experiment with potentiometer as in the figure, it was found that the deflection is one-sided and (*i*) the deflection decreased while moving from one-end (A) of the wire to the other end (B), (*ii*) the deflection increased while the jockey was moved towards end B.

(*i*) Which terminal positive or negative of cell E_1 is connected at X in case (*i*) and how is E_1 related to E?

(*ii*) Which terminal of cell E_1 is connected at X in case (*ii*)?

Main concept used: (*i*) V = IR, (*ii*) $K = \frac{V}{l}$.



Ans. (*i*) One-sided deflection in galvanometer decreases when jockey moves towards B. So the potential in galvanometer circuit decreases as compared to potential across AJ earlier or potential between AJ′ increases. It is possible when positive terminal of E_1 is at X and negative at Y. So $E_1 > E$.

(*ii*) One-sided deflection in galvanometer increases when jockey moves from end A to B. So the potential in galvanometer circuit increases as compared to potential across AJ earlier or potential between AJ′ decrease. It is possible when positive terminal of E is at Y and negative is at X. So $E_1 < E$.

Q3.21. A cell of emf E and internal resistance *r* is connected across an external resistance R. Plot a graph showing the variation of P.D. across R, verses R.

Main concept used: $V = \frac{E R}{R + r}$.

Ans. We know that $I = \frac{E}{R + r}$ and V = IR.

So $$V = \frac{ER}{R + r} \quad ...(I)$$

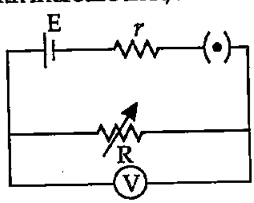
$$V = \frac{E}{1 + \frac{r}{R}} \quad ...(II)$$

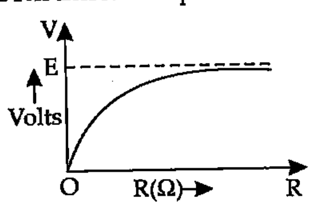
Here E, *r* are constants. So

$$V \propto \frac{1}{1 + \frac{r}{R}} \quad \text{(from II)}$$

and $$V \propto R \quad \text{(from I)}$$

With increase in R, P.D. across R is increased upto maximum value E.





SHORT ANSWER TYPE QUESTIONS

Q3.22. First set of n equal resistors of R each are connected in series to battery of emf E and internal resistance R. A current (I) is observed to flow. Then the n resistors are connected in parallel to the same battery. It is now observed that the current is increased 10 times. What is n?

Main concept used: (*i*) Series and parallel combination of resistances, (*ii*) $I = \dfrac{E}{R+r}$ for a battery.

Ans. When n resistances of each RΩ are connected in series and parallel then

$$R_S = R + R + R + \cdots n \text{ times} \quad \Rightarrow \quad R_S = nR$$

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \cdots n \text{ times} \quad \Rightarrow \quad \frac{1}{R_p} = \frac{n}{R}$$

$$\Rightarrow \quad \boxed{R_p = \frac{R}{n}}$$

When n resistors are connected in series connected with battery of emf E then current (I) flows. So

$$\frac{E}{R + nR} = I \qquad \ldots(I)$$

and now n resistances are connected in parallel combination then current in circuit increased to 10 times of I

$$\therefore \quad \frac{E}{R + \frac{R}{n}} = 10\,I$$

$$\frac{E}{R + \frac{R}{n}} = \frac{10\,E}{R + nR}$$

$$\frac{1}{R\left(1 + \frac{1}{n}\right)} = \frac{10}{R(1+n)}$$

$$10\left(1 + \frac{1}{n}\right) = 1 + n$$

$$10 + \frac{10}{n} - 1 - n = 0$$

$$-n + \frac{10}{n} + 9 = 0 \qquad \text{[Multiply by } -n \text{ to both sides]}$$

$$n^2 - 10 - 9n = 0$$

$$n^2 - 9n - 10 = 0$$

$$n^2 - 10n + 1n - 10 = 0$$

$$n(n - 10) + 1(n - 10) = 0$$

So $$(n + 1)\,(n - 10) = 0$$

or $n = -1$ is not possible or $n = 10$.

