5 Magnetism and Matter

MULTIPLE CHOICE QUESTIONS—I

Q5.1. A Toroid of n turns, mean radius R and cross-sectional area *a* carries a current I. It is placed on a horizontal table taken as *x-y* plane. It's magnetic moment *m* —

(*a*) is non-zero and points in z-direction by symmetry.
(*b*) points along the axis of toroid ($\vec{M} = m\hat{\varphi}$).
(*c*) is zero otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the toroid.
(*d*) is pointing radially outwards.

Main concept used: No magnetic field outside the toroid. Magnetic field only inside the toroid and $\vec{M} = I\vec{A}$.

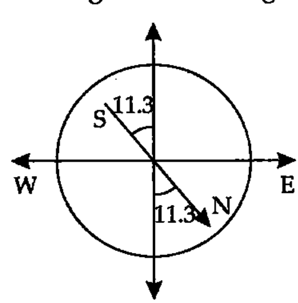
Ans. (*c*): We know that there is no magnetic field outside the toroid. So outside, the M.F. falls very rapidly as it is inversely proportional to third power of distance from centre of toroid.

By Amperian circuital law, there is no any current so no magnetic moment outside. Verifies answer (*c*).

Q5.2. The magnetic field of Earth can be modelled by that of a point dipole placed at the centre of Earth. The dipole axis makes an angle of 11.3° with the axis of Earth. At Mumbai, declination is nearly zero, then

(*a*) the declination varies between 11.3° W to 11.3° E.
(*b*) the least declination is zero.
(*c*) the plane defined by dipole axis and the earth axis passes through Greenwich.
(*d*) declination averaged over the earth must be always negative.

Main concept used: Magnetic declination is an angle between angle of magnetic meridian and the geographic meridian.



Ans. (*a*): As the earth's magnetic field can be considered similar to that of a hypothetical magnetic dipole located at the centre of earth. The axis of dipole is tilted 11.3° with axis of geographic of earth by 11.3° both side east and west (South pole of earth towards West and North pole towards east).

So the declination varies from East to West both side 11.3° on whole earth surface verify answer (*a*). Also see figure with answer Q.24.

Q5.3. In a permanent magnet at room temperature

(*a*) magnetic moment of each molecule is zero.

(*b*) the individual molecules have non-zero magnetic moment which are all perfectly aligned.

(*c*) domains are partially aligned.

(*d*) domains are all perfectly aligned.

Main concept used: In permanent magnet more domains are permanently aligned (not all).

Ans. (*c*): At room temperature permanent magnet behaves like a ferromagnetic substance.

When there is no magnetic field in ferromagnetic substance domains are randomly spread. So resultant magnetic moment is about zero.

But when this substance is placed in strong magnetic field some domains aligned in external field permanently even in absence of external magnetic field.

So domains partially aligned permanently verifies the option (*c*).

Q5.4. Consider two idealised systems (*i*) a parallel plate capacitor with large plates and small separation, (*ii*) a long solenoid of length L >> R, radius of cross-section. In (*i*) E is ideally treated as a constant between the plates and zero outside. In (*ii*) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental law as below

(*a*) case (*i*) contradicts Gauss's law of electrostatic fields.

(*b*) case (*ii*) contradicts Gauss's law of magnetic fields.

(c) case (*i*) agrees with $\oint_S \mathrm{E} \cdot dl = 0$

(*c*) case (*ii*) contradicts $\oint \mathrm{H}.dl = \mathrm{I}_{en}$

Main concept used: Electric field lines do not form continuous path while the magnetic field lines forms closed paths.

Ans. (*b*): According to Gauss's law of electrostatic field $\oint_s \mathrm{E} \cdot ds = \frac{q}{\varepsilon_0}$ so it does not contradicts for electrostatic field as the electric field lines do not form continuous path.

According to Gauss's law of magnetic field $\oint_s \mathrm{B} \cdot ds = 0$. It contradicts for magnetic field, because there is a magnetic field inside the solenoid, and no field outside the solenoid carrying current, but the magnetic field lines form the closed paths.

Q5.5. Paramagnetic sample shows a net magnetisation of 8 $\mathrm{Am^{-1}}$ when placed in external magnetic field of 0.6 T at a temperature of

4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be—

(*a*) $\frac{32}{3}\,Am^{-1}$, (*b*) $\frac{2}{3}\,Am^{-1}$, (*c*) $6\,Am^{-1}$, (*d*) $2.4\,Am^{-1}$

Main concept used: Curie's law,

$$\text{Magnetisation} \propto \frac{\text{B(Magnetic field induction)}}{\text{T(Absolute temperature)}}.$$

Ans. (*b*) According to the Curie's law of magnetisation, (I), for a substance is directly proportional to magnetic field induction (B) and inversely proportional to the absolute temperature T.

$$I \propto \frac{B}{T} \quad \text{or} \quad \frac{I_2}{I_1} = \frac{B_2T_1}{B_1T_2}$$

$I_1 = 8\,Am^{-1}$ $\quad B_1 = 0.6T$ $\quad T_1 = 4\,K$

$I_2 = ?$ $\quad B_2 = 0.2T$ $\quad T_2 = 16\,K$

$$I_2 = \frac{B_2T_1}{B_1T_2}I_1 = \frac{0.2\times 8\times 4}{0.6\times 16} = \frac{2}{3}\,Amp^{-1}$$

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q5.6. S is the surface of a lump of magnetic material.

