6 Electromagnetic Induction

MULTIPLE CHOICE QUESTIONS—I

Q6.1. A square of side L metres lies in X-Y plane in a region where the magnetic field is given by

$B = B_0(2i + 3j + 4k)$ Tesla, where B_0 is constant. The magnitude of flux passing through the square is

(*a*) $2B_0L^2$ Wb (*b*) $3B_0L^2$ Wb

(*c*) $4B_0L^2$ Wb (*d*) $\sqrt{29}\,B_0L^2$ Wb

Main concept used: $Q = \vec{B}.\vec{A}$ direction of A is perpendicular to the plane of square.

Ans. (*c*): Square lies in X-Y plane in $\vec{B}$ so $\vec{A} = L^2.\hat{k}$

$\therefore \quad Q = B.A$

$$= B_0(2i + 3j + 4k).(L^2\hat{k}) = B_0[2 \times i.k + 3 \times j.k + 4 \times k.k]$$

$$= B_0L^2[0 + 0 + 4] = 4B_0L^2 \text{ Wb.}$$

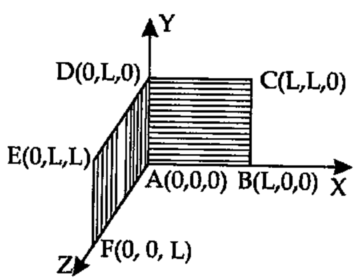
Q6.2. A loop made of straight edges has six corners at A(0, 0, 0), B(L, 0, 0), C(L, L, 0), D(0, L, 0), E(0, L, L) and F(0, 0, L). A magnetic field $B = B_0(i + k)$ Tesla is present in region. The flux passing through the loop ABCDE FA (in that order) is

(*a*) B_0L^2 Wb (*b*) $2B_0L^2$ Wb

(*c*) $\sqrt{2}\,B_0L^2$ Wb (*d*) $4B_0L^2$ Wb

Main concept used: Direction of $\vec{A}$ of loop.

Ans. (*b*): The loop can be considered in two planes.



(*i*) Plane of ABCDA is in X-Y plane

So its vector $\vec{A}$ is in Z-direction so

$$A_1 = |A|\hat{k} = L^2\hat{k}$$

(*ii*) Plane of DEFAD is in Y-Z plane

So $\quad A_2 = |A|\,i = L^2 i$

$\therefore \quad A = A_1 + A_2 = L^2(\hat{i} + \hat{k})$

$\quad B = B_0(i + k)$

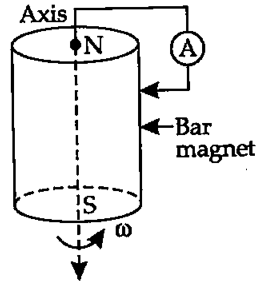
So $\quad Q = B.A = B_0(i + k).L^2(i + k) = B_0L^2[i.i + i.k + k.i + k.k]$

$$= B_0L^2[1 + 0 + 0 + 1] \quad \because \cos 90° = 0$$

$$= 2B_0L^2 \text{ Wb}$$

Verifies answer (*b*).

Q6.3. A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then

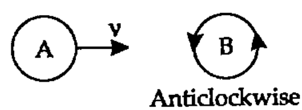


(*a*) a direct current flows in the Ammeter A.
(*b*) no current flows in ammeter A.
(*c*) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$
(*d*) a time varying non-sinusoidal current flows through the ammeter A.

Main concept used: Induced current flows only when circuit is complete and there is a variation of B about a circuit.

Ans. (*b*): Here, circuit with ammeter is complete. We know that as there is a symmetry in magnetic field of a bar magnet about its axial axis, so no change in magnetic field across the circuit when magnet is rotated, either alone or with circuit. No current flows in ammeter verifies answer (*b*).

Q6.4. There are two coils A and B as shown in figure. A current starts flowing in B as shown in figure, when A is moved towards B and stops when A stops moving. The current in A is counterclockwise. B is kept stationary when A moves. We can infer that

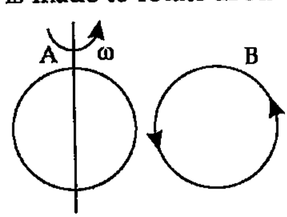


(*a*) there is a constant current in the clockwise direction in A.
(*b*) there is a varying current in A.
(*c*) there is no current in A.
(*d*) there is a constant current in the counter clockwise direction in A.

Main concept used: Due to the current in A and its motion the flux changes across B. So induced current is produced.

Ans. (*d*): When coil A moves towards coil B with constant velocity so rate of change of magnetic flux due to coil B in coil A will be constant gives constant current in A in same direction as in B by Lenz's law. Hence verifies answer (*d*).

Q6.5. Same as Question 6.4 except the coil A is made to rotate about a vertical axis as in figure. No current flows in B if A is at rest. The current in coil A when the current in B (at $t = 0$) is counter clockwise and the coil A is as shown at this instant, $t = 0$, is



(*a*) constant current clockwise

(*b*) varying current clockwise
(*c*) varying current counterclockwise
(*d*) constant current counterclockwise

Main concept used: Lenz's law.

Ans. (*a*): At $t = 0$ current in B is clockwise and coil A is considered above B. The counterclockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, the current in A is constant along clockwise direction by Lenz's rule. As flux changes across coil A by rotating it near the N-pole formed by flowing current in B in anticlockwise. So verifies ans. (*a*).

Q6.6. The self inductance L of a long solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(*a*) l and A increases
(*b*) l decreases and A increases
(*c*) l increases and A decreases
(*d*) Both l and A decreases.

Main concept used: Self inductance $L = \mu_r \mu_0 n^2 A.l$.

Ans. (*b*): As we know $L = \mu_r \mu_0 \frac{N^2}{ll}.A.l$

$$L = \mu_r \mu_0 \frac{N^2.A}{l}$$

as L is constt. for a coil

$$L \propto A \quad \text{and} \quad L \propto \frac{1}{l}$$

As μ_r and N are constant here so, to increase L for a coil, area A must be increased and l must be decreased so verify ans. (*b*).

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q6.7. A metal plate is getting heated. It can be because

(*a*) a direct current is passing through the plate.
(*b*) it is placed in a time varying magnetic field.
(*c*) it is placed in a space varying magnetic field but does not vary with time.
(*d*) a current (either direct or alternating) is passing through the plate.

Main concept used: Heating effect of current I^2Rt and eddy current.

Ans. (*a*) (*b*) (*d*): A plate or conductor is heated by heating effect of current ($H = I^2Rt$). Verify answers (*a*) (*d*).

When time dependent magnetic field varies across the metal plate due to production of eddy currents, heating of plate takes place verify (*b*).

Q6.8. An e.m.f. is produced in a coil, which is not connected to external voltage source. This can be due to

(*a*) the coil being in a time varying magnetic field.
(*b*) the coil moving in a time varying magnetic field.

