4 Moving Charges and Magnetism

MULTIPLE CHOICE QUESTIONS—I

Q4.1. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $B = B_0\hat{k}$.

(*a*) They have equal z-components of momenta.

(*b*) They must have equal charges.

(*c*) They necessarily represent a particle-antiparticle pair.

(*d*) The charge to mass ratio satisfy $\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$

Main concepts used: (*i*) Pitch, $P = \frac{2\pi mv\cos\theta}{Bq}$,

(*ii*) Law of conservation of momenta.

Ans. (*d*): For a given pitch, $P = \frac{2\pi mv\cos\theta}{Bq}$

$$\frac{q}{m} = \frac{2\pi v\cos\theta}{BP}$$ [θ is angle of velocity of charge particle with X-axis]

If motion is not helical, $\theta = 0$.

As path of both the particles is identical and helical but of opposite direction in same magnetic field so by law of conservation of momenta

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$$

So, verifies answer (*d*).

Q4.2. Biot-Savart law indicates that the moving electrons (with velocity *v*) produce a magnetic field B such that

(*a*) B is perpendicular to velocity *v*.

(*b*) B is parallel to *v*.

(*c*) it obeys inverse cube law.

(*d*) it is along the line joining the electron and point of observation.

Main concept used: Biot-Savart law.

Ans. (*a*): By Biot-Savart law, $dB = \frac{I dl \sin\theta}{r^2}$

or $$dB = \frac{I \times dl}{r}$$

I can be considered flow of charge.

So the magnetic field is perpendicular to the direction of flow of charge verifies answer '*a*'.

Q4.3. A current-carrying circular loop of radius R is placed in x-y plane with centre at origin. Half of the loop with $x > 0$ is now bent so that it now lies in the y-z plane.

(*a*) The magnitude of magnetic moment now diminishes.
(*b*) The magnetic moment does not change.
(*c*) The magnitude of B at (0, 0, z), $z \gg R$ increases.
(*d*) The magnitude of B at (0, 0, z), $z \gg R$ is unchanged.

Main concept used: Direction of magnetic field due to circular loop (Right-Hand Thumb-Rule).

Ans. (*a*): As the direction of magnetic field due to current-carrying circular loop is perpendicular and it is perpendicular to plane of loop and unidirectional.

In first case, direction of magnetic field is only in positive x-z direction but when it is bented then half of B is along z-x axis (due to unfolded loop) and half of B is along $+x$ direction so vector sum of B will decrease. Verifies answer (*a*).

Q4.4. An electron is projected with uniform velocity along the axis of a current-carrying long solenoid. Which of the following is true?

(*a*) Electron will be accelerated along the axis.
(*b*) The electron path will be circular about the axis.
(*c*) The electron will experience a force at 45° to the axis and hence execute a helical path.
(*d*) The electron will continue to move with uniform velocity along the axis of the solenoid.

Main concept used: Lorentz force.

Ans. (*d*): The Lorentz force acts on a charged particle in a magnetic and electric field is $F = q[\vec{E} + v \times B]$. As there is no E, force due to E.F. is zero and force due to B is perpendicular to the direction of v and B which will be perpendicular to the direction of motion (v), so will not affect the velocity of moving charge particle. So verifies answer (*d*).

Q4.5. In a cyclotron, a charged particle

(*a*) undergoes acceleration all the time.
(*b*) speeds up between the dees because of the magnetic field.
(*c*) speeds up in a dee.
(*d*) slows down within a dee and speeds up between dees.

Main concept used: Working of cyclotron and motion of charged particle in magnetic field, electric field or both.

Ans. (*a*): There is crossed electric and magnetic field between dees so the charged particle accelerates by electric field between Dee's towards other Dee.

Inside dees, there is no electric field due to shielding effect of charge or field. So only magnetic force keeps the circular motion of charged particle inside (any circular motion is also accelerated).

So the charged particle accelerates inside and between dees always verifies answer (*a*).

Q4.6. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field B. The work done to rotate the loop by 30° about an axis perpendicular to its plane is

(*a*) MB (*b*) $\frac{\sqrt{3}}{2}$MB (*c*) $\frac{MB}{2}$ (*d*) zero.

Main concept used: Work done by loop in orientation

$$= MB(\cos\theta_2 - \cos\theta_1). \quad \text{Where } M = N/A$$

Ans. (*b*) and (*d*): When the axis of rotation of loop is along B then angle between $\vec{B}$ and $\vec{A}$ is 90° always. So WD by loop to rotate i.e., WD = MB cos 90°. So WD is zero. Verifies option (*d*).

But when the axis of rotation of loop is not along the direction of B, then direction of vector B and A will change with time.

Work done by loop during orientation in uniform magnetic field

$$= MB(\cos\theta_2 - \cos\theta_1) = MB\cos\theta$$

$$= MB\cos 30° = MB\frac{\sqrt{3}}{2}$$

So answer (*b*) is verified.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q4.7. The gyro-magnetic ratio of an electron in an H-atom, according to Bohr's model is

(*a*) independent of which orbit it is in.
(*b*) negative.
(*c*) positive.
(*d*) increases with the quantum number *n*.

Main concept used: Gyro-magnetic ratio,

$$\mu_e = \frac{\text{Magnetic moment of } e}{\text{Angular momentum of } e}.$$

Ans. (*a*) and (*b*): Magnetic moment of

$$e = I \cdot A = \frac{-e}{T} \cdot \pi r^2$$

$$M_e = \frac{-e\pi r^2}{\frac{2\pi r}{v}} = \frac{-e\pi r^2 v}{2\pi r} = \frac{-evr}{2}$$

The angular momentum of $e = L = m\vec{v}r$

$\therefore$ Gyro-magnetic ratio, $\mu_e = \dfrac{\text{Magnetic moment of } e}{\text{Angular momentum of } e} = \dfrac{-evr}{2.mvr} = \dfrac{-e}{2m}$.

