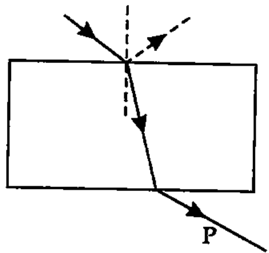
10 Wave Optics

MULTIPLE CHOICE QUESTIONS—I

Q10.1. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure



A Polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.

(*a*) For a particular orientation, there shall be darkness as observed through polaroid.

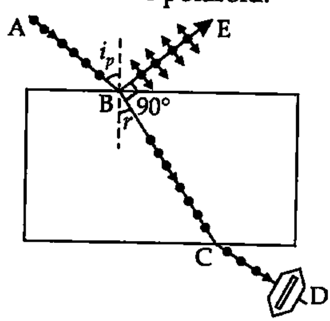
(*b*) The intensity of light as seen through the polaroid shall be independent of the rotation.

(*c*) The intensity of the light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid.

(*d*) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid.

Main concept used: Brewster's angle.

Ans. (*c*): When a ray ABCD of light passes through prism in such a way that angle between reflected ray BE and refracted ray BC is 90° then only reflected ray is plane polarised. So polaroid rotated in the way of CD the intensity will never be zero but varies in one complete rotation so, it verifies answer (*c*).



Q10.2. Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall

(*a*) be a fine sharp slit white in colour at the centre.

(*b*) a bright slit white at the centre diffusing to zero intensities at the edges.

(*c*) a bright slit white at the centre diffusing to regions of different colours.

(*d*) only be a diffused slit white in colour.

Ans. (*a*): Width of slit 10^4 Å = 10,000 Å.

Wavelength of visible light varies from 4000 to 8000 Å. As the width of slit 10000 Å is comparable to that of wavelength of visible light *i.e.*

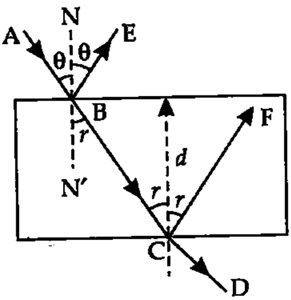
8000 Å. Hence the diffraction occurs with maxima at the centre. So at the centre all colours appear *i.e.* white colour at the centre appear.

Q10.3. Consider a ray of light incident from air onto a slab of glass (refractive index n) of width 'd' at an angle θ. The phase difference between the ray reflected by the top surface of the glass and the bottom surface is

(*a*) $\frac{2\pi\mu d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{-\frac{1}{2}}+\pi$ (*b*) $\frac{4\pi d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}$

(*c*) $\frac{4\pi d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}+\frac{\pi}{2}$ (*d*) $\frac{4\pi d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}+2\pi$

Ans. (*a*): Consider a ray of light ABCD through prism, and reflected rays BE and CF from incidence points B and C as shown in figure.



The time difference between two reflected rays BE and CF is equal to the time taken by ray to travel from B to C.

∴ Time difference dt between two reflected rays BE and CF are

$$dt = \frac{BC}{v_g} \quad ...(I)$$

$$\because \quad \mu = \frac{v_a}{v_g}$$

$$v_a = c$$

$$\therefore \quad v_g = \frac{c}{\mu} \quad ...(II)$$

$$\cos r = \frac{d}{BC}$$

$$BC = \frac{d}{\cos r} \quad ...(III)$$

Substitute (II) and (III) in (I),

$$dt = \frac{d/\cos r}{c/\mu} = \frac{\mu d}{c\cos r} \quad ...(IV)$$

$$\mu = \frac{\sin\theta}{\sin r} \quad \text{or} \quad \sin r = \frac{\sin\theta}{\mu}$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{\sin^2\theta}{\mu^2}}$$

From IV, $$dt = \frac{\mu d}{c\sqrt{1-\frac{\sin^2\theta}{\mu^2}}} = \frac{\mu d}{c}\left[1-\frac{\sin^2\theta}{\mu^2}\right]^{-1/2}$$

Phase difference $$d\phi' = \frac{2\pi}{T}\cdot dt = \frac{2\pi\mu d}{Tc}\left[1-\frac{\sin^2\theta}{\mu^2}\right]^{-1/2}$$

$$d\phi' = \frac{2\pi\mu d}{\frac{1}{v}.v\lambda}\left[1-\frac{1}{\mu^2}\sin^2\theta\right]^{-\frac{1}{2}} = \frac{2\pi\mu d}{\lambda}\left[1-\frac{1}{\mu^2}\sin^2\theta\right]^{-\frac{1}{2}}$$

The phase diff. between ray AB and BC after refraction is π

$\therefore$ Net phase difference = $d\phi' + \pi$

$$d\phi = \frac{2\pi\mu d}{\lambda}\left[1-\frac{\sin^2\theta}{\mu^2}\right]^{-1/2} + \pi$$ [it is very near to option (*a*).]

Q10.4. In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case

(*a*) there shall be alternate interference patterns of red and blue.
(*b*) there shall be an interference pattern for red distinct from that for blue.
(*c*) there shall be no interference fringes.
(*d*) there shall be an interference pattern for red mixing with one for blue.

Main concept used: Condition for interference is coherent soure or of same fequency.

Ans. (*c*): For sustained interference the source must be coherent and should emit the light of same frequency.

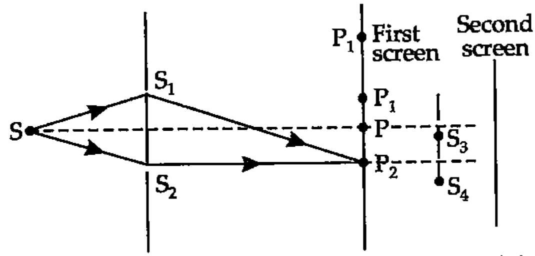
In this problem one hole is covered with red and other with blue, which has different frequency, so no interference takes place.

