12 Atoms

MULTIPLE CHOICE QUESTIONS—I

Q12.1. Taking Bohr's radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(*a*) 53 pm (*b*) 27 pm (*c*) 18 pm (*d*) 13 pm

Main concept used: Bohr's model, $r \propto \frac{1}{Z}$ or $r = \frac{r_0}{Z}$

Ans. (*c*): According to Bohr's model of atom, radius of an atom in its ground state is $r = r_0/Z$ where r_0 is Bohr's radius, and Z is atomic number. As $r_0 = 53$ pm and atomic number of Lithium atom is 3 so,

$r = \frac{53}{3} = 17.67$ pm ≈ 18 pm verifies option (*c*).

Q12.2. The binding energy of a Hydrogen atom, considering an electron moving around a fixed nuclei (proton) is

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2} \qquad (m = \text{mass of electron})$$

If one decides to work in a frame of reference, where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = \frac{-Me^4}{8n^2\varepsilon_0^2h^2} \qquad (M = \text{mass of proton})$$

The last expression is not correct because

(*a*) *n* would not be integral.

(*b*) Bohr-quantisation applies only to electron.

(*c*) the frame in which the electron is at rest is not inertial.

(*d*) the motion of the proton would not be in circular orbits, even approximately.

Main concept used: $m_p > m_e$ need more centripetal force for revolution.

Ans. (*c*): As the mass of an electron is negligible as compared to proton. So the centripetal force cannot provide the electrostatic force, $F_p = \frac{m_p v^2}{r}$. So the given expression is not true, as it form non-inertial frame of reference due to $m_e <<< m_p$ or centripetal force on $F_e <<< F_p$. So verifies answer (*c*).

Q12.3. The simple Bohr's model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(*a*) of the electrons not being subject to central force.
(*b*) of the electrons colliding with each other.
(*c*) of screening effects.
(*d*) the force between the nucleus and an electron will no longer be given by Coulomb's law.

Main concept used: How a centripetal force can be increased.

Ans. (*a*): As the mass of an electron is negligible as compared to a nucleon, so electron cannot being subject to central force. Verifies answer (*a*).

Q12.4. For the ground state, the electron in an H-atom has an angular momentum *h*, according to the simple Bohr's model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,

(*a*) because Bohr's model gives incorrect values of angular momentum.
(*b*) because only one of these would have a minimum energy.
(*c*) angular momentum must be in the direction of spin of electron.
(*d*) because electrons go around only in horizontal orbits.

Main concept used: Bohr's second postulate on atomic model.

Ans. (*a*): According to Bohr's second postulate of atomic model, angular momentum of revolving electron must be some integral multiple of $\frac{h}{2\pi}$ so the Bohr's model does not give correct value of angular momentum. Hence verified answer (*a*).

Q12.5. O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of two atoms

(*a*) is not important because nuclear forces are short ranged.
(*b*) is as important as electrostatic force for binding the two atoms.
(*c*) cancels repulsive electrostatic force between the nuclei.
(*d*) is not important because oxygen nucleus have equal number of neutrons and protons.

Main concept used: Properties of Nuclear and Coulombian forces.

Ans. (*a*): Nuclear forces is too much stronger. Only attractive force as compared to electrostatic repulsive force and nuclear force decreases to zero on increasing distance. So in case of oxygen molecule, the distance between atoms of oxygen is larger as compared to the distances between nucleons in a nucleus. So force between the nuclei of two oxygen atoms is not important as nuclear forces are short ranged forces. Hence verified answer (*a*).

Q12.6. Two H-atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is

(*a*) 10.20 eV (*b*) 20.40 eV (*c*) 13.6 eV (*d*) 27.2 eV

Main concept used: Electrons of K(n = 1) energy level are called ground state electrons. It has minimum energy *i.e.*, – 13.6 eV.

Ans. (*a*): Total energy of two H-atoms in ground state

$$= 2\,(-13.6) = -27.2 \text{ eV}$$

The maximum amount by which their combined kinetic energy is reduced when any one H-atom goes into first excited state after the inelastic collision *i.e.*, the total energy of two H-atom after inelastic collision

$$E = \frac{13.6}{n^2} + 13.6$$

$$= \frac{13.6}{2^2} + 13.6 \qquad [\because \text{ for excited state } (n = 2)]$$

$$= 3.4 + 13.6 = 17.0 \text{ eV}$$

So loss in KE due to inelastic collision

$$= 27.2 - 17.0 = 10.2 \text{ eV}$$

Q12.7. A set of atoms in an excited state decays

(*a*) in general to any of the states with lower energy.
(*b*) into a lower state only when excited by an external electric field.
(*c*) all together simultaneously into a lower state.
(*d*) to emit photons only when they collide.

Main concept used: By emitted a photon by an electron, it comes back in its lower energy.

Ans. (*a*): A set of atoms in an excited state decays in general to any of the states with lower energy.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q12.8. An ionised H-molecule consists of an electron and two protons. The protons are separated by a small distance of the order of Å. In the ground state,

(*a*) the electron would not move in circular orbits.
(*b*) the energy would be $(2)^4$ times that of h-atom.
(*c*) the electrons, orbit would go around the protons.
(*d*) the molecule will soon decay in a proton and a H-atom.

Main concept used: In H-molecule, 2 proton or 2 nucleus and 1 electron.

Ans. (*a*) and (*c*): In H-molecule two nucleus of 2 H-atoms are separated of the order of angstrom, *i.e.* of order of nuclear forces. One electron will revolve around the protons. Hence verifies answer (*a*) and (*c*).

Q12.9. Consider aiming a beam of free electrons towards free protons. When they scatter, an electron and a proton cannot combine to produce a H-atom,

(*a*) because of energy conservation.
(*b*) without simultaneously releasing energy in the form of radiation.