(c) the coil moving in a constant magnetic field.
(d) the coil is stationary in external spatially varying magnetic field which does not change with time.

Main concept used: e.m.f. produced by electromagnetic induction.

Ans. (a) (b) (c): e.m.f. produced in coil due to change in magnetic flux with time in options (a), (b) and (c). But in part (d) magnetic field does not change with time although varying magnetic field but rate of change of magnetic field does not change so rejects the option (d). So answers are (a) (b) and (c).

Q6.9. The mutual inductance M_{12} of coil 1 with respect to coil 2
(a) increases when they are brought nearer.
(b) depends on the current passing through coils.
(c) increases when one of them is rotated about an axis.
(d) is the same as M_{21} of coil 2 with respect to coil 1.

Main concept used: Mutual inductance depends on Geometry of coils which is constant for two coils.

Ans. (a) (d): As we know that $M_{12} = M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$ where $r_1 l$ is the common area of cross-section of coil so M_{12} does not depend on passing current and rotation. l is also common length. So answer (a) and (d) verified.

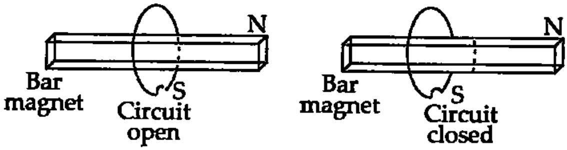
Q6.10. A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because
(a) the magnetic field is constant.
(b) the magnetic field is in the same plane as the circular coil and it may or may not vary.
(c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably.
(d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction.

Main concept used: Induced e.m.f. can produce only when there is change in magnetic flux with coil or inside coil.

Ans. (b) (d): When coil expands in constant magnetic field, the magnetic flux inside the coil (along its area vector) increase then induced current produce so rejects option (a) and (c).
As the component of magnetic field along area vector is zero so $\phi = B.A$ becomes zero. So, no induced current flows in coil verifies the options (b) and (d).

VERY SHORT ANSWER TYPE QUESTIONS

Q6.11. Consider a magnet surrounded by a wire with an ON/OFF switch S (in figure). If the switch is thrown from the off position (open circuit) to the on position (closed circuit) will a current flow in the circuit? Explain.



Main concept used: $\because \phi = BA\cos\theta$, so ϕ can be changed by changing either B, or A or θ.

Ans. As we know that induced e.m.f. or induced current can flow when there is change in magnetic flux with respect to time and magnetic flux $\phi = BA\cos\theta$. As there is no any change in B, *i.e.*, magnet, A, *i.e.*, area of circuit and no change in angle between $\vec{B}$ and $\vec{A}$. So no induced current will flow.

Q6.12. A wire in the form of a tightly wound solenoid is connected to a DC source and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

Main concept used: Lenz's law, leakage of flux and $L = \mu_r \mu_0 n^2 A l$.

Ans. When the coil is stretched the magnetic flux will leak through the gaps between two turns. According to Lenz's law, the e.m.f. is produced in nearby coil which will oppose the increase or decrease in flux in coil. So, on stretching the coil, the current will increase. When the coil is stretched then n (no. of turns per unit length) will decrease. So L decrease or reactance decrease which makes current increase.

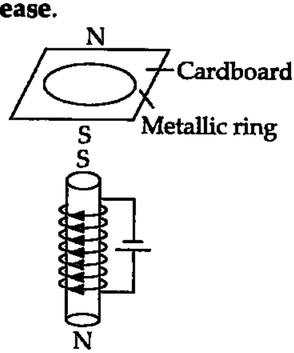
Q6.13. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid will the current increase or decrease? Explain.

Main concept used: Ferromagnetic core increases flux in solenoid which in turn decrease the current by Lenz's law.

Ans. When the ferromagnetic iron core is inserted inside the solenoid the magnetic flux will increase.

As flux increased, then by Lenz's law, to oppose increase in flux *i.e.*, to decrease flux **the current in coil will decrease.**

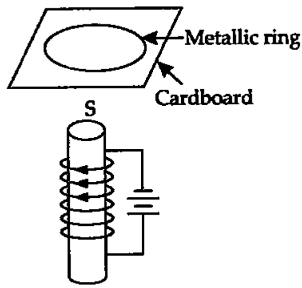
Q6.14. Consider a metal ring kept on the top of a fixed solenoid (say on a cardboard) as in figure. The centre of ring coincides with axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.



Main concept used: Lenz's law.

Ans. When the current is switched on in solenoid, magnetic flux increased across the ring so induced current produced in such direction so that it can decrease the flux or oppose the increased flux. So, the direction of magnetic flux between solenoid will be like that ring is repelled and jumps. Or when current is switched on, the upper end becomes south pole. By Lenz's law the induced current produce south pole lower side and North pole upper side so repelled and jumps up.

Q6.15. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying current I as in figure. The centre of the ring coincides with the axis of solenoid. If the current in the solenoid is switched off, what will happen to the ring?



Main concept used: Application of Lenz's law.

Ans. As current was already flowing through the solenoid so it behaves like a magnet and let S pole is upper side as flux in ring is constant. So there is no induced current in ring.

When current is switched off, magnetic flux decreases so induced current produced in ring in such a way so that it can increase the flux. So North pole is produced in ring in lower side, and attracted by solenoid. So, downward force of attraction between ring and solenoid acts but as cardboard and solenoid are fixed ring will not be able to move downward.

Q6.16. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe it takes more time to come down, than it takes for similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

Main concept used: Eddy current and Lenz's law.

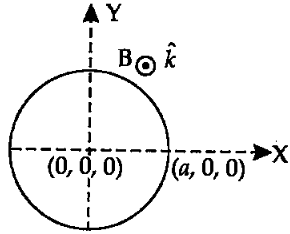
Ans. When a cylindrical bar magnet of radius 0.8 cm is dropped inside a hollow cylindrical metallic pipe of radius 1 cm the magnetic flux across the pipe changes, and due to induction, eddy current will produce.

The direction of eddy current will be in such a way so that it can oppose the cause *i.e.*, motion of magnet. So upward force acts on bar magnet and decreases the acceleration, which in turn takes more time to fall magnet through the pipe, as compared to simple metallic piece in place of magnet.

SHORT ANSWER TYPE QUESTIONS

Q6.17. A magnetic field in a certain region is given by $B = B_0 \cos \omega t\, \hat{k}$, and a coil of radius a with resistance R is placed in X-Y plane with its centre at the origin in the magnetic field as in figure. Find magnitude and direction of the current at $(a, 0, 0)$ at

$$t=\frac{\pi}{2\omega},\quad t=\frac{\pi}{\omega}\quad \text{and}\quad t=\frac{3\pi}{2\omega}$$



Main concept used: Faraday's law of EMF.

Ans. The direction of magnetic field is along Z-axis. So the magnetic flux passing through circular region of coil of radius a placed in X-Y plane.

