13 Nuclei

MULTIPLE CHOICE QUESTIONS—I

Q13.1. Suppose we consider a large number of containers each containing initially 10,000 atoms of a radioactive material with a half life of 1 year. After 1 year,

(*a*) all the containers will have 5000 atoms of the material.
(*b*) all the container, will contain the same number of atoms of the material but that number will only be approximately 5000.
(*c*) the containers will in general have different number of atoms of the material but their average will be close to 5000.
(*d*) none of the containers can have more than 5000 atoms.

Ans. (*c*): Half life time for a radioactive substance is defined as the time in which a radioactive atomic substance remains half of its original value of radioactive atom. So after one year means one half life *i.e.*, average atoms of radioactive substance remain after 1 year in each container is equal to 1/2 of 10,000 = 5000 atoms (average).

Q13.2. The gravitational force between a H-atom and another particle of mass m will be given by Newton's law: $F = G\frac{M \cdot m}{r^2}$, where r is in km and

(*a*) $M = m_{\text{proton}} + m_{\text{electron}}$.
(*b*) $M = m_{\text{proton}} + m_{\text{electron}} - \frac{B}{c^2}$ $(B = 13.6 \text{ eV})$.
(*c*) M is not related to mass of H-atom.
(*d*) $M = m_{\text{proton}} + m_{\text{electron}} - \frac{|V|}{c^2}$ [$|V|$ = magnitude of potential energy of electron in the H-atom.]

Ans. (*b*): During formation of H-atom some mass of nucleons convert into energy by $E = mc^2$, this energy is used to bind the nucleons along with nucleus. So mass of atom becomes slightly less than sum of actual masses of nucleons and electrons.

Actual mass of H atom $= M_p + M_e - \frac{\text{B.E.}}{c^2}$ $\left(\frac{B}{c^2} \text{ is binding energy}\right)$

B.E. (B) of H atom is 13.6 eV per atom.

Q13.3. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of atom

(*a*) do not change for any type of radioactivity.
(*b*) change for α and β-radioactivity but not for γ-radioactivity.

(*c*) change for α-radioactivity but not for others.
(*d*) change for β-radioactivity but not for others.

Ans. (*b*): β-particles carries one unit of negative charge, and α-particle carries 2 units of positive charge, and γ-particle carries no charge. So the electronic energy level of the atom changes in emission of α and β particle, but not in γ decay.

Q13.4. M_x and M_y denote the atomic masses of parent and the daughter nuclei respectively in a radioactive decay. The Q_1-value for a β^- decay is Q_1 and for a β^+ decay is Q_2. If m_e denotes the mass of an electron, then which of the following statements is correct?

(*a*) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y - 2m_e)c^2$
(*b*) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y)c^2$
(*c*) $Q_1 = (M_x - M_y - 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2m_e)c^2$
(*d*) $Q_1 = (M_x - M_y + 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2m_e)c^2$

Ans. (*a*): Let the parent nuclei $_Z X^A$ is radioactive atom and decay β^- as under, where Z and A have usual meanings.

$$_Z X^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + Q_1$$

$$Q_1 = \left[m_n\left({}_Z X^A\right) - m_n\left({}_{Z+1}Y^A\right) - m_e\right]c^2$$

$$= \left[m_n({}_Z X^A) + m_e Z - m_n({}_{Z+1}Y^A) - (Z+1)m_e\right]c^2$$

$$= [m({}_Z X^A) - m({}_{Z+1}Y^A)]C^2$$

Let the nucleus $= {}_Z X^A$ radiate β decay $_Z X^A \rightarrow {}_{Z-1}Y^A + {}_1e^0 + \nu + Q_2$

$$Q_2 = [m({}_Z X^A) - m({}_{Z-1}Y^A) - 2m_e]c^2$$

$$= [m_n({}_Z X^A) + m_e Z - m_n({}_{Z-1}Y^A) - (Z-1)m_e - 2m_e]c^2$$

$$= [m({}_Z X^A) - m({}_{Z-1}Y^A) - 2m_e]c^2$$

$$Q_2 = (M_x - M_y - 2m_e)c^2$$

Q13.5. Tritium is an isotope of hydrogen whose nucleus triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + \bar{e} + \bar{\nu}$, if one of the neutrons in triton decays, it would transform into $_2He^3$ nucleus. This does not happen. This is because

(*a*) Triton energy is less than that of He^3 nucleus.
(*b*) yhe electron created in the beta decay process cannot remain in the nucleus.
(*c*) both the neutrons in triton have to decay simultaneously, resulting in a nucleus with 3 protons which is not a He^3 nucleus.
(*d*) free neutrons decay due to external perturbation which is absent in triton nucleus.

Ans. (*a*): Triton ($_1H^3$) has 1 proton and 2 neutrons. If a neutron decays as $n \rightarrow p + \bar{e} + \bar{\nu}$, then nucleus will have 2 proton and 1 neutron, *i.e.* triton atom converts in $_2He^3$ (2 proton and 1 neutron).

