8 Electromagnetic Waves

MULTIPLE CHOICE QUESTIONS—I

Q8.1. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of appropriate electromagnetic radiation to achieve the dissociation lies in

(*a*) visible region. (*b*) infrared region.
(*c*) ultraviolet region. (*d*) microwave region.

Main concept used: (*i*) E = $h\nu$ and (*ii*) Range of electromagnetic spectrum.

Ans. (*c*): $E = 11\text{ eV} = 11 \times 1.6 \times 10^{-19}\text{ J}$

$h = 6.62 \times 10^{-34}\text{ J-S}$

$\therefore \quad E = h\nu$

$$\nu = \frac{E}{h} = \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = \frac{8.8 \times 10^{-19+34}}{3.31} = \frac{880}{331} \times 10^{15}\text{ Hz}$$

Q8.2. A linearly polarized electromagnetic wave given as $E = E_0\hat{i}\cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as

(*a*) $E_r = -E_o\hat{i}\cos(kz - \omega t)$. (*b*) $E_r = E_0\hat{i}\cos(kz + \omega t)$.
(*c*) $E_r = -E_o\hat{i}\cos(kz + \omega t)$. (*d*) $E_r = E_o\hat{i}\cos(kz - \omega t)$.

Main concept used: When a wave is reflected from a denser medium then its phase angle is changed by 180°.

Ans. (*b*): $E = E_o i\cos(kz - \omega t)$ (given)

As E is along +ive X axis so reflected ray will be along negative X-axis. Its electric component will also be opposite to earlier, *i.e.* in –z direction, phase will change by π ($z \to -z$) and ($i \to -i$)

$$E_r = E_o(-i)\cos[k(-z) - \omega t + \pi]$$

$$E_r = -E_o\hat{i}\cos[\pi - (kz + \omega t)]$$

$$\therefore \quad E_r = +E_o\hat{i}\cos(kz + \omega t)$$

Q8.3. Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2, the total momentum delivered (for complete absorption) during 30 minutes is

(*a*) 36×10^{-5} kg m/s. (b) 36×10^{-4} kg m/s.
(c) 108×10^{4} kg m/s. (d) 1.08×10^{7} kg m/s.

Main concept used: Momentum of incident light = $\frac{V}{C}$ (Total energy)

Ans. (*b*): Energy flux = ϕ = 20 W/cm^2

$A = 30\ cm^2$, $t = 30 \times 60$ sec

U = Total energy falling *t* sec = ϕ At

$U = 20 \times 30 \times 30 \times 60$ J

Momentum of the incident light $= \dfrac{U}{C} = \dfrac{20 \times 30 \times 30 \times 60}{3 \times 10^8}$

$= 36 \times 10^{-4}$ kg ms^{-1}

As no reflection from the surface and for complete absorption so momentum of reflected radiation is zero.

Momentum delivered to surface = Change in momentum

$= p_f - p_i = 0 - 36 \times 10^{-4} = -36 \times 10^{-4}$ kg m/s

(–) sign shows the direction of momentum.

Q8.4. The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is

(*a*) $\dfrac{E}{2}$. (*b*) 2E. (*c*) $\dfrac{E}{\sqrt{2}}$. (*d*) $\sqrt{2}E$.

Main concept used: E.F. intensity on a surface due to incidence radiation is $I_{av} \propto E_o^2$ and $\dfrac{P_{av}}{A} \propto E_o^2$

$P_{av} \propto E_o^2$ (as A is constant)

Ans. (*c*): $E_o \propto \sqrt{P_{av}}$

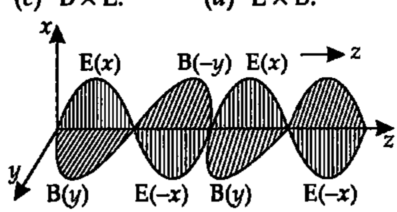
$$\frac{(E_o)_1}{(E_o)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}} = \sqrt{\frac{100\ W}{50\ W}} = \frac{\sqrt{2}}{1}$$

$\therefore$ $$(E_o)_2 = \frac{(E_o)_1}{\sqrt{2}}$$

Q8.5. If E and B represent the electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along

(*a*) $\vec{E}$. (*b*) $\vec{B}$. (*c*) $\vec{B} \times \vec{E}$. (*d*) $\vec{E} \times \vec{B}$.

Main concept used: The direction of e.m. wave can be find out by right hand grip rule or by $\vec{E} \times \vec{B}$.



Ans. (*d*): The direction of propagation of electromagnetic wave is perpendicular to both $\vec{E}$ and $\vec{B}$ and is given by $\vec{E} \times \vec{B}$ by right thumb rule.

The electric field E is along E(+*x*) and E(–*x*) axis and magnetic field B is along B(*y*) and B(–*y*) axis. So by cross product of E and B, direction is perpendicular to E and B, from $\vec{E}$ to $\vec{B}$ *i.e.* (E × B) in +z direction.

Q8.6. The ratio of contributions made by the electric field and magnetic field components to the intensity of an E.M. wave is

(*a*) $c : 1$ (*b*) $c^2 : 1$ (*c*) $1 : 1$ (*d*) $\sqrt{c} : 1$

Main concept used: (*i*) $I = U_{av}.c$, (*ii*) $U_{av} = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}\frac{B_o^2}{\mu_o}$,

(*iii*) $c = \frac{E_o}{B_o}$.

Ans. (*c*): Average energy by electric field E_o is U_{av}

$$U_{av} = \frac{1}{2}\varepsilon_o E_o^2 \quad \text{But } E_o = cB_o$$

$$(U_{av})_{\text{electric field}} = \frac{1}{2}\varepsilon_o (cB_o)^2 = \frac{1}{2}\varepsilon_o c^2 B_o^2$$

$$= \frac{1}{2}\varepsilon_o \cdot \frac{1}{\mu_o \varepsilon_o}(B_o)^2 \qquad \left[\because c^2 = \frac{1}{\mu_o \varepsilon_o}\right]$$

$$(U_{av})_{\text{electric field}} = \frac{1}{2\mu_o}B_o^2 = (U_{av})_{(\text{magnetic field})}$$

$$\text{Ratio} = \frac{(U_{av})_{\text{electric field}}}{(U_{av})_{\text{magnetic field}}} = \frac{1}{1}, \text{ i.e. } 1:1$$

Q8.7. An E.M. wave radiates outwards from a dipole antena, with E_o as the Amplitude of its electric field vector. The electric field E_o which transports significant energy from the source falls off as

(*a*) $\frac{1}{r^3}$ (*b*) $\frac{1}{r^2}$ (*c*) $\frac{1}{r}$ (*d*) remains constant

Main concept used: $E_o \propto \frac{1}{r}$ from dipole antenna.