So it is independent of velocity or orbit of *e* depends only on charge and is with negative sign, *i.e.* μ_e of *e* is opposite of any positive charge. So verified answers (*a*) and (*b*).

Q4.8. Consider a wire carrying a steady current I placed in a uniform magnetic field B perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do not work. This implies that,

(*a*) motion of charges inside the conductor is unaffected by B since they do not absorb energy.

(*b*) some charges inside the wire move to surface as a result of B.

(*c*) if the wire moves under the influence of B, no work is done by the force.

(*d*) if the wire moves under the influence of B, no work is done by the magnetic force on the ions, assumed fixed within the wire.

Main concept used: Force on current-carrying conductor placed in magnetic field B is equal to $BIl \sin\theta$, its direction is perpendicular to B and I (or *l*).

Ans. (*b, d*): Force (F) on current-carrying conductor by magnetic field B is perpendicular to $\vec{B}$. So by formula—

$$F = BIL \sin\theta = I \times B \times L$$

F is perpendicular to B and L both by Fleming's Left-hand Rule.

So work done by magnetic field is $W = F.l \cos\theta$.

$W = F.l \cos 90°$ will be equal to zero. Verifies the answer (*d*).

Due to magnetic induction some charges can move on the surface of conductor verifies answer (*b*).

Q4.9. Two identical current-carrying coaxial loops carry current I in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as C,

(*a*) $\oint_C B.dl = \mp 2\mu_0 I$.

(*b*) the value of $\oint_C B.dl$ is independent of sense of C.

(*c*) there may be a point on C where $\vec{B}$ and $\vec{dl}$ are perpendicular.

(*d*) B vanishes everywhere on C.

Main concept used: Ampere's circuital law.

Ans. (*b*) and (*c*): Loops are identical placed coaxially and carrying same current in opposite sense. So inside amperian loop of any type direction of current will be opposite by Ampere's circuital law

$$\oint_C B.dl = \mu_0(I - I) = \mu_0(0) = 0$$

As the magnetic field inside (over everywhere) the loop is perpendicular to the direction of plane of loop, so

$$\oint_C \mathbf{B}.dl = |\mathbf{B}.\vec{dl}| \cos 90° = 0$$

So, answers (*b*) and (*c*) are verified.

Q4.10. A cubical region of space is filled with some uniform electric and magnetic fields. An electron enters the cube across one of its faces with velocity **v**, and a positron enters via opposite face with velocity –**v**. At this instant,

(*a*) the electric forces on both the particles cause identical accelerations.

(*b*) the magnetic forces on both the particles cause equal accelerations.

(*c*) both particles gain or loose the energy at same rate.

(*d*) the motion of centre of mass (CM) is determined by 'B' alone.

Main concept used: Lorentz force, $F = q(E + v \times B)$.

Ans. (*b*), (*c*) and (*d*): As $F = qE$ here E is same but q is in opposite nature force. Electric force or acceleration is not identical. It discards answer (*a*).

As the $F = qv \times B$, *i.e.*, F is perpendicular to velocity and magnetic field, so particle revolves perpendicular to both $\vec{B}$ and $\vec{v}$ with uniform speed. But magnitude of acceleration by magnetic field is equal. It verifies answer (*b*).

As magnitudes of charge $\vec{v}$, $\vec{E}$ and B are constant, so gain or loose the energy at the same rate verifies answer (*c*).

As there is no change in centre of mass of particles therefore the motion of centre of mass is determined by B alone. It verifies answer (*d*).

Q4.11. A charged particle would continue to move with a constant velocity in a region wherein,

(*a*) $E = 0, B \neq 0,$ (*b*) $E \neq 0, B \neq 0$

(*c*) $E \neq 0, B = 0,$ (*d*) $E = 0, B = 0$

Main concepts used: (*i*) Lorentz force, (*ii*) How the net force on charged particle may be zero.

Ans. (*a*), (*b*) (*d*):

We know that $F_L = F_e + F_m = qE + qv \times B$

The velocity $\vec{v}$ of a charge (q) particle in magnetic field ($\vec{B}$) and electric field ($\vec{E}$) will be constant. If Lorentz force (F_L) on q is zero. As

$$\vec{F}_m = q\vec{E} + q\vec{v} \times \vec{B}$$

i.e., for constant v $\quad F_m = 0$

(*i*) When E = 0, but B ≠ 0, then for $F_m = 0$

$$q(0) + qv \times B = 0 \quad \text{or} \quad q(v \times B) = 0 \qquad \text{...I}$$

$q \times v \times B$ will be zero as the force F_m is ⊥ to the direction of $\vec{B}$ and $\vec{v}$ both (By Fleming's left hand rule) $F_m = 0$. So, verifies option (*a*).

(*ii*) If B ≠ 0, E ≠ 0, Consider $F_m = 0$ may or may not be as under

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

$$q(E + v \times B) = 0$$

$$q \neq 0 \text{ so } E + (v \times B) = 0$$

or

$$qE = -q(v \times B)$$

Above two forces F_m and F_e may be equal and opposite when the direction of E is opposite to direction of $(v \times B)$ and the magnitude of E and B must be in such a way that

$$(qE) = q(v \times B)$$

i.e.,

$$|E| = v \times B$$

⇒ E.F. must be $|v|$ times of B and perpendicular to B.

(*iii*) B = 0, E ≠ 0

$$qE \neq 0$$

It will change the velocity or direction of v and v cannot be constant discards option (*c*).

(*iv*) When E = 0 and B = 0. Then $qE + qv \times B = 0$

$$0 + 0 = 0$$

So no force acts on charge particle. Hence it will move with uniform velocity, verifies option (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q4.12. Verify that the cyclotron frequency $\omega = \dfrac{eB}{m}$ has the correct dimensions of $[T]^{-1}$.

Main concepts used: (*i*) For a circular motion, there must be centripetal force perpendicular to velocity. (*ii*) Magnetic force is perpendicular to motion of particle.

