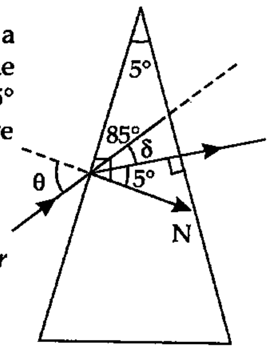
9 Ray Optics and Optical Instruments

MULTIPLE CHOICE QUESTIONS—I

Q9.1. A ray of light incident at an angle θ on a refracting surface of a prism emerges from the other face normally. If the angle of prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is



(*a*) 7.5°. (*b*) 5°.
(*c*) 15°. (*d*) 2.5°.

Main concept used: $\delta = (\mu - 1)A$ and $\delta = i - r$

Ans. (*a*): $\delta = (1.5 - 1)5° = 0.5 \times 5 = 2.5°$

$$\theta - r = \delta, \theta = \delta + r$$

$$\theta = 2.5 + 5 = 7.5°$$

Q9.2. A short pulse of white light is incident from air to glass slab at normal incidence. After travelling through the slab, the first colour to emerge is

(*a*) blue. (*b*) green. (*c*) violet. (*d*) red.

Main concept used: $c = \nu\lambda$, ν does not change during refraction.

Ans. (*d*): $\because$ $c = \nu\lambda$ and ν is constant during refraction so $c \propto \lambda$. The velocity of red colour is maximum in glass as the $\lambda_R > \lambda_V$.

Q9.3. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image

(*a*) moves away from the lens with uniform speed 5 m/s.
(*b*) moves away from the lens with uniform acceleration.
(*c*) moves away from the lens with non-uniform acceleration.
(*d*) moves towards the lens with a non uniform acceleration.

Main concept used: When $u = \infty$, then $v = f$ and when $u = f$, then $v = \infty$

Ans. (*c*): When an object approaches towards a lens with uniform speed, its image moves away from the lens to infinity with non uniform acceleration.

Q9.4. A passenger in an aeroplane shall

(*a*) never see a rainbow.
(*b*) may see a primary and a secondary rainbow as concentric circles.
(*c*) may see a primary and a secondary rainbow as concentric arcs.
(*d*) shall never see a secondary rainbow.

Ans. (*b*): A passenger in an aeroplane may see primary and secondary rainbow as concentric circles.

Q9.5. You are given four sources of light each one providing a light of a single colour, red, blue, green and yellow. Suppose the angle of refraction far a beam of yellow light corresponding to a particular angle of incidence at interface of two media is 90°. Which of the following statements is correct if source of yellow light is replaced with that of other lights without changing the angle of incidence?

(*a*) The beam of red light would undergo total internal reflection.

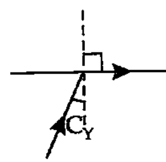
(*b*) The beam of red light would bend towards normal while it gets refracted through the second medium.

(*c*) The beam of blue light would undergo total internal reflection.

(*d*) The beam of green light would bend away from the normal as it gets refracted through the second medium.

Main concept used: Critical angle for yellow is greater than green and blue and smaller than red.

Ans. (*c*): We know that if angle of refraction is 90° for the length then incidence angle is called critical angle. So light rays are passing from denser to rarer medium.



As $\sin c = \frac{1}{\mu}$ so, $c \propto \frac{1}{\mu}$ and $\mu_v > \mu_g > \mu_Y > \mu_R$

So, critical angle for $C_v < C_g < C_Y < C_R$, *i.e.*, critical angle of blue and green light is smaller than that of yellow and it is greater for red colour light.

As the angle of refraction for yellow light is 90° for a particular incident angle. This incidence angle is critical angle for yellow let it be C_Y. As $C_R > C_v$. So it will not get total internal reflection and $C_V < C_Y$, $C_g < C_Y$.

So light of blue and green colour get total internal reflection. So correct answer is (*c*).

Q9.6. The radius of curvature of curved surface of plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will

(*a*) act as a convex lens only for the objects that lie on its curved side.

(*b*) act as a concave lens for the objects that lie on its curved side.

(*c*) act as a convex lens irrespective of side on which the object lies.

(*d*) act as a concave lens irrespective of side on which the object lies.

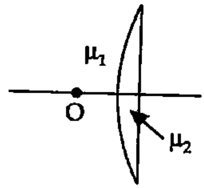
Main concept used: Lens Maker's formula.

Ans. (*c*): If object lies on curved side then R_1 = +20 cm and $R_2 = \infty$, $\mu_1 = 1, \mu_2 = 1.5$

$$\frac{1}{f} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{20} - \frac{1}{\infty}\right] = \frac{0.5}{20} = \frac{5}{200} = \frac{1}{40}$$

$f = +40$ cm



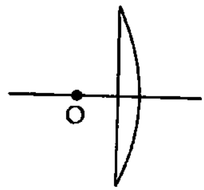
If object lies on plane side $R_1 = \infty$ and $R_2 = -20$ cm, $\mu_1 = 1, \mu_2 = 1.5$

$$\frac{1}{f} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{\infty} - \left(-\frac{1}{20}\right)\right] = 0.5 \times \left(+\frac{1}{20}\right)$$

$$\frac{1}{f} = \frac{5}{200}$$

$f = +40$ cm



So, lens will always act as a convex lens irrespective of side on which objects lie. So, answer is (*c*).

Q9.7. The phenomena involved in the reflection of radiowaves by ionosphere is similar to

(*a*) reflection of light by plane mirror.
(*b*) total internal reflection of light in air during a mirage.
(*c*) dispersion of light by water molecules during the formation of rainbow.
(*d*) scattering of light by the particles of air.

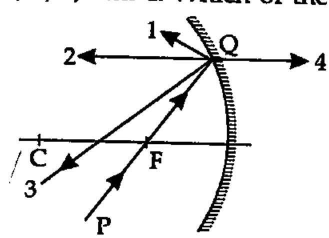
Main concept used: Refractive index of ionosphere is less than atmosphere for radiowaves. Although the refractive index of any material can never be less than one of vacuum.

Ans. (*b*): Ionosphere is transparent optical medium and radiowave is reflected back. Reflection through transparent surface is total internal reflection so that internal reflection of radiowave takes place.

Q9.8. The direction of light rays incidence on a concave mirror is shown by PQ, while the direction in which the ray would travel after reflection is shown by four rays marked 1, 2, 3, and 4. Which of the four rays correctly shows the direction of reflected ray?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

Main concept used: Normal at incidence point in spherical mirrors passes through centre of curvature of lens.



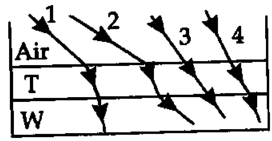
Ans. (*b*): Incidence ray PQ is coming through principal focus F so it must be parallel to principal axis, *i.e.* either 2 or 4.