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

The direction area vector A of coil is along Z-direction and B is also in Z-direction so $\theta = 0°$

$$\phi = (B_0 \cos\omega t).(\pi a^2)\cos 0°$$

$$\phi = B_0 \pi a^2 \cos\omega t$$

By Faraday's law of electromagnetic induction,

$$\varepsilon = \frac{d\phi}{dt} = B_0\pi a^2 \sin\omega t.\omega$$

$$\varepsilon = B_0\pi a^2 \omega\sin\omega t$$

Flow of induced current in coil $I = \frac{\varepsilon}{R}$ where R is resistance of coil.

$$\therefore \quad I = \frac{B_0\pi a^2\omega}{R}\sin\omega t$$

For currents at $t = \frac{\pi}{2\omega},\quad t=\frac{\pi}{\omega},\quad t=\frac{3\pi}{2\omega}$

$\therefore$ Current at $t = \frac{\pi}{2\omega}$

$$\therefore \quad I = \frac{B_0\pi a^2\omega}{R}\sin\omega.\frac{\pi}{2\omega} = \frac{B_0\pi a^2\omega}{R}\sin\frac{\pi}{2} \quad \left[\sin\frac{\pi}{2}=1\right]$$

So, at $t = \frac{\pi}{2\omega}$,

$$\text{Current } I = \frac{B_0\pi a^2\omega}{R} \qquad (\text{along } +\hat{j})$$

at $t = \frac{\pi}{\omega}$,

$$I = \frac{B_0\pi a^2\omega}{R}\sin\omega\frac{\pi}{\omega}$$

$$I = \frac{B_0\pi a^2\omega}{R}\sin\pi = \frac{B_0\pi a^2\omega}{R}\times 0 = 0$$

at $t = \frac{\pi}{\omega}$, induced current $I = 0$

at $t = \frac{3\pi}{2\omega}$, $\quad I = \frac{B_0 \pi a^2 \omega}{R} \sin \omega . \frac{3\pi}{\omega}$

$$I = \frac{B_0 \pi a^2 \omega}{R} \sin 3\pi \qquad [\because \sin(2\pi + \pi) = \sin \pi]$$

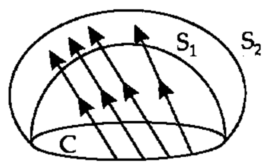
$$= \frac{B_0 \pi a^2 \omega}{R} \sin \pi$$

$$I = \frac{-B_0 \pi a^2 \omega}{R}$$

So, at $t = \frac{3\pi}{\omega}$,

Induced current $I = \frac{-B_0 \pi a^2 \omega}{R}$ (along $-\hat{j}$)

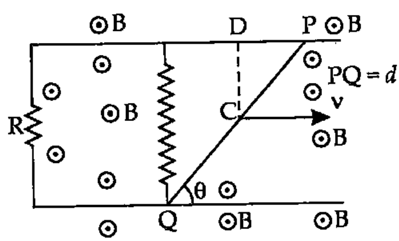
Q6.18. Consider a closed loop C in a magnetic field (as in figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with loop and using the formula $\phi = B_1.dA_1 + B_2.dA_2 + B_3.dA_3 + \ldots$. Now if we chose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux? Justify your answer.



Main concept used: Properties of magnetic lines.

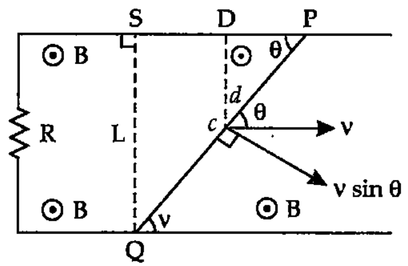
Ans. The magnetic flux linked with the surface can be considered as the number of magnetic field lines passing through the surface. So let $d\phi = \vec{B}.\vec{A}$ represents magnetic field lines in an area A to magnetic flux B. By the concept of continuity of line B cannot end or start in space, therefore, the number of lines passing through S_1 must be the same as the number of lines passing through S_2 surface. Therefore, in both the cases we get the same flux.

Q6.19. Find the current in the wire for the configuration shown in figure. Wire PQ has negligible resistance. $\vec{B}$, the magnetic field is coming out of the paper. θ is fixed angle made by PQ travelling smoothly over two conducting parallel wires separated by distance d.



Main concept used: Induced current due to change in area of loop.

Ans. The motional electric field E due to the motion along CD is $E = vB$.



The direction of E will be ⊥ to both v and B $\because F = q \times B$

F is force on free charge particle of PQ.

So the motional e.m.f. ε = E along PQ × effective length PQ.

$$\text{Electric field along PQ} = v \times B = v \sin\theta \cdot B = vB\sin\theta$$

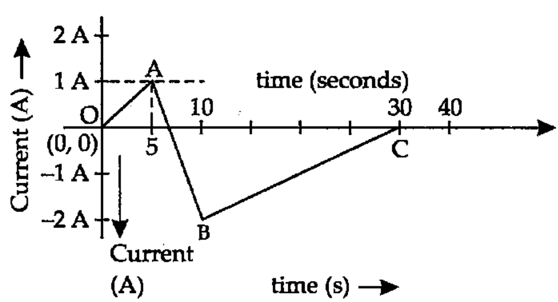
$$\varepsilon = (vB\sin\theta)\left(\frac{d}{\sin\theta}\right)$$

$$\varepsilon = vBd$$

$$\therefore \quad \text{Induced current} = \frac{\varepsilon}{R} = \frac{vBd}{R}$$

It is independent of q.

Q6.20. A (current versus time) graph of the current passing through solenoid is shown in figure. For which time is the back electromotive force (u) a maximum? If the back e.m.f. at $t = 3$s is e, find the back e.m.f. at $t = 7$s, 15 s and 40s. OA, AB, BC are straight line segments.



Main concepts used: (*i*) Current is variable so magnetic flux will change which in turns changes the emf (*ii*) Lenz's law.

Ans. The maximum back electromotive force (u) will be maximum when there is maximum rate of change of magnetic flux which is directly proportional to the rate of change of current.

Maximum change or rate of current will be where (t – I) graph makes maximum angle with time axis which is in part AB.

So the maximum back e.m.f. will occur between 5 s to 10 s. As the back e.m.f. at $t = 3$ s is 'e' (given)

Rate of change of current at $t = 3$ s = slope of OA graph with time axis

So rate of change of current at 3 s = $\frac{1}{5}$ A/s.

So back electromotive force at $t = 3$ s = $L \times \frac{1}{5} = \frac{L}{5} = e$ given)

$\because e = L.\frac{dI}{dt}$ and L = constant for solenoid.

Similarly **back e.m.f.** u_1 between 5 to 10 sec.

$$u_1 = L.\left(\frac{-3}{5}\right) = -3\frac{L}{5} = -3e$$

back e.m.f. between 10 to 30 sec

$$u_2 = L\frac{[0-(-2)]}{(30-10)} = \frac{+2L}{20} = \frac{+1}{2}\frac{L}{5}$$

$$u_2 = +\frac{1}{2}e$$

So back e.m.f. at 7 sec = –3e

back e.m.f. at 15 sec = $+\frac{1}{2}e$

At 40 sec graph is along time axis, i.e. its slope with time axis is zero.