Ans. In cyclotron, charged particles revolve in circular orbit due to magnetic force which acts perpendicular to the velocity of particle.

So it provides the centripetal force for revolution. So, $\dfrac{mv^2}{R} = qv \times B$.

Here, θ = 90° as θ is angle between $\vec{v}$ and $\vec{B}$

$$\therefore \quad \frac{mv^2}{R} = qvB \quad \Rightarrow \quad \frac{v^2}{Rv} = \frac{qB}{m} \quad \text{or} \quad \frac{qB}{m} = \frac{v}{R}$$

$$\because \quad \omega = \frac{qB}{m}$$ so dimensions of below must be equal.

So $$[\omega] = \left[\frac{qB}{m}\right] = \left[\frac{v}{R}\right]$$

$$\left[\frac{2\pi}{T}\right] = \left[\frac{LT^{-1}}{L}\right] = [T^{-1}]$$

So dimensions of ω is $[T^{-1}]$.

Q4.13. Show that a force that does no work must be a velocity dependent force.

Main concept used: W.D. = F.dl

Ans. As work done by force is zero, so

$$dW = F.dl = 0$$

$$\Rightarrow \quad F.\frac{dl}{dt} \times dt = 0$$

$$dW = F.v\, dt = 0$$

$$dt \neq 0 \qquad [\therefore F.v = 0]$$

So F must be velocity-dependent, *i.e.*, angle between F and v must be 90° always, then

For $$F.v = 0$$

$$Fv \cos\theta = \cos 90°$$

$$\theta = 90°$$

If v changes direction then to make $\theta = 90°$, F must change angle according to v. So F is dependent on v to make work done zero.

Q4.14. The magnetic force depends on $\vec{v}$ which depends on the inertial frame of reference. Does then magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?

Main concept used: Propagation of electromagnetic waves.

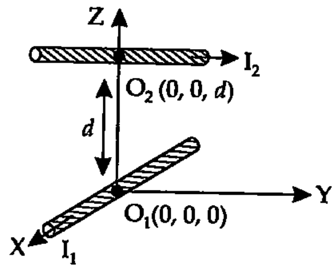
Ans. The magnetic force changes from inertial frame to frame, *i.e.* the magnetic force depends on frame of reference. So the net acceleration which comes into existence out of this is, however, frame independent (non-relativistic physics) for inertial frame.

Q4.15. Describe the motion of a charged particle in a cyclotron if the frequency of radio frequency (rf) field were doubled.

Main concept used: In cyclotron, frequency of charged particle is equal to the frequency of radio, $\left(T = \frac{2\pi}{\omega}\right)$.

Ans. When the frequency ω of electric field (oscillator) is doubled, the time-period $\left(T = \frac{2\pi}{\omega}\right)$ becomes half. So the charged particle will take half time to reach between dees. Hence, a charged particle accelerates as it moves in circular path between the dees during motion in Dee's the radius of moving charged particle remain same.

Q4.16. Two long wires carrying current I_1 and I_2 are arranged as shown in figure. The one carrying current I_1 is along the X-axis. The other carrying current I_2 is along a line parallel to Y-axis given by X = 0, Z = *d*. Find the force exerted at O_2 because of the wire along X-axis.



Main concept used: RHGR to find the direction of magnetic field and $F = BIl \sin\theta$.

Ans. We know that force on current (I) carrying conductor placed in magnetic field B is

$$F = B \times I.dl = BIdl \sin\theta.$$

The direction of magnetic field at O_2 due to the current I_1 is parallel to Y-axis and in –Y direction.

As wire of current I_2 is parallel to Y-axis, current in I_2 is also along Y-axis. So I_2 and B_1 (magnetic field due to current I_1) are also along Y-axis *i.e.*, angle between I_2 and B_1 is zero. So magnetic force F_2 on wire of current I_2 is $F_2 = B_1 I_2\, dl \sin 0° = 0$.

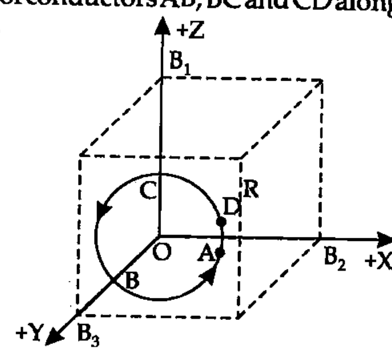
Hence, force on O_2 due to wire of current I_1 is zero.

SHORT ANSWER TYPE QUESTIONS

Q4.17. A current-carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of X-Y, Y-Z and Z-X planes with their centres at the origin, joined together. Find the direction and magnitude of B at the origin.

Main concepts used: (*i*) Direction of magnetic field in a current-carrying loop, (*ii*) Magnitude of magnetic field B due to an arc.

Ans. Consider in figure, 3 quadrants of conductors AB, BC and CD along positive X-Y, Y-Z and Z-X planes respectively. A and D are connected to a battery which is responsible to flow current I through the three quadrants of radius R coordinate of A or D (R, 0, 0), B(0, R, 0) and of C(0, 0, R). Now the direction of magnetic field by right-hand thumb rule due to quadrants AB, BC and CD are $+B_1$, B_2 and B_3 along +Z, +X and +Y directions respectively. So, at the centre of quadrant



$$\because \ \vec{B} = \frac{\mu_0 I}{2\pi R} \cdot \frac{\frac{\pi}{2}}{2\pi}$$

$$B = \frac{\mu_0 I}{8\pi R}$$

So M.F. due to quadrants AB, BC and CD at their centre O are B_1, B_2 and B_3 respectively.

$$B_1 = \frac{\mu_0 I}{8\pi R}\hat{k},\ B_2 = \frac{\mu_0 I}{8\pi R}\hat{i},\ \text{and}\ B_3 = \frac{\mu_0 I}{8\pi R}\hat{j}.$$

So net magnetic field at origin due to three current-carrying loops $B = B_1 + B_2 + B_3$.

$$B = \frac{\mu_0 I}{8R}[\hat{i} + \hat{j} + \hat{k}]$$

The resultant of B_1,B_2 and B_3 will be diagonal OR of cube of side B_1, B_2, B_3 as the $|B_1| = |B_2| = |B_3|$

Q4.18. A charged particle of charge e and mass m is moving in an electric field $\vec{E}$ and magnetic field $\vec{B}$. Construct dimensionless quantities and quantities of dimension $[T]^{-1}$.