(*a*) Lines of B are necessarily continuous across S.

(*b*) Some lines of B must be discontinuous across S.

(*c*) Lines of H are necessarily continuous across S.

(*d*) Lines of H cannot all be continuous across S.

Main concepts used: (*i*) Magnetic field lines are always continuous. (*ii*) Field lines of electric field of a dipole begins from positive charge and ends on negative charge or escape to infinity. (*iii*) Also magnetic intensity (H) outside the magnet is $H = \frac{B}{\mu_0}$ and for

$$\text{inside the magnet } H = \frac{B}{\mu_0\mu_r}.$$

Ans. (*a*) (*d*): Magnetic field lines for magnetic induction (B) form continuous lines so lines of B are necessarily continuous across S. Also magnetic intensity (H) to magnetise varies for inside and outside the lump. So lines of H cannot all be continuous across S.

Q5.7. The primary origin of magnetism lies in

(*a*) atomic currents (*b*) Paulie's exclusion principle

(*c*) polar nature of molecules (*d*) intrinsic spin of electron.

Main concept used: Motion of charge produces magnetic field, due to magnetic effect of current.

Ans. (*a*) (*d*): Motion of charge particle produces magnetism and nature of magnetism depends on the nation of motion of charge particle.

In atom, electrons revolve and spin around the nucleus which in turn produces current and due to magnetic effect of current, magnetism produces in the material.

Q5.8. A long solenoid has 1000 turns per metre and carries a current of 1 A. It has a soft iron core of $\mu_r = 1000$. The core is heated beyond the Curie temperature T_C.

(*a*) The H field in the solenoid is (nearly) unchanged but the B field decreases drastically.

(*b*) The H and B fields in the solenoid are nearly unchanged.

(*c*) The magnetisation in the core reverses the direction.

(*d*) The magnetisation in the core diminishes by a factor of about 10^8.

Main concept used: (*i*) Behaviour of magnetic material beyond Curie temperature, (*ii*) Magnetic intensity $H = nI$, (*iii*) The magnetic induction $B = \mu_r \mu_o nI$

Ans. (*a*) (*d*): $n = 1000$ turns per metre, $\mu_r = 1000$

$H = nI = 1000 \times 1 = 1000$ Amp. So H is constant verifies the answer (*a*).

$B = \mu_o \mu_r nI = (\mu_o nI)\mu_r = K \mu_r$ (K = constant)

So, $B \propto \mu_r$.

By Curie's law, when ferromagnetic substance is heated beyond Curie temperature it behaves like a paramagnetic substance. Where,

Susceptibility of $(\chi_m)_{ferro} = 10^3$

Susceptibility of $(\chi_m)_{para} = 10^{-5}$

$$\therefore \qquad \frac{B_2}{B_1} = \frac{\chi_2}{\chi_1} = \frac{10^{-5}}{10^3} = 10^{-8} \quad \text{or} \quad B_2 = 10^{-8} B_1$$

So, magnetisation becomes 10^{-8} times of earlier or diminished by 10^8 times. Verified answer (*d*).

Q5.9. Essential difference between electrostatic shielding by conducting shell and magnetic shielding is due to

(*a*) electrostatic field lines can end on charges and conductors have free charges.

(*b*) lines of B can also end but the conductors cannot end them.

(*c*) lines of B cannot end on any material and perfect shielding is not possible.

(*d*) shells of high permeability material can be used to divert the lines of B from the interior region.

Main concepts used: (*i*) Electric field lines start from positive charge and ends on negative charge and there is existence of positive and negative charge separately (*ii*) Non-existence of monopole, (*iii*) Magnetic field lines start from North pole and ends to South pole, (*iv*) The path of magnetic field lines can be affected (attracted or repelled) by magnetic materials.

Ans. (*a*) (*c*) (*d*): Conductors have free charge particles so the lines of force can be stopped by conductors gives shielding effect.

As non-existence of mono pole magnetic field lines cannot be stopped or shield.

Magnetic field lines are affected by magnetic materials so can be repelled by using a high permeability magnetic material to get the region of no magnetic field.

Q5.10. Let the magnetic field on the earth is modelled by that of a point magnetic dipole at the centre of the earth. The angle of dip at a point on the geographical equator

(*a*) is always zero (*b*) can be zero at specific points

(*c*) can be positive or negative (*d*) is bounded

Main concept used: Angle of dip specifies the direction of resultant magnetic field of earth.

Ans. (*b*) (*c*) (*d*): As the dipole is placed at an angle of 11.3° W to 11.3° E from geographical N-S axis. South and North pole of it are just like an imagined magnet of earth. South of dipole is in North direction towards 11.3° towards west and North pole in South direction 11.3° towards east.

The resultant magnetic field due to dipole will be zero at its equatorial plane of dipole but not on geographical equator verifies answer (*b*).