(*c*) because of momentum conservation.
(*d*) because of angular momentum conservation.

Ans. (*a*) and (*b*): The binding energy of H-atom is larger as compared to the energy of free electron. Large amount of energy is required so that electron can reach near the proton which is possible when nuclear force (attractive) acts between *e* and *p* become of order of nuclear force, it is not possible without energy conservation and releasing energy simultaneously. Hence verifies answers (*a*) and (*b*).

Q12.10. The Bohr's model for spectra of a H-atom
(*a*) will not be applicable to hydrogen in molecular form.
(*b*) will not be applicable as it is for a He atom.
(*c*) is valid only at room temperature.
(*d*) predicts continuous as well as discrete spectral lines.

Main concept used: Niel's and Bohr's atomic model is valid for H-atom only and explains the line of spectrum.

Ans. (*a*) and (*b*): Not applicable for H molecule or He atom. It does not depend on minor change in temperature.

Q12.11. The Balmer series for the H-atom can be observed
(*a*) if we measure the frequencies of light emitted when an excited atom falls to the ground state.
(*b*) if we measure the frequencies of light emitted due to transitions between excited states and the first excited state.
(*c*) in any transition in a H-atom.
(*d*) as a sequence of frequencies with the higher frequencies getting closely packed.

Main concept used: When an electron is excited and jumps from higher to lower energy level it emits photons for spectrum.

Ans. (*b*) and (*d*): When electron jumps from higher energy level to first energy level, the spectrum of light is called Balmer series. Spectrum lines become closer if electron jumps from higher energy level to ground level. Hence verifies answers (*b*) and (*d*).

Q12.12. Let $E_n = \dfrac{-1me^4}{8\varepsilon_0^2 n^2 h^2}$ be the energy of *n*th level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $\dfrac{(E_2 - E_1)}{h}$ falls on it,

(*a*) it will not be absorbed at all.
(*b*) some of the atom will move to the first excited state.
(*c*) all atoms will be excited to the $n = 2$ state.
(*d*) no atoms will make a transition to the $n = 3$ state.

Main concept used: An electron of an atom absorbs photons of same energy as required by electron to reach in next higher orbit only.

Ans. (*b*) and (*d*): When the energy of radiation of photons is $\frac{(E_2 - E_1)n}{h}$, then electron jumps in next energy level ($n = 2$) after receiving this energy equal to the $E_2 - E_1$ energy. The new state is its unstable state. Electron jumps from E_2 to E_1 by radiating the energy of same frequency *i.e.*, to $(E_2 - E_1)$. Electron can jump in next orbit. So electron from ground state will jump at $n = 2$ not $n = 3$.

Q12.13. The simple Bohr's atomic model is not applicable to He^4 atom because

(*a*) He^4 is an inert gas.
(*b*) He^4 atom has neutrons in the nucleus.
(*c*) He^4 has one more electron.
(*d*) electrons are not subject to central forces.

Main concept used: Bohr's model is valid for H atom only *i.e.* for 1 electron only.

Ans. (*c*) and (*d*): Bohr's atomic model is applicable only for one electron and in He^4 there are two electrons. Electrons are not subject to central forces due to longer distances than nuclear size verifies answers (*c*) and (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q12.14. The mass of H atom is less than the sum of the masses of a proton and electron. Why is this?

Main concept used: Mass defect

Ans. During formation of an atom with neucleon and electrons, it need the energy to bind the nucleons together in nucleus. This energy comes from Einstein mass-energy relation

$$E = \Delta m C^2$$

where $\Delta m = [Zm_p + (A - Z)\, m_n] - M$

So the mass of a H atom is

$$m_p + m_p - \frac{B.E.}{C^2}$$

where B.E. = 13.6 eV

Q12.15. Imagine removing one electron from ${}_2He^4$ and ${}_2He^3$. Their energy levels, as worked out on the basis of Bohr's atomic model will be very close. Explain why.

Main concept used: Bohr's model explains only the stability of H-atom.

Ans. If we remove one electron from the isotopes of $2He^4$ and $2He^3$, both atoms will have 1 electron as in H-atom and the nucleus is much (four times) heavier than H-atom. So stability can remain and these atoms are very close to the H atom so the energy levels are as of Hydrogen and these atoms will be very close.

Q12.16. When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of e.m. radiations. Why cannot it be emitted as other forms of energy?

Main concept used: When electron jumps from one higher energy level to another lower energy level then due to motion of charge an e.m. wave is produced.

Ans. When charge *i.e.,* electron jumps from higher to lower energy level; there is acceleration in charge particle. The accelerated charge particle can produce electromagnetic wave only.

Q12.17. Would the Bohr's formula for the H-atom remains unchanged if proton had a charge $\left(\frac{+4}{3}\right)e$ and electron a charge $\left(\frac{-3}{4}\right)e$, where $e = 1.6 \times 10^{-19}$ C. Give reasons for your answer.

Main concept used: Electrostatic force remains same as m_p and m_e does not change. Position of e and p does not change *i.e.,* p in nucleus and e revalue.

Ans. As there is no interchange in the position of proton and electron only the magnitude of charge changes as Coulombian force will be same as product of both charges $\frac{+4}{3}e \times \frac{-3}{4}e = -e^2$ is same as earlier. $p = +1e$ and $e = -1$, $+1e \times -1e = -e^2$. So electrostatic force does not change. So Bohr's formula for the new H-atom remain same.

Q12.18. Consider two different H-atoms. The electron in each atom is in an excited state. It is possible for the electrons to have different energies but the same orbital angular momentum according to Bohr's model?

Main concept used: Bohr's atomic model, $L = \frac{nh}{2\pi}$

Ans. In excited state of electrons of two H-atoms, electrons may be in orbit or energy level either $n = 2, 3, \ldots$ and can have same energy but angular momentum by Bohr's model is $L = \frac{nh}{2\pi}$. As n for both may be different so both H-atom will have different angular momentum.