Ans. (*c*): As we know that electromagnetic waves are radiated from dipole antena and radiated energy $E \propto \frac{1}{r}$.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q8.8. An electromagnetic wave travels in vacuum along +Z direction

$$E = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t).$$

Choose the correct options from the following:

(*a*) The associated magnetic field is given as

$$B = \frac{1}{C}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t).$$

(*b*) The associated magnetic field is given as

$$B = \frac{1}{C}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t).$$

(*c*) The given electromagnetic field is circularly polarised.

(*d*) The given electromagnetic wave is plane polarised.

Main concept used: From Maxwell's equations $|B_o| = \frac{|E_o|}{|C|}$.

Ans. (*a*), (*b*) and (*d*): Here, in electromagnetic wave, the electric field vector is given as $E = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t)$. In electromagnetic wave the associated magnetic field vector $\vec{B} = \frac{\vec{E}}{\vec{C}} = \frac{1}{C}(E_1 i + E_2 j)\cos(kz - \omega t)$ verify option (*a*) and (*b*) as $\vec{E}$ and $\vec{B}$ are along X- and Y-axis respectively so the direction of propagation is Z-axis and E.M. wave is plane polarised along X and Y direction $\vec{E}$ and $\vec{B}$ respectively verifies option (*d*).

Q8.9. An electromagnetic wave travelling along Z-axis is given as $E = E_0\cos(kz - \omega t)$. Choose the correct options from the following:

(*a*) The associated magnetic field is given as $B = \frac{1}{c}\hat{k} \times \vec{E} = \frac{1}{\omega}(\hat{k} \times \vec{E})$.

(*b*) The electromagnetic field can be written in terms of the associated magnetic field $\vec{E} = c(\vec{B} \times \hat{k})$.

(*c*) $\hat{k} \cdot E = 0, \quad \hat{k} \cdot B = 0 \cdot$

(*d*) $\hat{k} \times \vec{E} = 0, \quad \hat{k} \times \vec{B} = 0 \cdot$

Main concept used: $E = E_0\cos(kz - \omega t)$

Ans. (*a*), (*b*) and (*c*): E.M. wave is travelling in +Z direction. Its electric field is given by

$E = E_0 \cos(kz - \omega t)$ along X direction which is perpendicular to Z-axis *i.e.*, along X direction.

The associated magnetic field B is also perpendicular to +Y *i.e.*, $\hat{k} \times \vec{E}$.

As $B = \frac{E}{c} = \frac{1}{c}(\hat{k} \times \vec{E})$ (along Y-axis).

The associated electric field can be written in terms of magnetic field as $\vec{E} = c(\vec{B} \times \hat{k})$.

Angle between $\hat{k}$ and $\vec{E}$ is 90° ⇒ $E.B = EB\cos 0° = 1$

As we know that B, E and direction of propagation of E.M. wave are perpendicular to each other.

Here, $E = E_0\cos(kz - \omega t)$

As the direction of propagation of E.M. wave is in +Z direction *i.e.*, $v\hat{k}$, then E is in $E\hat{i}$ and $B\hat{j}$.

For option (*a*) $\quad B = \frac{1}{c}\hat{k} \times \vec{E}$ [given in option (*a*)]

$$Bj = \frac{1}{c}\hat{k} \times \vec{E}\hat{i} \text{ is true} \quad (\because \hat{k} \times \hat{i} = \hat{j})$$

$\frac{1}{\omega}(\hat{k} \times \vec{E}) = B$ [given in option (*a*)]

$$\frac{1}{\omega}(\hat{k} \times E\hat{i}) = B\hat{j} \text{ is true as } \hat{k} \times \hat{i} = \hat{j}$$ verifies answer (*a*).

For option (*b*) $E\hat{i} = c(B\hat{j} \times \hat{k})$ is true $\therefore \hat{} = \hat{j} \times \hat{k}$

For option (*c*) $\hat{k}.E(\hat{i}) = 0$ *is true* $\because \hat{k}.\hat{i} = 0$

$\hat{k}.B(\hat{j}) = 0$ *is true* $\because \hat{k}.\hat{j} = \cos 90° = 0$

For option (*d*) $\hat{k} \times E(\hat{i}) = 0$ is false as $\hat{k} \times \hat{i} = -\hat{j} \neq 0$

Q8.10. A plane electromagnetic wave propagating along X-direction can have the following pairs of E and B

(*a*) E_x, B_y (*b*) E_y, B (c) B_x, E_y (d) E_z, B_y

Main concept used: The direction of propagation $\vec{v}$, magnetic field $\vec{B}$ and electric field are perpendicular to each other.

Ans. (*b*) and (*d*): As the EM wave is plane polarised and its propagation is in +X direction. So direction of $\vec{E}$ and $\vec{B}$ will be in either Y and Z direction or Z and Y direction. So verifies answers (*b*) and (*d*).

Q8.11. A charged particle oscillates about its mean equilibrium position with a frequency 10^9 Hz. The electromagnetic waves produced:

(*a*) will have the frequency of 10^9 Hz.
(*b*) will have the frequency of 2×10^9 Hz.
(*c*) will have wavelength 0.3 m.
(*d*) fall in the region of radiowaves.

Main concept used: (*i*) $c = \nu\lambda$, (*ii*) the frequency of wave as the frequency due to which it is produced.

Ans. (*a*) (*c*) and (*d*): Vibrating particle produces electric and magnetic field, so will produce an E.M. wave of same frequency 10^9 Hz verifies answer (*a*).

$\because \nu = 10^9$ Hz, $c = 3 \times 10^8$ m/s

So, $$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^9} = \frac{3 \times 10^8}{10 \times 10^8} = 0.3 \text{ m}$$ verifies answer (*c*).