Q10.5. The given figure shows a standard two slit arrangement with slits S_1, S_2. P_1, P_2 are the two minima points on either side of P.

At P_2 on the screen there is a hole and behind P_2 is a second screen, 2-slit arrangement with slits S_3 and S_4 and a second screen behind them.

(*a*) There would be no interference pattern on the second screen but it would be lighted.
(*b*) The second screen would be totally dark.
(*c*) There would be a single bright point on the second screen.
(*d*) There would be a regular two slit pattern on the second screen.

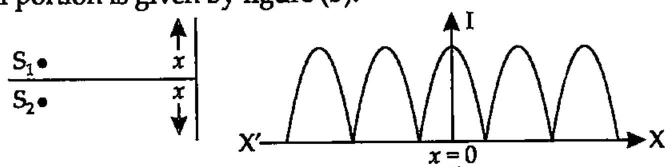
Main concept used: Each point on wavefront acts as source of secondary wavelets.



Ans. (*d*): At P_2 is minima due to two wavefronts in opposite phase coming from, two slits S_1 and S_2, but there is wavefronts from S_1, S_2 so P_2 will act as a source of secondary wavelets. Wavefront starting from P_2 reaches at S_3 and S_4 slits which will again acts as two monochromatic or coherent sources and will form pattern on second screen.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q10.6. Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [fig. (a)]. The pattern of intensity distribution seen in the central portion is given by figure (b).



(*a*) (*b*)

In this case which of the following statements are true?

(*a*) S_1 and S_2 have the same intensities.

(*b*) S_1 and S_2 have a constant phase difference.

(*c*) S_1 and S_2 have the same phase.

(*d*) S_1 and S_2 have the same wavelength.

Main concept used: Condition for sustained interference.

Ans. (*a*), (*b*) and (*c*): (*i*) As the intensity at dark fringe is zero so intensities of S_1 and S_2 are equal.

(*ii*) As the graph of maxima and minima is symmetric. So the waves from S_1 and S_2 are at same phase difference or zero phase difference.

Q10.7. Consider sunlight incident on a pinhole of width 10^3 Å. The image of the pinhole seen on a screen shall be

(*a*) a sharp white ring.

(*b*) different from a geometrical image.

(*c*) a diffused central spot white in colour.

(*d*) a diffused coloured region around a sharp central white spot.

Ans. (*b*) and (*d*): The width of pinhole 10^3 Å = 1000 Å and wavelength of visible light is 4000 Å to 8000 Å *i.e.*, size of slit less than (or comparable) with the wavelength of light.

So light from pinhole will diffract from the hole. Due to the diffraction pattern of fringes, the shape are quite different from hole.

Q10.8. Consider the diffraction pattern for a small pinhole. As the size of the hole is increased

(*a*) the size decreases.
(*b*) the intensity increases.
(*c*) the size increases.
(*d*) the intensity decreases.

Ans. (*a*) and (*b*): We know that width (B_0) of central maxima $B_0 = \frac{D\lambda}{d}$ and width of n^{th} secondary maxima $= \frac{\lambda}{d}$ here distance (D) between slit and screen, wavelength λ of source does not change.

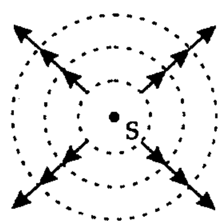
So on increasing width of hole of pinhole, '*d*' increase. Hence the size of central maxima decreases verifies the option (*a*).

As the energy passing through hole increased on increasing the size of hole. So the intensity of pattern will increase. Hence verifies the option (*b*).

Q10.9. For the light diverging from a point source

(*a*) the wavefront is spherical.
(*b*) the intensity decreases in proportion to the distance squared.
(*c*) the wavefront is parabolic.
(*d*) the intensity at wavefront does not depend on distance.

Main concept used: Properties of wavefront



Ans. (*a*) and (*b*): light from point source emits in all around the source with same speed so forms a **spherical surface of wavefront** or spherical wavefront.

As the intensity (I) always decreases as the reciprocal of square of distance.

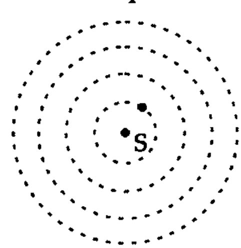
$$I = \frac{P}{4\pi r^2}$$

r = radius of spherical wavefront at any time ($r = vt$)

VERY SHORT ANSWER TYPE QUESTIONS

Q10.10. Is Huygen's principle valid for longitudinal sound waves?

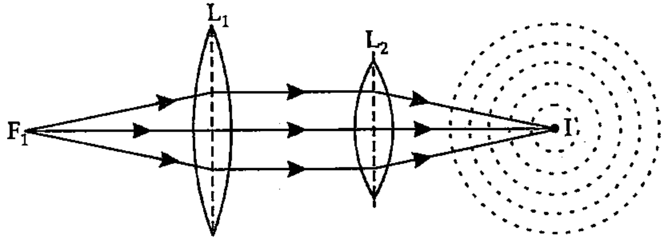
Ans. Consider a source of sound formed with the compressions and rarefactions forward in all directions with same velocity. So longitudinal waves propagate with spherical symmetry in all directions as the wavefront in light waves. So Huygen's principle is valid for longitudinal sound waves also.



On a surface of sphere there will be either compression or rarefaction and that part can also behave like a source of sound but with low intensity.