Main concept used: How a charged particle moves in a magnetic and electric field.

Ans. When a charged particle is placed in an electric and magnetic field, its motion will be circular, and centripetal force is applied by magnetic force $F_m = qvB \sin 90° = qvB$.

$\therefore$ Centripetal force $= \frac{mv^2}{R}$ is balanced by $F_m = qvB$

or $$\frac{mv^2}{R} = qvB$$

$$\frac{v}{R} = \frac{qB}{m}$$

$\because$ $v = \omega R$ and $q = e$

$\therefore$ $$\omega = \frac{v}{R} = \frac{eB}{m}$$

Dimensional formula for angular velocity ω

$$\omega = \left[\frac{eB}{m}\right] = \left[\frac{v}{R}\right] = [T^{-1}]$$

Q4.19. An electron enters with a velocity $v = v_0\hat{i}$ into a cubical region (faces parallel to coordinate planes), in which there are uniform electric and magnetic fields. The orbit of electron is found to spiral down inside the cube in the plane parallel to X-Y plane. Suggest a configuration of fields $\vec{E}$ and $\vec{B}$ that can lead to it.

Main concept used: Motion of charged particle in magnetic and electric field is helical.

Ans. The velocity of electron is $v = v_0\hat{i}$, *i.e.*, along X-axis so magnetic field is in Y direction.

$$B = B_0\hat{k}$$

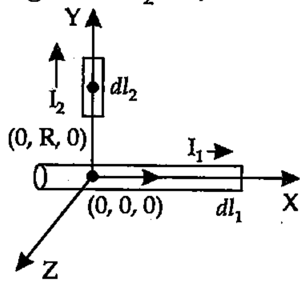
The moving electron enters into cubical region. The force on electron due to Lorentz force

$$F_m = -e[v_0\hat{i} \times B\hat{k}] = -ev_0B\hat{j}$$

which revolve the electron in X-Y plane.

The force due to electric field $F_e = e\vec{E}\hat{k}$ accelerates electron along Z-axis which in turn increases the radius of circular path. So the motion becomes **helical path.**

Q4.20. Do magnetic forces obey Newton's third law? Verify for two current elements $dl_1 = dl\hat{i}$ located at the origin and $dl_2 = dl\hat{j}$ located at (0, R, 0). Both carry current I.



Main concept used: The direction of magnetic field due to current-carrying conductor. And direction of force on current-carrying conductor placed in magnetic field.

Ans. The direction of magnetic field on dl_2 due to magnetic field B_1 by dl_1 will be along +Z direction by Right-Hand Grip Rule, *i.e.*, directions of I_2 and B_1 are perpendicular.

Force on dl_2 due to $dl_1 = B_1I_2dl_2 \sin 90° = B_1I_2l_2$

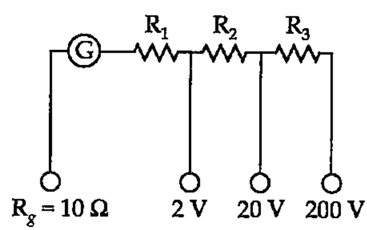
Similarly, angle between magnetic field B_2 and current I_1 is 0°. So the force acting on dl_1 due to dl_2

$$= B_2I_1\, dl_1 \sin 0° = 0.$$

$$\therefore \qquad |dl_1| = |dl_2| = |dl|$$

So magnetic force existing on dl_1 is zero but on dl_2 due to dl_1 is not zero so magnetic forces do not obey Newton's third law.

Q4.21. A multirange voltmeter can be constructed by using a galvanometer circuit, as shown in figure. We want to construct a voltmeter that can measure 2 V, 20 V, 200 V using galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.



Main concept used: Resistance of galvanometer R_g, R_1, R_2 and R_3 are in series and G can tolerate $I_g = 1$ mA.

Ans. For 2 V:

$$I_g(R_g + R_1) = 2\text{ V}$$

$$1 \times 10^{-3}[10 + R_1] = 2$$

$$R_1 = 2000 - 10 = \mathbf{1990\ \Omega.}$$

For 20 V: $\boxed{I(R_g + R) = V}$ $\because R = R_1 + R_2$

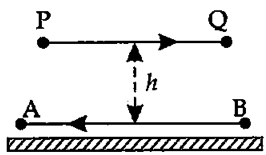
$$I_g[R_g + R_1 + R_2] = 20 \text{ V}$$
$$10^{-3}[10 + 1990 + R_2] = 20$$
$$R_2 = 20000 - 2000$$
$$R_2 = 18000\ \Omega = \mathbf{18\ k\Omega}.$$

For 200 V: $I_g[R_g + R_1 + R_2 + R_3] = 200 \text{ V}$ $\because R = (R_1 + R_2) + R_3$

$$10^{-3}[10 + 1990 + 18000 + R_3] = 200$$
$$20000 + R_3 = 200000$$
$$R_3 = 200000 - 20000$$
$$= 180000\ \Omega = \mathbf{180\ k\Omega}$$

Hence, $R_1 = 1990\ \Omega$; $R_2 = 18\ k\Omega$; $R_3 = 180\ k\Omega$

Q4.22. A long straight wire carrying current of 25 A rests on a table as shown in figure. Another wire PQ of length 1 m and mass 2.5 g carries the same current but in opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise?