So, $\frac{dI}{dt} = 0$.

or back e.m.f. at 40 sec = 0

Q6.21. There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and 1 A current passes through B what is flux through A?

Main concept used: Mutual inductance.

Ans. Let I_A current is passing through coil A having mutual inductance M_{AB} with respect to coil B.

N_A = number of turns in coil A

N_B = number of turns in coil B

ϕ_A = flux linked with coil A due to coil B

ϕ_B = flux linked with coil B due to coil A

M_{BA} = Mutual inductance of coil B with respect to coil A

Then

$$\text{Total flux through B} = M_2\phi_2 = M_{BA}I_1$$

$$10^{-2} = M_{BA} \times 2$$

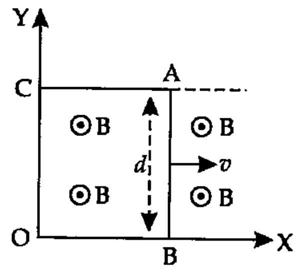
$$M_{BA} = \frac{10^{-2}}{2} = 5 \text{ mH}$$

Now total flux through A= $M_A\phi_A = M_{AB} I_2$ $[\because M_{AB} = M_{BA}]$

$= 5 \text{ mH} \times 1 \text{ Wb} = 5 \text{ mWb}$ **Ans.**

LONG ANSWER TYPE QUESTIONS

Q6.22. A magnetic field $B = B_0 \sin(\omega t)\hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (figure). The wires are in X-Y plane. The wire AB of length d has resistance R and parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in circuit? What is the force needed to keep the wire moving at constant velocity?



Main concept used: e.m.f. induced in AB will be due to change in area of loop within magnetic field and Force F = B I L sin θ, θ is angle between $\overline{B}$ and L.

Ans. In figure CA and OB are long parallel conducting wires, connected by d and conductor CO. The resistance of ACOB is negligible.

Let wire AB at $t = 0$ is at $x = 0$ *i.e.*, on Y-axis. Now AB moves with velocity $v\hat{i}$.

Let at any time t, position of conductor AB is $x(t) = v\hat{i}t$

Motional e.m.f. across AB

$$V_{AB} = \frac{W_{AB}}{q} = \frac{F.d}{q} = \frac{qvi \times B}{q}.d$$

$$V_{AB} = v\hat{i} \times B_0 \sin \omega t.\hat{k} \times d$$

$$\varepsilon_1 = B_0 \sin \omega t\, v(\hat{i} \times \hat{k})d = B_0 dv \sin \omega t\,(-\hat{j})$$

$$\varepsilon_1 = -B_0 vd \sin \omega t\,(\hat{j}) \text{ (along –ve Y direction in AB)}$$

e.m.f. due to change in magnetic field from $t = 0$ to at $t = 1$ when AB is at $(x, 0)$

$$\therefore \qquad \varepsilon_2 = \frac{-d\phi}{dt} = \frac{-d}{dt}\vec{B}.\vec{A} = \frac{-d}{dt}[B_0 \sin \omega t\,\hat{k}.(x.d)\hat{k}]$$

Area vector A is along $\hat{k}$

$$\therefore \qquad \varepsilon_2 = \frac{-d}{dt}(B_0 \sin \omega t\; x.d)$$

$$\varepsilon_2 = -B_0\, xd.\omega \cos \omega t \qquad \text{...(II)}$$

$\therefore$ Total magnitude of e.m.f. $= -B_0\, vd \sin \omega t - B_0\, xd\omega \cos \omega t$

$$\varepsilon = -B_0 d[v \sin \omega t + x\omega \cos \omega t]$$

The direction of electric induced current by Fleming Right Hand Rule is from (A to B) *i.e.*, in clockwise in loop.

$$I = \frac{\varepsilon}{R} = \frac{B_0 d}{R}[v \sin \omega t + \omega x \cos \omega t]$$

The force F acting on conductor AB = BI d sin θ

θ is angle between $B\hat{k}$ and $I(-\hat{j}) = 90°$

$$F = I \times B\, dl$$

$$= I(-\hat{j}) \times B(\hat{k})\, d \sin\theta \qquad [\because \theta = 90°]$$

$$= -I B\, (\hat{j} \times \hat{k})\, d \sin 90° = -I B(\hat{i}) d$$

$$F = -\hat{i} I B d$$

it shows the direction of force on AB is towards $-x$ direction, which can be verified by Fleming's left hand rule

$\therefore \qquad |F| = I B d$

∴ To keep wire in motion with constant velocity, it will be towards positive x direction $+\,i$ opp. to electromagnetic force

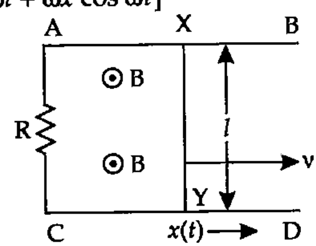
$$F = B\,.\,d\,I = |B| \cdot d \frac{\varepsilon}{R}$$

$$F = \frac{B d B_0 d}{R}[v \sin\omega t + \omega x \cos\omega t]$$

$$= \frac{B_0 \sin\omega t\, B_0 d^2}{R}[v \sin\omega t + \omega x \cos\omega t]$$

$$F = \frac{B_0^2 d^2 \sin\omega t}{R}[v \sin\omega t + \omega x \cos\omega t]$$

Q6.23. A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in figure. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $B = B(t)\hat{k}$.



(*i*) Write down the equation for the acceleration of the wire XY.

(*ii*) If B is independent of time, obtain $v(t)$ assuming $v(0) = u_0$.

(*iii*) For (*ii*) show that the decrease in kinetic energy of XY is equal to the heat lost in R.

Main concept used: Induced e.m.f. or e.m.i. magnetic force, power consumption.