As it is a concave mirror so, ray cannot go behind the mirror so ray (4) is discarded.

So ray 2 is the reflected ray. It is verifies answer (*b*)

We can verify it again by drawing normal Q_C and find that $\angle r = \angle i$. So ray (2) is the reflected ray.

Q9.9. The optical density of turpentine is higher than that of water while its mass density is lower than water. Figure below shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?



(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

Main Concept: Laws of refraction.

Ans. (*b*): $\mu_a < \mu_T > \mu_W$. Here, incidence ray passes from air to turpentine to water, *i.e*, from rare to denser then denser to rarer so first it bends towards normal then away from normal so the path shown is correct for ray (2).

Q9.10. A car is moving with constant speed of 60 km/hr on a straight road. Looking at the rear view mirror, the driver finds that car following him is at a distance of 100 m and is approaching with a speed of 5 km/h.

In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statements is /are correct?

(*a*) The speed of the car in the rear is 65 km/h.

(*b*) In the side mirror, the car, in the rear would appear to approach with a speed of 5 km h^{-1} to the driver of the leading car.

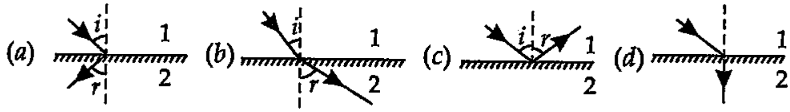
(*c*) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases.

(*d*) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases.

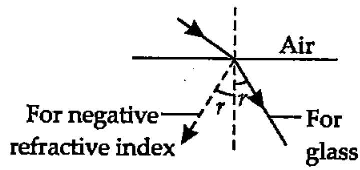
Main concept used: If object is at infinity, image in convex mirror is at focus so speed is zero when car is at infinity i.e., very far from centre of curvature of mirror.

Ans. (*d*): So when rear car approaches, initially it appear at rest as image is formed at focus. When car approaches nearer this speed will appear to increase so answer is (*d*).

Q9.11. There are certain materials developed in laboratories which have a negative refractive index. In figure below, a ray incidents from air (medium 1) into such a medium (medium 2) shall follow a path given by:



Ans. (*a*): The negative refractive index materials are those in which incident ray from air (medium 1) to them refract or bends differently or opposite and symmetric to normal to that of positive refractive index medium. So answer is (*a*).



MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q9.12. Consider an extended object immersed in water contained in a plane trough. When seen from close to the edge of the trough the object looks distorted because

(*a*) the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge.

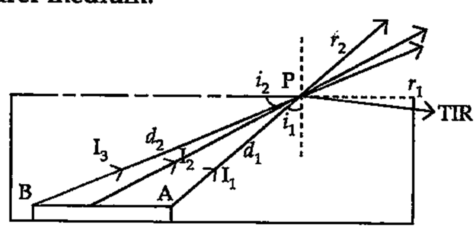
(*b*) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air.

(*c*) some of the points of the object far away from the edge may not be visible because of total internal reflection.

(*d*) water in a trough acts as a lens and magnifies the object.

Main concept used: Refraction and total internal reflection when light passes from denser to rarer medium.

Ans. (*a*), (*b*) and (*c*): We know that shifting (h) of image of an object immersed in liquid from object is directly proportional to the real distance of object from the surface of liquid



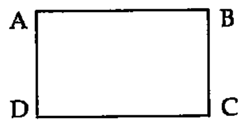
$$h = t\left(1 - \frac{1}{\mu}\right)$$

t = real depth or distance of the object from the surface of liquid of refractive index μ: If the object is seen from one edge of trough the relative differences of depth (distance) in H_2O between two ends of objects is larger than if it is seen from the top or away from edge. By above formula or distortion of nearer end is smaller than farther, verifies the option (*a*).

The angle subtended by an object is larger than its image in water, as its image shifts upward verifies option (*b*).

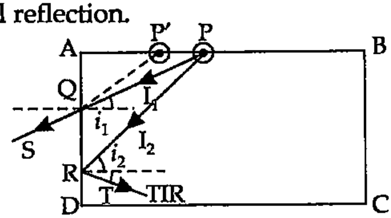
Rays coming out from object to observer passes from denser to rarer medium, and angle of incidence for rays from farther end B of object is larger than near end A. The incidence angles for rays coming from end 3 may have incidence angle more than critical angle and can cause total internal reflection, so will not reach to observer, cause nonvisible of farther end verifies option (*c*).

Q9.13. A rectangular block of glass ABCD has a refractive index 1.6. A pin is placed midway on the face AB (Fig). When observed from the face AD, the pin shall–



(*a*) appear to be nearer to A.
(*b*) appear to be nearer to D.
(*c*) appear to be at the centre of AD.
(*d*) not be seen at all.

Main concept used: Total internal reflection.



Ans. (*a*) (*b*): $\because \sin C = \dfrac{1}{\mu}$

$\therefore \sin C = \dfrac{1}{1.6} \Rightarrow C = 38.7°$

A point P is on the mid point of face AB. When seen through face AD, near to point A, the angle of incidence (i_1) will be smaller than critical angle $i_c = 38.7°$. So image of P will form at P′ and image can be seen at P′. P′ is nearer to both A and D as compared to P verifies option (*a*) and (*b*).

When seen near point D through face AD, angle of incidence $i_2 > i_c$ so total internal reflection takes place and object cannot be observed.

But object can be seen when viewed near to A so option (*d*) not verified.

Q9.14. Between the primary and secondary rainbows, there is a dark band, known as Alexandar's dark band. This is because

(*a*) light scattered into this region interfere destructively.
(*b*) there is no light scattered into this region.
(*c*) light is absorbed in this region.
(*d*) angle made at the eye by the scattered rays with respect to incident light of the sun, lies between approximately 42° and 50°.

Main concept used: Scattering and dispersion of light.

Ans. (*d*): Alexandar's dark band lies between the primary and secondary rainbow. This forms due to the light scattered into this region interfere destructively. Because the primary and secondary rainbows subtend angles (41° to 42°) and (51° to 54°) respectively at the observer's eye with respect to incident light ray, so the scattered rays with respect to the incident ray of the sun lies between approximately 42° to 50°.

Q9.15. A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in

(*a*) a larger angle to be subtended by the object at the eye and hence viewed in greater detail.

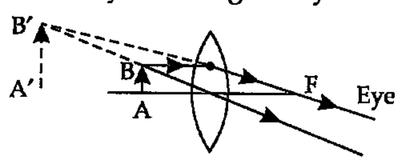
(*b*) the formation of virtual erect image.

(*c*) increase in the field of view.

(*d*) Infinite magnification at the near point.