Binding energy of ${}_1H^3$ is much smaller than ${}_2He^3$ so transformation is not possible energetically.

Q13.6. Heavy stable nuclei have more neutrons than protons. This is because of the fact that

(*a*) neutrons are heavier than protons.
(*b*) electrostatic force between protons are repulsive.
(*c*) neutrons decay into protons through beta decay.
(*d*) nuclear forces between neutrons are weaker than that between protons.

Ans. (*b*): Electrostatic force between proton-proton is repulsive which causes the unstability of nucleus. So nutrons are larger than protons.

Q13.7. In a nuclear reactor, moderators slow down the speed of neutrons which come out in the fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose because

(*a*) they will break up.
(*b*) elastic collision of neutrons with heavy nuclei will not slow them down.
(*c*) the net weight of reactor would be unbearably high.
(*d*) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature.

Main concept used: Mass of moderator must not be too much large for elastic collision.

Ans. (*b*): For elastic collision masses of both must be equal so that they can exchange the velocities. To slow down the speed of neutron substance should be made up of 1 proton for perfectly elastic *i.e.,* we need light nuclei not heavy. In heavy nuclei only direction will change not the speed.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q13.8. Fusion processes, like combining two deuterons to form a He nucleus are impossible at ordinary temperature and pressure. The reasons for this can be traced to the fact:

(*a*) nuclear forces have short range.
(*b*) nuclei are positively charged.
(*c*) the original nuclei must be completely ionised before fusion can take place.
(*d*) the original nuclei must first break up before combining with each other.

Ans. (*a*) and (*b*): Two deuteron can combine to form He atom when their nuclei come close to nuclear range where electrostatic repulsive force between positively charged deuterons does not act. Electrostatic force increases very high on decreasing their distance $\left(\because F \propto \frac{1}{r^2}\right)$.

To overcome this electrostatic repulsive force nuclei need very high temperature and pressure. Hence to combine two nuclei, they must reach closer of the range of where nuclear force acts and electrostatic repulsive force does not act verifies the answers (*a*) and (*b*).

Q13.9. Samples of two radioactive nuclides A and B are taken. λ_A and λ_B are the disintegration constants of A and B respectively. In which of the following cases the two samples can simultaneously have the same decay rate at any time?

(*a*) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$.

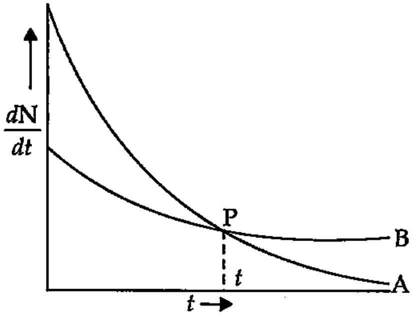
(*b*) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$.

(*c*) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$.

(*d*) Initial rate of decay of B is same as the rate of decay of A at $t = 2h$ and $\lambda_B < \lambda_A$.

Ans. (*b*) and (*d*): Both radioactive samples can have same rate of decay at any time from initial time, if the initial rate of decay of A is equal to the twice of B and $\lambda_A > \lambda_B$. Decay rate can be same if $\lambda_A > \lambda_B$ and initial rate of decay of both are equal at $t = 2h$.

Q13.10. The variation of decay rate of two radioactive samples A and B with time is shown in figure. Which of the following statements are true?



(*a*) Decay constant of A is greater than that of B hence A always decays faster than B.

(*b*) Decay constant of B is greater than that of A but its decay rate is always smaller than that of A.

(*c*) Decay constant of A is greater than that of B, but it does not always decay faster than B.

(*d*) Decay constant of B is smaller than that of A but still its decay rate become equal to that of A at later instant.

Ans. (*c*) and (*d*): From the given graph slope of A is greater than of B so rate of decay of A is greater than of B. $\frac{dN}{dt} = -\lambda t$ or at instant *t* or for a particular time *t*, $\frac{dN}{dt} \propto \lambda$ so $\lambda_A > \lambda_B$ at point P the intersecting point of two graphs at time *t* is same.

VERY SHORT ANSWER TYPE QUESTIONS

Q13.11. ${}_1He^3$ and ${}_2He^3$ nuclei have the same mass number. Do they have the same binding energy?

Ans. (*c*): In ${}_2He^3$ there are two protons which give electrostatic force of repulsion but in ${}_1He^3$ between nucleons there is only nuclear attractive force. So binding energy of ${}_1He^3$ is larger than ${}_2He^3$.

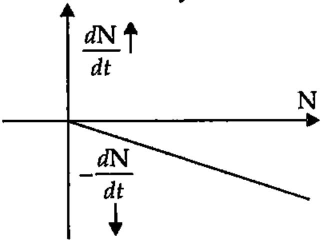
Q13.12. Draw a graph showing the variation of decay rate with number of active nuclei.