So, there are 10 resistors in combination.

Q3.23. Let there be n resistors $R_1 \dots R_n$ with $R_{max} = \max(R_1 \dots R_n)$ and $R_{min} = \min(R_1 \dots R_n)$. Show that when they are connected in parallel, the resultant resistance $R_p < R_{min}$ and when they are connected in series the resultant resistance $R_s > R_{max}$. Interpret the result physically.

Main concept used: (*i*) $R_s = R_1 + R_2 + \cdots$,

(*ii*) $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots.$$

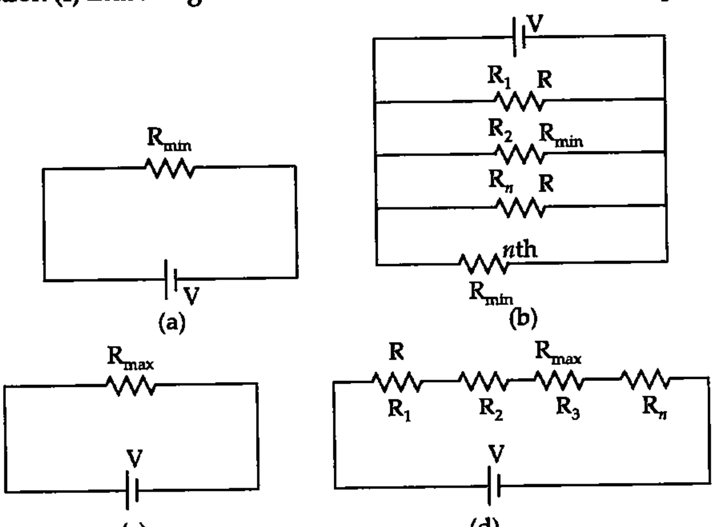
Ans. Let R_{min} and R_{max} are the minimum and maximum resistances among all resistances $R_1, R_2, \dots R_n$.

When resistors are connected in parallel then equivalent resistance R_p is $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$.

Multiplying both sides by R_{min} we get,

$$\frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + \cdots + \frac{R_{min}}{R_n} \qquad \dots I$$

Among $R_1, R_2, \dots, R_n$, there must be a resistor which is minimum. So there must be a term $\frac{R_{min}}{R_{min}}$ in R.H.S. of equation (I). So R.H.S. in equation (I) must be greater than 1 as all other terms are also positive.



or $$\frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + \cdots + \frac{R_{min}}{R_{min}} + \cdots + \frac{R_{min}}{R_n} > 1$$

or $$\frac{R_{min}}{R_p} > 1 \qquad \text{or} \qquad R_p < R_{min}.$$

So in parallel combination, the equivalent resistance R_p is always less than any smallest resistance in the combination.

When n resistors are connected in series then equivalent resistance –

$$R_s = R_1 + R_2 + R_3 + \cdots + R_n. \qquad ...II$$

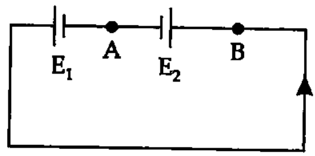
Here, in R.H.S. there must be a term R_{max} which has maximum value among R_1, R_1, ... R_n. As all terms in R.H.S. of equation II are positive so

$$R_s = R_1 + R_2 + R_3 + \cdots + R_{max} + \cdots + R_n > R_{max}$$

or $$R_s > R_{max}$$

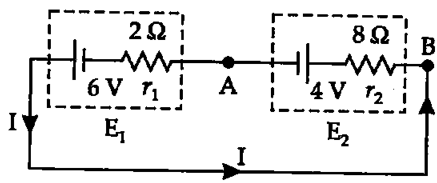
So in series combination equivalent resistance is always greater than the maximum resistance (R_{max}) among R_1, R_2, ..., R_n.

Q3.24. Here the circuit shows two cells connected in opposition to each other. Cell E_1 is of emf 6 V and internal resistance 2 Ω, the cell E_2 is of emf 4 V and internal resistance 8 Ω. Find the potential difference between points A and B.