Main concept used: An object is seen more clearly if subtends larger angle to the eye. Very far object subtends very small angle to eye.

Ans. (*a*) and (*b*): In magnifying glass, the object is placed within the focal length and the image formed is magnified and erect. As (A′B′) image is magnified so it subtends larger angle at the eye than object (AB), so can be seen more clearly.



Q9.16. An astronomical refractive telescope has an objective of focal length 20 m and an eye piece of focal length 2 cm.

(*a*) The length of the telescope tube is 20.02 m.

(*b*) The magnification is 1000.

(*c*) The image formed is inverted.

(*d*) An objective of larger aperture will increase the brightness and reduce the chromatic aberration of the image.

Main concept used: (*i*) $L = fo + fe$ (*ii*) $m = \frac{fo}{fe}$ and (*iii*) ray diagram of telescope.

Ans. (*a*), (*b*) and (*c*): The length of the telescope

$$L = fo + fe = 20 + 00.02 = 20.02 \text{ m}$$

and
$$m = \frac{fo}{fe} = \frac{20 \text{ m}}{0.02 \text{ m}} = \frac{2000}{2} = 1000$$

The final image formed in telescope (Refracting) is inverted, virtual and smaller than object.

VERY SHORT ANSWER TYPE QUESTIONS

Q9.17. Will focal length of a lens for a red light be more, same or less than that for blue light?

Main concept used: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Ans. As we know that ($\mu_V > \mu_R$), so $\frac{1}{f}$ will be large for blue light and smaller for Red coloured light. So f will be larger for red coloured light.

Q9.18. The near vision of an average person is 25 cm. To view an object with an angular magnification of 10, what should be the power of the microscope?

Main concept used: Magnification of microscope at distance of distinct vision.

Ans. To see the final image at distinct vision D = 25 cm

for eye lens $u = -f$ and $v = -25$

$$m = \frac{+v}{u}$$

$$10 = \frac{-25}{-f}$$

$$f = +\frac{25}{10} = +2.5 \text{ cm } = 0.025 \text{ m}$$

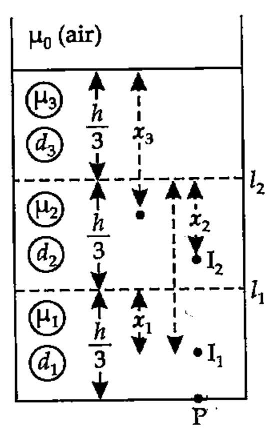
$$\therefore \quad P = \frac{1}{f} = \frac{1}{0.025} = \frac{1000}{25} = \mathbf{40\ D}$$

Q9.19. An unsymmetrical double convex, thin lens forms the image of a point object on its axis. Will the position of the image change if the lens is reversed?

Main concept used: Lens maker's formula

Ans. Position of image in convex lens changes either by changing u of f. Here, u is constant and $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Here, μ and curvature of the lens are same so f is constant. So on reversing the lens, position and nature of the image **will not change.**

Q9.20. Three immisible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is $\frac{h}{3}$. A dot is made at the bottom of the beaker. For near normal vision, find the apparent depth of the dot.



Main concept used: (*i*) $\mu = \frac{\text{real depth}}{\text{apparent depth}}$,

(*ii*) Image formed by one medium acts as an object for second medium.

Ans. The liquids are immisible, so liquids arranged from bottom to top d_1, d_2 and d_3 with their refractive indices μ_1, μ_2, μ_3 respectively separated by layers l_1, l_2 as shown in figure.

Consider the layer l_1. Let object is at its bottom at P then distance (x_1) of image of P by liquid of refractive index μ_1 from layer l_1

$$x_1 = \frac{h/3}{{}_2\mu_1} \qquad \because \quad {}_2\mu_1 = \frac{\mu_1}{\mu_2}$$

$$\therefore \qquad x_1 = \frac{h}{3\dfrac{\mu_1}{\mu_2}} = \frac{h}{3}\frac{\mu_2}{\mu_1}$$

This image I_1 acts as object for liquid of μ_2 and form the image at I_2. The distance x_2 below level l_2

$$\text{Real depth} = \left(\frac{h}{2} + x_1\right)$$

$$x_2 = \frac{\text{Real depth}}{{}_3\mu_2} = \frac{\text{Real depth}}{\mu_2/\mu_3}$$

$$x_2 = \frac{\mu_3}{\mu_2}\left[\frac{h}{3} + x_1\right] = \frac{\mu_3}{\mu_2}\left[\frac{h}{3} + \frac{h}{3}\frac{\mu_2}{\mu_1}\right] = \frac{h}{3}\frac{\mu_3}{\mu_2}\left[1 + \frac{\mu_2}{\mu_1}\right]$$

$$= \frac{h}{3}\left[\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_2}\frac{\mu_2}{\mu_1}\right] = \frac{h}{3}\left[\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right]$$

as the point P is seen from outside

$$x_3 = \frac{\mu_0}{\mu_3}\left[\frac{h}{3} + x_2\right] = \frac{1}{\mu_3}\left[\frac{h}{3} + \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right] \qquad [\because \mu_0 = 1 \text{ (refractive index of air)}]$$

$$= \frac{h}{3}\left[\frac{1}{\mu_3} + \frac{1}{\mu_3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right]$$

$$x_3 = \frac{h}{3}\left[\frac{1}{\mu_3} + \frac{\mu_3}{\mu_3}\left(\frac{1}{\mu_2} + \frac{1}{\mu_1}\right)\right] = \frac{h}{3}\left[\frac{1}{\mu_3} + \frac{1}{\mu_2} + \frac{1}{\mu_1}\right]$$

x_3 is apparent depth.

Q9.21. For a glass prism $(\mu = \sqrt{3})$ the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism.

Ans. The required relation for minimum angle of deviation

$$\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\dfrac{A}{2}} \qquad [\because \delta_m = A \text{ (given)}]$$

$$\therefore \qquad \mu = \frac{\sin A}{\sin\dfrac{A}{2}} = \frac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}} = 2\cos\frac{A}{2}$$

$$\sqrt{3} = 2\cos\frac{A}{2} \Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{A}{2} = \cos 30° \Rightarrow \frac{A}{2} = 30°$$

$A = 60°$ is angle of prism.

SHORT ANSWER TYPE QUESTIONS

Q9.22. A short object of length L is placed along the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f, what will be the length of the image? You may take $L \ll |v - f|$

Main concept used: The length of image is the difference between the image distance of extremities.