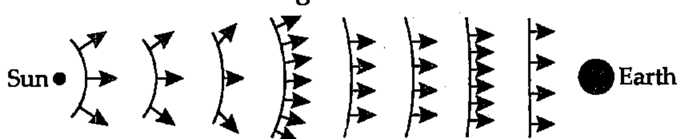
Q10.11. Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of wavefronts emerging from the final image?

Ans. Consider a point 'F' on focus of converging lens L_1. The light rays from F, becomes parallel after refraction through L_1. When these parallel rays falls on converging lens L_2 placed co-axial on the other side of F of L_1. L_2 converges the rays at it's focus at I. It now behave like a point source of rays and form a spherical wave front.



Q10.12. What is the shape of the wavefront on earth for sunlight?

Ans. As the sun is very-very far from the earth so can be considered at infinity and sun can be considered as a point source which gives spherical wavefront. The size of the earth is very small as compared to distance of sun from earth and size of the sun so the plane wavefront reaches on earth as shown in figure here.



Q10.13. Why is the diffraction of sound waves more evident in daily experience than that of light wave?

Ans. We know that frequencies of sound waves varies from 20 Hz to 20,000 Hz, so its corresponding wavelength varies from 15 m to 15 mm respectively. The size of slit (almost) becomes comparable to wavelength of sound so diffraction of sound wave takes place easily.

But the wavelength of visible light varies from 0.4 to 0.7 micron which is very small. So the size of most of the slits is not comparable with wavelength of visible light, due to this diffraction of light cannot take place.

Q10.14. The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ radian and a typical photoprinter prints a minimum of 300 dpi (dots per inch) (1 inch = 2.54 cm). At what minimum distance z should a printed page be held so that one does not see the individual dots?

Ans. Angular separation = 5.8×10^{-4} radian

The average distance between any two dots = $\dfrac{2.54}{300} = 0.85 \times 10^{-2}$ cm

At the distance z cm, angle subtended $= \frac{\text{arc}}{\text{rad}} = \frac{0.85 \times 10^{-2}}{z}$

Resolution angle for human $= 5.8 \times 10^{-2}$ rad $= \frac{0.85 \times 10^{-2}}{z}$

Maximum distance up to which human eye cannot see

$$\text{2 dots distinctly} = z = \frac{0.85 \times 10^{-2}}{5.8 \times 10^{-2}} = 14.5 \text{ cm}$$

which is less than distance of distinct vision.

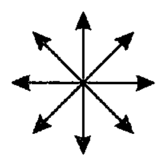
So a normal person cannot see the dots.

Q10.15. A polaroid I is placed in the front of a monochromatic source. Another polaroid II is placed in front of this polaroid I and rotated till no light passes. A third polaroid (III) is now placed in between Ist and IInd . In this case will the light emerge from II. Explain.

Main concept used: Sunlight unpolarised and Law of MALUS

Ans. Polaroid III is placed between two crossed polaroids I and II and no light passes through IInd polaroid.

Polaroid III is now placed between I, II and now rotates keeping I, II in no rotation. Let angle between polaroid I and III be θ and intensity of plane polarised light after Ist polaroid is I_0. Then



Unpolarised light

$I_1 = I_0$

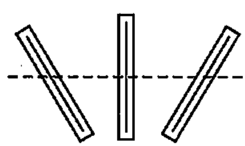
$I_3 = I_0 \cos^2 \theta$ is intensity of light after polaroid III

$$I_2 = I_3 \cos^2 \theta' = I_0 \cos^2 \theta . \cos^2 (90 - \theta)$$

$$I_2 = I_0 \cos^2 \theta . \sin^2 \theta$$

$$I_2 = I_0 (2 \cos \theta \sin \theta)^2 = I_0 (\sin 2\theta)^2$$

$$I_2 = \frac{I_0}{4} \sin^2 2\theta$$



θ is angle between I and III Polaroid.

No light will pass through II Polaroid from above eqn. when

$$\sin^2 2\theta = 0$$
$$\sin 2\theta = 0$$
$$\sin 2\theta = \sin 0$$
$$\theta = 0°$$

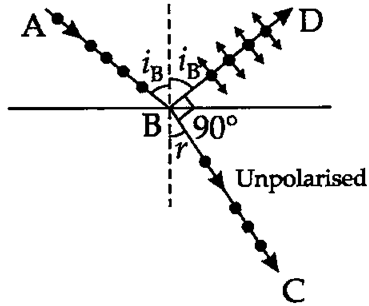
∴ No light will pass when the angle between I and III Polaroid is zero *i.e.*, plane of polaroid are parallel. In all other cases when III is not parallel to either I or II, light will pass.

SHORT ANSWER TYPE QUESTIONS

Q10.16. Can reflection result in plane polarised light if the light is incident on the interface from the side with higher refractive index?