Main concept used: Current in circuit from higher to lower potential, $I = \frac{E}{r}$.

Ans. The above figure can be redrawn as given here. The direction of current in circuit will be as shown in the figure.



So point B is at higher potential than A. So $V_B > V_A$.

Current (I) in circuit, $I = \frac{E_1 + E_2}{r_1 + r_2} = \frac{(6-4)V}{(2+8)\Omega} = 0.2$ amp

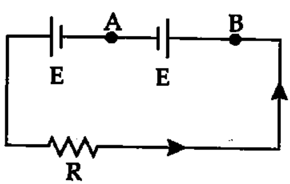
For positive potential A is near to positive terminal of E_2 so has +4 V. So potential across E_1 and E_2:

$$E_1 = V - Ir_1 = 6 - 0.2 \times 2 = 6 - 0.4 = 5.6 \text{ V}$$
$$E_2 = V + Ir_2 = 4 + 0.2 \times 8 = 4 + 1.6 = 5.6 \text{ V}$$

So potential between A and B = E_2 = 5.6 Volt.

As current is flowing from B to A. So potential at B is larger than A.

Q3.25. Two cells of same emf E but internal resistances r_1, r_2 are connected in series to an external resistor R (Figure). What should be the value of R so that potential difference across terminals of the first cell become zero?



Main concepts used: $I = \frac{E}{R + r}$

$V_0 = E - I_r$ and Kirchhoff's law, Ohms' law.

Ans. Current I flowing in the circuit $I = \dfrac{E + E}{R + r_1 + r_2}$.

$$V_1 = E - Ir_1 = E - \frac{2E \cdot r_1}{R + r_1 + r_2}$$

The net potential difference across Ist cell $V_1 = 0$ (Given)

$$\therefore \quad E - \frac{2Er_1}{R + r_1 + r_2} = 0$$

$$\text{or} \quad 1 - \frac{2r_1}{R + r_1 + r_2} = 0$$

$$\frac{2r_1}{r_1 + r_2 + R} = \frac{1}{1}$$

$$\text{or} \quad 2r_1 = r_1 + r_2 + R$$

$$\boxed{r_1 - r_2 = R}$$

It is the required condition for the potential difference across Ist cell to be zero.

Q3.26. Two conductors are made of the same material and have the same length. Conductor A is solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistances R_A to R_B.

Main concept used: $R_0 = \dfrac{\rho l}{A}$

Ans. Conductor A (solid wire R_A) **Conductor B (hollow tube R_B)**

Conductor A	Conductor B	
$l_1 = l$	$l_2 = l$	(Given)
$A_1 = \pi r_1^2$	$A_2 = \pi r_2^2 - \pi r_1^2$	
$r_1 = \frac{1}{2}\text{ mm} = 0.5 \times 10^{-3}\text{ m}$	$r_2 = \frac{2}{2}\text{ mm} = 1 \times 10^{-3}\text{ m}$	
$\rho_1 = \rho$	$\rho_2 = \rho$	

$$\frac{R_A}{R_B} = \frac{\frac{\rho_1 l_1}{A_1}}{\frac{\rho_2 l_2}{A_2}} = \frac{\rho_1 l_1}{A_1} \times \frac{A_2}{\rho_2 l_2} = \frac{\rho l}{A_1} \times \frac{A_2}{\rho l} = \frac{A_2}{A_1}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{A_2}{A_1} = \frac{\pi r_2^2 - \pi r_1^2}{\pi r_1^2} = \frac{\pi(r_2^2 - r_1^2)}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 - 1$$

$$= \left(\frac{1 \times 10^{-3}}{0.5 \times 10^{-3}}\right)^2 - 1 = (2)^2 - 1 = 4 - 1 = \frac{3}{1}$$

$$\therefore \quad R_A : R_B = 3 : 1.$$

Q3.27. Suppose there is a circuit consisting of only resistors and batteries. Suppose one is to double (or increase it to n-times) all voltage and resistances. Show that currents are unaltered.

(See NCERT Textbook Example 3.7)

Main concept used: $I = \dfrac{E}{R + r}$.