As the range of radiowaves are between 10 Hz to 10^{12} Hz and 10^9 Hz lies between this range verifies answer (*d*).

Q8.12. The source of electromagnetic waves can be a charge

(*a*) moving with constant velocity. (*b*) moving in a circular orbit.
(*c*) at rest. (*d*) falling in an electric field.

Main concept used: An E.M. wave can be produced either by accelerated or oscillating charge.

Ans. (*b*) and (*d*): Motion of a particle in circular orbit is accelerated motion verifies answer (*b*).

When a charge particle falls in electric field the velocity of charge particle changes so its motion becomes accelerated and can produce E.M. wave. It verifies answer (*d*).

Q8.13. An E.M. wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it. Which of the following are true?

(*a*) Radiation pressure is I/c if the wave is totally absorbed.
(*b*) Radiation pressure is I/c if the wave is totally reflected.
(*c*) Radiation pressure is $\frac{2I}{c}$ if the wave is totally reflected.
(*d*) Radiation pressure is in the range $\frac{I}{c} < p < \frac{2I}{c}$ for real surface.

Main concept used: Due to dual nature of the wave, E.M. wave also has particle nature.

Ans. (*a*) (*c*) and (*d*): Radiation pressure is the force exerted by particles (dual nature of particle) on unit area, due to the change in momentum of radiated particles per unit area per sec = $\frac{I}{c}$.

I = intensity of radiation
c = velocity of radiation

Radiations are absorbed, so momentum per unit area per second = $\frac{I}{c}$ verify the answer (*a*).

When radiation is reflected back, the momentum becomes double as in earlier case, so discards answer (*b*) and verifies answer (*c*).

So variation of radiation pressure p comes between the range $\frac{I}{c} < p < \frac{2I}{c}$ verifies answer (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q8.14. Why is the orientation of the portable radio with respect to broadcasting station important?

Ans. Transmitted carrier wave signals are plane polarised and if the intensity of signal is poor then receiving antenna of radio must be parallel to the component of either electric or magnetic field. Because energy is only due to amplitudes of electric and magnetic components in EM wave, magnitude of amplitude is in particular direction perpendicular to each other and perpendicular to wave propagation.

Q8.15. Why does the microwave oven heats up a food item containing water molecule most efficiently?

Main concept used: Resonance phenomenon.

Ans. In Microwave oven, molecules of food item starts to vibrate by driven force due to microwaves with the frequency of microwave. But the natural frequency of water molecules matches with microwave frequency which causes resonance (more amplitude) which further causes increase in temperature.

Q8.16. The charge on parallel plate capacitor varies as $q = q_0 \cos 2\pi\nu t$. The plates are very large and close to each other. Separation between plates is d and common area of plates is A. Neglecting the edge effects, find the displacement current through the capacitor.

Ans. The displacement current I_d in capacitor is

$$I_d = I_C = \frac{dq}{dt}, \text{ where } q = q_o \cos(2\pi v t) \qquad \text{(Given)}$$

$$\therefore \quad I_d = I_C = \frac{d}{dt} q_o \cos(2\pi v t)$$

$$I_d = -q_o 2\pi v \sin 2\pi v t$$

$$I_d = -2\pi v q_o \sin(2\pi v t)$$

Q8.17. A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency?

Main concept used: $X_c \propto \frac{1}{v}$ and $I = \frac{V}{X_C}$

Ans. As we know that $X_C = \frac{1}{2\pi v C}$ and $I_C = \frac{V}{X_C}$

so reactance of capacitor increases on decreasing frequency $X_c \propto \frac{1}{v}$.

So $\qquad I = 2\pi v CV$

As reactance of capacitor increases, the current by Ohm's law will decrease. $\left[\because I \propto \frac{1}{X_C}\right]$

So the displacement current decreases frequency decreases when the conduction current is equal to displacement current.

Q8.18. The magnetic field of a beam, emerging from a filter facing a floodlight, is given by $B = 12 \times 10^{-8} \sin(1.2 \times 10^7 z - 3.6 \times 10^{15} t)$ T.

Find the average intensity of the beam.

Main concept used: $I_{av} = \frac{B_o^2}{2\mu_o} C$

$$B = B_o \sin(kx - 2\pi v t) \qquad \left(\because k = \frac{2\pi}{\lambda}\right)$$

Ans. $B = 12 \times 10^{-8} \sin[1.2 \times 10^7 Z - 3.6 \times 10^{15} t]$ ...I

Standard equation $B = B_o \sin(kx - \omega t)$ ...II

Comparing the different terms in above two (I, II) equations

$$B_o = 12 \times 10^{-8} \text{ Tesla}$$

$$I_{av} = \frac{B_o^2}{2\mu_o} C = \frac{12 \times 10^{-8} \times 12 \times 10^{-8} \times 3 \times 10^8}{2 \times 4\pi \times 10^{-7}}$$

$$= \frac{12 \times 12 \times 3 \times 10^{-8-8+8+7}}{8 \times 3.14} = \frac{54}{3.14 \times 10} = \mathbf{1.72\ W/m^2}$$

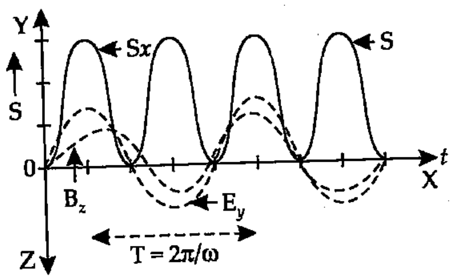
Q8.19. Poynting vector S is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction

of the wave propagation. Mathematically, it is given by $S = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$.
Show the nature of S versus t graph.

Ans. Consider an electromagnetic wave. Let electric field (E) of EM wave varies along Y-axis the propagation of wave is along X-axis, then $\vec{E} \times \vec{B}$ will give the direction of flow of energy in electromagnetic wave along X-axis.

$$\vec{E} = E_o \sin(\omega t - kx)\hat{j}$$
$$\vec{B} = B_o \sin(\omega t - kx)\hat{k}$$
$$S = \frac{1}{\mu_o}(\vec{E} \times \vec{B}) = \frac{1}{\mu_o} \sin^2(\omega t - kx)\hat{j} \times \hat{k}$$
$$S = \frac{1}{\mu_o} \sin^2(\omega t - kx)\hat{i}$$

Variation of |S| with time will be as given in figure.