Ans. By law of radioactive decay

$$-\frac{dN}{dt} \propto \lambda N$$

N is the number of radioactive nuclei in the sample.

So, $\frac{dN}{dt}$ can be negative.



The variation of decay rate with number of active nuclei is shown by the above graph.

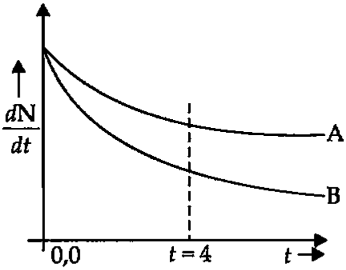
Q13.13. Which sample A or B shown in figure has shorter mean life?

Ans. Initially at $t = 0$ from figure given

$$\left(\frac{dN_0}{dt}\right)_A = \left(\frac{dN_0}{dt}\right)_B$$

so $$(N_o)_A = (N_o)_B$$

i.e., initially both samples has equal number of radioactive atoms. Considering at any instant $t = t$ from figure,



$$\left(\frac{dN}{dt}\right)_A > \left(\frac{dN}{dt}\right)_B$$

$$\lambda_A N_A > \lambda_B N_B$$

$$N_A > N_B$$

$$\lambda_A < \lambda_B \qquad \left(\frac{-dN}{dt} = \lambda N\right)$$

$$\therefore \qquad \tau = \frac{1}{\lambda}$$

$$\therefore \qquad \frac{1}{\tau_A} < \frac{1}{\tau_B}$$

$$\tau_A > \tau_B$$

So mean life time of sample A is greater than of B.

Q13.14. Which one of the following cannot emit radiation and why? Excited nucleus, excited electron.

Ans. Excited electron has energy in eV while excited nucleus in M eV. Energy of γ-rays have energy of the order of M eV. So excited *e* cannot emit radiation when nucleus is excited.

Q13.15. In pair annihilation, an electron and a positron destroy each other to produce gamma radiations. How is the momentum conserved?

Ans. When an electron and positron combine together coming from opposite directions they destroy each other by the emission of two γ-rays in opposite direction to conserve linear momentum as below

$$_0e^{-1} + {_0e^{+1}} \rightarrow 2\gamma\text{-ray photons.}$$

SHORT ANSWER TYPE QUESTIONS

Q13.16. Why do stable nuclei never have more protons than neutrons?

Ans. If in a stable nucleus number of protons are larger than neutrons the repulsive electrostatic force between proton-proton becomes larger in place of nuclear force of attraction between nucleons.

So for stability, repulsive force between proton-proton must be smaller than nuclear attractive force between nucleons.

Q13.17. Consider a radioactive nucleus A which decays to a stable nucleus C, through the following sequence

$$A \rightarrow B \rightarrow C$$

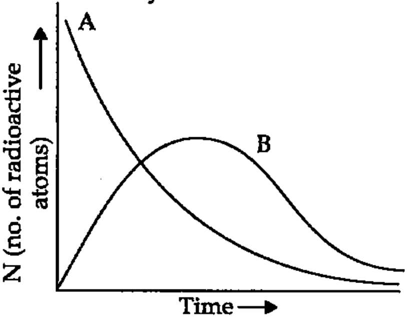
Here B is an intermediate nuclei which is also radioactive. Considering that there are N_0 atoms of A initially, plot the graph showing the variation of number of atoms of A and B versus time.

Main concept used: Law of radioactive decay

Ans. Let us consider that in sample A initially at $t = 0$, there are N_0 atoms of AB and atoms of B are zero initially.

When A decay to B the number of radioactive atoms of A decreases and of B increases.

When rate of decay of A decreases to a lower value, then the number of radioactive atoms of B becomes maximum. After this the radio-active atoms and rate of decay of B decreases. Finally, the number of radioactive atoms of sample A and B becomes very low near to zero.



Q13.18. A piece of wood from the ruins of an ancient building was found to have a C^{14} activity of 12 disintegrations per minute per gram of its carbon content. The carbon C^{14} activity of the living wood is 16 disintegrations per minute per gram. How long ago did the tree, from which the wooden sample came die? Given half life of C^{14} is 5760 years.

Main concept used: Carbon dating.