Ans. (*i*) Let wire XY at $t = 0$ is at $x = 0$

and at $t = t$ is at $x = x(t)$

Magnetic flux is a function of time $\phi(t) = B(t) \times A$

$\therefore \qquad \phi(t) = B(t)\,[l.x(t)]$

$$\varepsilon = -\frac{d\phi(t)}{dt} = -\frac{dB(t)}{dt} l.x(t) - B(t)\, l.\frac{dx(t)}{dt}$$

$$\varepsilon = \frac{-\,dB(t)}{dt} l.x(t) - B(t)\, l\, v(t)$$

The direction of induced current by Fleming's Right Hand Rule or by Lenz's law is in clockwise direction in loop XY < AX.

$$I = \frac{\varepsilon}{R} = \frac{-l}{R}\left[x(t)\frac{dB(t)}{dt} + B(t)\,v(t)\right] \qquad ...(I)$$

The force acting on the conductor is $F = B(t)\, I\, l \sin 90°$

$$F = B(t)\, I.l$$

$$F = \frac{B(t)\, l\varepsilon}{R} = \frac{-B(t)l^2}{R}\left[\frac{-dB(t)}{dt}.x(t) - B(t).v(t)\right]$$

$$\frac{md^2x}{dt^2} = \frac{-B(t)l^2}{R}\left[x(t)\frac{dB(t)}{dt} + B(t)\,v(t)\right]$$

or
$$\frac{d^2x}{dt^2} = \frac{-l^2}{mR}B(t)\left[x(t)\frac{dB(t)}{dt} + B(t)\cdot v(t)\right] \qquad ...(II)$$

(*ii*) Now B is independent of time *i.e.* B does not change with time or it is constant

$$\therefore \quad \frac{dB}{dt} = 0,\ B(t) = B \text{ and } v(t) = v \qquad ...(III)$$

Put (III) in (II) we get

$$\frac{d^2x}{dt^2} = \frac{-l^2}{mR}[0 + Bv]$$

$$\frac{d^2x}{dt^2} + \frac{B^2l^2}{mR}\frac{dx}{dt} = 0 \qquad (\div \text{ by } m)$$

$$\frac{dv}{dt} + \frac{B^2\,l^2}{mR}v = 0$$

Integrating using variables separable from differential equation we have

$$v = A \exp\left(\frac{-l^2\,B^2\,t}{mR}\right)$$

at $t = 0, \quad v = u$

$$\therefore \quad v(t) = u \exp\left(\frac{-l^2\,B^2\,t}{mR}\right) \qquad ...(IV)$$

(*iii*) Heat lost per second in (*ii*) where $\frac{dB}{dt} = 0$

$$H = I^2R$$

Magnitude of current from equation I in (*i*) part

$$I = \frac{Blv}{R} = \frac{-l}{R}[0 + Bv] \qquad \left[\because \frac{dB}{dt} = 0\right]$$

Heat produced per second $H = I^2R$

$$\therefore \qquad H = \frac{B^2 l^2 v^2}{R^2}.R$$

$$H = \frac{B^2 l^2}{R} u^2 \exp\left[\frac{-2l^2 B^2 t}{mR}\right]$$

[v from eqn. (IV) in (*iii*) part]

$$\text{Power lost} = \int_0^t I^2 R dt = \frac{B^2 l^2 u^2}{R} \int_0^t e^{\frac{-2l^2B^2t}{mR}} dt$$

$$\left[\because v^2 = u^2 \exp^{\frac{-2l^2B^2t}{mR}}\right]$$

$$\text{Power lost} = \frac{B^2 l^2 u^2}{R} \frac{mR}{2l^2 B^2}\left[1 - e^{(-2l^2B^2t/mR)}\right]$$

$$= \frac{mu^2}{2}\left[1 - e^{\frac{-2l^2B^2t}{mR}}\right]$$

$$= \frac{mu^2}{2} - \frac{m}{2} u^2 e^{\frac{-2l^2B^2t}{mR}}$$

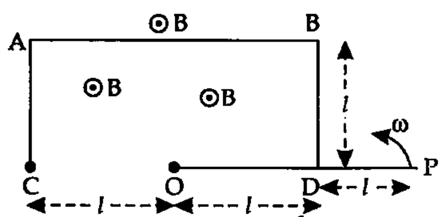
$$= \left[\frac{mu^2}{2} - \frac{mv^2(t)}{2}\right]$$

= initial K.E. – final K.E.

Power lost = decrease in kinetic energy

This proves that decrease in K.E. of XY is equal to the heat lost in R.

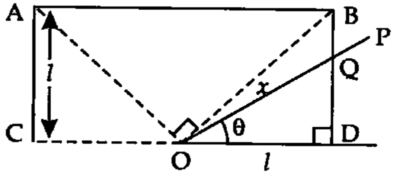
Q6.24. ODBAC is a fixed rectangular conductor of negligible resistance (CO is not connected) and OP is a conductor which rotates anticlockwise with an angular velocity ω (figure). The entire system is in a uniform magnetic field B whose direction is along the normal to the surface of the rectangular conductor ABDC.



The conductor OP is in electric contact with ABDC. The rotating conductor has resistance of λ per unit length. Find the current in rotating conductor as it rotates by 180°.

Main concept used: Induced e.m.f. produced by change in area.

Considering the position of OP between $0 < \theta < \frac{\pi}{4}, \frac{\pi}{4} < \theta < \frac{3\pi}{4}$ and $\frac{3\pi}{4} < \theta < \frac{4\pi}{4}$ and O is mid point of CD and OD = BD = l



Ans. (*i*) Let rotating conductor is in contact with BD at Q making angle $0° < \theta < 45°$

Magnetic flux in ΔODQ is ϕ

$$\phi = B.A$$

The direction of B and A are 0° or 180°

$$Q = B \times \frac{1}{2} l \times QD \qquad \left[\because \frac{QD}{l} = \tan\theta \quad QD = l \tan\theta\right]$$

$$\therefore \quad \phi = B.A \cos 0 = BA = B\frac{1}{2} \times l.l \tan\theta$$

$$\phi = \frac{1}{2} Bl^2 \tan\theta = \frac{1}{2} Bl^2 \tan \omega t \qquad (\because \theta = \omega t)$$

$$\text{Induced e.m.f. } \varepsilon = \frac{-d\phi}{dt} = \frac{d}{dt}\frac{1}{2} Bl^2 \tan \omega t$$

$$\varepsilon = \frac{1}{2} Bl^2 \omega \sec^2 \omega t$$

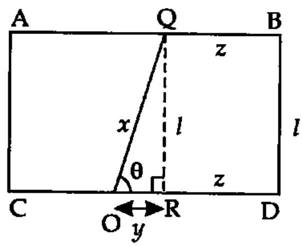
$$I = \frac{\varepsilon}{R} = \frac{Bl^2}{2R} \omega \sec^2 \omega t \qquad \left[\because \cos\theta = \frac{l}{x}, x = \frac{l}{\cos\theta}\right]$$

$$R \text{ of } OQ = \lambda x = \frac{\lambda l}{\cos\theta}$$

$$R = \frac{\lambda l}{\cos \omega t}$$

$$I = \frac{Bl^2}{2.\lambda l} \omega \cos \omega t \sec^2 \omega t = \frac{Bl\omega}{2\lambda \cos \omega t}$$

(*ii*) Now rotating conductor rotates from B to A *i.e.*, 45° to 135° or $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$.



$$\phi = B.A = B \text{ or } ODBQ$$
$$= B.$$

$$\text{Area of } \Delta ORQ = \frac{1}{2} y \times l$$

$$\tan\theta = \frac{l}{y}$$

$$y = \frac{l}{\tan\theta}$$

$\therefore$ Area of $\Delta ORQ = \frac{1}{2}\frac{l^2}{\tan\theta} = \frac{l^2}{2\tan^2\omega t}$