Ans. As the mean distance of object from mirror is u

$$\therefore \qquad u_1 = u - \frac{L}{2} \quad \text{and} \quad u_2 = \left(u + \frac{L}{2}\right)$$

Let the image of the two ends of object form at distance v_1 and v_2 $(v_1 > v_2)$. So length of image on principal axis is $L' = (v_1 - v_2)$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{u-f}{uf} \Rightarrow v = \frac{uf}{u-f}$$

$$\text{So} \qquad L' = v_1 - v_2 = \frac{\left(u - \frac{L}{2}\right)f}{\left(u - \frac{L}{2}\right) - f} - \frac{\left(u + \frac{L}{2}\right)f}{\left(u + \frac{L}{2}\right) - f}$$

$$\Rightarrow \qquad L' = f\left[\frac{u - \frac{L}{2}}{\left(u - f - \frac{L}{2}\right)} - \frac{u + \frac{L}{2}}{\left(u - f + \frac{L}{2}\right)}\right]$$

$$\Rightarrow \qquad L' = f\left[\frac{\left(u - \frac{L}{2}\right)\left(u - f + \frac{L}{2}\right) - \left(u + \frac{L}{2}\right)\left(u - f - \frac{L}{2}\right)}{\left(u - f - \frac{L}{2}\right)\left(u - f + \frac{L}{2}\right)}\right]$$

$$= \frac{f\left[u^2 - uf + \frac{uL}{2} - \frac{uL}{2} + \frac{fL}{2} - \frac{L^2}{4} - \left(u^2 - uf - \frac{uL}{2} + \frac{uL}{2} - \frac{fL}{2} - \frac{L^2}{4}\right)\right]}{(u-f)^2 - \frac{L^2}{4}}$$

$$\therefore \quad L \ll (u - f) \quad \therefore \quad \frac{L^2}{4} \lll (u - f)$$

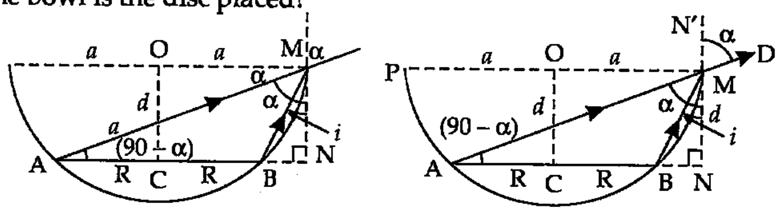
So neglecting the terms $\frac{L^2}{4}$

$$L^2 = f\left[\frac{\frac{fL}{2}+\frac{fL}{2}}{(u-f)^2}\right]$$

$$L' = \frac{f\,fL}{(u-f)^2} \Rightarrow L' = \frac{Lf^2}{(u-f)^2}$$

It is the length of image f.

Q9.23. A circular disc of radius R is placed co-axially and horizontally inside an opaque hemispherical bowl of radius 'a' as in figure. The far edge of the disc is just visible when viewed from the edge of the bowl. The bowl is filled with transparent liquid of refractive index μ, and the near edge of the disc becomes just visible. How far below the top of the bowl is the disc placed?



(*a*) Bowl filled with air (*b*) Bowl filled with liquid

Main concept used: Snell's law and Geometry.

Ans. In figure AM and BM are the rays from the ends of disc AB reaching at one end of bowl at M. MN is tangent at M, so MN ⊥ AB *i.e.*, ∠N = 90°

Taking incidence ray BM and refracted ray MD

$$BN = CN - CB = OM - CB = a - R$$

$$MB = \sqrt{d^2 + (a - R^2)}$$

$$\therefore \qquad \sin i = \frac{BN}{BM} = \frac{(a-R)}{\sqrt{d^2+(a-R)^2}}$$

$$\angle r = \angle \alpha = \angle AMN$$

$$\sin r = \cos(90° - \alpha) = \frac{AN}{AM} = \frac{a+R}{\sqrt{d^2+(a+R)^2}}$$

For incidence ray BM to the horizontal level of liquid MP, MN will be normal at M. ∠i and ∠r will be incidence and refracted angles when ray BM passes from liquid (μ) to air. By Snell's law, as ray passes from liquid to air

$${}_1\mu_0 = \frac{\sin i}{\sin r} \Rightarrow \frac{\mu_0}{\mu_1} = \frac{\sin i}{\sin r} \qquad [\mu_0 \text{ for air} = 1] \quad [\mu_1 = \mu \text{ for liquid}]$$

$$\frac{1}{\mu} = \frac{\sin i}{\sin r}$$

$$\frac{1}{\mu} = \frac{\dfrac{a-R}{\sqrt{d^2+(a-R)^2}}}{\dfrac{(a+R)}{\sqrt{d^2+(a+R)^2}}} = \frac{(a-R)\sqrt{d^2+(a+R)^2}}{(a+R)\sqrt{d^2+(a-R)^2}}$$

$$d = \frac{\mu(a^2-d^2)}{\sqrt{(a+r)^2-\mu(a-r)^2}}$$

It is required expression.

Q9.24. A thin convex lens of focal length 25 cm is cut into two pieces 0.5 cm above the principal axis. The top part is placed at (0, 0) and an object is placed at (– 50 cm, 0). Find the co-ordinates of the image.

Main concept used: There is no effect on focal length if a lens is cut by plane parallel to principal axis.

Ans. Object is placed 0.5 cm above principal axis

$u = -50$ cm f = +25 cm, $v = ?$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

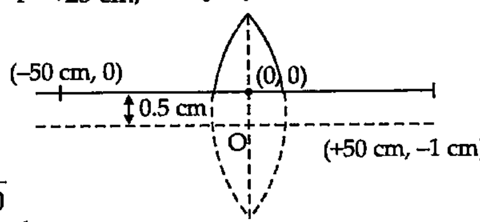
$$\frac{1}{v} - \frac{1}{-50} = \frac{1}{25}$$

$$\frac{1}{v} = \frac{1}{25} - \frac{1}{50}$$

$$= \frac{2-1}{50} = \frac{1}{50}$$

$$v = 50 \text{ cm}$$

$$m = \frac{+v}{u} = \frac{+(50)}{-50} = -1$$



So the size of image is equal to that of object, m is negative so image is inverted.

So image is at (50 cm, –1 cm) and 0.5 cm below the X – X′ axix.

Q9.25. In many experimental set ups, the source and the screen are fixed at a distance say D and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of image sizes for these two points.