SHORT ANSWER TYPE QUESTIONS

Q12.19. Positronium is just like a H-atom with the proton replaced by the positively charged antiparticle of the electron (called the positron) which is as massive as electron. What would be the ground state energy of positronium?

Main concept used: $E_n = -\frac{m_e e^4}{8\varepsilon_0 n^2 h^2} = -13.6\text{ eV}$ for H atom in ground state.

Ans. As in the new H-atom (positronium) proton is replaced by positron of mass $m = m_e/2$ as under

$$\text{Mass of positronium} = m = m_e^- + m_e^+$$

$$m = m_e + m_e = 2m_e$$

$$m_e^+ = m_e^{-1} = \frac{m_e}{2}$$

as $E_n = -13.6$ and so energy of positron

$$E'_n = \frac{-m_e^+ e^4}{8\varepsilon_0 n^2 h^2} = \frac{-\left(\frac{m_e}{2}\right)e^4}{8\varepsilon_0 n^2 h^2} = \frac{-13.6}{2}$$

So $\quad E'_n = \dfrac{-13.6}{2} \qquad \left(\because\ m_e = \dfrac{m}{2}\right)$

$E'_n = -6.8 \text{ eV}$

Q12.20. Assume that there is no repulsive force between the electrons in an atom, but force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances calculate the ground state energy of a He-atom.

Main concept used: Energy of an electron revolving in stable nth orbit $E_n = Z\dfrac{m_e e^4}{8\varepsilon_0^2 n^2 h^2}$, m_e is mass of electron.

Ans. For H atom $\qquad Z = 1$ and $n = 1$

$$\therefore \qquad E_n = \frac{-m_e e^4}{8\varepsilon_0^2 1^2 h^2} = -13.6 \text{ eV}$$

For He-atom, $\qquad Z = 4$ and $n = 1$

$$\therefore \qquad E_n = \frac{-4m_e e^4}{8\varepsilon_0^2 1^2 h^2} = -4 \times 13.6 = -54.4 \text{ eV}$$

Q12.21. Using Bohr's model, calculate the electric current created by the electron when the H-atom is in the ground state.

Main concept used: $I = \dfrac{-e}{T} = -e\nu$

Ans. Let for an electron of H atom velocity in orbit = v m/s

Radius of orbit = a_0 = Bohr's radius

So the number of revolutions per second $\nu = \dfrac{2\pi a_0}{v}$

$$\therefore \qquad I = -e\nu \qquad \left(\because\ \nu = \frac{2\pi a_0}{v}\right)$$

$$= \frac{-e2\pi a_0}{v}$$

(–) sign shows that the direction of current is opposite to the direction of motion of electron.

Q12.22. Show that first few frequencies of light that is emitted when electrons fall to nth level from levels higher than n, are approximate harmonics. (*i.e.*, in the ratio 1 : 2 : 3) when $n >> 1$.

Main concept used: Spectrum of H atom.

Ans. When an electron falls from $(n + n')^{th}$ to nth energy level, the frequency of radiations in spectrum of H-atom like atoms is given as

$$\nu = CRZ^2 \left[\frac{1}{(n+n')^2} - \frac{1}{n^2} \right]$$

here $n >> n'$

$n' = 1, 2, 3, \ldots$

R = Rydberg's constant

$$\nu = CRZ^2 \left[\frac{1}{n^2\left[1+\frac{n'}{n^2}\right]^2} - \frac{1}{n^2} \right] = CRZ^2 \left[\frac{1}{n^2}\left[1+\frac{n'}{n}\right]^{-2} - \frac{1}{n^2} \right]$$

Neglecting the higher terms as $n >> n'$

$$= CRZ^2 \left[\frac{1}{n^2}\left(1 - \frac{2n'}{n}\right) - \frac{1}{n^2} \right] = CRZ^2 \left[\frac{1}{n^2} - \frac{2n'}{n^3} - \frac{1}{n^2} \right]$$

$$= \frac{-CRZ^2 2n'}{n^3} = \left(\frac{2CRZ^2}{n^3} \right) n'$$

So the first few frequencies of light that is emitted when electrons fall from $(n + n')$ to nth energy level are in the ratio of $n' = 1:2:3, \ldots$ when $n >>> 1$.

Q12.23. What is the minimum energy, that must be given to a H atom in ground state so that it can emit an H_γ line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?

Main concept used: (*i*) H_γ line in Balmer series corresponds to $n = 5$ to $n = 2$ (*ii*) Energy of electron in ground state of H atom = –13.6 eV, (*iii*) Energy of electron in nth energy level = $\frac{-13.6}{n^2}$.

Ans. We know that H_γ spectral line in Balmer series formed when electron falls from $n = 5$ to $n = 1$.

Here the electron is in ground state *i.e.*, $n = 1$ and must be taken to $n = 5$ for H_γ line. So the energy of

$$H_\gamma = E_5 - E_1 = \left(-\frac{13.6}{5^2} \right) - (-13.6) = -0.54 + 13.6 = 13.06 \text{ eV}$$

Since angular momentum is conserved, so the angular momentum of

H_γ = change in angular momentum of electron

$= L_5 - L_2 = 5h - 2h$

$= 3h = 3 \times 6.63 \times 10^{-34} = 19.89 \times 10^{-34} \text{ kg m}^2/\text{s}$

LONG ANSWER TYPE QUESTIONS

Q12.24. The first four spectral lines in the Lyman series of a H-atom are λ = 1218Å, 1028Å, 974.3Å and 951.4Å. If instead of Hydrogen, we consider Deuterium, calculate the shift in the wavelength of these lines.

Main concept used: Reduced mass of 2 particle system $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$.