Q8.20. Professor CV Raman surprised his students by suspending freely a tiny light ball in transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

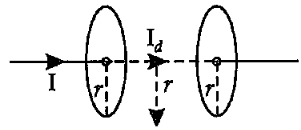
Ans. We know the dual nature of radiation and matter. EM wave carries energy and momentum. Due to this change in momentum (by direction or velocity of wave), EM wave, exert pressure on the surface, by reflection and refraction. This property of EM waves helped professor CV Raman to surprise his students by suspending freely a tiny light ball in transparent vacuum chamber by shining a laser beam on it. The tails of the comets are also due to radiation pressure.

Electromagnetic radiations can pass even through vacuum and has particle nature.

Mobile phone placed in evacuated transparent chamber can ring up which can be seen through transparent chamber, but sound of ring tone cannot be heard, proves the propagation of electromagnetic wave in vacuum.

SHORT ANSWER TYPE QUESTIONS

Q8.21. Show that the magnetic field B at a point in between plates of a parallel plate capacitor during charging is $\frac{\mu_o \varepsilon_o r}{2} \frac{dE}{dt}$ (symbols having usual meaning).



Ans. Let I_d be the displacement current in the region of magnetic field between two plates of a parallel plate capacitor.

The magnetic field induction at a point in a region between two plates of a capacitor at a perpendicular distance from the axis of plate is

$$B = \frac{\mu_o I}{2\pi r} = \frac{\mu_o}{2\pi r} I_d = \frac{\mu_o}{2\pi r}\left(\varepsilon_o \frac{d\phi}{dt}\right)$$

$$B = \frac{\mu_o \varepsilon_o}{2\pi r} \frac{d}{dt}(E\pi r^2) \quad \because \boxed{\phi_E = \vec{E}\,.\,\vec{A}}$$

$$B = \frac{\mu_o \varepsilon_o r}{2} \frac{dE}{dt}$$

Q8.22. Electromagnetic waves with wavelength

(*i*) λ_1 is used in satellite communication.

(*ii*) λ_2 is used to kill germs in water purifiers.

(*iii*) λ_3 is used to detect leakage of the oil in underground pipelines.

(*iv*) λ_4 is used to improve visibility in runways during fog and mist conditions.

(*a*) Identify and name the part of electromagnetic spectrum to which these radiations belong.

(*b*) Arrange these wavelengths in ascending order of their magnitude.

(*c*) Write one more application of each.

Ans. (*a*) (*i*) Microwave is used in satellite communications so, λ_1 is the wavelength of microwave. It is used in microwave oven.

(*ii*) Ultraviolet rays are used to kill germs in water purifier so, λ_2 is the wavelength of ultraviolet rays.

λ_2 are UV rays that can be focused into very narrow beam for high precision application such as LASIK (Laser—assisted in situ kerotomileusis) eye surgery.

(*iii*) X-rays are used to detect leakage of oil in underground pipelines so λ_3 is in X-ray region. It is also used to detect cracks in machinery and to detect fracture in bones of body.

(*iv*) Infrared rays λ_4 are used to improve visibility due to larger wavelength of low scattering.

Infrared rays are used in optical communication.

(*b*) The arrangement of the wavelengths in ascending order are

$$\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$$

(*c*) Uses are along with part (*a*).

Q8.23. Show that average value of radiant flux density S over a single period T is given by $S = \frac{1}{2c\mu_o}E_o^2$.

Ans. Radiant flux density $S = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

or
$$c^2 = \frac{1}{\mu_o \varepsilon_o}$$

$$\frac{1}{\mu_o} = \varepsilon_o c^2$$

$$\therefore \quad S = \varepsilon_o c^2 (\vec{E} \times \vec{B}) \quad \text{...(I)}$$

Let electromagnetic waves be propagated along X-axis so its electric and magnetic field vectors are along Y and Z axis.

$$\therefore \quad \vec{E} = E_o \cos(kx - \omega t)\hat{j}$$

$$\vec{B} = B_o \cos(kx - \omega t)\hat{k}$$

$$\vec{E} \times \vec{B} = (E_o B_o) \cos^2(kx - \omega t)\ (\hat{j} \times \hat{k})$$

Put E × B in I

$$\therefore \quad S = \varepsilon_o c^2 E_o B_o \cos^2(kx - \omega t)\hat{i}$$

So average value of the magnitude of radiant flux density over complete cycle is

$$S_{av} = c^2 \varepsilon_o (E_o B_o) \frac{1}{T}\int_0^T \cos^2(kx - \omega t)\, dt\, \hat{i}$$

$$= \frac{c^2 \varepsilon_o E_o B_o}{T}\left[\frac{T}{2}\right] = \frac{c^2}{2}\varepsilon_o E_o \left(\frac{E_o}{c}\right) \quad \left[\because c = \frac{E_o}{B_o} \text{ or } B_o = \frac{E_o}{c}\right]$$

$$= \frac{c\varepsilon_o E_o^2}{2} \quad \left[c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \text{ or } \varepsilon_o = \frac{1}{\mu_o c^2}\right]$$

$$= \frac{c}{2} \cdot \frac{1}{\mu_o c^2} E_o^2$$

$$S_{av} = \frac{E_o^2}{2\mu_o c} \quad \textbf{Hence proved}$$

Q8.24. You are given a 2 μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?

Ans. $C = 2\ \mu F = 2 \times 10^{-6}$ F, $I_d = 1$ mA $= 10^{-3}$ A

$$\because \quad q = CV$$

$$I_d dt = C.dV$$

$$I_d = C \cdot \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{I_d}{C} = \frac{10^{-3}\,A}{2\times 10^{-6}} = \frac{1000}{2}\text{ Volts}$$

$$\frac{dV}{dt} = 500\text{ V/s}$$

Q8.25. Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is $\frac{I}{c}$.