Flux through OQBD

$$\phi = B.\left(lz + \frac{l^2}{2\tan\theta}\right)$$

$$= Blz + \frac{1}{2}Bl^2\cot\theta = Blz + \frac{1}{2}Bl^2\cot\omega t$$

$$\therefore \quad \frac{d\phi}{dt} = \frac{d}{dt}Blz + \frac{1}{2}Bl^2(-\operatorname{cosec}^2\omega t)\,\omega$$

as l B and z are constant $\therefore \frac{d(l\,B\,z)}{dt} = 0$

$$\because \quad \varepsilon = \frac{-d\theta}{dt}$$

$$\therefore \quad -\varepsilon = 0 - \frac{1}{2}\frac{Bl^2\omega}{\sin^2\omega t}$$

$$\varepsilon = \frac{1}{2}\frac{Bl^2\omega}{\sin^2\omega t}$$

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{\varepsilon\sin\omega t}{\lambda l} \qquad \left[\because \sin\theta = \frac{l}{x}, x = \frac{l}{\sin\omega t}\right]$$

$$= \frac{\sin\omega t}{\lambda l}\cdot\frac{1}{2}\frac{Bl^2\omega}{\sin^2\omega t}$$

$$\boxed{I = \frac{Bl\omega}{2\lambda\sin\omega t}}$$

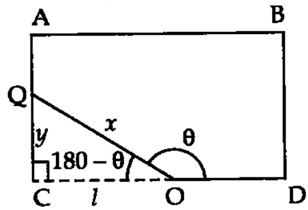
(*iii*) When $\frac{3\pi}{4} < \theta < \frac{2\pi}{2}$, the flux through OQABD = ϕ = B . A

$$\phi = B.\left(2l^2 + \frac{1}{2}ly\right)$$

$$\phi = B.\left(2l^2 + \frac{l^2\tan\omega t}{2}\right) \quad \ldots\left(\begin{aligned}&\tan(180-\theta) = \frac{y}{l}\\ &y = l(-\tan\theta)\\ &y = -l\tan\theta\end{aligned}\right)$$

$$\frac{d\phi}{dt} = \frac{d}{dt}\left[2l^2 + \frac{l^2}{2}(\tan\omega t)\right]B$$

$$-\varepsilon = 0 + \frac{Bl^2}{2}\frac{d}{dt}\tan\omega t$$



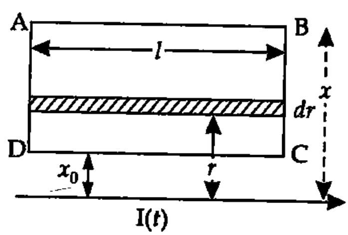
$$-\varepsilon = +\frac{Bl^2\omega}{2}\sec^2\omega t = -\frac{Bl^2\omega}{2\cos^2\omega t}$$

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x}$$

$$I = \frac{-Bl^2\omega}{2\cos^2\omega t}\,\frac{\cos\omega t}{\lambda(-l)} \qquad \left[\because \frac{l}{x} = \cos(180-\theta)\right.$$

$$\boxed{I = \frac{Bl\omega}{2\lambda\cos\omega t}} \qquad \left.\frac{l}{x} = -\cos\theta \Rightarrow x = \frac{-l}{\cos\omega t}\right]$$

Q6.25. Consider an infinitely long wire carrying a current I(t) with $\frac{dI}{dt} = \lambda$ = constant. Find the current produced in rectangular loop of wire ABCD of resistance R.



Main concept used: Total magnetic flux across rectangle can be find out by integration.

Ans. Consider a strip of width dr and length l inside a rectangle at distance r from the surface of current carrying conductor. The magnetic field across strip of length l = $B(r) = \frac{\mu_0 I}{2\pi r}l$. B($r$) is perpendicular to the paper upward.

$\therefore$ Flux in strip $\phi = \frac{\mu_0 I}{2\pi} l \int_{x_0}^{x} \frac{dr}{r}$

$$\phi = \frac{\mu_0 I l}{2\pi}[\log_e r]_{x_0}^{x} = \frac{\mu_0 I l}{2\pi}\log_e \frac{x}{x_0}$$

$$\varepsilon = \frac{-d\phi}{dt}$$

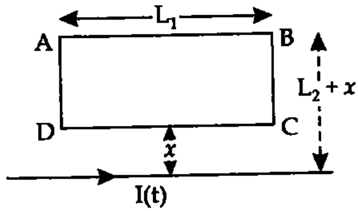
So $$IR = \frac{d\phi}{dt}$$

$$I = \frac{1}{R}\frac{d}{dt}\left[\frac{\mu_0 I l}{2\pi}\log_e\frac{x}{x_0}\right] = \frac{\mu_0 l}{2\pi R}.\log_e\frac{x}{x_0}\frac{dI}{dt}$$

$$I = \frac{\mu_0 \lambda l}{2\pi R}\log_e\frac{x}{x_0} \qquad \left[\because \frac{dI}{dt} = \lambda \text{ (given)}\right]$$

Q6.26. A rectangular loop of wire ABCD is kept close to an infinitely long wire carrying a current $I(t) = I_0\left(1 - \frac{t}{T}\right)$ for $0 \le t \le T$ and $I(0) = 0$

for $t > T$ (figure). Find the total charge passing through a given point in the loop in time T. The resistance of the loop is R.



Main concept used: Relation between instantaneous current and instantaneous magnetic flux.

Ans. If I(t) is instantaneous current then,

$$I(t) = \frac{1}{R}\frac{d\phi}{dt} \quad \text{...(I)}$$

If Q is charge passing in time t

$$\therefore \quad I(t) = \frac{dQ}{dt} \quad \text{...(II)}$$

From (I) and (II)

$$\frac{dQ}{dt} = \frac{1}{R}.\frac{d\phi}{dt}$$

or

$$dQ = \frac{1}{R}.d\phi \quad \text{...(III)}$$

Integrating both sides,

$$\int_{Q_1}^{Q_2} dQ = \frac{1}{R}\int_{\phi_1}^{\phi_2} d\phi$$

$$Q_2(t) - Q_1(t) = \frac{1}{R}[\phi_2(t) - \phi_1(t)]$$

For magnetic flux in rectangle:

Magnetic flux due to current carrying conductor at a distance x'

$$Q(t) = \frac{\mu_0 I(t)}{2\pi x'}$$

If length of strip is L_1 so total flux on strip of length L_1 at distance x' is

$$Q(t) = \frac{\mu_0 I(t)}{2\pi x'} L_1$$

x' varies from x to $(x + L_2)$ so total flux in strip

$$\phi(t) = \frac{\mu_0}{2\pi} L_1 \int_x^{x+L_2} \frac{dx}{x'} I(t) = \frac{\mu_0 L_1}{2\pi}.I(t_1)\log_e\frac{(L_2 + x)}{x}$$