Main concept used: Principle of reversibility

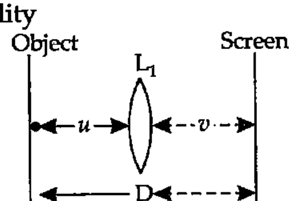
Ans.

$$u = -(D - v)$$

$$v = v$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{v} + \frac{1}{(D-v)}$$

$$= \frac{D - v + v}{v(D-v)} = \frac{D}{vD - v^2}$$



$$fD = vD - v^2$$

$$v^2 - vD + fD = 0$$

$$v = \frac{+D \pm \sqrt{D^2 - 4Df}}{2} = \frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2} \quad \text{...I}$$

$$u = -(D - v) = -\left[D - \left\{\frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}\right\}\right]$$

$$u = -\left[\frac{D}{2} \mp \frac{\sqrt{D^2 - 4Df}}{2}\right] \quad \text{...II}$$

From II, when the position of object $u_2 = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$

[infront of l_2 (lower sign)]

Then from I, the position of image $v_2 = \frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$ [lower sign]

Similarly when position of object $u_1 = \frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$

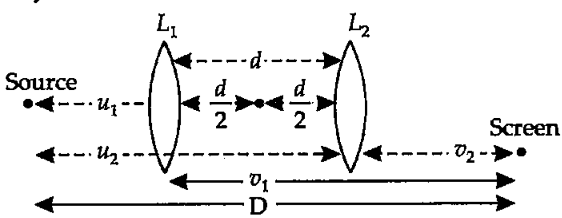
[from II upper sign]

then the position of image $v_1 = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$ [from I upper sign]

The distance between two positions of lens $d = v_1 - v_2$

$$d = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2} - \left[\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}\right]$$

$d = \sqrt{D^2 - 4Df}$ is the distance between two positions of lenses.



In first case of L_1

$$u_1 = \frac{D}{2} - \frac{d}{2}$$

$$v_1 = \frac{D}{2} + \frac{d}{2}$$

$$\therefore \qquad m_1 = \frac{v_1}{u_1} = \frac{\frac{D}{2}+\frac{d}{2}}{\frac{D}{2}-\frac{d}{2}} = \frac{D+d}{D-d}$$

In second case

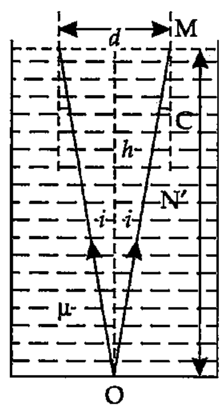
$$u_2 = \frac{D}{2}+\frac{d}{2}$$

$$v_2 = \frac{D}{2}-\frac{d}{2}$$

$$\therefore \qquad m_2 = \frac{v_2}{u_2} = \frac{\frac{D}{2}-\frac{d}{2}}{\frac{D}{2}+\frac{d}{2}} = \frac{D-d}{D+d}$$

$$\frac{m_2}{m_1} = \frac{\frac{D-d}{D+d}}{\frac{D+d}{D-d}} = \left(\frac{D-d}{D+d}\right)^2$$

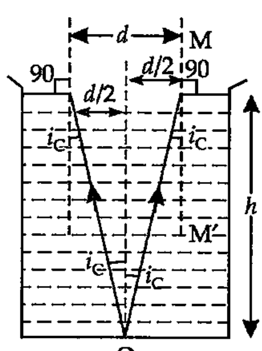
$\boxed{\frac{m_2}{m_1} = \left(\frac{D-d}{D+d}\right)^2}$ is the required ratio of size of images in two cases.



Q9.26. A jar of height h is filled with a transparent liquid of refractive index μ (figure). At the centre of the jar on the bottom surface is a dot. Find the minimum diameter of a disc, such that when placed on the top surface symmetrically about the centre, the dot is invisible.

Main concept used: Total internal reflection

Ans. The point O will be invisible if the light ray coming from O does not come out or it get graze the surface of liquid or gets total internal reflection as in figure here.



Ray OA is incident at A with critical angle i_c and it's angle of refraction will be 90°.

For other ray if incident angle is more than i_c it will get total internal reflection from the surface so will not come out from liquid.

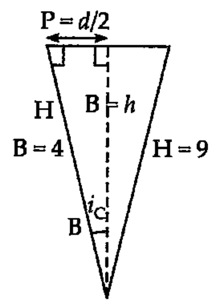
So disc of diameter d is required to stop the rays from 'O' out of liquid.

$$\tan i_c = \frac{d/2}{h} \quad \text{or} \quad \tan i_c = \frac{d}{2h} \quad \Rightarrow \quad d = 2h \tan i_c$$

$$\sin i_c = \frac{1}{\mu} = \frac{P}{H} \quad \text{[From figure]}$$

$$B^2 = H^2 - P^2$$

$$B = \sqrt{\mu^2 - 1}$$



$$\therefore \quad \tan i = \frac{P}{B} = \frac{1}{\sqrt{\mu^2 - 1}}$$

$$\therefore \quad d = 2h \times \frac{1}{\sqrt{\mu^2 - 1}}$$

Q9.27. A myopic adult has a far point at 0.1 m. His power of accommodation is 4D.

(*i*) What power of lens required to see the distant objects?

(*ii*) What is his near point without glasses?

(*iii*) What is his near point with glasses? (Take the image distance from the lens of the eye to the retina to be 2 cm).

Main concept used: $P = P_1 + P_2$ and $P = \frac{1}{f}$

Ans. (*i*) Power of lens required to see clearly the object placed at infinity. $u = -\infty$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{\infty}$$

$$\frac{1}{f} = \frac{1}{-10}$$

$$f = -10 \text{ cm} = -0.1 \text{ m}$$

$$P = \frac{1}{f} \text{m}$$

$$P = \frac{1}{-0.1} \text{m}$$

$$P = -10 \text{ Diopter}$$

(*ii*) **When no corrective lens used:** Let powers of eye when object is at far point, near point are P_f and P_n respectively and power of accommodation $P_a = +4D$

$$\therefore \quad P_n = P_f + P_a$$

When object is at far point its clear image is formed at retina 2 cm from eye lens

$$\therefore \quad u = -10 \text{ cm} = -0.1 \text{ m}, \qquad v = 2 \text{ cm} = 0.02 \text{ m}$$

If f is focal length of eye lens focused at far point then

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{(-0.1)}$$

$$\frac{1}{f} = 50 + 10 = 60$$

$$P_f = 60 \text{ D}$$

$$\therefore \quad P_n = P_f + P_a = 60 + 4 = 64 \text{ D}$$

Let the near point be x_n

$$u = -x_n \qquad (v = 2 \text{ cm} = 0.02 \text{ m})$$

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{0.02} + \frac{1}{x_n} = \text{Power } (P_n)$$

$$50 + \frac{1}{x_n} = 64$$

$$\frac{1}{x_n} = 64 - 50 = 14 \text{ D}$$

Near point without glass $x_n = \frac{1}{14} \text{m} = \frac{100}{14} \text{cm} = \mathbf{7\ cm}$ (Approx.)