Ans. Case I: Consider a circuit having external $R_1, R_2, ...,$ connected with some batteries E_1, E_2, E_3 ... having their internal resistances $r_1, r_2, r_3, ...$.

Let the equivalent resistance, emf and internal resistance of above combination is R_{eq}, E_{eq}, and r_{eq} respectively. So the current passing in the circuit, $I_1 = \dfrac{E_{eq}}{R_{eq} + r_{eq}}$.

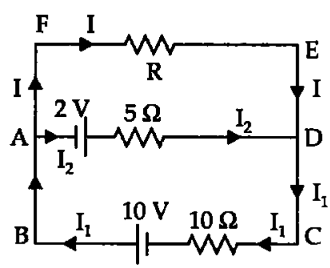
Now the resistances and cells are again connected in a manner that their equivalent resistance, emf and internal resistances are nR_{eq}, nE_{eq} and nr_{eq} respectively. So again current in new circuit $I_2 = \dfrac{nE_{eq}}{nR_{eq} + nr_{eq}}$.

$$I_2 = \frac{nE_{eq}}{n[R_{eq} + r_{eq}]} = \frac{E_{eq}}{R_{eq} + r_{eq}} = I_1$$

So the current remains same if the R, E and r of a circuit is increased by n times, *i.e.* nR, nE, nr.

LONG ANSWER TYPE QUESTIONS

Q3.28. Two cells of voltage 10 V and 2 V and internal resistances 10 Ω and 5 Ω respectively are connected in parallel with the positive end of 10 V battery connected to negative pole of 2 V battery as in figure. Find the effective voltage and effective resistance of combination.



Main concepts used: V = IR, Kirchhoff's law.

Ans. Applying junction rule at A

$$\boxed{I_1 = I + I_2} \qquad ...I$$

Apply Kirchhoff's loop rule on loop B CEF and loop ADEF

$$10 = IR + 10\,I_1 \qquad ...II$$
$$2 = -IR + 5\,I_2 \qquad ...III$$
$$2 = -IR + 5(I_1 - I) \qquad \text{[from I]}$$
$$2 = -IR + 5\,I_1 - 5\,I \qquad ...IV$$
$$4 = -2IR + 10\,I_1 - 10\,I \qquad \text{[on multiplying IV by 2] ...V}$$
$$10 = IR + 10\,I_1$$
$$-\quad -\quad -$$
$$-6 = -3\,IR - 10\,I \qquad \text{[on subtracting (II) from (V)]}$$

or $3 IR + 10 I = 6$

$$I(3R + 10) = 2 \times 3$$

$$\frac{I(3R+10)}{3} = 2$$

$$2 = I\left(R + \frac{10}{3}\right) \qquad \text{...VI}$$

Let the effective potential difference due to both batteries is V_{eq}. It will be across resistance R. So

$$V_{eq} = I(R + R_{eq}) \qquad \text{...VII}$$

Where R_{eq} is the resistance of circuit except R

Comparing (VI) and (VII),

$$V_{eq} = \textbf{2 Volts and } R_{eq} = \frac{10}{3}\ \Omega$$

Q3.29. A room has AC (air-conditioner) that runs 5 hours a day at voltage 220 V. The wiring of the room consists of Cu wire of 1 mm radius and length of 10 m. Power consumption per day is 10 commercial units. What fraction of it goes in the Joule heating in wires? What would happen if the wiring is made of the aluminium of the same dimensions?

$[\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega\text{-m}, \quad \rho_{Al} = 2.7 \times 10^{-8}\ \Omega\text{-m}]$

Main concepts used: $P = I^2R,\ R = \frac{\rho l}{A}$.