Ans. Pressure on surface due to particle nature of wave $P = \frac{F}{A}$

Force = Rate of change of momentum

If mass (m) of radiant particle for wave of velocity c then

$$E = mc^2$$

$$E = U \text{ (let)}$$

$$U = mc \, . \, c$$

$$U = p \, . \, c$$

where, momentum $p = mc$

Now differentiating w.r.t. time (t) we get

$$\frac{dU}{dt} = c \, . \, \frac{dp}{dt}$$

$$\frac{1}{c} \cdot \frac{dU}{dt} = \frac{dp}{dt} \qquad \left[\because \frac{dp}{dt} = F \text{ (by Newton's second law)}\right]$$

$$\therefore \qquad F = \frac{1}{c}\frac{dU}{dt} \quad \text{or} \quad \frac{F}{A} = \frac{1}{A} \cdot \frac{1}{c} \cdot \frac{dU}{dt}$$

Pressure (P) on surface due to e.m. wave radiation and $P = \frac{F}{A}$

$$P = \frac{1}{c} \cdot \left[\frac{dU}{Adt}\right]$$

We know that intensity of radiation is equal to the radiant energy (U) falling on unit surface per second.

$$\therefore \qquad I = \frac{1}{A} \cdot \frac{dU}{dt} \quad \text{or} \quad \boxed{P = \frac{I}{c}} \text{ Hence proved.}$$

Q8.26. What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of a room its intensity essentially remains constant.

What geometrical characteristic of LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?

Ans. Bulb spreads its light in all around spherically and symmetrically. So if the distance from the bulb is doubled, the surface area covered by radiations changes from $4\pi r^2$ to $4\pi(2r)^2$ *i.e.*, 2^2 or four times decreased in straight laser. But in a bulb it decreased by $4(4\pi)$, i.e. 16π decreased. So the intensity becomes one-fourth the initial value in straight line. Since $\left(I \propto \frac{1}{r^2}\right)$ and for spherical source $\left(I \propto \frac{1}{4\pi r^2}\right)$.

In case of laser, it does not spread in all directions. It passes only along a straight line. So its intensity remains same almost.

Geometrical characteristics of LASER beam which is responsible for the constant intensity are:

(*i*) Monochromatic, (*ii*) Coherent, (*iii*) Highly collimated from all around source, (*iv*) Unidirectional.

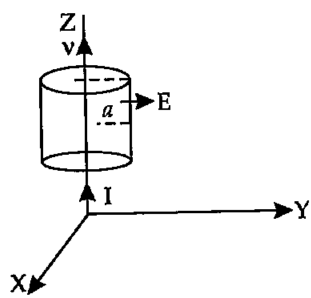
These characteristics are missing in the case of light from the bulb.

Q8.27. Even though an electric field E exerts a force qE on a charged particle, yet electric field of EM wave does not contribute to the radiation pressure (but transfers energy). Explain.

Ans. In electromagnetic wave, the electric field is oscillating, so the resultant electric force on the particle will be zero, as the direction of electric force changes every half of time period. As the electric field vibrates, the radiation of energy will takes place only due to vibrating electric and magnetic field.

LONG ANSWER TYPE QUESTIONS

Q8.28. An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the +Z-axis (figure). The wire is set into motion along its length with a uniform velocity $\vec{v} = v\hat{k}_z$. Calculate the poynting vector $S = \frac{1}{\mu_0}(E \times B)$.



Ans. Consider a cylindrical Gaussian surface in such a way that the axis of cylinder lies on wire. Electric field intensity due to long straight wire at a distance a and charge density λ c/m

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 a} = \frac{\lambda}{2\pi\varepsilon_0 a}\hat{j}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{i}$$

$$I = \frac{q}{t} = \frac{\lambda l}{t} = \lambda.v$$

$$\therefore \quad \vec{B} = \frac{\mu_0 \lambda v}{2\pi a}\hat{i}$$

$$S = \frac{1}{\mu_0}(E \times B) = \frac{1}{\mu_0}\left[\frac{\lambda}{2\pi\varepsilon_0 a}\hat{j} \times \frac{\mu_0 \lambda v}{2\pi a}\hat{i}\right]$$

$$= \frac{\lambda^2 v}{4\pi^2 a^2 \varepsilon_o}(\hat{j} \times \hat{i}) = \frac{\lambda^2 v}{4\pi^2 a^2 \varepsilon_o}(-\hat{k})$$

$$S = \frac{-\lambda^2 v}{4\pi^2 a^2 \varepsilon_o}\hat{k}$$

Q8.29. Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\varepsilon = 80\varepsilon_o$, permeability $\mu \approx \mu_o$ and resistivity $\rho = 0.25\ \Omega$-m. Imagine a parallel plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_o \sin(2\pi\nu t)$. What fraction of the conduction current density is the displacement current density?

Ans. Suppose distance between the parallel plates of capacitor is 'd' and the applied voltage

$$V(t) = V_o \sin(2\pi\nu t)$$

$$\therefore \quad E = \frac{V(t)}{d} = \frac{V_o \sin(2\pi\nu . t)}{d}$$

By Ohm's conduction current density

$$\vec{J}_C = \frac{1}{\rho}\vec{E} = \frac{1}{\rho}\frac{V_o \sin(2\pi\nu t)}{d}$$

Let $$J_o^C = \frac{V_o}{\rho d}$$

$$\therefore \quad \boxed{J_C = J_o^C \sin 2\pi\nu t}$$

The displacement current

$$J_d = \varepsilon\frac{dE}{dt} = \varepsilon\frac{d}{dt}\frac{V_o}{d}\sin(2\pi\nu t) = \frac{\varepsilon V_o}{d}\cdot\cos(2\pi\nu t)\,(2\pi\nu)$$

$$J_d = \frac{2\pi\nu\varepsilon V_o}{d}\cos(2\pi\nu t)$$

Let $$J_o^d = \frac{2\pi\nu\varepsilon V_o}{d}$$

Then $$J_d = J_o^d \cos(2\pi\nu t)$$

$$\frac{J_o^d}{J_o^c} = \frac{\frac{2\pi\nu\varepsilon V_o}{d}}{\frac{V_o}{\rho d}} = 2\pi\nu\varepsilon_o\rho$$