Main concept used: Brewster's angle

Ans. When a ray of light passes from a medium (air) of refractive index μ_1 to another medium of refractive index μ_2, more than μ_1 with incidence angle is equal to Brewster's angle (i_B), then transmitted ray BC will be unpolarised light and the reflected light BD will be plane polarised. Angle DBC between refracted and reflected ray is 90°.



$$\angle r + \angle i_B + 90 = 180°$$

So $\quad \angle r + \angle i_B = 90° \quad$ or $\quad \angle r = 90 - i_B$

By Snell's law, $\mu_2 = \dfrac{\sin i_B}{\sin r} = \dfrac{\sin i_B}{\cos i_B} = \tan i_B$

$\therefore \quad \dfrac{\mu_2}{\mu_1} = \tan i_B \qquad \left(\begin{array}{c} \because \mu_2 > \mu_1 \\ \therefore \tan i_B > 1 \\ i_B > 45° \end{array}\right) \qquad$ ...(I)

$\sin i_C = \dfrac{1}{\mu_2}$ when light passes from medium 2 to 1

$\sin i_C = \dfrac{\mu_1}{\mu_2} \qquad (\because \mu_2 > \mu_1)$

so $\quad \sin i_C < 1 \qquad$ ...(II)

or $\quad i_C < 90°$

From I $\quad \tan i_B > 1$

From II $\quad 1 > \sin i_C$

or $\quad \tan i_B > 1 > \sin i_C \quad$ or $\quad |\tan i_B| > |\sin i_C|$

Because $\quad 45 < i_B$

$i_C < 90°$

So $\quad 45° < i_B < 90°$

$0 < i_C < 90°$

So $\quad i_B > i_C$

Thus, polarisation by reflection takes place.

Q10.17. For the same objective, find the ratio of the least separation between two points to be distinguished by a microscope for light of 5000 Å and electrons accelerated through 100 V used as the illuminating substance.

Main concept used: R.P. (microscope) $= \dfrac{1}{d} = \dfrac{2 \sin \beta}{1.22\,\lambda}$

and $\quad \lambda_D = \dfrac{1.22}{\sqrt{V}}$ nm

Ans. $\lambda = 5000\ \text{Å} = 5000 \times 10^{-10}\ m$

In microscope, $\text{R.P.} = \frac{1}{d} = \frac{2 \sin \beta}{1.22\ \lambda}$

Limit of resolution by light of 5000 Å

$$d_{min} = \frac{1.22\ \lambda}{2 \sin \beta} \quad \text{or} \quad d_{min} = \frac{1.22 \times 5000 \times 10^{-10}}{2 \sin \beta}$$

The de Broglie wavelength λ_d of illuminated light $= \frac{1.22}{\sqrt{V}}$ nm

$$\lambda_d = \frac{1.22}{\sqrt{100}} \text{nm} = \frac{1.22}{10} \times 10^{-10} \text{m}$$

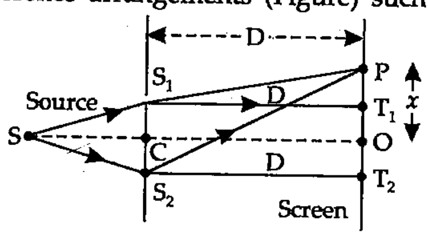
The limit of resolution by 100 V light $d'_{min} = \frac{1.22\ \lambda_d}{2 \sin \beta}$

$$d'_{min} = \frac{1.22 \times 1.22 \times 10^{-10}}{2 \sin \beta}$$

$$\text{The required ratio} = \frac{d'_{min}}{d_{min}} = \frac{\frac{1.22 \times 1.22 \times 10^{-10}}{2 \sin \beta}}{\frac{1.22 \times 5000 \times 10^{-10}}{2 \sin \beta}} = \frac{1.22}{5000}$$

The required ratio $= \frac{122}{500} \times 10^{-3} = \mathbf{0.244 \times 10^{-3}}$

Q10.18. Consider two slit interference arrangements (Figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the **first minima** on the screen falls at a distance D from the centre O.



Main concept used: For nth minima the path difference $= (2n-1)\frac{\lambda}{2}$ in Y D.S.E.

Ans. According to θ,

$x = \text{D}$ (Given) ...(I)

$\text{D} = \frac{1}{2}d$ (Given) ...(II)

$d = 2\text{D}$

Path difference at $P = S_2P - S_1P$

$$\text{Path diff. } p = \sqrt{D^2 + \left(x + \frac{d}{2}\right)^2} - \sqrt{D^2 + \left(x - \frac{d}{2}\right)^2}$$

Substitute the value of d and x from I and II

$$= \sqrt{D^2 + (D + D)^2} - \sqrt{D^2 + (D - D)^2}$$

$$= \sqrt{5D^2} - \sqrt{D^2}$$

$$p = D(\sqrt{5} - 1) \quad \text{...(III)}$$

The path difference for nth dark fringe from central maxima 'O' is $(2n - 1)\frac{\lambda}{2}$.

$\therefore$ For Ist minima $p = \frac{\lambda}{2}$

Put the value of p in (III)

$$\frac{\lambda}{2} = D(\sqrt{5} - 1)$$

$$D = \frac{\lambda}{2(\sqrt{5} - 1)}$$

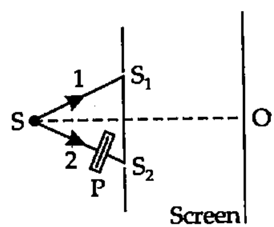
Rationalising the denominator, we get,

$$D = \frac{\lambda}{2(\sqrt{5} - 1)} \times \frac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)} = \frac{(2.236 + 1)}{2 \times (5 - 1)}\lambda = \frac{3.236}{2 \times 4}\lambda$$

$$= \frac{3.236}{8}\lambda = 0.404\,\lambda$$

LONG ANSWER TYPE QUESTIONS

Q10.19. Figure shows the two slit arrangement with a source which emits unpolarised light. P is a polariser with axis whose direction is not given. If I_0 is the intensity of principal maxima, when no polariser is present, calculate in the present case, the intensity of the principal maxima as well as of the first minima.