Ans. Rate of disintegration in old wood sample of C-14 radioactive atoms is 12 atoms per min per gm. Initially rate of disintegration of C-14 when the tree was live = 16 atoms per min per gm.

$$T_{1/2} \text{ of C–14 nuclei} = 5760 \text{ years}$$

According to radioactive decay law,

$$N = N_0 e^{-\lambda t} \quad \text{or} \quad R = R_0 e^{-\lambda t}$$

$$12 = 16 e^{-\lambda t}$$

$$e^{\lambda t} = \frac{16}{12}$$

$$\log_e e^{\lambda t} = \log_e \frac{4}{3}$$

$$\lambda t = \log_e \frac{4}{3}$$

$$t = \frac{2.303 \times \log_{10}\left(\frac{4}{3}\right)}{\lambda} \text{ half life} \qquad \left(\because \quad \lambda = \frac{0.6931}{T_{1/2}}\right)$$

$$= \frac{2.303(\log 4 - \log 3) \times 5760}{0.6931} \text{ years}$$

$$\therefore \qquad t = \frac{2.303\,(0.6020 - 0.4771)\,5760}{0.6931} = \mathbf{2391.20\ years}$$

Q13.19. Are the nucleons fundamental particles, or do they consist of still smaller parts? One way to find out is to probe a nucleon just as Rutherford probed an atom. What should be the kinetic energy of an electron for it to be able to probe a nucleon? Assume the diameter of a nucleon to be approximately 10^{-15} m.

Main concept used: de-Broglie wavelength

Ans. To detect the properties of nucleons inside the nucleus the wavelength of particle which may detect nucleons that must be of size of nucleons (10^{-15} m). So the wavelength of particle which can detect the nucleons must be equal to or less than 10^{-15}.

$$\lambda = 10^{-15}\, m$$

$$\lambda = \frac{h}{p}$$

$$\because \quad E = h\nu = \frac{hc}{\lambda} \qquad [\because\ c = \nu\lambda]$$

$$\text{K.E.} = \text{P.E. of (electron)} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{10^{-15}} \text{ J}$$

$$\text{K.E.} = \frac{6.6 \times 3 \times 10^{-34+8+15}}{1.6 \times 10^{-19}} \text{ eV} = \frac{99 \times 10^{-34+23+19}}{8}$$

$$= 12.4 \times 10^{-34+42} = 1.24 \times 10^{1} \times 10^{+8}$$

$$\text{K.E.} = 1.24 \times 10^{9}\ \text{eV}$$

So the K.E. of particle which may detect nucleon inside the nucleus must be of 1.24×10^9 eV per particle.

Q13.20. A nuclide 1 is said to be the mirror isobar of nuclide 2, if $Z_1 = N_2$ and $Z_2 = N_1$.

(*a*) What nuclide is a mirror isobar of $_{11}Na^{23}$?

(*b*) Which nuclide out of the two mirror isobars have greater binding energy and Why?

Main concept used: Mirror isobar and Binding energy difference in neutrons and protons.

Ans. (*a*): Here Z is atomic number and N is no. of neutron in $_{11}Na^{23}$

$$Z_1 = 11$$

$$N_1 = 23 - 11 = 12$$

Mirror isobar of $_{11}Na^{23}$ is

$$Z_2 = N_1 = 12$$

So Mg is isobar of $_{11}Na^{23}$

So $_{12}Mg^{23}$ is the mirror isobar of $_{11}Na^{23}$.

(*b*) As the neutrons in $_{12}Mg^{23}$ are '11' and in $_{11}Na^{23}$ are '12' so, the number of neutrons in Na is larger than Mg and hence nuclear short range attractive forces in Na will be larger than repulsive electrostatic forces between proton-proton.

So, $_{11}Na^{23}$ has more binding energy than $_{12}Mg^{23}$.

LONG ANSWER TYPE QUESTIONS

Q13.21. Sometimes a radioactive nucleus decays into a nucleus which itself is radioactive. An example is:

$$^{38}\text{Sulphur} \xrightarrow[= 2.48\ \text{hrs.}]{\text{half life}} {}^{38}\text{Cl} \xrightarrow[= 0.62\ \text{h}]{\text{Half life}} {}^{38}\text{Ar (stable)}$$

Assume that we start with 1000 S^{38} nuclei at time $t = 0$. The number of Cl^{38} is of count zero at $t = 0$ and will again be zero at $t = \infty$. At what value of t, would the number of counts be a maximum?

Main concept used: Radioactive law of decay.

Ans. $$S^{38} \xrightarrow[2.48\text{h}]{\text{H.Life}} Cl^{38} \xrightarrow[0.62\text{h}]{\text{H.Life}} Ar^{38}$$

Initially at $t = 0$, number of radioactive atoms of $S^{38} = N_1$ and of Cl^{38} are zero.