The angle of dip. will change on changing the equatorial plane of dipole, *i.e.*, where angle of dip. is zero. It is not zero at all points of geographic equator. Angle of dip on geographical equator will be zero where it meets with magnetic equator. So verifies answers (*c*) and (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q5.11. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?

Main concept used: (*i*) Magnetic dipole moment of a charge particle $\boxed{M = \frac{eh}{4\pi m}}$, (*ii*) Mass of proton is 1836 times larger than electron.

Ans. As we know that magnetic moment of a charged particle of charge e and mass m is

$$M_e = \frac{eh}{4\pi m_e} \quad \text{and} \quad M_p = \frac{eh}{4\pi m_p}$$

As charge on proton and electron are equal in magnitude so $M \propto \frac{1}{m}$ or $\frac{M_e}{M_p} = \frac{m_p}{m_e}$.

As the mass of proton is 1836 times as of electron, so magnetic moment of proton $M_p = \frac{M_e}{m_p} m_e = \frac{M_e \cdot m_e}{1836\, m_e}$, $M_p = \frac{1}{1836}$ of M_e.

So magnetic moment of proton is $\frac{1}{1836}$ times of electron, so can be neglected.

Q5.12. A permanent magnet in the shape of a thin cylinder of length 10 cm has $M = 10^{+6}$ A/m. Calculate the magnetisation current I_M.

Main concept used: Magnetic moment $= \frac{I}{l}$.

Ans. $M = 10^{+6}$ ampre/metre, $l = 10$ cm $= 0.10$ m

I_M = Magnetisation current

$$M = \frac{I_M}{l}$$

So, $I_M = M \times l = 10^6 \times 0.1 = 10^5$ ampere.

Q5.13. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of $N_2(\sim 5 \times 10^{-9})$ (at S.T.P) and of $Cu(\sim 10^{-5})$.

Main concept used: Magnetic susceptibility is the property of substance which show how a material behaves in external magnetic field.

Ans.

$$\rho_N = \frac{28\text{ g}}{22.4\text{ lit}} = \frac{28\text{ g}}{22400\text{ ml}} = \frac{28}{22400}\text{ g per cm}^3$$

$$\rho_{cu} = 8\text{ g/cm}^3$$

$$\therefore \quad \frac{\rho_N}{\rho_{Cu}} = \frac{28}{22400 \times 8} = 1.6 \times 10^{-4}$$

Also,

$$\frac{\chi_N}{\chi_{Cu}} = \frac{5 \times 10^{-9}}{10^{-5}} = 5 \times 10^{-4}$$

$$\chi = \frac{\text{Intensity of magnetisation (M)}}{\text{Magnetising field intensity (H)}}$$

$$\chi = \frac{\frac{\text{Magnetic moment } (m)}{\text{Volume (V)}}}{\text{H}}$$

$$\chi = \frac{m}{\text{HV}}$$

If m', V, ρ are the mass, volume and density of magnetic material, then

$$\frac{m'}{\rho} = V$$

$$\chi = \frac{m \times \rho}{H.m'}$$

$\chi \propto \rho$ as mass, m' and H are constant and magnetic material are same.

So $$\frac{\chi_N}{\chi_{Cu}} = \frac{\rho_N}{\rho_{Cu}} = 1.6 \times 10^{-4}$$

The major difference between diamagnetic susceptibility of N_2 gas and solid Cu is due to their difference between densities.

Q5.14. From molecular point of view, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.

Main concept used: Susceptibility χ is the ratio of intensity of magnetisation (M) of magnetic material to the intensity of magnetising field (H).

$$\chi = \frac{M}{H}.$$

Magnetic moment of diamagnetism due to orbital motion is opposite to applied field.

Ans. The direction of external magnetic field H and magnetism M due to orbital motion of electrons of diamagnetic substances are opposite so net magnetism becomes zero. Hence, the susceptibility (χ) of *diamagnetism is not much affected by temperature.*

The direction of magnetism due to orbital motion of electrons in paramagnetism and ferromagnetism material and external applied field are in same direction so net magnetism increased and much affected by temperature. *As temperature raised the alignment of atomic magnetism is disturbed resulting in decrease in susceptibility.*

Q5.15. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet.

(*i*) In which direction will it move

(*ii*) What will be the direction of its magnetic moment?

Main concept used: Superconducting material and liquid nitrogen are diamagnetic materials.

Ans. Liquid nitrogen and a superconducting material are diamagnetic materials and remain diamagnetic when superconducting material is dipped in liquid nitrogen.

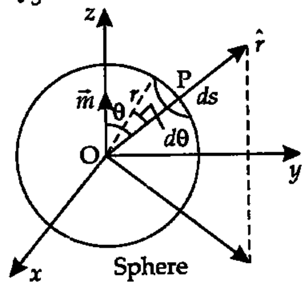
So when external magnetic field is applied on superconducting material dipped in liquid nitrogen it will *be repelled by external* magnetic field and direction of motion will be opposite to direction of magnet or magnetic field.

SHORT ANSWER TYPE QUESTIONS

Q5.16. Verify the Gauss's law for magnetic field of a point dipole (magnetic) of dipole moment $\vec{m}$ at the origin for surface which is a sphere of radius R.

Main concept used: Gauss's law in magnetism.