Ans. Let the amplitudes of ray 1 and 2 are A_1 and A_2 respectively. A_2 has constant phase diff. ϕ

$$A_1 = A_{\perp}^1 + A_{||}^1$$

$$A_2 = A_{\perp}^2 + A_{||}^2$$

where $$A_{\perp}^1 = A_{\perp}^0 \,(\sin kx - \omega t)$$

$$A_{\perp}^2 = A_{\perp}^0 \,(kx - \omega t + \phi)$$

Maximum amplitude of $A_\perp$ and $A_{||}$ are same = $A_\perp^0$
∴ Similarly for

$$A_{||}^1 = A_{||}^0 \sin(kx - \omega t)$$
$$A_{||}^2 = A_{||}^0 \sin(kx - \omega t + \phi)$$

When ray 2 is **polarised** by P then its $A_{||}^2$ vector stopped.

Case I: When there is no polaroid.

$$A = \text{Resultant amplitude}$$
$$= A_\perp + A_{||} = [A_\perp^1 + A_\perp^2] + [A_{||}^1 + A_{||}^2]$$
$$= [A_\perp^0 \sin(\omega t - kx) + A_\perp^0 \sin(\omega t - kx + \phi)]$$
$$+ [A_{||}^0 \sin(\omega t + \phi) + A_{||}^0 \sin(\omega t - kx + \phi)] \quad ...I$$

The amplitudes of S_1, S_2 when there is no polariser then perpendicular and parallel components are equal, as the wavefront is coming from same source S.

$$\therefore \quad A_\perp^1 = A_{||}^1 = A_\perp^2 = A_{||}^2 = A_0$$

Where $A_\perp^1$ and $A_{||}^1$ are the amplitudes of electric and magnetic field vector respectively and $A_\perp^2$ and $A_{||}^2$ are the amplitudes of electric and magnetic field vector, respectively.

From equation I, we have

$$A = A_0 \sin(\omega t - kx) + A_0 \sin(\omega t - kx + \phi) + A_0 \sin(\omega t - kx) + A_0 \sin(\omega t - kx + \phi)$$
$$A = 2A_0[2 \sin \omega t - kx + \sin(\omega t - kx + \phi)].$$

We know that the intensity of maxima in Young's double slit experiment

$I_1 = I_2 = I$ a.id the resultant intensity at a point = $\boxed{I = I_0(1 + \cos\phi)}$

$$\because \quad I = kA^2 \quad \text{or} \quad I_0 = kA_0^2$$

Intensity at a point in Young's double slit without polariser

$$\therefore \quad I = 2I_0(1 + \cos\phi) = 2 \,.\, kA_0^2(1 + \cos\phi)$$

For maxima $\cos\phi = 1$

$I = 2kA_0^2(1 + 1) = 4kA_0^2$ leaving the k (constant) then

$$I = 4A_0^2$$

Case II: When polariser P is placed in the path of ray 2: Let $\perp^r$ vector of ray 2 is blocked by polariser

$$A_1 = A_\perp^1 + A_{||}^1$$
$$A_2 = A_{||}^2 \qquad (\because A_\perp^2 \text{ vector blocked})$$

Resultant amplitude

$$A = A_1^2 + A_2^2$$

$$I = (A_{||}^1 + A_{||}^2)^2 + |A_\perp^1|^2$$

$$|A_{||}^0|_{av} = |A_\perp^0|_{av} = A_0$$

$$I = kA_0^2 (1 + \cos\phi) + A_0^2 \frac{1}{2} \qquad ...(I)$$

$$= kA_0^2 \left[1 + \cos\phi + \frac{1}{2}\right]$$

$$= kA_0^2 \left[\frac{3}{2} + \cos\phi\right] \qquad (\cos\phi = 1 \text{ for maxima})$$

$$= kA_0^2 \times \frac{5}{2} \qquad ...(II)$$

$$I_0 = 4kA_0^2 \qquad \text{(when no polarisation of ray 2)}$$

$$A_0^2 = \frac{I_0}{4k} \qquad ...(III)$$

Intensity of principal maxima with polariser

$$I = \frac{I_0}{4}\frac{5}{2} \qquad \text{[from II and III]}$$

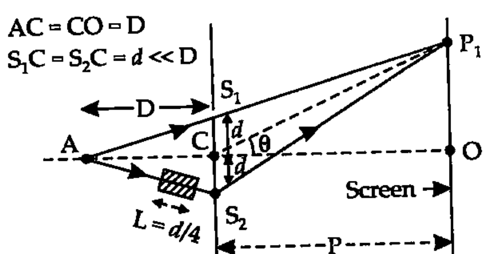
$$\boxed{I = \frac{5}{8} I_0}$$

Intensity at the Ist minima $\cos\phi = -1$

$$\therefore \qquad I = |A_0|^2 (1 - 1) + \frac{A_0^2}{2} \qquad \text{[from I]}$$

$$I = \frac{I_0}{4} \cdot \frac{1}{2} = \frac{I_0}{8}$$

Q10.20. A small transparent slab containing material of $\mu = 1.5$ is placed along AS_2 (Figure). What will be the distance from 'O' of the principal maxima and of the Ist minima on either side of the principal maxima, obtained in the absence of the glass slab.



Main concept used: When ray passes through the slab it gets lateral displacement, then fringes shifts by $(\mu - 1)$.