Main concept used: Direction and magnitude of magnetic field due to current-carrying conductor and force on a current-carrying conductor placed in magnetic field.

Ans. Wire PQ must experience a repulsive force due to magnetic field by wire AB.

Let the wire is balanced at height h thus, magnetic force due to wire AB on PQ = m g

$$F_m = m\,g$$

Let magnetic field due to AB at height h is B_1 and length of PQ is l_2, balanced at height h. The angle between B_1 and I in PQ is 90°.

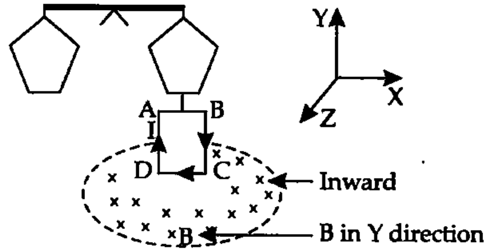
$$\therefore \quad B_1 I_2 l_2 \sin\theta = m\,g$$
$$\frac{\mu_0 I_1}{2\pi h} \cdot I_2 l_2 \sin 90° = m\,g \qquad [I_1 = I_2 = I = 25 \text{ A}]$$
$$\frac{\mu_0 I^2 l_2}{2\pi h} = m\,g$$
$$h = \frac{\mu_0 I^2 l_2}{2\pi\, mg} = \frac{4\pi \times 10^{-7} \times 25 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8}$$
$$= \frac{2 \times 625}{2.5 \times 9.8} \times 10^{-7+3} = \frac{20 \times 25}{9.8} \times 10^{-4} \text{ m} = 51 \times 10^{-4} \text{ m}$$
$$h = \mathbf{0.51\ cm}$$

LONG ANSWER TYPE QUESTIONS

Q4.23. A 100 turn rectangular coil ABCD (in X-Y plane) is hung from one arm of a balance as in figure.

A mass 500 g is added to the other arm to balance the weight of coil. A current of 4.9 A passes through the coil and a constant magnetic field of

0.2 T acting inward (in X-Z plane) is switched on such that only arm CD of length 1 cm lies in the field. How much additional mass m must be added to regain the balance?



Main concept used: (i) Force experienced by current carrying conductors due to M.F. (ii) Weight of coil measured by beam balance is 500 gm.

Ans. The magnetic field is perpendicular to arms BC and AD, so torque will act on CD and AB arms due to it, coil rotate.

When current of 4.9 A does not pass through the coil the balance measures mass of coil 500 g.

On arm AD and BC of rectangular coil magnetic force due to M.F. will be equal and opposite so it will rotate the coil horizontally not vertically up or down. So does not affect the balance.

When current of 4.9 A passes through the coil, downward force acts on arm CD due to magnetic field. Length of arm CD is 1 cm (10^{-2} m).

$\therefore$ Force acting on arm CD $= F_m = B \times I.l = BIl \sin\theta$

[θ is angle between B and I in CD]

$= 0.2 \times 4.9 \times \sin 90° \times 10^{-2}$

or $F_m = 0.98 \times 10^{-2}$ N

Now let the weight mg is added on other side of beam balance to balance the coil

$$mg = F_m$$

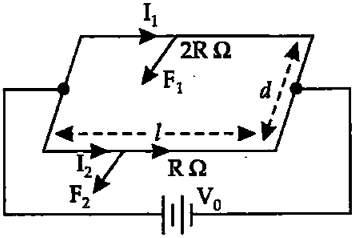
$$m \times 9.8 = 0.98 \times 10^{-2}$$

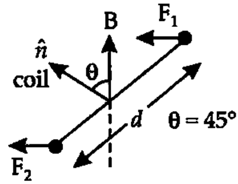
$$m = \frac{0.98}{9.8} \times 10^{-2} = 10^{-3} \text{ kg} = \mathbf{1\ g}.$$

Q4.24. A rectangular conducting loop consists of two wires on two opposite sides of length 'l' joined together by rods of length 'd'. The wires are each of the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance R and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in a uniform magnetic field B at 45° to its plane. Find the torque (τ) exerted by the magnetic field on the loop about an axis through the centres of rods.

Main concepts used: $R = \frac{\rho l}{A}$, $F_m = BIL \sin\theta$, $V = IR$.

Ans.





As $R = \frac{\rho l}{A}$, the thicker wire has resistance R Ω then resistance of thinner wire 2R Ω as both the wires are of the same material with same length but differing in area of cross-section by factor 2 (as given).

$$V_0 = I_1R_1 \Rightarrow I_1 = \frac{V_0}{R_1} = \frac{V_0}{R} \text{ and } I_2 = \frac{V_0}{2R}$$

So
$$F_1 = BI_1 l \sin\theta = \frac{BV_0 l}{R}\sin 45° = \frac{BV_0 l}{\sqrt{2}R}$$

$$F_2 = BI_2 l \sin\theta = \frac{BV_0 l \sin 45°}{2R} = \frac{BV_0 l}{2\sqrt{2}R}$$

$$\tau_1 = F_1 d = \frac{BV_0 l}{\sqrt{2}R}\cdot d \quad \text{and} \quad \tau_2 = \frac{BV_0 ld}{2\sqrt{2}R}$$

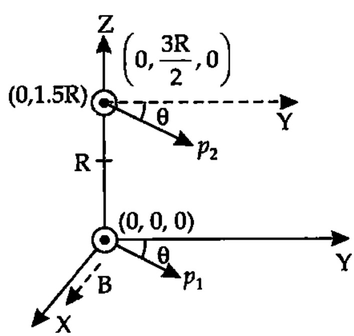
∴
$$\text{Net torque} = \tau = \tau_1 - \tau_2 = \frac{BV_0 ld}{\sqrt{2}R} - \frac{BV_0 ld}{2\sqrt{2}R}$$

$$\tau = \frac{BV_0 ld}{\sqrt{2}R}\left[1 - \frac{1}{2}\right] = \frac{BV_0 ld}{2\sqrt{2}R}.$$

Q4.25. An electron and a positron are released from (0, 0, 0) and (0, 0, 1.5 R) respectively in a uniform magnetic field $B = B_0\hat{i}$, each with an equal momentum of magnitude $\vec{p} = e\vec{B}R$. Under what conditions on the direction of momentum will the orbits be non-intersecting circles?