Ans. Reduced mass of H atom (mass defect) = μ_H then

$$\frac{1}{\mu_H} = \frac{1}{me} + \frac{1}{M} \qquad \text{(M is mass of H atom)}$$

$$\frac{1}{\mu_H} = \frac{M + m_e}{M \cdot m_e} = \frac{M\left[1 + \frac{m_e}{M}\right]}{M \cdot m_e} \qquad (\because \quad M >>> m_e)$$

$$\therefore \qquad \mu_H = m_e\left[1 + \frac{m_e}{M}\right]^{-1} = m_e\left[1 - \frac{m_e}{M}\right]$$

For Deuterium M = 2 M

reduced mass of Deuterium = μ_D

$$\mu_D = m_e\left[1 - \frac{m_e}{M}\right]\left[1 + \frac{m_e}{2M}\right]$$

Total energy E_n of the electron revolving in an n^{th} stationary orbit,

$$E_n = \frac{-me^4}{8n^2\varepsilon_0^2h^2}$$

m is the reduced mass of the electron and proton in H atom.

So $\qquad h\nu = En_i - En_f$

$$\nu = \frac{me^4}{8\varepsilon_0^2h^2}\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right] = \frac{c}{\lambda}$$

$\nu \propto m$ (reduced mass)

$$\frac{1}{\lambda} \propto \mu \quad \Rightarrow \quad \lambda \propto \frac{1}{\mu}$$

For hydrogen and deuterium = $\frac{\lambda_H}{\lambda_D}$

So
$$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D} = \frac{m_e\left[1 - \frac{m_e}{M}\right]}{m_e\left[1 - \frac{m_e}{M}\right]\left[1 + \frac{m_e}{2M}\right]}$$

$$\lambda_D = \left[1 + \frac{m_e}{2M}\right]^{-1}\lambda_H = \left(1 - \frac{m_e}{M}\right)\lambda_H$$

$$\lambda_D = \lambda_H\,(0.99973)$$

$$\lambda_H = 1218\text{Å}, 1028\text{ Å}, 974\text{Å and } 954\text{Å} \qquad \text{(Given)}$$

$$\lambda_{D1} = 0.9973 \times 1218\text{Å} = 1214\text{Å}$$

$$\lambda_{D2} = 0.9973 \times 1028 = 1025\text{Å}$$

$$\lambda_{D3} = 0.9973 \times 974 = 971\text{Å}$$

$$\lambda_{D4} = 0.9973 \times 954 = 951\text{Å}$$

Q12.25. Deuterium was discovered in 1932 by Harold Urey by measuring the small change in wavelength for the particular transition in ${}_1H^1$ and ${}_1H^2$. This is because, the wavelength of transition depend to a certain extent on the nuclear mass. If nuclear motion is taken into account, then the electrons and nucleus revolve around their common centre of mass.

Such a system is equivalent to a single particle with a reduced mass μ revolving around the nucleus at a distance equal to the electron-nucleus separation. Here

$$\mu = \frac{m_e M}{(M + m_e)}$$

where M is the nuclear mass and m_e is the electronic mass.

Estimate the percentage difference in wavelength for the first line of the Lyman series in ${}_1H^1$ and ${}_1H^2$ (mass of ${}_1H^1$ nucleus = 1.6725×10^{-27} kg and mass of ${}_1H^2$ nucleus is 3.3374×10^{-27} kg. Mass of electron = 9.109×10^{-31} kg)

Main concept used: Percent wavelength difference

$$\frac{\Delta\lambda}{\Delta H} \times 100 = \frac{(\lambda_D - \lambda_H)}{\lambda_H} \times 100$$

Ans. Total energy of electron in n^{th} stable orbit in H or like atom

$$E_n = \frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2 n^2}$$

μ = reduced mass of electron, proton and neutron (mass defect)

$$E_H = \frac{\mu_H (1)^2 e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{1} - \frac{1}{2^2}\right] = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left[\frac{3}{4}\right]$$

$$E = h\nu = \frac{h}{\lambda} \quad \text{or} \quad \lambda_H = \frac{h}{E_H}$$

$$\therefore \qquad h\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\,\frac{3}{4}$$

$$\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^3} \cdot \frac{3}{4}$$

The percentage difference in the wavelength = $\frac{(\lambda_D - \lambda_H)}{\lambda_H} \times 100$

Percent change in wavelength

$$\% \text{ change} \quad \Delta E = \left[\frac{\lambda_D}{\lambda_H} - 1\right] \times 100 \qquad (\because \Delta E = E_1 - E_2) \quad ...(I)$$

$$h\nu = \frac{\mu e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_2^2} - \frac{1}{n_2^2}\right]$$

$$\nu = \frac{\mu e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{c}{\lambda} = \frac{\mu e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = \frac{\mu e^4}{8\varepsilon_0^2 c h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

as μ = mass defect, e, ε_0 c, and h are constants for an atom.

$$\therefore \qquad \lambda \propto \frac{1}{h}$$

So eqn. Ist can be written as percentage change in the wavelength

$$= \left[\frac{\mu_H}{\mu_D} - 1\right] \times 100$$

$$\because \qquad \mu = \frac{m_e M}{(M + m_e)} \qquad \text{(Given)}$$

$\therefore$ Percentage change in wavelength $= \left[\dfrac{\dfrac{m_e M_H}{(M_H + m_e)}}{\dfrac{m_e M_D}{(M_D + m_e)}} - 1\right] 100$

$$\frac{\Delta\lambda}{\lambda_H} \times 100 = \left[\frac{M_H}{M_D}\frac{(M_D + m_e)}{(M_H + m_e)} - 1\right] \times 100$$

$$= \left[\frac{M_H}{M_D} \frac{M_D\left(1 + \dfrac{m_e}{M_D}\right)}{M_H\left(1 + \dfrac{m_e}{M_H}\right)} - 1\right] \times 100$$