Ans. We know by Gauss's law in magnetism $\oint_S \vec{B}.\vec{ds} = 0$.



Magnetic moment (m) of dipole at origin O is $\vec{m} = m\hat{k}$.

Let P be a point at a distance r from O and OP makes an angle θ with z axis. Component of $\vec{m}$ along OP is equal to $\vec{m}$ cos θ.

Now the magnetic field of induction at P due to dipole of moment $\vec{m}$ cos θ is

$$B = \frac{\mu_0}{4\pi}\frac{2\vec{m}\cos\theta}{r^3}\hat{r}$$

Here r is the radius of sphere with centre 'O' lying in X-Y-Z plane. Centre at origin.

Take an elementary area ds at P then

$$\vec{ds} = r(r\sin\theta)\,d\theta\,\hat{r} = r^2\sin\theta\,d\theta\,\hat{r} \quad \left[\because d\theta = \frac{ds}{r^2} \text{ or } ds = r^2 d\theta\right]$$

$$\oint_S \vec{B}.\vec{ds} = \oint \frac{\mu_0}{4\pi}\frac{2\vec{m}\cos\theta}{r^3}\hat{r}\cdot(r^2\sin\theta\cdot d\theta\,\hat{r})$$

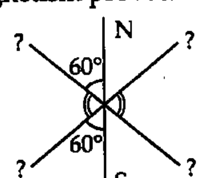
$$= \frac{\mu_0\vec{m}}{4\pi r}\int_0^{2\pi} 2\sin\theta\cos\theta\cdot d\theta = \frac{\mu_0\vec{m}}{4\pi r}\int_0^{2\pi}\sin 2\theta\cdot d\theta$$

$$= \frac{\mu_0\vec{m}}{4\pi r}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} = \frac{\mu_0}{4\pi r.2}[\cos 4\pi - (-\cos 0)]$$

$$= \frac{\mu_0}{4\pi r\times 2}[-\cos 0 + 1] = \frac{\mu_0}{8\pi r}[-1+1]$$

$\oint B.ds = 0$ Hence the Gauss's law in magnetism proved.

Q5.17. Three identical bar magnets are riveted together at centre in the same plane as shows in figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet is shown in figure. Determine the poles of the remaining two.

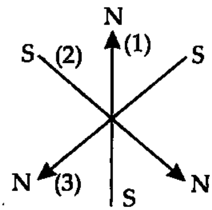


Main concept used: The resultant magnetic force on magnets must be zero for no motion.

Ans. The poles must be symmetric to each other or a magnet. It is possible only when the poles of the remaining two magnets are as in given figure.

The north pole of magnet (1) is equally attracted by south poles of (2) and (3) magnets placed at equal distance.

Similarly one pole of any one magnet is attracted by opposite poles of other two magnets so resultant force or moment on each magnet is zero and will not be in motion placed on table.



Q5.18. Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of (*i*) electric dipole $\vec{p}$ in an electrostatic field $\vec{E}$ and (*ii*) magnetic dipole $\vec{m}$ in a magnetic field $\vec{B}$. Write down a set of conditions on $\vec{E}$, $\vec{B}$, $\vec{p}$, and $\vec{m}$ so that the two motions are verified to be identical. (Assume identical initial conditions).

Main concept used: Force on dipole (electric) in electric field and torque on magnetic dipole in magnetic field.

Ans. Let θ is the angle between $\vec{m}$ and $\vec{B}$.

∴ Torque on magnetic dipole in a magnetic field B is

$$\tau = \vec{m}\vec{B}\sin\theta \quad \text{...I}$$

Similarly if θ is the angle between electric dipole moment $\vec{p}$ and electric field E then torque on electric dipole in E is

$$\tau' = \vec{p}\vec{E}\sin\theta \quad \text{...II}$$

For if motion in I and II of electric and magnetic dipole are identical then $\tau' = \tau$

$$\vec{p}\vec{E}\sin\theta = \vec{m}\vec{B}\sin\theta$$

or

$$\vec{p}\vec{E} = \vec{m}\vec{B} \quad \text{...III}$$

We know that $\vec{E} = c\vec{B}$ (relation between E and B) ...IV

c is velocity of light.

Put the value of E from IV in III

$$\vec{p}cB = \vec{m}\vec{B}$$

$$\boxed{\vec{p} = \frac{\vec{m}}{c}}$$

It is the required relation.

Q5.19. A bar magnet of magnetic moment $\vec{m}$ and moment of inertia I (about axis passing through centre and perpendicular to length) is cut into two equal pieces perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid point, perpendicular to length, in magnetic field B. What would be the similar period T′ for each piece?

Main concept used: $T = 2\pi\sqrt{\frac{I}{mB}}$ and $\boxed{nm' = m}$

Ans. If a magnet of magnetic moment m is cut into n equal parts then magnetic moment m' of all equal parts is $nm' = m$. So magnetic moment of each 2 parts of magnet $= m' = \frac{m}{2}$.

$$I = \frac{ml^2}{12}$$

as the length of new magnet $= l' = \frac{l}{2}$

So original time period $T = 2\pi\sqrt{\frac{I}{mB}}$.