The magnitude of charge is given on length L_1

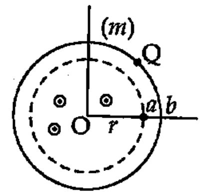
$$\int dQ = \frac{1}{R}\int d\phi \quad \text{[from (III)]}$$

$$\int_0^Q dQ = \frac{1}{R}.\frac{\mu_0 L_1}{2\pi}\log_e\left(\frac{L_2 + x}{x}\right)\int_0^1 I(t_1)$$

$$Q = \frac{\mu_0 L_1}{R\,2\pi}\log_e\left(\frac{L_2 + x}{x}\right)(I - 0) = \frac{\mu_0 L_1 I_1}{2\pi R}\log\left(\frac{L_2 + x}{x}\right)$$

Q6.27. A magnetic field B is confined to a region $r \le a$ and points out of the paper (in the Z-axis), $r = 0$ being the centre of circular region. A charged ring (charge Q) of radius b ($b > a$) and mass m lies in the X-Y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

Main concept used: Change in magnetic flux causes induced e.m.f. in turn electric field around the ring. The torque experienced by the ring produces change in angular momentum.



Ans. As the magnetic field is reduced to zero in $\Delta(t)$, so magnetic flux linked with the ring reduces from maximum to zero. Change of magnetic flux across the conducting ring induces e.m.f. The induced e.m.f. causes the electric field across the ring.

The induced e.m.f. in metallic ring = $(E \times 2\pi b)$ $\quad (\because V = Ed)$ ...(I)

By Faraday's law of e.m.i.

The induced e.m.f. = rate of change of magnetic flux

$$= \frac{B\pi a^2}{\Delta t} \quad \text{...(II)}$$

From (I) and (II)

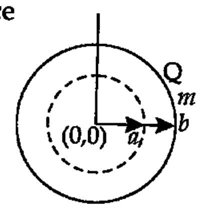
$$2\pi b E = \frac{B\pi a^2}{\Delta t}$$

Since the charged ring experiences an electric force = QE

This electric force try to rotate the ring given by

= Force × Perpendicular distance

$$= QE \times 2b = Qb\frac{B\pi a^2}{2\pi b \Delta t}$$



$$\text{Torque on ring} = Q.\frac{Ba^2}{2\Delta t}$$

Change in angular momentum = Torque × Δt

$$= Q.\frac{Ba^2}{2\Delta t}.\Delta t$$

$$\text{Change in angular momentum} = Q.\frac{Ba^2}{2}$$

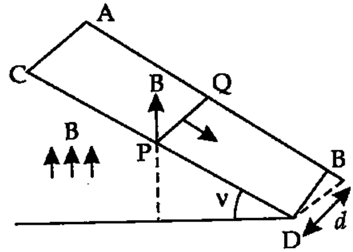
As initial momentum of ring was zero.

So, final momentum of ring = $Q.\frac{Ba^2}{2}$.

$$mb^2\omega = Q.\frac{Ba^2}{2} \qquad (\because L = mr^2\omega)$$

$$\omega = \frac{Q.Ba^2}{2mb^2}$$

Q6.28. A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to horizontal (figure). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field B along the vertical direction. If the rod is initially at rest, find the velocity of rod as a function of time.



Main concept used: EMI, friction, motion on slope.

Ans. From free body diagram the component of B perpendicular to wire PQ or line F_m is $\vec{B}\cos\theta$:

Angle between B cos θ and PQ = 90°

$$d\phi = \vec{B}\cdot\vec{d}A \text{ (Angle between, } \cos\theta \text{ and } ar \text{ PQCD is zero)}$$

$$d\phi = (\vec{B}\cos\theta)(d \times v \times dt)$$

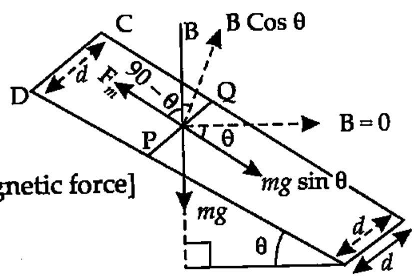
$$\frac{d\phi}{dt} = Bvd\cos\theta$$

$$-\varepsilon = Bvd\cos\theta$$

$$\varepsilon = -Bvd\cos\theta$$

$$I = \frac{-Bvd}{R}\cos\theta$$

$$P = F_m . v \quad [F_m \text{ magnetic force}]$$



$$F_m = \frac{P}{v} = \frac{\varepsilon . I}{v}$$

$$F_m = \frac{\not{-}B \not{v} d\cos\theta}{\not{v}} \cdot \frac{Bvd\cos\theta}{P}$$

$$\therefore \quad F_m = \frac{vB^2d^2\cos^2\theta}{R}$$

This Lorentz force opposes the motion of sliding wire PQ

∴ Net force acting on wire PQ of mass m,

∴ By Newton's second law $m\frac{d^2x}{dt^2} = mg\sin\theta - F_m$

$$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{vB^2d^2\cos^2\theta}{R}$$

$$\frac{dv}{dt} = g\sin\theta - \frac{vB^2d^2\cos^2\theta}{mR}$$

$$\frac{dv}{dt} + \frac{vB^2d^2\cos^2\theta}{mR} = g\sin\theta$$

It is a linear differential equation,

$$v = \frac{g \sin\theta}{\frac{B^2d^2\cos^2\theta}{mR}} + Ae^{\left(\frac{-B^2d^2\cos^2\theta}{mR}\right)t}$$

here, A is constant,

$$v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - Ae^{\left(\frac{-B^2d^2\cos^2\theta}{mR}\right)t}$$

Initially at $t = 0$, $v = 0$

$$\therefore \quad 0 = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - A.e^0$$

$$A = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}$$

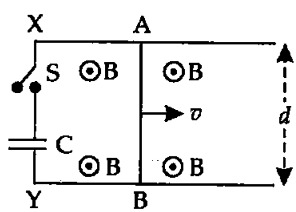
$$\therefore \quad v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}e^{-\left(\frac{B^2d^2\cos^2\theta}{mR}\right)t}$$

$$v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left[1 - e^{-\left(\frac{B^2d^2\cos^2\theta}{mR}\right)t}\right]$$

Let us consider a new constant $\frac{mR}{B^2.d^2.\cos^2\theta} = \alpha$

$$\boxed{v = \alpha g\sin\theta\left[1 - e^{-\frac{t}{\alpha}}\right]}$$

Q6.29. Find the current in the sliding rod AB of resistance R for the arrangement shown in figure. $\vec{B}$ is a constant and is out of the paper. Parallel wires have no resistance. $\vec{v}$ is constant. Switch S is closed at time $t = 0$.



Main concept used: Properties of capacitor, e.m.f.