$$= \left[\left(1 + \frac{m_e}{M_D}\right)\left(1 + \frac{m_e}{M_H}\right)^{-1} - 1\right] \times 100$$

$$= \left[\left(1 + \frac{m_e}{M_D}\right)\left(1 - \frac{m_e}{M_H}\right) - 1\right] \times 100$$

me << M_D so neglecting the higher degree term

$$\frac{\Delta\lambda}{\lambda_H}\times 100 = \left[1-\frac{m_e}{M_H}+\frac{m_e}{M_D}-\frac{m_e\, m_e}{M_D\cdot M_H}-1\right]\times 100$$

$$= m_e\left[\frac{1}{M_D}-\frac{1}{M_H}\right]\times 100$$

$$= 9.1\times 10^{-31}\left[\frac{1}{3.3374\times 10^{-27}}-\frac{1}{1.6725\times 10^{-27}}\right]\times 100$$

$$= \frac{9.1\times 10^{-31+2}}{10^{-27}}\left[\frac{1.6725-3.3374}{3.3374\times 1.6725}\right]$$

$$\frac{\Delta\lambda\times 100}{\lambda_H} = \frac{-9.1\times 10^{-29+27}\times 0.6649}{3.3374\times 1.6725} = \frac{-6.05059\times 10^{-2}}{5.5180}$$

$$\frac{\Delta\lambda\times 100}{\lambda_H} = -1.084\times 10^{-2}\ \%\ \text{decrease in wavelength.}$$

(–) sign shows that $\lambda_D < \lambda_H$.

Q12.26. If a proton had a radius R and the charge was uniformly distributed, calculate using Bohr theory, the ground state energy of a H-atom when (*i*) R = 0.1Å (*ii*) R = 10Å.

Main concept used: Energy of H atom when (*i*) point nucleus (*ii*) spherical nucleus of radius R.

Ans. (*i*) Consider in H atom nucleus as a point charge electron is revolving around nucleus with speed v and radius r_A. The Coulombian force provides centripetal force to revolve around nucleus.

$$\therefore \qquad \frac{m_e v^2}{r_A} = \frac{-Ke^2}{r_A^2} \qquad ...(I)$$

Here $\qquad K = \dfrac{1}{4\pi\varepsilon_0}$

(–) sign shows the force of attraction.

By Bohr's postulate, angular momentum $= \dfrac{nh}{2\pi}$

$$mv\,r_A = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi m r_A}$$

$$\frac{m\,n^2h^2}{4\pi^2 m^2 r_A^2 r_A} = \frac{Ke^2}{r_A^2} \qquad \text{[From I]}$$

$$r_A = \frac{n^2h^2}{4\pi^2 m\,Ke^2} \qquad ...(II)$$

For ground state $n = 1$

$$r_A = \frac{h^2}{4\pi^2 mKe^2}$$

$$= \frac{6.63 \times 10^{-34} \times 6.63 \times 10^{-34}}{(2 \times 3.14)^2 \times 9.1 \times 10^{-31} \times 9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}$$

$$= \frac{6.63 \times 6.63 \times 10^{-68+38+31-9}}{9.1 \times 9 \times 1.6 \times 1.6 \times 4 \times 3.14 \times 4 \times 3.14}$$

$$= r_A = 0.53 \text{ Å} \quad \Rightarrow \quad 0.53 \times 10^{-10} \text{ m}$$

$$\text{P.E.} = \frac{-Ke^2}{r_A} = \frac{-9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}} \text{J}$$

$$= \frac{-9 \times 1.6 \times 1.6 \times 10^{-19} \times 10^{-19} \times 10^9}{0.53 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{eV} = \frac{-9 \times 1.6}{0.53} \times 10^{-19+9+10}$$

$$\text{P.E.} = \frac{14.4}{0.53} = 27.17 = 27.2 \text{ eV}$$

$$\text{K.E} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \frac{m \cdot n^2 h^2}{4\pi^2 m^2 r^2} = \frac{1}{2} \frac{h^2}{4\pi^2 mr^2} \qquad (n = 1 \text{ for ground state})$$

$$= \frac{1}{2} \frac{6.62 \times 10^{-34} \times 6.62 \times 10^{-34}}{4 \times 3.14 \times 3.14 \times 9 \times 10^{-31} \times 0.53 \times 10^{-10} \times 0.53 \times 10^{-10}} \text{J}$$

$$= \frac{6.62 \times 6.62 \times 10^{-68+31+10+10}}{4 \times 3.14 \times 3.14 \times 18 \times 0.53 \times 0.53 \times 1.6 \times 10^{-19}} \text{eV}$$

$$= \frac{6.62 \times 6.62 \times 10^{-68+51+19}}{4 \times 3.14 \times 3.14 \times 18 \times 0.53 \times 0.53}$$

$$\text{K.E.} = 0.1373 \times 10^2 \text{ eV} = 13.7 \text{ eV}$$

$$\text{P.E.} = 27.2 \text{ eV}$$

(*ii*) Now for spherical nucleus of radius, R, electron moves charge inside the nucleus $R >> r_b$ then electron moves inside the nucleus. Then (r_b is radius of new Bohr's orbit of revolving electron)

$$\text{Charge} = \frac{e \cdot \left(\frac{4}{3}\pi r_b^3\right)}{\frac{4}{3}\pi R^3}$$

$$e' = q_2 = \frac{er_b^3}{R^3}$$

$$q_1 = e$$

$$\frac{mv^2}{r_b} = \frac{Kee'}{r_b^2} \qquad \text{(By Coulomb's law)}$$

$$mvr_b = \frac{nh}{2\pi} \Rightarrow v_b = \frac{nh}{2\pi m r_b} \qquad \text{(By Bohr's postulate)}$$