Ans. When ray AS_2 passes through glass slab of thickness L and refractive index μ the path difference caused by slab is $(\mu - 1)L$ and path difference caused by Young's double slit experiment is $2d \sin \theta$. Total path difference at P_2 is

$$\therefore \quad \Delta x = 2d \sin \theta + (\mu - 1) L$$

for principal maxima, path difference $\Delta x = 0$

$\therefore 2d \sin \theta_0 + (\mu - 1) L = 0$ (For central maxima $\theta = \theta_0$)

$$\sin \theta_0 = \frac{-(\mu - 1) L}{2d} = \frac{-(1.5-1)L}{2 \times 4L} \quad \left[\text{as } L = \frac{d}{4}, d = 4L\right]$$

$$\sin \theta_0 = \frac{-0.5 \cdot L}{2 \times 4L} = \frac{-1}{16}$$

For central maxima, $OP = D \tan \theta_0 \approx D \sin \theta_0 = \frac{-D}{16}$

For small $\angle\theta_0$, $\sin \theta_0 = \theta_0$ and $\tan \theta_0 = \theta_0$

For the first minima the path difference $= \frac{+\lambda}{2}$

$$\therefore \quad 2d \sin \theta_1 + (\mu - 1)L = \frac{\pm\lambda}{2}$$

(For both upper and lower side from O)

$$2d \sin \theta_1 + (1.5 - 1)L = \frac{\pm\lambda}{2}$$

$$2d \sin \theta_1 = \frac{\pm\lambda}{2} - 0.5\,L \quad \left(\because \; L = \frac{d}{4}\right)$$

$$\sin \theta_1 = \frac{\pm\frac{\lambda}{2} - 0.5\frac{d}{4}}{2d} = \frac{\pm\frac{\lambda}{2} - \frac{d}{8}}{2d}$$

$$\sin \theta_1 = \frac{(\pm 4\lambda - d)/8}{2d}$$

For diffraction, $\lambda = d$ (half the slit dist.)

$$\therefore \quad \sin \theta_1 = \frac{(\pm 4\lambda - \lambda)}{16\lambda} = \frac{(\pm 4 - 1)\lambda}{16\lambda}$$

$$\sin \theta_1 = \frac{\pm 4 - 1}{16}$$

For positive direction side, $\sin \theta_1^+ = \frac{3}{16}$

For negative direction side, $\sin \theta_1^- = \frac{-5}{16}$

The distance of first minima from principal maxima on either side x_1^+ and x_1^-, are

$$x_1^+ = D\tan\theta_1^+ = D\cdot\frac{\sin\theta_1^+}{\cos\theta_1^+}$$

$$x_1^+ = D\tan\theta_1^+ = D\frac{\sin\theta_1^+}{\sqrt{[1-\sin^2\theta_1^+]}} = \frac{D3/16}{\sqrt{1-\frac{9}{256}}}$$

$$x_1^+ = D\tan\theta_1 = \frac{\frac{3D}{16}}{\frac{\sqrt{256-9}}{16}} = \frac{3D}{\sqrt{247}} \text{ [above point O on screen]}$$

$$x_1^+ = \frac{3D}{\sqrt{247}}$$

The first minima starts after the end of central maxima. So the Ist minima starts at distance of $D\tan\theta = x_1^+ = \frac{3D}{\sqrt{247}}$ above O in positive direction.

The distance of first minima on the negative side is

$$x_1^- = \frac{D\sin\theta_1^-}{\sqrt{1-\sin^2\theta_1^-}} = \frac{\frac{-5D}{16}}{\frac{\sqrt{16^2-5^2}}{16}}$$

$$x_1^- = \frac{-5D}{\sqrt{256-25}} = \frac{-5D}{\sqrt{231}}$$

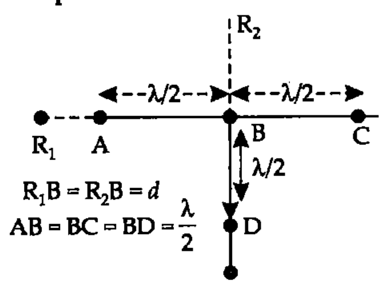
Q10.21. Four identical monochromatic sources A, B, C, D as shown in the figure produce waves of the same wavelength λ and are coherent. Two receivers R_1 and R_2 are at great but equal distances from B.

(*i*) Which of the two receivers picks up the larger signal?

(*ii*) Which of the two receivers pick up the larger signal when B is turned off?

(*iii*) Which of the two receivers pick up the larger signal when D is turned off?

(*iv*) Which of the two receivers can distinguish which of the sources B or D has been turned off?



Main concept used: The resultant disturbance at a point is equal to sum of disturbances due to individual sources.

Ans. Consider all the disturbances at R_1

Let the wave from source A has zero path difference at R_1

$\therefore \qquad y_A = a\cos\omega t \qquad$ (at R1)

Path diff. between A and B is $\lambda/2$ or π so wave of B at R_1 is

$$yB = -a \cos \omega t$$

Path difference between wave A and C is λ or 2π.
So the wave from source C is

$$y_C = a \cos (\omega t - 2\pi)$$

or

$$y_C = a \cos \omega t$$

Path difference between A and D = $R_1 D - AR_1$

$$= \sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - \left(d - \frac{\lambda}{2}\right) = \left(d^2 + \frac{\lambda^2}{4}\right)^{1/2} - d + \frac{\lambda}{2}$$

$$= d\left[1 + \frac{\lambda^2}{4d^2}\right]^{1/2} - d + \frac{\lambda}{2} = d\left[1 + \frac{\lambda^2}{8d^2}\right] - d + \frac{\lambda}{2}$$

Neglecting the term $\frac{\lambda^2}{8d^2}$, $(\because \lambda^2 << 8d^2)$

$\therefore$ Path difference between A and D $= d + 0 - d + \frac{\lambda}{2} = \frac{\lambda}{2}$ or π

$$\therefore \quad y_D = a \cos (\omega t - \pi) = -a \cos \omega t$$

So all the signals picked up by R_1 from A, B, C and D

$$y_{R_1} = y_A + y_B + y_C + y_D$$

$$= a \cos \omega t - a \cos \omega t + a \cos \omega t - a \cos \omega t$$

$$y_{R_1} = 0 \text{ so } \langle I_{R_1} \rangle \approx 0$$

So resultant signal at R_1 due to A, B, C and D sources is zero or no signal.