If M is the mass of original magnet then the mass of each two magnets m' will be $\frac{M}{2}$.

So, $$I = \frac{Ml^2}{12} \quad \text{and} \quad I' = \frac{\frac{M}{2}\cdot\left(\frac{l}{2}\right)^2}{12} = \frac{Ml^2}{8\times 12}$$

$$\frac{T}{T'} = \frac{2\pi\sqrt{\frac{I}{mB}}}{2\pi\sqrt{\frac{I'}{m'B}}} = \sqrt{\frac{I}{m}\cdot\frac{m'}{I'}} \quad \text{or} \quad \frac{T'}{T} = \sqrt{\frac{m}{m'}\cdot\frac{I'}{I}}$$

$$\frac{I'}{I} = \frac{\frac{m'l'^2}{12}}{\frac{ml^2}{12}} = \frac{\frac{m}{2}\cdot\left(\frac{l}{2}\right)^2}{ml^2}$$

$$\frac{I'}{I} = \frac{\frac{m}{2}\cdot\frac{l^2}{4}}{ml^2} = \frac{1}{8}$$

$$\frac{m}{m'} = \frac{m}{\frac{m}{2}} = \frac{2}{1}$$

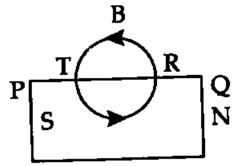
$$\therefore \quad \frac{T}{T'} = \sqrt{\frac{2}{1}\times\frac{1}{8}} = \sqrt{\frac{1}{4}}$$

$$\frac{T}{T'} = \frac{1}{2} \quad \text{or} \quad T' = \frac{T}{2}\ \text{sec}$$

Q5.20. Use (*i*) Ampere's law for $\vec{H}$ and (*ii*) continuity of lines of $\vec{B}$ to conclude that inside a bar magnet, (*a*) lines of $\vec{H}$ run from the N-pole to S-pole while (*b*) lines of $\vec{B}$ must run from S-pole to N-pole.

Main concept used: Ampere's circuital law, angle between $\vec{B}$ and $\vec{dl}$ inside or outside the magnet.

Ans. Consider an Amperian loop C inside and outside the magnet NS on side PQ of magnet then



$$\int_P^Q \vec{H}.\vec{dl} = \int_Q^P \frac{\vec{B}}{\mu_0}\cdot\vec{dl}$$

where B is magnetic field and m_0 is dipole moment. As angle between B and dl varies from 90°, 0, 90° from R to T in figure, so cos θ is greater than 1. So $\int_P^Q \vec{H}.\vec{dl} = \int_Q^P \frac{\vec{B}}{\mu_0}\cdot\vec{dl} > 0$ *i.e.* positive.

Hence, the value of B must be varied from south pole to north pole inside the magnet. According to Ampere's law $\oint_{PQP} \vec{H}.\vec{dl} = 0$

$$\oint_{PQP} \vec{H}.\vec{dl} = \int_P^Q \vec{H}.\vec{dl} + \int_Q^P \vec{H}.\vec{dl} = 0$$

As $\int_P^Q H.dl > 0$ (outside the magnet) and $\int_Q^P H.dl < 0$ (inside the magnet). It is due to the angle between H and dl is more than 90° inside the magnet so cos θ is negative. It means the lines of H must run from north pole to south pole.

LONG ANSWER TYPE QUESTIONS

Q5.21. Verify the Ampere's law for magnetic field of a point dipole of dipole moment $\vec{m} = m\hat{k}$. Take C as the closed curve running clockwise along

(*i*) the Z-axis from Z = a > 0 to Z = R,

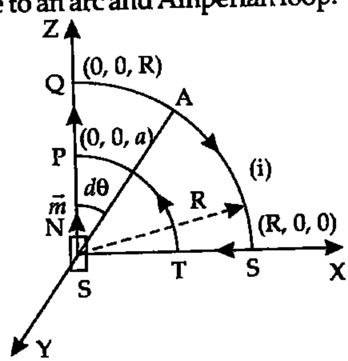
(*ii*) along the quarter circle of radius R and centre at the origin in first quadrant of X-Z plane,

(*iii*) Along the X-axis from X = R to X = a and

(*iv*) along the quarter circle of radius a and centre at the origin in the first quadrant of X-Z plane.

Main concept used: Magnetic field due to an arc and Amperian loop.

Ans. In figure from P to Q every point on Z-axis lies on dipole is in $\hat{k}$ direction so all points on Z-axis lies on axial dipole NS placed at origin.