At any time t, $$\frac{dN_1}{dt} = -\lambda_1 N_1$$

and $$N_1 = N_0 e^{-\lambda_1 t}$$

It is the rate of formation of Cl^{38} from S^{38}. Let N_2 is the number of Cl^{38} atoms (radioactive):

$$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1$$

$$= \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2 \qquad \text{...(I)}$$

Multiplying both sides by $e^{+\lambda_2 t}dt$

$$e^{\lambda_2 t}dN_2 = \lambda_1 N_0 e^{-\lambda_1 t+\lambda_2 t}dt - \lambda_2 N_2 e^{+\lambda_2 t}dt$$

$$e^{\lambda_2 t}dN_2\, dt + \lambda_2 N_2 e^{+\lambda_2 t}dt = \lambda_1 N_0 e^{(\lambda_2-\lambda_1)t}dt$$

Integrating both sides

$$N_2 e^{\lambda_2 t} = \frac{N_0\lambda_1}{(\lambda_2-\lambda_1)} e^{(\lambda_2-\lambda_1)t} + C \qquad \left[\because e^{\lambda_2 t}dN_2 \cdot dt = 0\right]$$

∴ Cl^{38} atom is formed after disintegration of S^{38}, so initially number of Cl^{38} atoms are $N_2 = 0$.

at $t = 0$, $N_2 = 0$,

$$0 \times e^0 = \frac{N_0\lambda_1}{\lambda_2-\lambda_1} e^0 + C \qquad \text{...(II)}$$

∴ $$0 \times 1 = \frac{N_0\lambda_1}{\lambda_2-\lambda_1}(1) + C$$

or $$C = \frac{-N_0\lambda_1}{\lambda_2-\lambda_1}$$

∴ $$N_2 e^{\lambda_2 t} = \frac{N_0\lambda_1}{\lambda_2-\lambda_1} e^{(\lambda_2-\lambda_1)t} - \frac{N_0\lambda_1}{\lambda_2-\lambda_1}$$

$$\frac{N_2}{e^{-\lambda_2 t}} = \frac{N_0\lambda_1}{(\lambda_2-\lambda_1)}\left[e^{(\lambda_2-\lambda_1)t} - 1\right] \qquad \text{...(III)}$$

Multiplying $e^{-\lambda_2 t}$ to both sides we get

$$N_2 = \frac{N_0\lambda_1}{\lambda_2-\lambda_1}\left[e^{(\lambda_2-\lambda_1-\lambda_2)t} - e^{-\lambda_2 t}\right] \qquad [\because e^0 = 1]$$

$$N_2 = \frac{N_0\lambda_1}{\lambda_2-\lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

$$N_2\lambda_2 - N_2\lambda_1 = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_0 e^{-\lambda_2 t}$$

N_o are the number of S^{38} atoms

No. of Cl^{38} atoms after time t will be $N_2 = N_0 e^{-\lambda_2 t}$

For $N_{2\,max} \cdot \dfrac{dN_2}{dt} = 0$

∴ $$N_2\lambda_2 - N_2\lambda_1 = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_2$$

$$N_2\lambda_2 - \lambda_1 N_2 + \lambda_1 N_2 = \lambda_1 e^{-\lambda_1 t}$$

$$N_2\lambda_2 = \lambda_1 e^{-\lambda_1 t}$$
$$N_2\lambda_2 = \lambda_1 N_0 e^{-\lambda_1 t}$$
$$N_2 = \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$$

Put the value of N_2 in (III)

$$\frac{\lambda_1}{\lambda_2} N_0\, e^{-\lambda_1 t}\, e^{\lambda_2 t} = \frac{N_0\lambda_1}{(\lambda_2 - \lambda_1)}\left[e^{(\lambda_2-\lambda_1)t} - 1\right]$$
$$e^{(\lambda_2-\lambda_1)t} = \frac{\lambda_2}{(\lambda_2 - \lambda_1)}\left[e^{(\lambda_2-\lambda_1)t} - 1\right]$$

By cross multiplication and multiplying both sides by $e^{-(\lambda_2-\lambda_1)t}$

$$\frac{\lambda_2 - \lambda_1}{\lambda_2} = 1 - e^{-(\lambda_2-\lambda_1)t}$$
$$1 - \frac{\lambda_1}{\lambda_2} = 1 - e^{(\lambda_1-\lambda_2)t}$$
$$\frac{\lambda_1}{\lambda_2} = e^{(\lambda_1-\lambda_2)t}$$
$$\log_e\left(\frac{\lambda_1}{\lambda_2}\right) = \log_e e^{(\lambda_1-\lambda_2)t}$$

or
$$\log_e\left(\frac{\lambda_1}{\lambda_2}\right) = (\lambda_1 - \lambda_2)\, t$$

$$t = \frac{\log_e\left(\frac{\lambda_1}{\lambda_2}\right)}{\lambda_1 - \lambda_2} \qquad \left(\because \ \lambda = \frac{0.6931}{T/2}\right) \quad \text{...(IV)}$$

$$\therefore \quad \left(\frac{\lambda_1}{\lambda_2}\right) = \frac{\frac{0.6931}{2.48}}{\frac{0.6931}{0.62}} = \frac{0.62}{2.48} = \frac{1}{4}$$

$$(\lambda_1 - \lambda_2) = \frac{0.6931}{2.48} - \frac{0.6931}{0.62} = \frac{0.6931(-1.86)}{2.48 \times 0.62} = \frac{-0.6931 \times 1.86}{2.48 \times 0.62}$$

$$\therefore \quad t = \frac{\log_e\left(\frac{1}{4}\right) \times 2.48 \times 0.62}{-0.6931 \times 1.86} = \frac{-\log_e 4 \times 2.48 \times 0.62}{-0.6931 \times 1.86}$$
$$= \frac{2.303 \times 0.3010 \times 2.48 \times 0.62}{0.6931 \times 1.86} = \frac{1.06586}{1.2892} = \mathbf{0.8267\ hrs.}$$

Number of Cl^{38} radioactive atoms will be maximum at $N_2 = 0.8267$ hrs.