Main concept used: The boundaries of two circles of radius r_1 and r_2 will not overlap if the distance (d) between their centres is greater than $(r_1 + r_2)$.

Ans. As $B = B_0\hat{i}$ so magnetic field is along +X-axis. The circular motion of the momenta of both an electron and a positron are in Y-Z plane. Let p_1 and p_2 are the momentum of the electron and positron respectively, the magnitude of charge and momentum of both are equal so they revolve in Y-Z plane due to MFB = *Boi.* but in opposite sense with same radius R as the direction and p_1 and p_2 are opposite.



Let p_1 and p_2 make an angle +θ (–θ) with Y-axis, as shown in the above figure.

The centres of the respective circles must be perpendicular to the momentum and at a distance R. Let the centre of revolving electron and positron are C_e and C_p respectively so co-ordinates of

C_e will be (0, + R sin θ, R cos θ) and that of C_p will be

$$\left(0, -R\sin\theta, \frac{3}{2}R - R\cos\theta\right).$$

The planes of circular paths are in Y-Z plane.

The condition that the circular path of the electron and positron are non-intersecting circles is that the distance (d) between their centres must be greater than 2R

$$id > d > 2R$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$= (0-0)^2 + (+R\sin\theta + R\sin\theta)^2 + \left(R\cos\theta - \frac{3}{2}R + R\cos\theta\right)^2$$

$$= (2R\sin\theta)^2 + \left(2R\cos\theta - \frac{3}{2}R\right)^2$$

$$= 4R^2\sin^2\theta + 4R^2\cos^2\theta + \frac{9}{4}R^2 - 6R^2\cos\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta \quad \text{...(I)}$$

$$d > 2R$$

$$d^2 > 4R^2$$

$$4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta > 4R^2$$

$$\frac{9}{4}R^2 - 6R^2\cos\theta > 0$$

$$3R^2\left(\frac{3}{4} - 2\cos\theta\right) > 0$$

$$3R^2 > 0 \text{ rejected}$$

$$-2\cos\theta > \frac{3}{4}$$

$\cos\theta < \frac{3}{8}$ is the condition that two circular paths do not intersect; θ is angle of momentum of electron or positron with Y-axis.

Q4.26. A uniform conducting wire of length 12 a and resistance R is wound up as current-carrying coil in the shape of (i) an equilateral triangle of side a (ii) a square of side a, (iii) a regular hexagon of side a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

Main concept used: Magnetic moment of a coil of n turns, $M = nIA$,

$$\text{Number of turns in coil} = \frac{\text{Length of wire}}{\text{Perimeter of coil}}.$$

Ans. (*i*) The coil of shape equilateral Δ is made up with side a. So number of turns in coil $= \frac{12a}{3a} = 4$ turns.

$\therefore$ Magnetic moment $= nIA = 4 \times I \times \frac{\sqrt{3}}{4}a^2$

Magnetic moment of triangular coil $= \sqrt{3}\,\mathbf{I}a^2$.

(*ii*) Number of turns in coil of square-shape of side $a = \frac{12a}{4a} = 3$ turns

So magnetic moment due to square shaped coil

$$= nIA = 3I \times a^2 = 3Ia^2$$

(*iii*) For a regular hexagon-shaped coil of side a, number of turns in coil

$$= \frac{12a}{6a} = 2$$

$$\therefore \qquad \text{Magnetic moment} = nIA = 2I.\left(\frac{\sqrt{3}}{4}a^2\right)\cdot 6 = 3\sqrt{3}Ia^2.$$

Q4.27. Consider a circular current-carrying loop of radius R in the X-Y plane with centre at origin. Consider the line integral

$$\mathfrak{I}(L) = \left|\int_{-L}^{L} B.dl\right| \text{ taken along Z-axis.}$$

(*a*) Show that $\mathfrak{I}(L)$ monotonically increases with L.

(*b*) Use an appropriate Amperian loop to show that $\mathfrak{I}(\infty) = \mu_0 I$, where I is the current in the wire.

(*c*) Verify directly the above result.

(*d*) Suppose we replace the circular coil by a square coil of side R carrying the same current I. What can you say about $\mathfrak{I}(L)$ and $\mathfrak{I}(\infty)$?

Main concept used: Ampere's circuital law.

Ans. (*a*) B(z) point in the same direction of Z-axis and hence $\mathfrak{I}(L)$ is monotonical function of L as B and dl are along the same direction. So

$$B.dl = Bdl\cos\theta = Bdl\cos 0° = Bdl.$$

(*b*) $\mathfrak{I}(L)$ + Contribution from large distance on contour $C = \mu_0 I$.

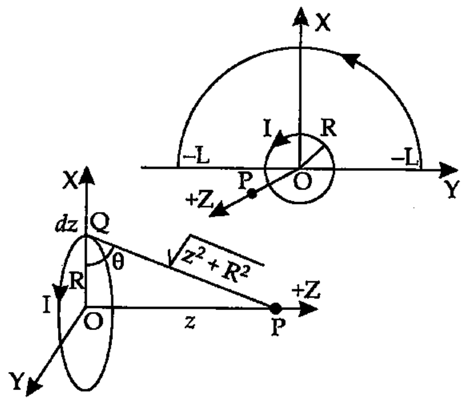
$\therefore$ As $\qquad L \to \infty$

Contribution from large distance $\to 0$. $\qquad \left(\text{As } B \propto \frac{1}{r^3}\right)$

$$\therefore \qquad \mathfrak{I}(\infty) - \mu_0 I.$$

(*c*) The magnetic field due to circular current-carrying loop of radius R in X-Y plane with centre at origin at any point lying at distance *a* from origin.