So magnetic field induction (B) at a point (0, 0, z) from magnetic dipole (centre at origin) and having magnetic moment $\vec{m}\hat{k}$ of magnitude ($\vec{m}$).

$$|B| = \frac{\mu_0}{4\pi}\frac{2(\vec{m})}{z^3} = \frac{\mu_0\vec{m}}{2\pi z^3}$$

(*i*) Ampere's law along Z axis from Z = *a* to Z = R *i.e.* from P to Q

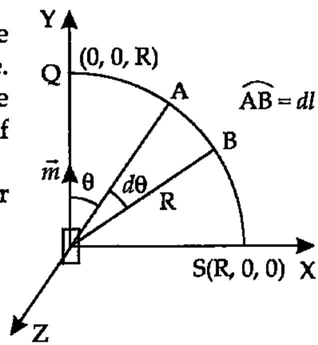
$$\int_P^Q \vec{B}.\vec{dl} = \int_P^Q B dl \cos 0° = \int_P^Q B\, dz$$

$$= \int_P^Q \frac{\mu_0 \vec{m}}{2\pi z^3} dz = \frac{\mu_0 \vec{m}}{2\pi}\int_a^R z^{-3} dz$$ [∵ distance of P and Q from origin are *a* and R respectively]

$$= \frac{\mu_0 \vec{m}}{2\pi}\left[\frac{z^{-2}}{-2}\right]_a^R = \frac{\mu_0 \vec{m}}{2\pi(-2)}\left[\frac{1}{R^2} - \frac{1}{a^2}\right]$$

$$\int_P^Q B.dl = \frac{\mu_0 \vec{m}}{4\pi}\left[\frac{1}{a^2} - \frac{1}{R^2}\right]$$

(*ii*) Ampere's law along the quarter circle QS of radius R as given in figure here. Point A can be considered on the equitorial line of magnetic dipole of moment $\vec{m}$ sin θ.



Magnetic field at point A on the circular arc is

$$B = \frac{\mu_0}{4\pi}\frac{\vec{m}\sin\theta}{R^3}$$

$$d\theta = \frac{dl}{R} \Rightarrow dl = R.d\theta$$

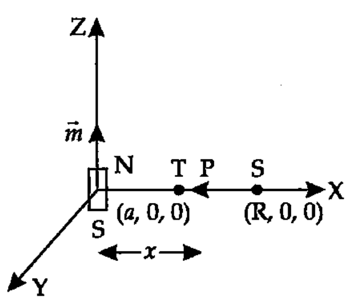
∴ By Ampere's law $\int B.dl = \int B.dl \cos\theta$

$$\int \vec{B}.\vec{dl} = \int_0^{\frac{\pi}{2}} \frac{\mu_0}{4\pi}\frac{\vec{m}\sin\theta}{R^3} \cdot R\, d\theta$$

$$\int \vec{B}.\vec{dl} = \frac{\mu_0 \vec{m}}{4\pi R^2}[-\cos\theta]_0^{\pi/2} = \frac{\mu_0 \vec{m}}{4\pi R^2}[-\cos 90° + \cos 0°]$$

$$\int B.dl = \frac{\mu_0 \vec{m}}{4\pi R^2}$$

(*iii*) Ampere's law along the X axis from *x* = R to *x* = *a* as in given figure here.
As all points from S to T lies on equitorial line of magnetic dipole N-S so magnetic field induction at a point P at a distance *x* from the dipole is

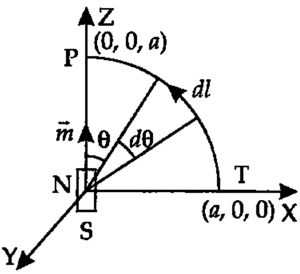


$$B = \frac{\mu_0}{4\pi}\frac{\vec{m}}{x^3} = \frac{\mu_0}{4\pi}\frac{\vec{m}\hat{k}}{x^3}$$

$$\int \vec{B}.\vec{dl} = \int_R^a \frac{-\mu_0 \vec{m}\hat{k}}{4\pi x^3}.\vec{dl} \quad \because \text{ angle between } dl \text{ and } m \text{ is } 90°$$

So $$\int \vec{B}.\vec{dl} = \int_R^a \frac{-\mu_0 |\vec{m}|\, dl \cos 90°}{4\pi x^3} = 0$$

(*iv*) Ampere's law along the quarter circle of radius '*a*' and centre at the origin in the quadrant X-Z plane as in figure here as in part (*ii*).



$$\int B.dl = \int_{\pi/2}^{0} \frac{\mu_0}{4\pi}\frac{\vec{m}\sin\theta}{a^3}.\,ad\theta$$

$$\int B.dl = \frac{\mu_0 \vec{m}}{4\pi a^2}[-\cos\theta]_{\pi/2}^{0}$$

θ = angle from axial line of dipole

$$\int B.dl = \frac{\mu_0 \vec{m}}{4\pi a^2}\left[-\cos 0 + \cos\frac{\pi}{2}\right] = \frac{\mu_0 \vec{m}}{4\pi a^2}(-1+0) = \frac{-\mu_0 \vec{m}}{4\pi a^2}$$

∴ Applying Ampere's law along close path starting from P to Q, Q to S and S to P.

$$\oint_{PQST} B.dl = \int_P^Q \vec{B}.\vec{dl} + \int_Q^S \vec{B}.\vec{dl} + \int_S^T \vec{B}.\vec{dl} + \int_T^P B.dl$$

From parts (*i*), (*ii*), (*iii*) and (*iv*) substituting the values

$$\oint_{PQST} B.dl = \frac{\mu_0 \vec{m}}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right) + \frac{\mu_0 \vec{m}}{4\pi R^2} + 0 + \frac{-\mu_0 \vec{m}}{4\pi a^2}$$

$$= \frac{\mu_0 \vec{m}}{4\pi a^2} - \frac{\mu_0 \vec{m}}{4\pi R^2} + \frac{\mu_0 \vec{m}}{4\pi R^2} - \frac{\mu_0 \vec{m}}{4\pi a^2}$$

$$\oint_{PQST} B.dl = 0$$

Proves the Ampere's law is magnetism.