(*iii*) **When used corrective lens:** When corrective lens is used then eye can see the object at infinity. Power of eye lens in this situation is P_∞

$u = \infty$ and $v = 2 \text{ cm} = 0.02 \text{ m}$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$P_\infty = \frac{1}{0.02} - \frac{1}{\infty} = 50$$

$$P_\infty = 50 + 0$$

$$P_\infty = 50 \text{ D}$$

If P_n' = Power of eye at near point when corrective lens is used

$$P_n' = P_\infty + P_a = 50 + 4 = 54 \text{ D}$$

Let near point in this situation is x_n'

$$u = -x_n' \text{ m}$$

$$v = +2 \text{ cm} = 0.02 \text{ m}$$

$$\frac{1}{f} = 54$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{0.02} + \frac{1}{x_n'} = 54 \qquad \text{(all distances are in m)}$$

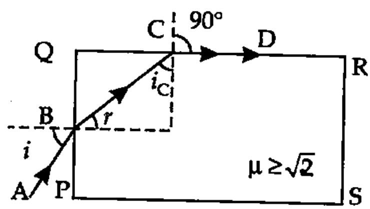
$$50 + \frac{1}{x'_n} = 54 \qquad \left(\frac{1}{x'_n} = 4\right)$$

$$x'_n = \frac{1}{4}\text{ m} = 0.25 \text{ m}$$

LONG ANSWER TYPE QUESTIONS

Q9.28. Show that for a material with refractive index $\mu \geq \sqrt{2}$, light incident at any angle shall be guided along the length perpendicular to incident face.

Main concept used: Snell's law and Total internal reflection



Ans. Consider a rectangular slab of refractive index $\mu \geq \sqrt{2}$. An incidence ray incidence at angle i on face PQ at incidence point. Refracted ray BC strike at face QR which is perpendicular to PQ with incidence angle i_c so that refracted ray CD passes normal to the face PQ as per required in question. So i_c must be critical angle

$$\mu = \frac{1}{\sin i_c} \qquad \text{(Snell's law at } c)$$

$$\sin i_c \geq \frac{1}{\mu} \qquad \left[\sin i_c = \frac{1}{\mu}\right]$$

$$\sin(90 - r) \geq \frac{1}{\mu} \qquad [\because \; r + 90 + i_c = 180]$$

$$\cos r \geq \frac{1}{\mu}$$

$$\cos^2 r \geq \frac{1}{\mu^2} \qquad \text{(squaring both sides)}$$

$$-\cos^2 r \leq -\frac{1}{\mu^2}$$

$$1 - \cos^2 r \leq 1 - \frac{1}{\mu^2}$$

$$\sin^2 r \leq 1 - \frac{1}{\mu^2} \qquad \text{...(I)}$$

$$\frac{\sin i}{\sin r} = \mu \qquad \text{(by Snell's law)}$$

$$\sin i = \mu \sin r$$

or

$$\sin^2 i = \mu^2 \sin^2 r \qquad \text{(squaring both sides)}$$

$$\frac{1}{\mu^2}\sin^2 i = \sin^2 r \qquad \text{...(II)}$$

Put (II) in (I)

$$\frac{1}{\mu^2}\sin^2 i \le 1-\frac{1}{\mu^2}$$

$$\sin^2 i \le \mu^2 - 1 \quad \text{[on multiplying by } \mu^2 \text{ on both sides]}$$

For smallest angle *i.e.*, $i = 90°$

$$\therefore \quad \sin^2 90 \le \mu^2 - 1 \qquad [\because \mu \ge \sqrt{2}]$$

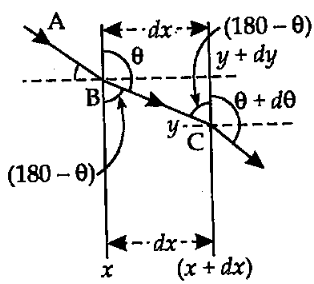
$$1 + 1 \le \mu^2$$

$$2 \le \mu^2$$

Taking square root

$$\sqrt{2} \le \mu \qquad \text{Hence proved.}$$

Q9.29. The mixture of a pure liquid and a solution in a long vertical column (*i.e.*, horizontal dimensions is very-very less than vertical dimensions) produces diffusion of solute particles and hence a refractive index gradient along the vertical dimension. A ray of light entering the column at right angles to the vertical is deviated from its original path. Find the deviation in travelling a horizontal distance $d << h$, the height of the column.



Ans. Consider a long vertical column of transparent liquid of infinite height (h) and thickness (dx). Consider a ray AB that enters at an angle θ into liquid of height y in column of liquid and emerges at an angle $(\theta + d\theta)$ at height $(y + dy)$. From Snell's law,

$$\mu(y)\sin\theta = \mu(y+dy)\sin(\theta + d\theta)$$

$$\mu(y)\sin\theta \cong \left[\mu(y) + \frac{d\mu}{dy}\cdot dy\right](\sin\theta\cos d\theta + \cos\theta\sin d\theta)$$

As $d\mu$, $d\theta$ are very small tends to zero

$$\therefore \quad \sin d\theta = d\theta, \quad \cos d\theta \cong 1 \quad \mu(y) = \mu \quad (\mu \text{ at height y constant})$$

$$\therefore \quad \mu\sin\theta \cong [\mu + d\mu](\sin\theta + \cos\theta\,.\,d\theta)$$

$$\mu\sin\theta \cong \mu\sin\theta + \mu\cos\theta\,d\theta + d\mu\sin\theta + \cos\theta\,.\,d\theta\,d\mu$$

Again $\quad d\theta\,d\mu \cong 0$

$$\mu\cos\theta\,.\,d\theta = -d\mu\sin\theta$$

$$d\theta = -\frac{1}{\mu}d\mu\frac{\sin\theta}{\cos\theta} = -\frac{1}{\mu}d\mu\tan\theta$$

From figure $\quad \tan(180-\theta) = \dfrac{dx}{dy}$

$$\tan\theta = \frac{dx}{dy}$$

$\therefore \qquad d\theta = -\frac{1}{\mu}\frac{d\mu}{dy} \cdot dx$

By integration on both sides

$$\int_0^\theta d\theta \cong \frac{-1}{\mu}\int_0^d \frac{d\mu}{dy} dx = -\frac{1}{\mu}\frac{d\mu}{dy} \cdot \int_0^d dx$$ [$\because$ μ and y does not change horizontally $\because$ $d\mu$, dy are constant horizontally.]