Consider a loop of current-carrying conductor placed in X-Y plane. A point P is in +Z direction at distance *z*, *i.e.* OP = z.



Again consider an element *dz* on loop of conductor as shown in figure below.

Let angle between R and QP = θ then magnetic field at P due to loop is

$$B\hat{z} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$

[Integrating both sides w.r.t.z and z can vary from –*d* to +*d*]

$$\int_{-\infty}^{\infty} B\hat{z}dz = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2 dz}{2(z^2+R^2)^{3/2}}$$

$$\tan\theta = \frac{z}{R}$$

$$z = R\tan\theta$$

Differentiating both sides:

$$dz = R.\sec^2\theta.d\theta$$

$$\cos\theta = \frac{R}{\sqrt{z^2+R^2}}$$

$$\cos^2\theta = \frac{R}{\sqrt{z^2+R^2}}$$

$$\int_{-\infty}^{\infty} Bzdz = \frac{\mu_0 I}{2}\int_{-\infty}^{+\infty} \frac{R^2}{(z^2+R^2)}\frac{dz}{\sqrt{(z^2+R^2)}}$$

$$= \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \cos^2\theta \cdot \frac{R\sec^2\theta.d\theta}{\sqrt{z^2+R^2}} \quad \left[\because \cos\theta = \frac{R}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \frac{R\,d\theta}{\sqrt{z^2+R^2}} = \frac{\mu_0 I}{2}\int_{-\pi/2}^{\pi/2} \cos\theta\,d\theta$$

$$= \frac{\mu_0 I}{2}[\sin\theta]_{-\pi/2}^{\pi/2} = \frac{\mu_0 I}{2}\left[\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}\right]$$

$$= \frac{\mu_0 I}{2}[1+1] = \mu_0 I = \boxed{\int_{-\infty}^{+\infty} B\hat{z}.dz = \mu_0 I}$$

(*d*) Because the area of square loop is smaller than the area of circular loop, for the same length of conducting wires, hence loop $B(z)_{\text{square loop}} <$ loop $B(z)_{\text{circular loop}}$

$\Im_S(L)_{sq.} = \Im_S(L)_{circular}$ [$\because$ length of conducting wire are equal]

By using arguments as in (*b*) part, Bz does not depend on length of wire

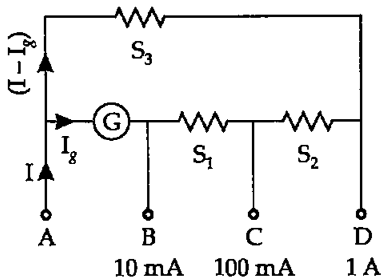
$\therefore\ \Im(\infty)_{\text{sq. loop}} = \Im(\infty)_{\text{circular loop}}$

Magnetic field due to circular or square loops remain same, *i.e.*

$$(Bz)_{\text{circular loop}} = (Bz)_{\text{square loop}} = \mu_0 I$$

So, $\Im(\infty)_{\text{sq. loop}} = \Im(\infty)_{\text{circular loop}} = \mu_0 I$ Hence proved.

Q4.28. A multi-range current meter can be constructed by using a galvanometer circuit as shown in figure. We want a current meter that can measure 10 mA, 100 mA and 1 A using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find S_1, S_2 and S_3 that have to be used.



Main concept used: Galvanometer can be converted into ammeter by lowering its net resistance by using shunt of proper value and using Kirchhoff's law.

Ans. We can measure the currents of magnitude 10 mA, 100 mA and 1 A by connecting ammeter A with B, C and D respectively. So,

for 10 mA $\rightarrow$ $I_g G = (I - I_g)(S_1 + S_2 + S_3)$ ...(I)

for 100 mA $\rightarrow$ $I_g(G + S_1) = (I - I_g)(S_2 + S_3)$ ...(II)

for 1 A $\rightarrow$ $I_g(G + S_1 + S_2) = (I - I_g) S_3$ ...(III)

$I_g = 1\text{ mA} = 10^{-3}$ A and $G = 10\ \Omega$

$$10^{-3} \times 10 = (10^{-2} - 10^{-3})(S_1 + S_2 + S_3) \quad \text{[from (I)]}$$

$$\Rightarrow \quad 10 = (10 - 1)(S_1 + S_2 + S_3)$$

$$\Rightarrow \quad 10 = 9(S_1 + S_2 + S_3) \quad \text{...(IV)}$$

$$10^{-3}(10 + S_1) = (10^{-1} - 10^{-3})(S_2 + S_3) \quad \text{[from (II)]}$$

$$\Rightarrow \quad 10 + S_1 = (100 - 1)(S_2 + S_3)$$

$$\Rightarrow \quad 10 + S_1 = 99(S_2 + S_3) \quad \text{...(V)}$$

$$10^{-3}(10 + S_1 + S_2) = (1 - 10^{-3})(S_3) \quad \text{[from (III)]}$$

$$\Rightarrow \quad 10 + S_1 + S_2 = (1000 - 1)S_3$$

$$\Rightarrow \quad 10 + S_1 + S_2 = 999\,S_3 \quad \text{...(VI)}$$

$$10 + S_1 = 99(S_3 + S_2) \quad \text{[from (V)]}$$

$$10 = 9(S_3 + S_2 + S_1) \quad \text{[from (IV)]}$$

$$\frac{10}{9} = S_1 + S_2 + S_3 \quad \text{[from (IV)]}$$

$$\underset{-}{\frac{10}{99}} + \underset{-}{\frac{S_1}{99}} = \underset{-}{S_2} + \underset{-}{S_3} \quad \text{[from (V)]}$$