$$\frac{J_o^d}{J_o^c} = 2\pi\nu\varepsilon\rho = 2\pi \times 4 \times 10^8 \times 80\varepsilon_o \times 0.25$$

$$= 4\pi\varepsilon_o \times 2 \times 80 \times 0.25 \times 10^8$$

$$= \frac{2 \times 80 \times 0.25 \times 10^8}{9 \times 10^9} = \frac{160 \times 25}{9 \times 10 \times 100}$$

$$\frac{J_o^d}{J_o^c} = \frac{4}{9}$$

Q8.30. A long straight cable of length l is placed symmetrically along Z-axis and has radius a ($<<< l$). The cable consists of a thin wire and a co-axial conducting tube. An alternating current $I(t) = I_o \sin(2\pi\nu t)$ flows down the central thin wire and returns along the co-axial conducting tube. The induced electric field at a distance s from the wire inside the cable is

$$\vec{E}(s, t) = \mu_o I_o \nu \cos(2\pi\nu t) \log_e\left(\frac{s}{a}\right)\hat{k}.$$

(*i*) Calculate the displacement current density inside the cable.

(*ii*) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I_d.

(*iii*) Compare the conduction current I_o with the displacement current I_o^d.

Main concept used: $\boxed{I_d = \varepsilon_o \dfrac{dE}{dt}}$.

Ans. (*i*) Induced electric field E(*s*, *t*) at distance *s*(*s* < radius of co-axial cable) is given as $E(s, t) = \mu_o I_o \nu \cos 2\pi\nu t \log_e\left(\frac{s}{a}\right)\hat{k}$. Displacement current density J_d is given by,

$$J_d = \varepsilon_o \frac{dE}{dt} = \varepsilon_o \mu_o I_o \nu \frac{d}{dt}\left[\cos 2\pi\nu t . \log_e \frac{s}{a}\right]\hat{k}$$

$$J_d = \varepsilon_o \mu_o I_o \nu \left[(-\sin 2\pi\nu t) \cdot 2\pi\nu . \log_e \frac{s}{a}\right]\hat{k}$$

[$\because$ *s* and *a* are constant]

$$J_d = -\varepsilon_o \mu_o I_o 2\pi\nu^2 \log_e\left(\frac{s}{a}\right)\sin(2\pi\nu t)\,\hat{k}$$

$$= -\frac{1}{C^2} I_o 2\pi\nu^2\left[-\log_e\left(\frac{a}{s}\right)\right]\sin(2\pi\nu t)\,\hat{k} \quad \left[\because C = \frac{1}{\sqrt{\mu_o \varepsilon_o}}\right]$$

$$= +\frac{2\pi\nu^2}{C^2} I_o \log_e\left(\frac{a}{s}\right)\sin 2\pi\nu t\,\hat{k}$$

$$= \frac{2\pi\nu^2}{\nu^2\lambda^2} I_o \log_e\left(\frac{a}{s}\right)\sin 2\pi\nu t\,\hat{k}$$

$$\boxed{J_d = \frac{2\pi I_o}{\lambda^2}\log_e \frac{a}{s}\sin(2\pi\nu t)\,\hat{k}}$$

(*ii*)
$$I_d = \int J_d . s\, ds\, d\theta = \int_{s=0}^{a} J_d \cdot s\, ds \int_0^{2\pi} d\theta = \int_{s=0}^{a} J_d\, s.ds\,[2\pi]$$

$$= 2\pi \int_{s=0}^{a} \frac{2\pi I_o}{\lambda^2}\left[\log_e\left(\frac{a}{s}\right)\sin(2\pi\nu t)\hat{k}\right].s\,ds$$

$$I_d = \left(\frac{2\pi}{\lambda}\right)^2 I_o \sin(2\pi\nu t)\hat{k}\int_{s=0}^{a}\log\left(\frac{a}{s}\right).s\,ds$$

Integration of $\int_{s=0}^{a} \log\left(\frac{a}{s}\right) \cdot s\, ds$

$$= \left[\log\left(\frac{a}{s}\right) \cdot \int_0^a s\, ds\right]_0^a - \int_{s=0}^{a} \left[\frac{d}{ds}\left[\log\left(\frac{s}{a}\right)\right] \cdot \int s\, ds\right] ds$$

$$= \left[\log\left(\frac{a}{s}\right) \frac{s^2}{2}\right]_0^a - \int_{s=0}^{a} \left(\frac{s}{a}\right) \cdot \frac{s^2}{2} ds$$

$$= \left[\log\left(\frac{a}{a}\right) \cdot \frac{a^2}{2} - 0\right] - \frac{1}{2a} \int_{s=0}^{a} s^3 ds$$

$$= 0 - \frac{1}{2a} \cdot \left[\frac{s^4}{4}\right]_0^a = -\frac{a^3}{8} \qquad [\because \log_e 1 = 0]$$

$$\therefore\ I_d = -\frac{a^3}{8} \cdot \left(\frac{2\pi}{\lambda}\right)^2 . I_o \sin 2\pi\nu t\, \hat{k} = -\frac{a}{2} \cdot \frac{a^2}{4} \left(\frac{2\pi}{\lambda}\right)^2 I_o \sin(2\pi\nu t)\, \hat{k}$$

The negative sign shows that the displacement current I_d is opposite to the conduction current I_C.

$$\therefore \qquad I_d = \frac{a}{2}\left(\frac{2\pi a}{2\lambda}\right)^2 I_o \sin(2\pi\nu t)\, (-\hat{k})$$

$$\boxed{I_d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_o \sin(2\pi\nu t)\, (-\hat{k})}$$

I_d is in –Z direction as I_C is in +Z direction.

(*iii*) $I_d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_o \sin(2\pi\nu t)(-\hat{k})$

$I_d = I_o^d \sin(2\pi\nu t)$

where $I_o^d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_o$

$$\text{Required ratio } \frac{I_o^d}{I_o} = \frac{\frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 \cdot I_o}{I_o} = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2$$

$$\boxed{\frac{I_o^d}{I_o} = \frac{\pi^2 a^3}{2\lambda^2}}$$

Q8.31. A plane EM wave travelling in vacuum along Z-direction is given by $\vec{E} = E_o \sin(kz - \omega t)\hat{i}$ and $\vec{B} = B_o \sin(kz - \omega t)\hat{j}$.