Ans. Induced current I in loop ABYX = $I_i = \frac{\varepsilon}{R} = \frac{-1}{R}\frac{dQ}{dt}$

$$I_i = \frac{1}{R}.\frac{d}{dt}\vec{B}.\vec{A}$$

$$I_i = \frac{vBd}{R} \qquad ...(I)$$

angle between $\vec{B}$ and $\vec{A}$ is zero. Direction of I from A to B is given by Fleming's right hand rule.

As switch S is closed at $t = 0$.

Current of first equation will charge the capacitor. Let $Q(t)$ be the charge on capacitor.

$\therefore$ $$Q(t) = C.V$$
$$Q(t) = C.I_c R$$

$\therefore$ $$I_c = \frac{Q(t)}{RC}$$

The capacitor opposes the flow of charge, so net current in circuit

$$I = I_i - I_c$$
$$I = \frac{Bvd}{R} - \frac{Q(t)}{RC}$$
$$\frac{dQ(t)}{dt} = \frac{Bvd}{R} - \frac{Q(t)}{RC}$$
$$\frac{Bvd}{R} = \frac{dQ(t)}{dt} + \frac{Q(t)}{RC}$$

or $$\frac{dQ(t)}{dt} + \frac{Q(t)}{RC} = \frac{Bvd}{R}$$

So the solution of linear differential equation is

$$Q(t) = \frac{\frac{Bvd}{R}}{\frac{1}{RC}} + Ae^{\frac{-t}{RC}}$$
$$Q(t) = Bvd\,C + A\,e^{-t/RC}$$

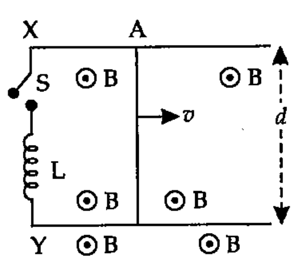
Initially at $t = 0$, $Q = 0$

So $$A = -Bvd\,C$$

So $$Q(t) = BvdC - Bvd\,C\,e^{-\frac{t}{RC}} \qquad [\because e^0 = 1]$$
$$Q(t) = BvdC\left[1 - e^{-\frac{t}{RC}}\right]$$

Current in circuit $I = \frac{dQ(t)}{dt} = \frac{Bvd}{R} e^{-\frac{t}{RC}}$ or $\frac{dQ(t)}{dt} = \frac{BvdC}{RC} e^{-\frac{t}{RC}}$

Q6.30. Find the current in the sliding rod AB of resistance R for the arrangement shown in figure. $\vec{B}$ is constant and is out of the paper. Parallel wires have no resistance, $\vec{v}$ is constant. Switch S is closed at $t = 0$.



Main concept used: Kirchhoff's law, induced current and current in inductor.

Ans. Conductor AB moves towards right with speed v and magnetic field $\vec{B}$ is perpendicularly upward so angle between B and v is 90° and an induced e.m.f., ε flows in loop AXYB.

$$\varepsilon = \frac{d}{dt}\vec{B}\vec{A} = \frac{d}{dt}BA\cos 0 = \frac{d}{dt}B(d \times v \times t) = Bvd$$

$\because$ angle between $\vec{B}$ and $\vec{A}$ is 0°.

$\therefore \quad \varepsilon = B.v.d$

at $t = 0$, current starts increasing in loop along with inductor L due to potential difference

By Kirchhoff's law in loop ABYX,

$$-L\frac{dI(t)}{dt} + Bvd = IR$$

$$\frac{L\,dI(t)}{dt} + RI(t) = Bvd$$

It is a differential equation.

$$I(t) = \frac{Bvd}{R} + Ae^{-\frac{Rt}{2}} \qquad ...(I)$$

at $t = 0$, $I = 0$

$$\therefore \quad 0 = \frac{Bvd}{R} + Ae^{0} \qquad [\because e^0 = 1]$$

$$A = \frac{-Bvd}{R}$$

$$\therefore \quad I(t) = \frac{Bvd}{R} - \frac{Bvd}{R}e^{-\frac{Rt}{2}} \qquad \text{[from I]}$$

$$\boxed{I = \frac{Bvd}{R}\left[1 - e^{-\frac{Rt}{2}}\right]}$$

This is the required current expression.

Q6.31. A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z component of magnetic field is $B_z = B_0(1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with velocity v, find the energy lost in resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m B, λ and acceleration due to gravity g.

Main concept used: Relation between induced current, power and velocity of free falling ring.

Ans. Magnetic flux across the ring of mass 'm' and radius 'l' falling under gravity in a region of having magnetic field $B_z = B_0(1 + \lambda z)$ is

$$\phi = \vec{B}_z.\vec{A} = B_0(1 + \lambda z).\pi l^2.$$

The angle between $\vec{B}$ and $\vec{A}$ is 0°

$$\varepsilon = \frac{d}{dt}[B_0(1 + \lambda z)]\,\pi l^2$$

$$IR = (B_0\pi l^2)\left[0 + \lambda\frac{dz}{dt}\right]$$

$$I = \frac{B_0\pi\lambda l^2}{R}\frac{dz}{dt} = \frac{B_0\pi\lambda l^2}{R}v$$

$$\text{Energy lost} = I^2R = \frac{B_0^2\pi^2\lambda^2 l^4}{R^2}v^2.R$$

$$\text{Energy lost} = \frac{B_0^2\pi^2\lambda^2 l^4 v^2}{R}$$

The energy must come from decrease in P.E $= mg\frac{dz}{dt} = mgv$

$$\therefore \quad mgv = \frac{B_0^2\pi^2\lambda^2 v^2 l^4}{R}$$

$$v = \frac{mgR}{B_0^2\pi^2\lambda^2 l^4} \quad \text{or} \quad v = \frac{mgR}{(\pi l^2\lambda B_0)^2}$$

It is the required relation.

Q6.32. A long solenoid S has n turns per metre with diameter a. At the centre of this coil, we place a smaller coil of N turns and diameter b (where $b < a$). If the current in solenoid increases linearly, with time, what is the induced e.m.f. appearing in the smaller coil. Plot graph showing nature of variation in e.m.f., if current varies as a function of $mt^2 + C$.

Main concept used: When varying current is passed through solenoid the varying magnetic field can induce the current in another coil (smaller).

Ans. Varying magnetic field B(t) in solenoid is

$$B_1(t) = \mu_0 n\, I(t)$$

This varying magnetic field changes flux in the smaller coil.

Magnetic flux in IInd coil

$$\phi_2 = B_1(t).A$$
$$= \mu_0 n\, I(t).\pi b^2$$

Induced e.m.f. in second coil due to solenoid's varying magnetic field in 1 turn

$$\varepsilon' = \frac{-d\phi_2}{dt} = \frac{-d}{dt}\mu_0 n\pi b^2 I(t)$$
$$= -\mu_0 n\pi b^2\frac{d}{dt}(mt^2 + C)$$
$$= -\mu_0 n\pi b^2 . 2mt$$

So net e.m.f. produced in N turns of smaller coil

$$\boxed{\varepsilon = -\mu_0 Nn\pi b^2\, 2mt}$$

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