$$\frac{10}{9} - \frac{10}{99} - \frac{S_1}{99} = S_1$$

$$\frac{110 - 10}{99} = S_1 + \frac{S_1}{99}$$

$$\frac{100}{99} = \frac{99\,S_1 + S_1}{99} \Rightarrow \frac{100}{99} = \frac{100}{99}S_1$$

$$S_1 = 1\,\Omega$$

So

$$\frac{10}{9} = 1 + S_2 + S_3 \quad \text{[from (IV)]}$$

or

$$\frac{1}{9} = S_2 + S_3$$

$$S_3 = \frac{10}{999} + \frac{S_1}{999} + \frac{S_2}{999} \quad \text{[from (VI)]}$$

$$S_3 = \frac{10}{999} + \frac{1}{999} + \frac{S_2}{999} \quad [\because S_1 = 1\,\Omega]$$

$$\Rightarrow \quad S_3 = \frac{11}{999} + \frac{S_2}{999}$$

$$S_2 = \frac{1}{9} - S_3 \quad \left[\because \frac{1}{9} = S_2 + S_3 \text{ from above}\right]$$

$$= \frac{1}{9} - \frac{11}{999} - \frac{S_2}{999} \quad \text{(from III)}$$

$$S_2 + \frac{S_2}{999} = \frac{111 - 11}{999}$$

$$\frac{1000}{999}S_2 = \frac{100}{999}$$

$$S_2 = 0.1\,\Omega$$

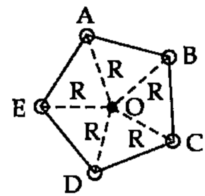
$$S_1 + S_2 + S_3 = \frac{10}{9} \quad \text{(from IV)}$$

$$\Rightarrow \qquad 1 + 0.1 + S_3 = \frac{10}{9}$$

$$\Rightarrow \qquad S_3 = \frac{10}{9} - \frac{11}{10} = \frac{100 - 99}{90} = \frac{1}{90}$$

Hence, $S_1 = 1\ \Omega$; $S_2 = 0.1\ \Omega$; $S_3 = 0.010\ \Omega$

Q4.29. Five long wires A, B, C, D and E each carrying current I are arranged to form edges of a pentagonal prism as shown in figure. Each carries current out of the plane of paper.



Front side

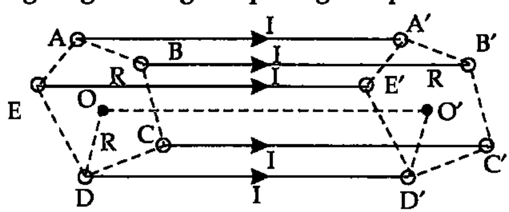
(*a*) What will be magnetic induction at a point on the axis O? Axis is at a distance R from each wire.

(*b*) What will be the field if current in one of the wires (say A) is switched off?

(*c*) What if current in one of the wire (say) A is reversed?

Main concept used: Magnetic field due to current-carrying conductor at distance R is $B = \dfrac{\mu_0 I}{2\pi R}$.

Ans. (*a*) Figure shows that five conductors AA′, BB′, CC′, DD′ and EE′ are along height of regular pentagonal prism ABCDE.



It is given that five identical conducting wires are along the heights of regular pentagon, represented in figure above by AA′, BB′, CC′, DD′, and EE′. Axis of regular pentagon is OO′ will be equidistant (R) from all five conductors, the current is passing through all five conductors are equal let (I).

As the current in all 5 conductors are equal to I and the distance of O from conductors is also equal to R then magnitude of magnetic field due to each conductor will be equal, *i.e.*

$$|B_1| = |B_2| = |B_3| = |B_4| = |B_5| = B = \frac{\mu_0 I}{2\pi R}$$

The direction of magnetic field induced can be find out by right hand grip rule, then direction of induced magnetic field at 'O' due to AA' will be perpendicular to both AA' and AO. Angles between any two consecutive magnetic field is $\frac{360°}{5} = 72°$

As shown in figures given here

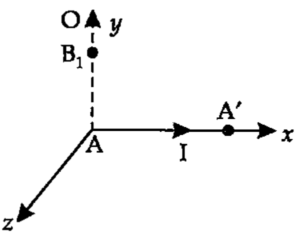


Fig. shows the direction of magnetic field B_1 due to AA' conducting wire

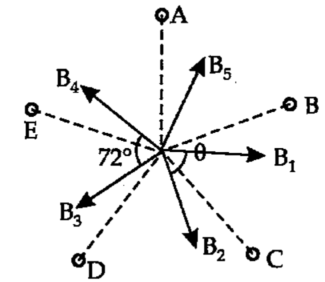


Fig. shows the direction of B_1, B_2, B_3, B_4 and B_5 due to conductors AA', BB', CC', DD', and EE' respectively

As $B = B_1 = B_2 = B_3 = B_4 = B_5$ and angle between consecutive magnetic fields is 72° or symmetric in 360° so their resultant at O will be zero, *i.e.*

$$\vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = 0$$

Hence, the induced magnetic induction at O due to five conductors as shown in figure is zero.

(*b*) When current in AA' is switched off, then $B_1 = 0$ and resultant becomes $R = B_2 + B_3 + B_4 + B_5$

But from (*a*) part $B_1 + B_2 + B_3 + B_4 + B_5 = 0$

or $$\vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = -\vec{B}_1$$

$$R = -B_1$$

$$R = -\frac{\mu_0 I}{2\pi R}$$

i.e. direction of resultant is opposite to $\vec{B}_1$.

(*c*) Here, on reversing the current in AA', direction of magnetic field due to AA' becomes $-\vec{B}_1$,

$$R = B_2 + B_3 + B_4 + B_5$$

$\because$ $$|-B_1| = |B_2| = |B_3| = |B_4| = |B_5| = B$$

$\therefore$ net induced magnetic field at O becomes

$$-B + B + B + B + B = 3B$$

$$R = \frac{3\mu_0 I}{2\pi R}$$

❑❑❑