(*i*) New distance $BR_2 = d$

Let the signal picked up by R_2 from B = $a_1 \cos \omega t$

The path diff. between B and D = $\frac{\lambda}{2}$ or π so

The signal picked up by R_2 from D = $a \cos (\omega t - \pi)$

$$y_D = -\cos \omega t$$

The path diff. between A and D signals at $R_2 = AR_2 - R_2B$

$$\text{Path diff.} = \sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - d$$

$$\Delta x = \left[d^2 + \frac{\lambda^2}{4}\right]^{1/2} - d = d\left[1 + \frac{\lambda^2}{8d^2}\right] - d$$

$$\therefore \quad \Delta x = d + \frac{d\lambda^2}{8d^2} - d = \frac{\lambda^2}{8d}$$

$$\text{Phase diff} = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\cdot\frac{\lambda^2}{8d} = \frac{2\pi\lambda}{8d} = \phi$$

So the signals received by R_2 from A and C are

$$y_A = a\cos(\omega t - \phi)$$
$$y_C = a\cos(\omega t - \phi)$$

Signals picked up by R_2 from A, B, C and D

$$y_{R_2} = y_A + y_B + y_C + y_D$$
$$= a\cos(\omega t - \phi) + a\cos\omega t + a\cos(\omega t - \phi) - a\cos\omega t$$
$$= 2a\cos(\omega t - \phi) \qquad \left(\because \langle I\rangle = \frac{1}{2}(a)^2\right)$$
$$\langle I_{R_2}\rangle = \frac{1}{2}[4a^2] \quad \Rightarrow \quad \langle I_{R_2}\rangle = 2a^2 \qquad \boxed{\because I = A^2}$$

Thus, R_2 picks up the larger signal than R_1.

(*ii*) If B is switched off,

(*a*) Signals picked up by

$$R_1 = y_A + y_C + y_D$$
$$R_1 = a\cos\omega t + a\cos\omega t - a\cos\omega t = a\cos\omega t$$
$$\therefore \qquad (I_{R_1})_{av} = \frac{1}{2}a^2 \qquad \text{...(I)}$$

(*b*) Signals picked up by

$$R_2 = y_A + y_C + y_D$$
$$= a\cos(\omega t - \phi) + a\cos(\omega t - \phi) - a\cos\omega t$$

Net signals at

$$R_2 = 2a\cos(\omega t - \phi) - a\cos(\omega t) = a[2\cos(\omega t - \phi) - \cos(\omega t)]$$
$$\langle I_{R_2}\rangle = \frac{a^2}{2} \qquad \text{...(II)}$$
$$\langle I_{R_1}\rangle = \langle I_{R_2}\rangle$$

So R_1 and R_2 picks up the signals of the same intensities.

(*iii*) If D is switched off

(*a*) Signals picked up by receiver R_1

$$R_1 = y_B + y_C + y_A = -a\cos\omega t + a\cos\omega t + a\cos\omega t$$
$$y_{R_1} = a\cos\omega t$$
$$\therefore \quad \langle I_{R_1}\rangle = \frac{1}{2}a^2$$

(*b*) Signals picked up by receiver R_2

$$R_2 = y_A + y_B + y_C$$
$$= a\cos(\omega t - \phi) + a\cos\omega t + a\cos(\omega t - \phi)$$
$$= 2a\cos(\omega t - \phi) + a\cos\omega t$$

$\therefore \quad \dfrac{2\pi\lambda}{a} = \phi$ is very small so can be neglected.

So signal received by

$$R_2 = 2a \cos \omega t + a \cos \omega t = 3a \cos \omega t$$

$$\langle I_{R_2} \rangle = \frac{9a^2}{2}$$

$$\langle I_{R_1} \rangle < \langle I_{R_2} \rangle$$

So R_2 picks up larger signal compared to R_1.

(*iv*) A signal at R_1 indicates that B has been switched off.
A signal at R_2 indicates that D has been switched off.

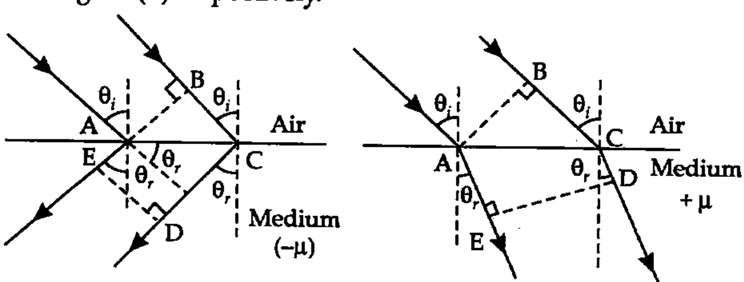
Q10.22. The optical properties of a medium are governed by the relative permittivity (ε_r) and relative permeability (μ_r). The refractive index is defined as $\sqrt{\varepsilon_r \mu_r} = \mu$. For ordinary material $\varepsilon_r > 0$ and $\mu_r > 0$ and the positive sign is taken for the square root.

In 1964, a Russian scientist V. Veselago postulated the existence of a material with $\varepsilon_r < 0$ and $\mu_r < 0$. Since then such metamaterials have been produced in the laboratories and their optical properties studied. For such materials $\mu = -\sqrt{\mu_r \varepsilon_r}$. As light enters a medium of such refractive index the phases travel away from the direction of propagation.