$$\therefore \quad \frac{m}{r_b}\frac{n^2h^2}{4\pi^2m^2r_b^2} = \frac{kee'}{r_b^2}$$

$$r_b = \frac{n^2h^2}{4\pi^2 mkee'}$$

Now for ground state of H, n = 1 and $e' = \frac{er_A^3}{R^3}$, then

$$\therefore \quad r_b = \frac{h^2}{4\pi^2 mK \cdot e \cdot e \cdot \frac{r_b^3}{R^3}} = \left(\frac{h^2}{4\pi^2 mKe^2}\right) \times \frac{R^3}{r_b^3} = r_A \times \frac{R^3}{r_b^3}$$

$$\left[\because r_A = \frac{h^2}{4\pi^2 mKe^2} = 0.53 \text{ Å calculated in part } (i)\right]$$

$$r_b = r_A\left(\frac{R}{r_b}\right)^3$$

$$r_b^4 = r_A R^3 = 0.53\text{Å} \times (10\text{Å})^3$$

$$r_b^4 = 0.53 \times 1000\ (\text{Å})^4 \qquad (\because r_A = 0.53 \text{ Å})$$

$$r_b = [530 \text{ A}^4]^{1/4} = 4.8 \text{ Å} < R = 10 \text{ Å}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{m}{2}\cdot\frac{h^2}{4\pi^2 m^2 r_b^2} = \frac{h^2}{8\pi^2 m r_b^2} \qquad \left[\because v = \frac{n^2h^2}{4\pi^2 m^2 r_b^2}\right]$$

$$= \frac{6.62 \times 6.62 \times 10^{-34} \times 10^{-34}}{8 \times 3.14 \times 3.14 \times 9.1 \times 10^{-31} \times 4.8 \times 4.8 \times 10^{-20} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{6.62 \times 6.62 \times 10^{-68+31+20+19}}{8 \times 3.14 \times 3.14 \times 9.1 \times 4.8 \times 4.8 \times 1.6} \text{ eV}$$

$$= \frac{43.8244}{26460.2} 10^{-68+70} = 0.001656 \times 10^2 \text{ eV}$$

$$\text{K.E.} = 0.167 \text{ eV}$$

$$\text{P.E.} = \frac{e^2}{4\pi\varepsilon_0}\frac{(r_b^2 - 3R^2)}{R^3} \qquad \left(\text{P.E.} = \frac{Kq_1q_2}{r}\right)$$

$$\text{P.E.} = \left[\frac{e^2}{4\pi\varepsilon_0 r_A}\right]\frac{r_A(r_b^2 - 3R^2)}{R^3} \qquad \left[\text{multyplying by } \frac{r_A}{r_A}\right]$$

From part (*i*)

$$\text{P.E.} = \frac{e^2}{4\pi\varepsilon_0 r_A} = 27.2 \text{ eV}$$

$$\therefore \text{ P.E.} = 27.2\left[\frac{0.53(\sqrt{530} - 300)}{1000}\right] \qquad [\because\ r_b = (530)^{1/4}\text{ Å and R} = 10\text{Å}]$$

$$= \frac{27.2 \text{ eV} \times 0.53(23.02 - 300)\text{ Å}^3}{1000 \text{ Å}^3} = 27.2 \times \frac{0.53(-276.9)}{1000}\text{ eV}$$

$$\text{P.E.} = \frac{3992.9}{1000} = \mathbf{-3.99\ eV}$$

$$\text{K.E.} = \mathbf{0.167\ eV}$$

Q12.27. In the Auger process, an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to outer electron, which may be ejected by the atom (this is called an Auger electron). Assuming the nucleus to be massive, calculate the Kinetic energy of an $n = 4$ Auger electron emitted by chromium by absorbing the energy from a $n = 2$ to $n = 1$ transition.

Main concept used: As chromium nucleus is massive recoil of the atom by emitted electron is negligible and the entire energy of transition may be considered to be ejected (Auger) electron. As there is a single valence electron in Cr the energy states may be thought of as given by the Bohr's model.

Ans. The energy E_n of the *n*th state

$$E_n = +Z^2R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = Z^2R\left(\frac{1}{1} - \frac{1}{4}\right) \qquad (\text{for } n_1 = 1,\quad n_2 = 2)$$

$Z = 24$

R = Rydberg constt.

$$\therefore \qquad E_n = \frac{3}{4}Z^2R$$

The energy required to eject an electron from $n = 4$ state is

$$E_4 = Z^2R\frac{1}{4^2} = \frac{1}{16}Z^2R$$

Energy given to electron is converted into K.E. of ejected electron.

Hence, the K.E. of Auger (ejected) electron = $E_n - E_4$

$$K.E. = Z^2R\frac{3}{4} - \frac{1}{16}Z^2R = \frac{11}{16}Z^2R = \frac{11}{16} \times 24 \times 24 \times 13.6 \text{ eV}$$

$$K.E. = 11 \times 36 \times 13.6 = \mathbf{5385.6\ eV}$$

Q12.28. The inverse square law of electrostatics is $|F| = \frac{e^2}{(4\pi\varepsilon_0)r^2}$ for the force between an electron and a proton. The $1/r$ dependence of $|F|$ can be understood in quantum theory as being due to the fact that the particle of light (photon) is massless. If the photon had a mass m_p, force would be modified to $|F| = \frac{e^2}{(4\pi\varepsilon_0)r^2}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]e^{-\lambda r}$ where $\lambda = \frac{m_p c}{\hbar}$ and $\hbar = \frac{h}{2\pi}$.

Estimate the change in the ground state energy of a H-atom if $m_p = 10^{-6}$ times the mass of an electron.