Q5.22. What are the dimensions of χ, the magnetic susceptibility? Consider an H atom. Guess an expression for χ upto a constant by constructing a quantity of dimensions of χ out of parameters of atom *e*, *m*, *v*, R and μ_0. Here *m* is electronic mass, v is the electronic velocity, R is the Bohr's radius. Estimate the number so obtained and compare with the value of $|\chi| \sim 10^{-5}$ for many solid materials.

Main concept used: (*i*) $\chi = \frac{M}{H} = \frac{\text{Intensity of magnetisation}}{\text{Magnetising field}}$, (*ii*) Biot-Savart's law, (*iii*) dimension analysis.

Ans. As the intensity of magnetisation (M) and magnetising field (H) both has the same unit, *i.e.* ampere per metre and $\chi = \frac{M}{H}$ so χ (susceptibility) has no unit.

We have to relate χ with e, v, $\tilde{m}$, R and μ_0. We will relate these physical quantities by using dimension and Biot-Savart's law.

(*i*) By Biot-Savart's law $dB = \frac{\mu_0}{4\pi}\frac{Idl \sin\theta}{r^2}$ can be used to find out the dimension of μ_0.

$$\mu_0 = \frac{dB.4\pi r^2}{Idl \sin\theta} \quad \text{for dB}$$

for dimension of dB

$$F = Bqv \sin\theta$$

$$B = \frac{F}{qv\sin\theta} = \frac{MLT^{-2}}{QLT^{-1}} = [ML^0T^{-1}Q^{-1}]$$

$$\therefore \qquad \mu_0 = \frac{[MT^{-1}Q^{-1}]L^2}{QT^{-1}L} = [MLQ^{-2}]$$

Where Q is dimension of charge.

χ depends on magnetic moment induced when H is turned on. H couples to atomic electrons through its charge e. The effect on m is via current I which involves another factor of e. The combination $\mu_0 e^2$ does not depend on the "charge" Q. Dimension $\chi = \mu_0^a e^2 m^b v^c R^d$

$$[M^0L^0T^0Q^0] = \left[MLQ^{-2}\right]^a Q^2M^b[LT^{-1}]^c\,[L]^d$$

$$[M^0L^0T^0Q^0] = M^{a+b} \quad L^{a+c+d} \quad T^{-c} \quad Q^{-2a+2}$$

Comparing the powers

$$c = 0, \qquad a + b = 0 \qquad -2a + 2 = 0 \qquad a + c + d = 0$$

$$1 + b = 0 \qquad -2a = -2 \qquad 1 + 0 + d = 0$$

$$b = -1 \qquad a = +1 \qquad d = -1$$

$$\chi = (\mu_0)e^2\, m^{-1}\, v^0\, R^{-1}$$

$$\chi = \frac{\mu_0 e^2}{mR}$$

$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} $\qquad e = 1.6 \times 10^{-19}$

$m = 9.1 \times 10^{-31}$ kg $\qquad R = 10^{-10}$ m

$$\chi = \frac{(4\pi \times 10^{-7})(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(9.1 \times 10^{-31}) \times 10^{-10}}$$

$$= \frac{4 \times 3.1 \times 1.6 \times 1.6}{9.1} \times 10^{-7-19-19+31+10}$$

$$= \frac{124 \times 256 \times 10^{-45+41}}{9.1} = \frac{317.4}{91} \times 10^{-4} = 3.5 \times 10^{-4}$$

$|\chi'| = 10^{-5}$ (given)

$$\frac{\chi}{|\chi'|} = \frac{3.5 \times 10^{-4}}{10^{-5}} = \frac{3.5 \times 10^{-4}}{10^{-1} \times 10^{-4}} = 35$$

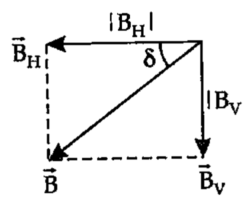
$$\chi = 35|\chi'|$$

Q5.23. Assume the dipole model for the earth's magnetic field B which is given by B_V = Vertical component of magnetic field $= \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}$ and B_H = Horizontal component of magnetic field $= \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}$. θ is equal to 90° latitude as measured from magnetic equator.

Find the loci of the points for which (*i*) |B| is minimum, (*ii*) dip angle is zero and (*iii*) dip angle is ±45°.

Main concept used: (*i*) $B^2 = B_H^2 + B_V^2$, (*ii*) angle of dip $\tan\alpha = \frac{B_V}{B_H}$.