Q13.22. Deuteron is a bound state of a neutron and a proton with a binding energy B = 2.2 MeV. A γ-ray of energy E is aimed at a deuteron nucleus to try to break it into a (neutron + proton) such that the n and p move in the direction of the incident γ-ray. If E = B, show that this cannot happen. Hence calculate how much bigger than B must E be for such a process to happen.

Main concept used: Laws of conservation of energy and momentum.

Ans. Binding energy (B) of deuteron = 2.2 MeV

Some part of energy of γ-ray is used up against binding energy B = 2.2 MeV and the rest part will impart K.E. to neutron and proton.

$$\mathrm{E} - \mathrm{B} = \mathrm{K}_n + \mathrm{K}_p \qquad \left(\begin{aligned} &\text{K.E.} = \frac{1}{2}mv^2 \times \frac{m}{m} \\ &\text{K.E.} = \frac{p^2}{2m} \end{aligned}\right)$$

$$\mathrm{E} - \mathrm{B} = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \qquad \text{...(I)}$$

By law of conservation of momentum,

$$p_n + p_p = \text{momentum of } \gamma\text{-ray of Energy E}$$

$$\left[\lambda = \frac{h}{p} \quad \text{or} \quad p = \frac{h\nu}{\lambda\nu} = \frac{h\nu}{c} = \frac{\mathrm{E}}{c}\right]$$

$$\therefore \qquad p_n + p_p = \frac{\mathrm{E}}{c} \qquad \text{...(II)}$$

Case I: If E = B then from

$$\therefore \qquad \frac{p_n^2}{2m} + \frac{p_p^2}{2m} = 0 \quad \text{or} \quad p_n^2 + p_p^2 = 0 \qquad \text{...(III)}$$

It can be possible if $p_n = p_p = 0$ because square of non zero number can never be zero.

If $p_n = p_p = 0$ then equation IInd cannot be satisfied and the process cannot take place.

From II, $0 + 0 = \dfrac{\mathrm{E}}{\mathrm{c}}$ or E = 0 but energy E of γ ray cannot be zero.

Case II: If E > B or E = B + λ where λ will be very small than B then from (I),

$$(\mathrm{B} + \lambda) - \mathrm{B} = \frac{p_n^2}{2m} + \frac{p_p^2}{2m}$$

$$\lambda = \frac{1}{2m}\left(p_n^2 + p_p^2\right)$$

$$\lambda = \frac{1}{2m}\left[\left(\frac{\mathrm{E}}{c} - p_p\right)^2 + p_p^2\right] \qquad \left[\because p_n = \frac{\mathrm{E}}{c} - p_p\right]$$

$$2m\lambda = \frac{\mathrm{E}^2}{c^2} + p_p^2 - \frac{2\mathrm{E}}{c}p_p + p^2$$

$$2p_p^2 - \frac{2E}{c}p_p + \left(\frac{E^2}{c^2} - 2m\lambda\right) = 0$$

It is a quadratic equation so its solution by quadratic formula

$$a = 2, b = \frac{-2E}{c}, c = \left(\frac{E^2}{c^2} - 2m\lambda\right)$$

$$p_p = \frac{+\frac{2E}{c} \pm \sqrt{\frac{4E^2}{c^2} - 4 \times 2\left(\frac{E^2}{c^2} - 2m\lambda\right)}}{4}$$

For a real and equal value of p_p discriminant must be zero as the value of p_p must be one.

$$\therefore \qquad \frac{4E^2}{c^2} - 8\left[\frac{E^2}{c^2} - 2m\lambda\right] = 0$$

$$\frac{4}{c^2}[E^2 - 2E^2 + 4mc^2\lambda] = 0$$

$$\therefore \qquad -E^2 + 4mc^2\lambda = 0$$

$$\lambda = \frac{E^2}{4mc^2}$$

$\because$ λ is very small

so $$E = B$$

$$\boxed{\lambda \cong \frac{B^2}{4mc^2}}$$

Q13.23. The deuteron is bound by nuclear forces just as H-atom is made up of p and e bound by electrostatic forces. If we consider the force between neutron and proton in deuteron as given in the form of a Coulomb potential but with an effective charge e'

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e'^2}{r}$$

estimate the value of $\left(\frac{e'}{e}\right)$ given that the binding energy of a deuteron is 2.2 MeV.