Ans. Mass of photon $= 9.1 \times 10^{-31} \times 10^{-6}$ kg

$= 9.1 \times 10^{-37}$ kg

Wavelength associated with a photon $= \frac{h}{m_p c}$

$$\lambda = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-37} \times 3 \times 10^8}$$

$$= \frac{6.62}{9.1 \times 3} \times 10^{-34+37-8} = 2.4 \times 10^{-7} >> r_A \qquad \text{(see Q.26)}$$

$$\lambda << \frac{1}{r_A} < e.\lambda_{r_A} << 1$$

$$U(r) = \frac{-e^2 e^{-\lambda_r}}{\pi\varepsilon_0 r}$$

$$mvr = \frac{h}{2\pi} = \hbar \quad \text{or} \quad v = \frac{\hbar}{mr} \qquad ...(I)$$

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right] \qquad \left[\because F = \frac{e^2}{(4\pi\varepsilon_0)r^2}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]e^{-\lambda r} \text{ given}\right]$$

$$\frac{m}{r} \cdot \frac{\hbar^2}{m^2r^2} = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1 + \lambda r}{r^2}\right]$$

$$\frac{\hbar^2}{mr} = \frac{e^2}{4\pi\varepsilon_0}(1 + \lambda r)$$

$$\frac{\hbar^2 4\pi\varepsilon_0}{me^2} = (r + \lambda r^2)$$

If $\lambda = 0, r = r_A = \dfrac{\hbar^2 4\pi\varepsilon_0}{me^2}$ [neglecting r^2]

$$\frac{\hbar^2}{m} = \frac{e^2}{4\pi\varepsilon_0} r_A \qquad (r_A \approx r + \lambda r^2)$$

$\because \quad \lambda_A >> r_B$ and $r = r_A + \delta$

taking $r_A = r + \lambda r^2$ $[\because r = (r_a + \delta)]$

$$r_A = (r_A + \delta) + \lambda(r_A + \delta) \qquad \text{(put } r = r_A + \delta)$$

$$= r_A + \delta + \lambda(r_A^2 + \delta^2 + 2r_A\delta)$$

$$0 = \delta + \lambda r_A^2 + 2r_A\delta\lambda \quad \text{(neglecting small term } \delta^2)$$

$$0 = \delta + 2r_A\delta\lambda + \lambda r_A^2$$

$$\Rightarrow \qquad \delta[1 + 2r_A\lambda] = -\lambda r_A^2$$

$$\delta = \frac{-\lambda r_A^2}{(1 + 2r_A\lambda)} = -\lambda r_A^2 (1 + 2r_A\lambda)^{-1}$$

$$\delta = -\lambda r_A^2[1 - 2r_A\lambda] = -\lambda r_A^2 + 2r_A^3\lambda^2$$

$\therefore$ λ and $r_A << 1$ so $r_A^3\lambda^2$ is very small so by neglecting it we get,

$$\boxed{\delta = -\lambda r_A^2}$$

$$V(r) = \frac{-e^2}{4\pi\varepsilon_0} = \frac{e^{(-\lambda\delta - \lambda r_A)}}{(r_A + \delta)} = \frac{-e^2}{4\pi\varepsilon_0} \cdot \frac{e^{-\lambda(\delta + r)}}{r_A\left(1 + \dfrac{\delta}{r_A}\right)}$$

$$= \frac{-e^2}{4\pi\varepsilon_0 r_A} e^{-\lambda r}\left(1 + \frac{\delta}{r_A}\right)^{-1} \qquad (\because \quad r = r_A + \delta)$$

$$= \frac{-e^2 e^{-\lambda r}}{4\pi\varepsilon_0 r_A}\left(1 - \frac{\delta}{r_A}\right)$$

$V(r) = -27.2$ eV remains unchanged

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{\hbar}{mr}\right)^2 = \frac{1}{2}\frac{\hbar^2}{mr^2} \qquad \left(\text{From I, } v = \frac{\hbar}{mr}\right)$$

$$= \frac{-\hbar^2}{2m(r_A + \delta)^2} = \frac{-\hbar^2}{2mr_A^2\left(1 + \dfrac{\delta}{r_A}\right)^2} = \frac{\hbar^2}{2mr_A^2}\left(1 - \frac{2\delta}{r_A}\right)$$

$$= \frac{\hbar^2}{2mr_A}\left(1 - \frac{-\lambda r_A^2}{r_A}\right) = \frac{\hbar^2}{2mr_A}(1 + 2\lambda r_A)$$

$$= 13.6 \text{ eV } (1 + 2\lambda r_A)$$

$$\text{Total energy} = \frac{-e^2}{4\pi\varepsilon_0 r_A} + \frac{\hbar^2}{2mr_A^2}(1 + 2\lambda r_A)$$

$$= [-27.2 + 13.6\,(1 + 2\lambda r_A)]\ \text{eV}$$

$$= -27.2 + 13.6 + 27.2\ \lambda r_A\ \text{eV}$$

$$\text{Total E} = -13.6 + 27.2\ \lambda r_A$$

$$\text{Change in energy} = -13.6 + 27.2\ \lambda r_A - (-13.6) = \mathbf{27.2\ \lambda r_A\ eV}$$

Q12.29. The Bohr's model for H-atom relies on the Coulomb's law of electrostatics. Coulomb's law has not directly been verified for very short distances of the order of angstroms. Supposing Coulomb's law between two opposite charges $+q_1, -q_2$ is modified to

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad (r \geq R_0)$$

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 R_0^2}\left[\frac{R_0}{r}\right]^{\varepsilon} \qquad (r \leq R_0)$$

Calculate in such a case, the ground state energy of a H-atom if $\varepsilon = 0.1$, $R_0 = 1$ Å