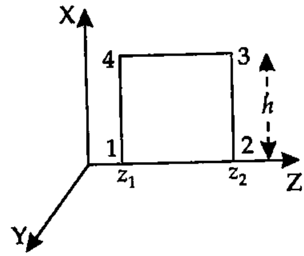
(*i*) Evaluate $\oint \vec{E} . \vec{dl}$ over the rectangular loop 1234 shown in figure.

(*ii*) Evaluate $\int \vec{B} . \vec{ds}$ over the surface bounded by loop 1234.

(*iii*) Use equation $\oint E.dl = \frac{-d\phi_B}{dt}$ to prove $\frac{E_o}{B_o} = C$.

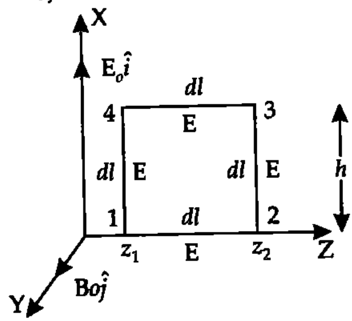
(*iv*) By using similar process and the equation $\oint B.dl = \mu_o I + \varepsilon_o \frac{d\phi_E}{dt}$.

Prove that $C = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$.



Ans. As the electromagnetic wave is propagating along Z-axis then its electric and magnetic field vectors are along X and Y axis.

(*i*) $\vec{E} = E_o\hat{i}$ and $\vec{B} = B_o\hat{j}$ as in figure below.



Line integral of E over loop 1234,

$$\oint \vec{E}.\vec{dl} = \int_1^2 \vec{E}.\vec{dl} + \int_2^3 \vec{E}.\vec{dl} + \int_3^4 \vec{E}.\vec{dl} + \int_4^1 \vec{E}.\vec{dl}$$

$$= \int_1^2 E.dl \cos 90° + \int_2^3 E.dl \cos 0° + \int_3^4 E.dl \cos 90° + \int_4^1 E.dl \cos 180°$$

$$\oint \vec{E}.\vec{dl} = \int_1^2 0 + \int_2^3 E.dl + \int_3^4 0 + \int_4^1 E dl(-1)$$

$$= 0 + [E.h]_{z_2} - [E.h]_{z_1} = E_o h \sin[kz_2 - \omega t) - E_o h \sin(kz_1 - \omega t)$$

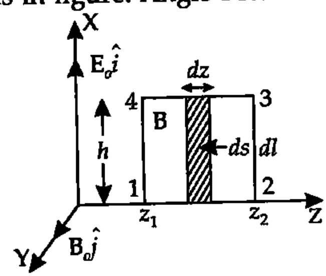
$$= hE_o[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)]$$

(*ii*) Consider a strip of area $ds = h.dz$ as in figure. Angle between $\vec{ds}$ and $\vec{B}$ is zero.

$$\therefore \int \vec{B}.\vec{ds} = \int \vec{B}.\vec{ds} \cos 0 \,(\hat{j})$$

$$= \int_{z_1}^{z_2} B.ds\,.\,\hat{j}$$

$$= \int_{z_1}^{z_2} B_o \sin(kz - \omega t)\, h.dz$$



$$= \frac{-B_o h}{k}\left[\cos(kz-\omega t)\right]_{z_1}^{z_2}$$

$$= \frac{-B_o h}{k}\left[\cos(kz_2-\omega t)-\cos(kz_1-\omega t)\right]$$

$$\int \vec{B}.\vec{ds} = \frac{B_o h}{k}\left[\cos(kz_1-\omega t)-\cos(kz_2-\omega t)\right]$$

(*iii*) Given $\oint \vec{E}.\vec{dl} = \frac{-d\phi_B}{dt} = -\frac{d}{dt}\oint B.ds$ $\quad [\because B = B_o \sin(kz-\omega t)\hat{j}]$

$$= \frac{d}{dt}\left[\frac{B_o h}{k}\{\cos(kz_1-\omega t)-\cos(kz_2-\omega t)\}\right]\hat{j}$$

$$= \frac{B_o h}{k}[-\sin(kz_1-\omega t)(-\omega)+\sin(kz_2-\omega t)(-\omega)]$$

$$= \frac{B_o h}{k}[\omega\sin(kz_1-\omega t)-\omega\sin(kz_2-\omega t)]$$

$$E = \frac{B_o h\omega}{k}[\sin(kz_1-\omega t)-\sin(kz_2-\omega t)]$$

$$E = E_0[\sin(kz_1-\omega t)-\sin(kz_2-\omega t)]$$

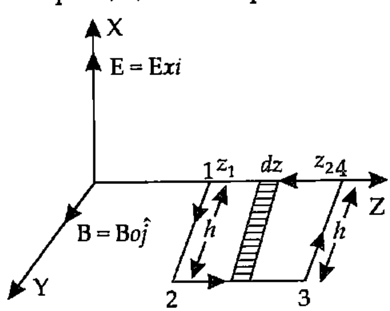
$$E_0 = \frac{B_o\omega}{k} \qquad \left[\because \frac{\omega}{k} = C\right]$$

$$\therefore \quad E_0 = B_Y C$$

$$\frac{E_0}{B_0} = \frac{E_0}{B_Y} = \frac{B_Y C}{B_Y} \qquad [\because B = B_0\hat{j} \Rightarrow B = B_Y]$$

$$\boxed{\frac{E_0}{B_0} = C}$$

(*iv*) Consider a loop 1.2, 3, 4 in Y-Z plane as in figure.