Ans. The binding energy of H atom in ground state

$$E = \frac{me^4}{8\pi\varepsilon_0^2 h^2} = 13.6\,\text{eV} \qquad \text{...(I)}$$

If proton and neutron had charge e' each and governed by the same electrostatic force, then in the above equation we would need to replace electronic mass m by the reduced mass m' of proton-neutron (as some mass of proton and neutron is used by binding energy) and electronic charge e is replaced by e′.

$$\frac{1}{m'} = \frac{1}{M} + \frac{1}{N} \qquad \begin{pmatrix} M = \text{mass of proton} \\ N = \text{mass of neutron} \end{pmatrix}$$

$$m' = \frac{M \cdot N}{M + N} \qquad \text{(take M = N)}$$

$$= \frac{M}{2} \qquad \text{(if } m = \text{mass of electron)}$$

$$m' = \frac{1836\,m}{2} = 918\,m$$

$$\therefore \quad \text{Binding energy (E)} = \frac{918\,m\,e'^4}{8\pi\varepsilon_0^2 h^2} \qquad \text{...(II)}$$

Dividing (II) by (I) we get,

$$\frac{E'}{E} = \frac{918\,e'^4}{e^4} = \frac{2.2\,\text{MeV}}{13.6\text{eV}}$$

$$\left(\frac{e'}{e}\right)^4 = \frac{2.2 \times 10^6\,\text{eV}}{918 \times 13.6\text{ eV}}$$

$$\frac{e'}{e} = \frac{2200000}{1248.48} = (176.21)^{1/4}$$

Required ratio $\dfrac{e'}{e}$ = **3.64**

Q13.24. Before the neutrino hypothesis, the β-decay process was thought to be the transition $n \rightarrow p + \overline{e}$. If this was true show that if the neutron was at rest, the proton and electron would emerge with fixed energies and calculate them. Experimentally the electron energy was found to have a large range.

Ans. Neutron was at rest before β decay from neutron. Hence energy of neutron = $E_n = m_n c^2$ and momentum of neutron $p_n = 0$ as its velocity is zero.

By the law of conservation of momentum,

$$p_n = p_p + p_e \text{ (Beta)}$$
$$0 = p_p + p_e$$

Let $p_e = p_p$ then

$$\Rightarrow \qquad |p_p| = |p_e| = p\text{(eV)}$$

$$\text{Energy of proton} = E_p = \sqrt{(m_p^2 c^4 + p_p^2 c^2)}$$

Energy of electron (β) = $E_e = \sqrt{(m_e^2c^4 + p_e^2c^2)}$ $\qquad (\because \ |p_e| = p_p)$

From conservation,

$$\therefore \qquad E_p = \sqrt{m_p^2c^4 + p^2c^2}$$

$$E_e = \sqrt{m^2c^4 \quad p^2c^2}$$

∴ By the law of conservation of energy,

$$(m_p^2c^4 + p^2c^2)^{1/2} + (m_e^2c^4 + p^2c^2)^{1/2} = m_nc^2$$

$$\because \qquad m_pc^2 = 936 \text{ MeV}$$

and $\qquad m_nc^2 = 938$ MeV

and $\qquad m_ec^2 = 0.5$ MeV

As the energy difference in neutron and proton is very small, pc will be small $pc << m_pc^2$ while pc may be greater than m_ec^2 so by neglecting $(m_ec^2)^2 = (0.5)^2$ (Given)

$$\Rightarrow \qquad m_pc^2 + \frac{p^2c^2}{2m_p^2c^4} = m_nc^2 - pc$$

$$m_pc^2 + \frac{p^2c^2}{2m_p^2c^4} + pc = m_nc^2$$

Again $pc << m_pc^2$ so neglecting $\frac{p^2c^2}{2m_p^2c^4}$ we get

$$pc = m_nc^2 - m_pc^2 = 938 \text{ MeV} - 936 \text{ MeV}$$

$pc = 2$ MeV is the momentum

$$\because \qquad E = mc^2$$

$$E^2 = m^2c^4$$

E is the energy of either proton or neutron then

$$E_p = \sqrt{m_p^2c^4 + p^2c^2} = \sqrt{(936)^2 + 2^2} = 936 \text{ MeV}$$

$$E_e = \sqrt{m_e^2c^4 + p^2c^2} = \sqrt{(a)^2 + 2^2} = \mathbf{2.06 \text{ MeV}}$$

Q13.25. The activity R of an unknown radioactive nuclide is measured at hourly intervals. The results found are tabulated as follows

t (hours)	0	1	2	3	4
R (mega B*q*)	100	35.36	12.51	4.42	1.56

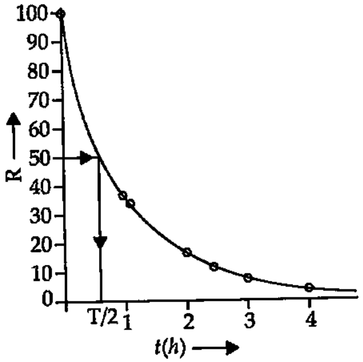
(*i*) Plot the graph of R versus *t* and calculate half-life from the graph.