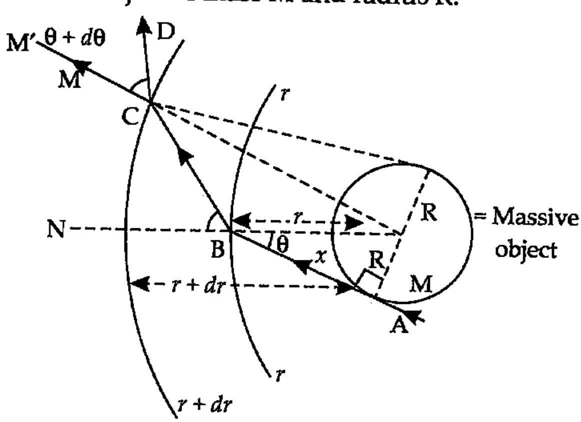
$$\theta \cong \frac{-1}{\mu}\frac{d\mu}{dy} d$$

Q9.30. If the light passes near a massive object, the gravitational interaction causes a bending of ray. This can be thought of as happening due to a change in the effective refractive index of the medium given by

$$n(r) = 1 + \frac{2GM}{rc^2}$$

where r is the distance of the point of consideration from the centre of the mass of the massive body, 'G' is the universal gravitational constant: M is the mass of the body and c is the speed of light in vacuum. Considering a spherical object find the deviation of the ray from the original path as it grazes the object.

Ans. Consider two spherical surfaces at r and $(r + dr)$ distance from the centre of massive object of mass M and radius R.



A ray ABCD incident at B and C on two surfaces at r and $(r + dr)$ then, by Snell's law

$$\mu(r) \sin\theta \cong \mu(r + dr) \sin(\theta + d\theta)$$

$$\mu(r) \sin\theta \cong \left[\mu(r) + \frac{d\mu}{dr} dr\right][\sin\theta \cdot \cos d\theta + \cos\theta \sin d\theta]$$

[$\because$ $d\theta \to 0$ then $\cos d\theta = 1$ and $\sin d\theta = d\theta$]

$\therefore \quad \mu(r)\sin\theta \cong [\mu(r)+d\mu][\sin\theta+\cos\theta\, d\theta]$

$\mu(r)\sin\theta \cong \mu(r)\sin\theta + d\mu\sin\theta + \mu(r)\cos\theta\cdot d\theta + d\mu\, d\theta\cdot\cos\theta$

$d\mu.\, d\theta\cos\theta$ is very small as $d\mu$ and $d\theta$ are very small

$\therefore \quad d\mu\, d\theta\cos\theta \cong 0$

$\therefore \quad \mu(r)\sin\theta \cong \mu(r)\sin\theta + d\mu\sin\theta + \mu\cos\theta\cdot d\theta$

$$0 \cong d\mu\sin\theta + \mu(r)\cos\theta\, d\theta$$

$$-d\mu\sin\theta = \mu(r)\cos\theta\cdot d\theta$$

Dividing both side by dr

$$\frac{-d\mu}{dr}\sin\theta = \mu(r)\cos\theta\frac{d\theta}{dr} \quad \text{...(I)}$$

$$\mu(r) = 1+\frac{2GM}{c^2}\cdot\frac{1}{r} \quad \text{(given)}$$

$$\frac{d\mu}{dr} = 0+\frac{2GM}{c^2}(-1)r^{-2}$$

$$-\frac{d(\mu)}{dr} = \frac{+2GM}{c^2r^2} \quad \text{...(II)}$$

$$\frac{2GM}{r^2c^2}\sin\theta = \left[1+\frac{2GM}{rc^2}\right]\cos\theta\cdot\frac{d\theta}{dr}$$

As the G is very small and c^2 is very large so

$$\frac{2GM}{rc^2}\to 0$$

$$\frac{2GM}{r^2c^2}\frac{\sin\theta}{\cos\theta} = \frac{d\theta}{dr}$$

$$\frac{2GM}{r^2c^2}\tan \cong \frac{d\theta}{dr}$$

$$\frac{2GM}{c^2}\int\frac{\tan\theta}{r^2}dr = \int_0^{\theta_0} d\theta$$

$$r^2 = x^2+R^2 \quad \text{(From figure)}$$

Differentiate w.r.t. r both sides

$$2r dr = 2x\, dx$$

$$dr = \frac{x}{r}dx$$

$$\therefore \quad \int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\alpha}^{\alpha}\frac{1}{r^2}\frac{R}{r}dx \quad \left(\tan\theta = \frac{R}{x}\right)$$

$$= \frac{2GM}{c^2}\int_{-\alpha}^{+\alpha}\frac{R}{\cancel{x} r^3}\cancel{x}\,dx = \frac{2GM}{c^2}\int_{-\alpha}^{\alpha}\frac{R}{(x^2+R^2)^{3/2}}dx$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\alpha}^{\alpha} R\frac{dx}{(x^2+R^2)^{3/2}}$$

$\because$ $\quad x = R\tan\theta$

$dx = R\sec^2\theta \cdot d\theta$

$$\theta_0 = \frac{2GM}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R\,R\sec^2\theta\,d\theta}{(R^2\tan^2\theta+R^2)^{3/2}} = \frac{2GM}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R^2\sec^2\theta\,d\theta}{[R^2(\tan^2\theta+1)]^{3/2}}$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R^2\sec^2\theta}{R^3\sec^3\theta}d\theta$$

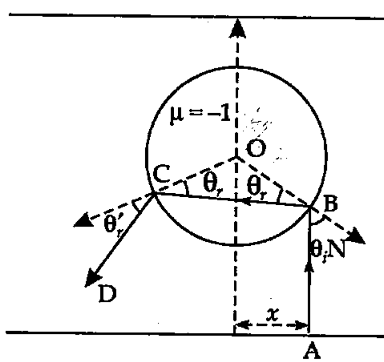
$$= \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\frac{1}{\sec\theta}d\theta = \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{2GM}{Rc^2}[\sin\theta]_{-\pi/2}^{\pi/2} = \frac{2GM}{Rc^2}\left[\sin\frac{\pi}{2}-\sin\left(\frac{-\pi}{2}\right)\right]$$

$$\theta_0 = \frac{2GM}{Rc^2}\left[1+\sin\frac{\pi}{2}\right] = \frac{2GM}{Rc^2}[1+1]$$

$\boxed{\theta_0 = \frac{4GM}{Rc^2}}$ is required rotation.

Q9.31. An infinitely long cylinder of radius R is made of an unusual exotic material with refractive index –1 (figure). The cylinder is placed between two planes whose normals are along the y direction. The centre of the cylinder 'O' lies along the y-axis. A narrow laser beam is directed along the y direction from the lower plate. The laser source is at a horizontal distance x from the diameter in the y direction. Find the range of x-such that light emitted from the lower plane does not reach the upper plane.



Main concept used: $\mu_r = -1$ and $\mu_d = 1$, when light passes from μ to $-\mu$ then reflection takes place from normal at incidence point.