Ans. Total energy consumed in 5 hrs a day by AC and wiring

$$= 10 \text{ kWh}$$

∴ Energy consumed in 1 hr by AC and wiring

$$= 2 \text{ kWh}$$

So total power of AC and wire = 2000 W

$$P = VI$$

$$I = \frac{P}{V} = \frac{2000}{220} \cong 9.0 \text{ A}$$

Let P_0 is power of wiring then,

$$P_0 = I^2R_w \qquad [R_w = \text{resistance of wiring}]$$

$$= 9 \times 9 \cdot \frac{\rho \cdot l}{A}$$

$$= \frac{9 \times 9 \times 1.7 \times 10^{-8} \times 10}{3.14 \times 1 \times 10^{-3} \times 1 \times 10^{-3}} = \frac{81 \times 17 \times 10^{-8+6}}{3.14}$$

$$= \frac{1377}{3.14} \times 10^{-2} = 4.38 = 4.4 \text{ Watt}$$

So loss of energy in wiring ≅ 4.4 J/sec

The fractional loss due to heating of wires $= \frac{4.4}{2000} \times 100\% = 0.22\%$

$$\frac{P_{Al}(\text{wiring})}{P_{Cu}(\text{wiring})} = \frac{I^2R_{Al}}{I^2R_{Cu}} = \frac{\rho_{Al}\dfrac{l_{Al}}{A_{Al}}}{\rho_{Cu}\dfrac{l_{Cu}}{A_{Cu}}} \quad \text{as } l_{Al} = l_{Cu} \text{ and } A_{Al} = A_{Cu}$$

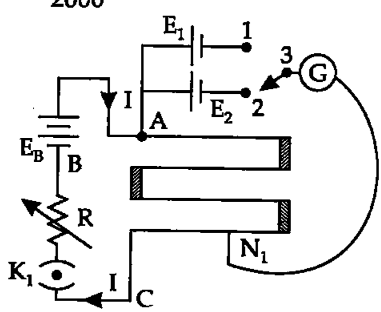
$$\frac{P_{Al}}{P_{Cu}} = \frac{\rho_{Al}}{\rho_{Cu}} \quad \text{or} \quad P_{Al} = \frac{2.7 \times 10^{-8}}{1.7 \times 10^{-8}} \times 4.4 \text{ Watt}$$

$$P_{Al} = 7 \text{ Watt}$$

So power loss in Al wiring = 7 Watt

The fractional loss due to Al wiring $= \dfrac{7}{2000} \times 100\% = 0.35\%$.

Q3.30. In an experiment with potentiometer, V_B = 10 V. R is adjusted to 50 Ω in the figure. A student wants to measure voltage E_1 of battery (approx. 8 V), finds no null point possible. He then diminishes R to 10 Ω and is able to locate the null point on the last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and potential drop per unit length across the wire in the second case.



Main concepts used: (*i*) $I = \dfrac{V}{R}$, (*ii*) Potentials at A and jockey point N_1 are equal at balance condition, (*iii*) Null point is obtained if E_1 and $E_2 < V_{AB}$ (wire of potentiometer).

Ans. Let R′ be the resistance of potentiometer wire.

∵ Variable resistance, R = 50 Ω

I is the current in primary circuit which is at E_B = 10 V.

∴ $$I = \frac{V_B}{R + R'} \Rightarrow \frac{10}{50 + R'} = I \text{ (in primary circuit)} \quad \text{...I}$$

Potential difference across the wire of potentiometer

$$V' = IR'$$

From I $$V' = \frac{10\,R'}{50 + R'} \quad \text{...II}$$

As with R = 50 Ω resistance, null point cannot be obtained by 8 Volt.

So $$V' < 8 \text{ Volt.}$$

$$\frac{10\,R'}{50 + R'} < 8 \quad \text{(No balance point)}$$

As 50 + R′ is positive so we can multiply above equation by positive number and we get

$$10\,R' < 400 + 8\,R'$$
$$2\,R' < 400$$

$$R' < 200 \qquad ...III$$

Similarly, null point is obtained by R = 10 Ω. then V″ > 8 at balance point

So it is possible when $\frac{10\,R'}{10+R'} > 8$ (as from I, R = 10)

Similarly, multiply above equation by positive number 10 + R′ to both sides

$$10\,R' > 80 + 8\,R'$$
$$2\,R' > 80$$
$$R' > 40 \qquad ...IV$$

As the null point is obtained on 4th segment or at $\frac{3}{4}$ of total length so at $\frac{3}{4}R'$ (No balance point)

or $$\frac{10\times\frac{3}{4}R'}{10+R} < 8 \qquad \text{(At balance point)}$$

So $$7.5\,R' < 80 + 8\,R$$
$$-0.5\,R' < 80$$
$$-R' < 160$$
$$R' > -160$$

R′ can never be negative so, –160 Ω is considered 160 Ω

So $$\boxed{160 < R' < 200} \qquad ...V$$

Any R′ between 160 Ω and 200 Ω will achieve null point. Since the null point is on last 4th segment of potentiometer wire, so the potential drop across 400 cm wire > 8 Volt.