(*ii*) Plot the graph of log $\frac{R}{R_0}$ versus *t* and obtain the value of half life from the graph.

Ans. (*i*) Graph between R versus t is exponential curve. From the graph at slightly more than $t = \frac{1}{2}h$ the R should be 50% so

at R = 50% the

$$t(h) = 0.7\ h$$
$$= 0.7 \times 60 \text{ min}$$
$$= 42 \text{ min}$$



(*ii*) For Graph between $\log\left(\frac{R}{R_0}\right)$ versus $t(h)$

at $t = 0$, $\qquad \log_e \frac{R}{R_0} = \log \frac{100}{100} = \log 1 = 0$

at $t = 1$ hour, $\qquad \log_e \frac{35.36}{100} = \log_e 0.3536 = -1.04$

$$= 2.302 \log_{10} 0.3536 = -1.04$$

at $t = 2$ hours, $\qquad \log_e \frac{12.5}{100} = \log_e 0.125$

$$= 2.303 \log_{10} 0.125 = -2.08$$

at $t = 3$ hour, $\qquad \log_e \frac{4.42}{100} = -3.11$

at $t = 4$ hour, $\qquad \log_e \frac{1.56}{100} = -4.16$

t (hours)	1	2	3	4
$\log_e \frac{R}{R_0}$	–1.04	–2.08	–3.11	–4.16

The graph showing the variation of $\log_e \frac{R}{R_0}$ versus $t(h)$ as follows:

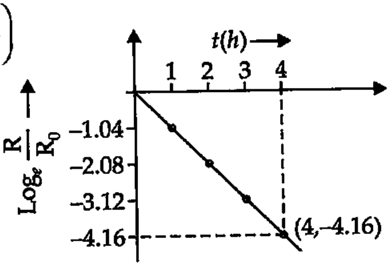
We know that disintegration constant

$$\lambda = \frac{\log_e \frac{R}{R_0}}{t_{1/2}} = -\left(\frac{4.16 - 3.11}{1}\right)$$

$$\lambda = -1.05 \text{ per hour}$$

$$t_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{1.05}$$

$$t_{1/2} = 42 \text{ min}$$



Q13.26. Nuclei with magic number of proton Z = 2, 8, 20, 28, 50, 52 and magic number of neutron N = 2, 8, 20, 28, 50, 82, and 126 are found to be very stable.

(*i*) Verify this by calculating the proton separation energy S_p, for $_{50}Sn^{120}$ and $_{51}Sb^{121}$.

The proton separation energy for a nuclide is the minimum energy required to separate the least tightly bound proton from a nucleus of that nuclide. It is given by

$$S_p = [M_{Z-1,N} + M_H - M_{ZN}]\,c^2$$

Given $\quad {}^{119}In = 118.9058\ u;\ {}_{50}Sn^{120} = 199.902199\ u;$

$${}_{51}Sb^{121} = 120.903824\ u \text{ and } {}_1H^1 = 1.0078252\ u$$

(*ii*) What does the existance of magic number indicate?

Ans. (*i*) $S_p = [M_{Z-1,\ N} + M_H - M_{Z,\ N}]c^2$

Here in this formula M_{Z-1} is the mass of atom of Z − 1 atomic number.

M_Z is the mass of atom of mass number Z

$\therefore \quad M_{Z-1}$ = Mass of atom whose atomic number is 50 − 1 = 49.

It is $_{49}In^{119}$ in this case $M_{Z-1} = {}_{49}In^{119} = 118.9058$ and N = 119 − 49 = 70.

$$S_p \text{ for } {}_{50}Sn^{120} = c^2[118.9058 + 1.0078252 - 199.902199]$$

$$S_p \text{ for } {}_{50}Sn^{120} = 0.0114362\ c^2$$

Now for S_p of $_{51}Sb^{121}$

$$S_p = [M_{Z-1,\ N} + M_H - M_{Z,N}]c^2$$

$\Rightarrow \quad Z = 51, Z - 1 = 50$ for S_n

$$M_{Z-1} = \text{mass of } {}_{50}Sn = 199.902199\ u$$

$$\therefore \quad S_p \text{ for } {}_{51}Sb^{121} = [199.902199 + 1.0078252 - 120.903824]c^2$$

$$= 0.0059912\ c^2$$

$$\therefore \quad S_p({}_{50}Sn^{120}) > S_p({}_{51}Sb^{121})$$

(*ii*) The existence of magic numbers indicates that the shell structure of nucleus is similar to the shell structure of atom. This also explains the peaks in binding energy per nucleon curve.

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