Ans. As the cylinder is made of refractive index (–1) and is placed in air of $\mu = 1$ so, when ray AB is incident at B to cylinder, θ_r will be negative *i.e.*, refracted ray will get reflection from normal. Similar thing happens at incidence point C and angle of refraction and incident at B and C will be equal (θ_r) as OB = OC = R and of refraction at C is 'θ_r''.

$$\theta_1 = |\theta_i| = |\theta_r| = |\theta_r'| \text{ as reflection takes place}$$

the total deviation of outcoming ray from the incoming ray $4\theta_1$. Rays shall not reach the receiving plane if $\frac{\pi}{2} \le 4\theta \le \frac{3\pi}{2}$ angles measured clockwise from the y axis.

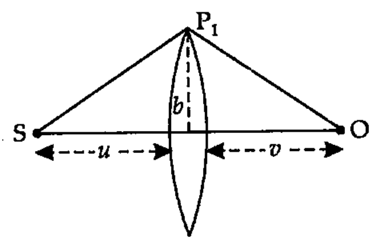
or $$\frac{\pi}{8} \le \theta \le \frac{3\pi}{8} \quad \text{(on dividing by 4 to all sides)}$$

$$\sin\theta_1 = \frac{x}{R}$$

$$\frac{\pi}{8} \le \sin^{-1}\frac{x}{R} \le \frac{3\pi}{8} \quad \text{or} \quad \frac{\pi}{8} \le \frac{x}{R} \le \frac{3\pi}{8}$$

Thus for light emitted from the source shall not reach the receiving plane if $\frac{R\pi}{8} \le x \le \frac{3R\pi}{8}$

Q9.32. (*i*) Consider a thin lens placed between a source(S) and an observer (O) (figure). Let the thickness of the lens vary as $W(b) = W_0 - \frac{b^2}{\alpha}$, where b is the vertical distance from the pole, W_0 is a constant. Using Fermat's principle *i.e.*, the time of transit for a ray between the source and observer is an extremum, find the condition that all paraxial rays starting from the source will converge at a point 'O' on the axis. Find the focal length.



(*ii*) A gravitational lens may be assumed to have a varying width of the form

$$W(b) = K_1 \log\left(\frac{K_2}{b}\right) \quad (b_{min} < b < b_{max})$$

$$= K_1 \log\left(\frac{K_2}{b_{min}}\right) \quad (b < b_{min})$$

show that an observer will see an image of a point object as a ring about the centre of the lens with an angular radius

$$\beta = \sqrt{\frac{(n-1)K_1 \frac{u}{v}}{u+v}}$$

Ans. The time taken by ray from S to $P_1 = \frac{SP_1}{c}$

$$t_1 = \frac{\sqrt{u^2 + b^2}}{c} = \frac{u}{c}\left(1 + \frac{b^2}{u^2}\right)^{1/2}$$

$$t_1 = \frac{u}{c}\left[1 + \frac{b^2}{2u^2}\right] \qquad \text{(assuming } b \lll u\text{)}$$

Similarly, time required by ray from P_1 to $O = \frac{P_1O}{c}$

$$t_2 = \frac{v}{c}\left[1 + \frac{b^2}{2v^2}\right]$$

Time required by ray to travel through the lens

$$t_3 = \frac{(\mu - 1)\,W(b)}{c}$$

Thus total time by ray from S to $O = t = t_1 + t_2 + t_3$

$$t = \frac{u}{c}\left(1 + \frac{b^2}{2u^2}\right) + \frac{v}{c}\left(1 + \frac{b^2}{2v^2}\right) + \frac{(\mu - 1)\,W(b)}{c}$$

$$= \frac{1}{c}\left[u + \frac{b^2}{2u} + v + \frac{b^2}{2v} + (\mu - 1)\,W(b)\right]$$

$$= \frac{1}{c}\left[u + v + \frac{b^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) + (\mu - 1)\,W(b)\right]$$

if $$\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$$

$\therefore$ $$t = \frac{1}{c}\left[u + v + \frac{b^2}{2D} + (\mu - 1)\left(W_0 - \frac{b^2}{\alpha}\right)\right]$$

By Fermat's Principle

$$\frac{dt}{db} = \frac{1}{c}\left[0 + 0\,\frac{2b}{2D} + (\mu - 1)\left(0 - \frac{2b}{\alpha}\right)\right] = \frac{b}{cD} - (\mu - 1)\frac{-2b}{\alpha c}$$

But time from S to O remains constant, so, $\frac{dt}{db} = 0$

or $$\frac{b}{cD} - \frac{2(\mu - 1)b}{\alpha c} = 0$$

$$\frac{2(\mu - 1)b}{\alpha c} = \frac{b}{cD}$$

$$\alpha = 2(\mu - 1)D$$

Thus, the convergence lens is formed $\alpha = 2(\mu - 1)D$.

It is independent of b and hence all paraxial rays from S will converge at O (*i.e.*, for rays $b <<< u$ and $b << v$).

Since $\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$ it is equivalent to focal length f

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\because \quad t = \frac{1}{c}\left[u + v + \frac{b^2}{2D} + (\mu - 1)\,W(b)\right]$$

here $$W(b) = K_1 \log_e\left(\frac{K_2}{b}\right)$$

$$t = \frac{1}{c}\left[u + v + \frac{b^2}{2D} + (\mu - 1)\,K_1 \log\frac{K_2}{b}\right]$$

$$\frac{dt}{db} = \frac{1}{c}\left[0 + 0 + \frac{2b}{2D} + (\mu - 1)\,K_1 \frac{b}{K_2}\right]\frac{d}{db}\left(\frac{K_2}{b}\right)$$

$$= \frac{1}{c}\left[\frac{b}{D} + (\mu - 1)\frac{K_1}{K_2}\, b \cdot K_2(-1)\frac{1}{b^2}\right]$$

as time from S to O is constant, so, $\frac{dt}{db} = 0$

so $$0 = \frac{1}{c}\left[\frac{b}{D} - (\mu - 1)\frac{K_1}{b}\right]$$

$$\therefore \quad (\mu - 1)\frac{K_1}{b} = \frac{b}{D} \qquad \left(\because \frac{1}{c} \neq 0\right)$$

$$b^2 = K_1\, D(\mu - 1)$$

$$b = \sqrt{(\mu - 1)\,K_1 D}$$

Thus all the rays passing at a height b shall contribute to the image. The ray path makes an angle

$$\beta = \frac{b}{v} = \frac{\sqrt{(\mu - 1)\,K_1 D}}{v} = \sqrt{\frac{(\mu - 1)\,K_1 D}{v^2}} \qquad \left(\because \frac{1}{D} = \frac{1}{v} + \frac{1}{u}\right)$$

$$= \sqrt{\frac{(\mu - 1)\,K_1 uv}{v^2\,(u + v)}} \qquad \left(D = \frac{uv}{u + v}\right)$$

$$\beta = \sqrt{\frac{(\mu - 1)\,K_1 u}{v(u + v)}}$$

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