Ans. Case I: when $r \leq R_0 = 1$ Å

Let $\varepsilon = 2 + \delta$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 R_0^2}\frac{R_0^{2+\delta}}{r^{2+\delta}} = \left(\frac{e\cdot(-e)}{4\pi\varepsilon_0}\right)\frac{R^\delta}{r^{2+\delta}} \qquad [q_1 = e, q_2 = -e]$$

$$|F| = -(1.6\times10^{-19})^2 \times 9\times10^9\ \frac{R^\delta}{r^{2+\delta}}$$

[(–)ve sign shows force of attraction]

$$|F| = 23.04\times10^{-29}\ \frac{R^\delta}{r^{2+\delta}}$$

The electrostatic force of attraction between positively charged nucleus and negatively charged electron provides necessary centripetal force.

$$\frac{mv^2}{r} = (23.04\times10^{-29})\ \frac{R_0^\delta}{r^{2+\delta}}$$

Let $\quad 23.04\times10^{-29} = \Lambda$

$$\therefore \qquad v^2 = \Lambda\frac{R_0^\delta}{r^{2+\delta}}\cdot\frac{r}{m} = \frac{\Lambda R_0^\delta}{mr^{1+\delta}}$$

By Bohr's IInd postulate, angular momentum

$$L = \frac{nh}{2\pi} \quad \text{and} \quad \hbar = \frac{h}{2\pi} \qquad \text{[Given in last Q.]}$$

$\therefore$
$$L = n\hbar$$
$$mvr = n\hbar$$
$$r = \frac{n\hbar}{mv} = \frac{n\hbar}{m}\sqrt{\frac{mr^{1+\delta}}{\Lambda R_0^\delta}} = \frac{n\hbar}{m}\left[\frac{m}{\Lambda R_0^\delta}\right]^{1/2} r^{1/2+\delta/2}$$
$$r^{1-\frac{1}{2}-\frac{\delta}{2}} = \frac{n\hbar}{m}\left[\frac{m}{\Lambda R_0^\delta}\right]^{1/2}$$
$$r^{\frac{1}{2}(1-\delta)} = \frac{n\hbar}{m}\left[\frac{m}{\Lambda R_0^\delta}\right]^{1/2}$$
$$r_n = \left[\frac{n^2\hbar^2}{m\Lambda R_0^\delta}\right]^{\frac{1}{1-\delta}}.$$
$$= \left[\frac{\hbar^2}{m\Lambda R_0^\delta}\right]^{\frac{1}{1-\delta}} \qquad (n = 1 \text{ for ground state})$$
$$r_1 = \left[\frac{(1.05\times10^{-34})^2}{9.1\times10^{-31}\times23.04\times10^{-29}\times10^{19}}\right]^{\frac{1}{2.9}}$$
$$\left[\because \hbar = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\times3.14} = (1.05\times10^{-34})\right]$$

where $\hbar = 1.05 \times 10^{-34}\ \text{JS}^{-1}$

$R_0 = 10^{19}$ and $1 - \delta = 2.9$

$\mathbf{r_1 = 8 \times 10^{-11}\ m = 0.08\ nm}$

This is a radius of orbit of electron in ground state of hydrogen atom.

Velocity of electron in ground state

By IInd postulate of Bohr's Atomic model
$$mv_n r_n = \frac{nh}{2\pi} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$
$$v_n = \frac{n\hbar}{mr_n} = \frac{n\hbar}{m}\left[\frac{m\Lambda R_0^\delta}{n^2\hbar^2}\right]^{\frac{1}{1-\delta}}$$

Case II: $n = 1$, $v_1 = \dfrac{\hbar}{mr_1} = \dfrac{1.05\times10^{-34}}{9.1\times10^{-31}\times0.08\times10^{-9}} = \dfrac{1.05}{0.728}\times10^{-34+31+9}$

$$= 1.44 \times 10^{-34+40} = 1.44 \times 10^6\ \text{m/s}$$
$$v_1 = 1.44\times10^6\ \text{m/s}$$

$$KE = \frac{1}{2} m v_1^2 - 9.43 \times 10^{-19}$$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.44 \times 10^6)^2 - 9.43 \times 10^{-19} J$$

$$= 5.9 \text{ eV}$$

P.E. from R_0 to $\frac{-\Lambda}{R_0}$

$$\text{P.E. from } R_0 \text{ to } r = +\Lambda R_0^\delta \int_{R_0}^{r} \frac{dr}{r^{2+\delta}}$$

$$\text{P.E.} = \frac{\Lambda R_0^\delta}{-(1+\delta)} \left[\frac{1}{r^{1+\delta}} \right]_{R_0}^{r}$$

This is the P.E. of electron in ground state R_0 to $r = \frac{-\Lambda}{R_0}$

$$\text{P.E.} = -\frac{\Lambda R_0^\delta}{(1+\delta)} \left[\frac{1}{r^{1+\delta}} - \frac{1}{R_0^{1+\delta}} \right]$$

$$= \frac{-\Lambda}{(1+\delta)} \left[\frac{R_0^\delta}{r^{1+\delta}} - \frac{1}{R_0} \right] = \frac{-\Lambda}{(1+\delta)} \left[\frac{R_0^\delta}{r^{1+\delta}} - \frac{1}{R_0} + \frac{1+\delta}{R_0} \right]$$

Put $\delta = -1.9$

$$\text{P.E.} = \frac{-\Lambda}{(1+(-1.9))} \left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1}{R_0} - \frac{0.9}{R_0} \right]$$

$$= \frac{-\Lambda}{-0.9} \left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1.9}{R_0} \right] = \frac{23.04 \times 10^{-2.9}}{0.9} [(0.8)^{0.9} - 1.9] J$$

$$= -17.3 \text{ eV}$$

$$\text{Total E} = \text{P.E.} + \text{K.E.}$$

$$= (-17.3 + 5.9) \text{ eV}$$

$$\text{Total energy} = -11.4 \text{ eV}$$

This is the required total energy of electron in ground state of H-atom.

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