Ans. (*i*) We know from fig.



$$B^2 = B_V^2 + B_H^2$$

$$= \left[\frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}\right]^2 + \left[\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}\right]^2$$

(substituting the value of B_V and B_H from question)

$$= \left[\frac{\mu_0}{4\pi}\right]^2 \frac{m^2}{r^6}[4\cos^2\theta + \sin^2\theta]$$

$$B^2 = \left(\frac{\mu_0}{4\pi}\right)^2 \times \frac{m^2}{r^6}[3\cos^2\theta + 1]$$

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^3}[3\cos^2\theta + 1]^{1/2} \quad ...(i)$$

From equation (*i*), the value of B will be minimum when $[3\cos^2\theta + 1]^{1/2}$ is minimum which will be at $\theta = \frac{\pi}{2}$. So magnetic equator lies at $\theta = \frac{\pi}{2}$, i.e., from magnetic dipole axis $\theta = \frac{\pi}{2}$ for magnetic equator.

(*ii*) For angle of dip δ

$$\tan\delta = \frac{B_V}{B_H} = \frac{\frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}}{\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}}.$$

$$\tan\delta = 2\cot\theta$$

for $\delta = 0$, $\cot\theta = 0$, $\theta = \frac{\pi}{2}$

So angle of dip will lie at magnetic equator.

(*iii*) $\tan\delta = \frac{B_V}{B_H}$ = angle of dip $\delta = \pm 45°$

or $\frac{B_V}{B_H} = \tan \pm 45° \Rightarrow \frac{B_V}{B_H} = \tan 45°$

$\frac{B_V}{B_H} = 1$ or $\boxed{B_V = B_H}$

$\delta = \pm 45°$

$\tan \pm 45° = 2\cot\theta$ [$\because \tan\delta = 2\cot\theta$]

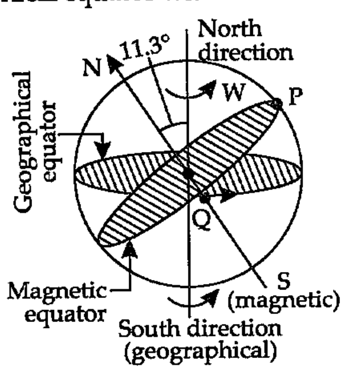
$\cot\theta = \frac{1}{2}$ or $\tan\theta = 2$

$\theta = \tan^{-1} 2$ is the locus of points where the angle of dip $\delta = \pm 45°$

Q5.24. Consider the plane S formed by the dipole axis and axis of earth. Let P be the point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. Obtain the angle of declination and dip at P and Q.

Main concept used: (*i*) Angles of magnetic axis with geographical axis, (*ii*) Angles of magnetic and geographical equator with their axis.

Ans. Q is the point of intersection between geographical meridian and magnetic meridian. So angle dip at P and Q will be zero with horizontal as magnetic middle stay horizontally.



As angle between axis of rotation of earth and magnetic axis is 11.3° and their respective equators are at 90° with their respective axis.

So angle between the plane of magnetic and geographical planes *i.e.* declination will be 11.3° at P and Q both.

Q5.25. There are two current carrying planar coils made each from identical wires of length L. Coil C_1 is circular of radius R and coil C_2 is square of side a. They are so constructed that they have same frequency of oscillation when they are placed in same uniform field $\vec{B}$ and carry the same current. Find a in terms of R.

Main concept used: (*i*) Magnetic moment $m = nIA$, (*ii*) Time period of oscillation.

Ans. As the frequencies (ω) for both coil are given same

$$\therefore \quad \omega_1 = \omega_2$$

$$\frac{2\pi}{T_1} = \frac{2\pi}{T}$$

So the time period of both the coils C_1 and C_2 are equal so

$$T_1 = T_2$$

$$2\pi\sqrt{\frac{I_1}{m_1 B}} = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

$$\sqrt{\frac{I_1}{m_1}} = \sqrt{\frac{I_2}{m_2}}$$

as B is same in both coils so squaring both sides we get,

$$\frac{I_1}{I_2} = \frac{m_1}{m_2} \qquad \text{...I,}$$

I_1, I_2 are the moment of inertia of coils C_1 and C_2 placed in same magnetic field B.

$$I_1 = \frac{mR^2}{2} \quad \text{...II,} \qquad I_2 = \frac{ma^2}{12} \quad \text{...III}$$

as the length of wire is same and identical so masses $m_1 = m_2 = m$

For circular shaped coil, magnetic moment

$$m_1 = n_1 I A_1 [\because \text{ current (I) in both are same, i.e. } I_1 = I_2 = I]$$

$$m_1 = \frac{L}{2\pi R} \cdot I \cdot \pi R^2 \qquad [\because L = 2\pi R n_1]$$

$$m_1 = \frac{LIR}{2} \qquad \text{...IV}$$

For square shaped coil, magnetic moment

$$m_2 = n_2 I A_2 \text{ as current } I_1 = I_2 = I \text{ (given)} = \frac{L}{4a} I . a^2$$

$$m_2 = \frac{LIa}{4} \qquad \text{...V}$$

Substitute II, III, IV, V in I

$$\frac{mR^2/2}{ma^2/12} = \frac{LIR/2}{LIa/4}$$

or

$$\frac{R^2/2}{a^2/12} = \frac{R/2}{a/4}$$

$$\frac{R^2}{2} \times \frac{a}{4} = \frac{R}{2} \cdot \frac{a^2}{12}$$

$$\frac{R}{8} = \frac{a}{24}$$

$$24R = 8a$$

$$3R = a$$

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