(*i*) According to the description above, show that if rays of light enter such a medium from air ($\mu = 1$) and an angle θ in 2nd quadrant then the refracted beam is in the 3rd quadrant.

(*ii*) Prove that Snell's law holds for such a medium.

Ans. (i) Let the postulates $-\sqrt{\mu_r \varepsilon_r} = \mu$ and $+\sqrt{\mu_r \varepsilon_r} = \mu$ are true then two parallel rays would proceed as shown in figure (a) and figure (b) respectively.



(a) (b)

Let two parallel rays at incidence angle θ_i from air would proceed in medium as shown in figures above. ED shows a wavefront, then all the points on ED will remain in same phase. All the points with the same optical path length must have the same phase.

so $$-\sqrt{\mu_r \varepsilon_r}\, AE = BC - \sqrt{\mu_r \varepsilon_r}\, CD$$

$$BC = \sqrt{\mu_r \varepsilon_r}\,(CD - AE)$$
$$BC > 0;\ CD > AE$$

As showing that the postulate is reasonable if however, the light proceeded in the sense it does for ordinary material *e.g.*, refracted rays are in IV quadrant in figure (b) then

$$-\sqrt{\varepsilon_r \mu_r}\,AE = BC - \sqrt{\mu_r \varepsilon_r}\,CD$$
$$BC = \sqrt{\mu_r \varepsilon_r}\,(CD - AE)$$
$$AE > CD \text{ hence } BC < 0$$

Figure showing that BC < 0 this is not possible. Hence the given postulate is correct.

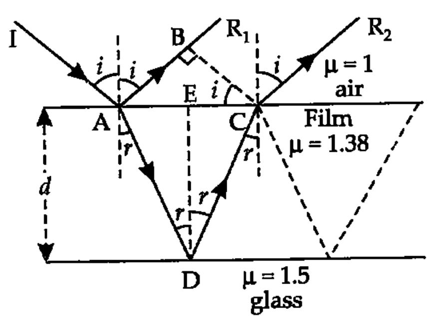
(*ii*) From fig. (a)

$$BC = AC \sin \theta_i$$
$$CD - AE = AC \sin \theta_r$$
$$BC = \sqrt{\mu_r \varepsilon_r} \qquad \text{(From Figure CD – AE = BC)}$$
$$AC \sin \theta i = \sqrt{\mu_r \varepsilon_r}\,AC \sin \theta_r$$
$$\frac{\sin \theta_i}{\sin \theta_r} = \sqrt{\mu_r \varepsilon_r}$$

which proves the Snell's law.

Q10.23. To ensure almost 100% transmittivity, photographic lenses are often coated with thin layer of dielectric material. The refractive index of this material is intermediated between that of air and glass (which makes the optical element of the lens). A typically used dielectric film is Mg F_2 (μ = 1.38). What should be the thickness of the film so that at the centre of the visible spectrum (5500 Å) there is maximum transmission?

Ans. In the given figure, incidence ray IA incident at point A from air to film surface with incident angle *i*. Here at A, it gets partial reflection and refraction, passes through paths AR_1 and AD respectively. At D it again gets partial reflection (and refraction) from the glass and film interface surface. At C the interface surface of film and air and finally after refraction from C pass through path CR_2 parallel to AR_1.



The amplitude (intensity) of wave during refraction and reflection decreases.

If the interference due to two reflected rays AR_1 and CR_2 is destructive interference, then the reflected rays AR_1 and CR_2 will not dominant.

Both reflections are from lower to higher refractive index surfaces so, optical path difference between AR_1 and CR_2 will be

$$\mu(AD + CD) - AB \quad \text{...(I)}$$

If d is the thickness of film then,

$$AD = AC = \frac{d}{\cos r} \quad \text{...(II)}$$

$$AB = AC \sin i$$

or $$\tan r = \frac{\frac{AC}{2}}{d}$$

∴ $$d \tan r = \frac{AC}{2}$$

$$AC = 2d \tan r$$

or $$AB = 2d \tan r \sin i \quad \text{...(III)}$$

So the optical path difference from (I)

$$= \mu\, AD + \mu\, AD - AB \qquad (\because AD = CD)$$

$$= 2\,\mu\, AD - AB$$

$$= \frac{2\mu d}{\cos r} - 2d \tan r \sin i \qquad \text{(From II and III)}$$

$$= 2 \cdot \frac{\sin i}{\sin r} \cdot \frac{d}{\cos r} - 2d \cdot \frac{\sin r}{\cos r} \cdot \sin i = \frac{2d \sin i}{\cos r}\left[\frac{1}{\sin r} - \frac{\sin r}{1}\right]$$

$$= \frac{2d \sin i\,(1 - \sin^2 r)}{\cos r \sin r} = \frac{2d \sin i \cos^2 r}{\sin r \cos r}$$

optical path difference between AR_1 and AR_2

$$= 2d\,\mu \cdot \cos r \qquad \left(\because \mu = \frac{\sin i}{\sin r}\right)$$

For two rays AR_1 and CR_2 to interfere destructively, path difference should be $\frac{\lambda}{2}$.

∴ $$2d\,\mu \cos r = \frac{\lambda}{2}$$

$$\mu d \cos r = \frac{\lambda}{4}$$

For photographic lenses the sources are vertical planes *i.e.,* rays incident at very small angle.

so $$i = r \approx 0$$

∴ $$\mu d \approx \frac{\lambda}{4} \qquad (\because \cos 0 = 1)$$

$$d = \frac{\lambda}{4\mu} = \frac{5500\ \text{Å}}{1.38 \times 4}$$

$$d = 1000\ \text{Å}$$

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