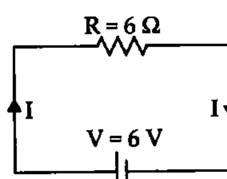
So $$K.400\text{ cm} > 8\text{ V} \qquad \text{(At balance point)}$$
$$K > \frac{8}{400}\text{ Volt/cm}$$
$$K > \frac{8}{4}\text{ Volt/m}$$
$$K > 2\text{ Volt/m}$$

As balance point is at 4th wire, so no balance point at 3 m,

i.e., $$K.3 < 8 \qquad \text{(No balance point)}$$
$$K < \frac{8}{3}\text{ Volt/m}$$
$$K < 2\frac{2}{3}\text{ Volt/m}$$

So $$\boxed{2\frac{2}{3}\text{ V/m} > K > 2\text{ Volt/m}}$$

Q3.31. (*a*) Consider circuit in figure. How much energy is absorbed by electrons from the initial state of no current (ignore thermal motion) to the state of drift velocity?



(*b*) Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would one associate with energy in problem (*a*)?

n = Number of free electrons/volume = $10^{29}/m^3$

Length of circuit = 10 cm

Cross-sectional area = A = 1 mm^2.

Main concepts used: $I = Anev_d$, KE = $\frac{1}{2}$ m v_d^2 per electron, ohmic loss of energy per sec = RI^2, V = RI.

Ans. $A = 1\ mm^2 = 10^{-6}\ m^2$ $\qquad$ $R = 6\ \Omega$

$n = 10^{29}/m^3$ $\qquad$ $V = 6\ V$

$e = 1.6 \times 10^{-19}$ C $\qquad$ $I = \frac{V}{R} = \frac{6}{6} = 1$ amp

$m_e = 9.1 \times 10^{-31}$ kg $\qquad$ $l = 10$ cm $= 10^{-1}$ m

(*a*) $\because$ $I = Anev_d$

$$\therefore\ v_d = \frac{I}{Ane} = \frac{1}{10^{-6} \times 10^{29} \times 1.6 \times 10^{-19}} \text{ m/s}$$

$$= \frac{1}{1.6} \times 10^{+6+19-29} = \frac{1}{1.6} \times 10^{-4} \text{ m/s}$$

$$KE = \frac{1}{2}mv_d^2 \text{ per electron}$$

Number of electrons (free) in wire = n (volume of wire)

$= n \times Al$

$$\therefore \text{ KE of all electrons} = \frac{1}{2}mv_d^2\ Anl$$

$$KE = \frac{1}{2} \times 9.1 \times 10^{-31} \times \frac{10^{-4} \times 10^{-4}}{1.6 \times 1.6} \times 10^{-6} \times 10^{29} \times 10^{-1}$$

$$= \frac{9.1}{2 \times 2.56} \times 10^{-31-8-7+29} = 1.78 \times 10^{-46+29}$$

$$= 1.78 \times 10^{-17} J.$$

So, to start flow of current I, the electrons will take energy from cell

= KE of all electrons = 1.78×10^{-17} J

(*b*) Loss of energy during current flowing = I^2R.

$P = 1 \times 1 \times 6 = 6$ Joule per second

$\because$ Energy = P.t

or $\quad t = \frac{E}{P} = \frac{1.78 \times 10^{-17}}{6}$

$\cong 0.29 \times 10^{-17}$ sec $\cong 0.3 \times 10^{-17}$

$= \mathbf{3 \times 10^{-18}}$ **second.**

❑❑❑