$$\oint \vec{B}.\vec{dl} = \int_1^2 \vec{B}.\vec{dl} + \int_2^3 \vec{B}.\vec{dl} + \int_3^4 \vec{B}.\vec{dl} + \int_4^1 \vec{B}.\vec{dl} \quad ...I$$

$$= \int_1^2 \vec{B}.\vec{dl}\cos 0° + \int_2^3 \vec{B}.\vec{dl}\cos 90° + \int_3^4 \vec{B}.\vec{dl}\cos 90° + \int_4^1 \vec{B}.\vec{dl}\cos 90° \quad ...II$$

$$\oint \vec{B}.\vec{dl} = \int_1^2 B.dl + \int_4^1 \vec{B}.(-dl) = \int_1^2 B.dl - \int_4^1 B.dl$$

$$= [B\,h]_{z_1} - [B\,h]_{z_2} \qquad [\because dl = h \text{ in figure}]$$

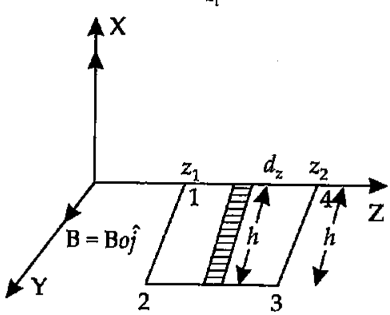
$$\oint \vec{B}.\vec{dl} = [B_0 h \sin(kz - \omega t)]_{z_1} - [B_0 h \sin(kz - \omega t)]_{z_2}$$

$$\oint B.dl = B_0 h[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad ...III$$

Now to calculate $\phi_E = \int E.ds$. Let us consider the rectangular strip of loop 1, 2, 3, 4 of area ds each $ds = hdz$.

$$\phi_E = \int \vec{E}.\vec{ds} = \int E.ds \cos 0° = \int E.ds = E_0 \int_{z_1}^{z_2} \sin(kz - \omega t).h\,dz$$

$$\phi_E = E_0 h\left[\frac{-\cos(kz - \omega t)}{k}\right]_{z_1}^{z_2}$$



$$\phi_E = \frac{E_0 h}{k}[\cos(kz_1 - \omega t) - \cos(kz_2 - \omega t)]$$

$$\frac{d\phi_E}{dt} = \frac{-E_0 h\omega}{k}[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad ...IV$$

By Ampere's circuital law

$$\int B.dl = \mu_0\left[I_C + \varepsilon_0 \frac{d\phi_E}{dt}\right] \qquad [I_C = \text{conduction current}]$$

$I_C = 0$ in vacuum

$$\therefore \quad \int B.dl = \mu_0\,\varepsilon_0 \frac{d\phi}{dt}$$

Using relations obtained in eqn. (III) and (IV)
B_0h [sin $(kz_1 - \omega t)$-sin $(kz_2 - \omega t)$]

$$= \mu_0 \varepsilon_0 \frac{E_0 h\omega}{k} [\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)]$$

$$B_0 h = \frac{\mu_0 \varepsilon_0 h\omega E_0}{k}$$

$$B_0 = E_0 \frac{\omega \mu_0 \varepsilon_0}{k}$$

$$\frac{E_0}{B_0}\frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0}$$

$$\frac{C\omega}{k} = \frac{1}{\mu_0 \varepsilon_0} \qquad \left[\because \frac{E_0}{B_0} = C\right]$$

$$\frac{CCk}{k} = \frac{1}{\mu_0 \varepsilon_0} \qquad [\omega = Ck]$$

$$C^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ Hence proved.

Q8.32. A plane EM wave travelling along Z-direction is described by $\vec{E} = E_0 \sin(kz - \omega t)\hat{i}$ and $\vec{B} = B_0 \sin(kz - \omega t)\hat{j}$. Show that

(*i*) The average energy density of the wave is given by

$$U_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

(*ii*) The time averaged intensity of the wave is given by

$$I_{av} = \frac{1}{2} C \varepsilon_0 E_0^2$$

Ans. (*i*) The electromagnetic wave carry energy which is due to electric field vector and magnetic field vector. In electromagnetic wave, E and B varies with time.

The energy density due to electric field $\vec{E}$ is $U_E = \frac{1}{2}\varepsilon_o E^2$

The energy density due to magnetic field B is $U_B = \frac{1}{2\mu_0} B^2$

Total average energy density of electromagnetic wave

$$U = U_E + U_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$E = E_0 \sin(kz - \omega t)\, \hat{i}$$
$$B = B_0 \sin(kz - \omega t)\, \hat{j}$$

average value of E^2 over a cycle $= \dfrac{E_0^2}{2}$

The average value of B^2 over a cycle $= \dfrac{B_0^2}{2}$

$$\therefore \qquad U_{av} = \frac{1}{2}\varepsilon_0 \frac{1}{2} E_0^2 + \frac{1}{2\mu_0}\frac{(B_0^2)}{2}$$

$$U_{av} = \frac{1}{4}\left[\varepsilon_0 E_o^2 + \frac{B_0^2}{\mu_o}\right]$$

(*ii*) We know that $E_0 = CB_0$ and $C = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2/C^2}{\mu_0} = \frac{E_0^2}{4\mu}\mu_0\varepsilon_0 = \frac{1}{4}\varepsilon_0 E_0^2$$

$$U_B = U_E$$

$$U_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

$$\because \qquad \frac{E_0}{B_0} = C \qquad \Rightarrow \qquad \frac{E_0^2}{B_0^2} = C^2$$

$$\Rightarrow \qquad \frac{E_0^2}{B_0^2} = \frac{1}{\mu_0 \varepsilon_0} \qquad \Rightarrow \qquad B_0^2 = \mu_0 \varepsilon_0 E_0^2$$

$$\therefore \qquad U_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{\mu_0 \varepsilon_0 E_0^2}{4\mu_0} = \frac{1}{4}\varepsilon_0 E_o^2 + \frac{1}{4}\varepsilon_0 E_0^2$$

$$U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$$

$$U_{av} = \frac{1}{4}\varepsilon_0 \cdot \frac{B_0^2}{\mu_0 \varepsilon_0} + \frac{B_0^2}{4\mu_0} \qquad \left[\because\ E_0^2 = \frac{B_0}{\mu_0 \epsilon_0}\right]$$

$$= \frac{B_0^2}{4\mu_0} + \frac{B_0^2}{4\mu_0}$$

$$U_{av} = \frac{B_0^2}{2\mu_0}$$

$$\therefore \qquad U_E = U_B$$

Time average intensity of wave

$$I_{av} = U_{av} C = \frac{1}{2}\frac{B_0^2 C}{\mu_o} = \frac{1}{2}\varepsilon_0 E_o^2 C$$

❑❑❑