1 Electric Charges and Fields

Lesson at a Glance

• Electric Charge

Charge is the property associated with matter due to which it produces and experiences electric and magnetic effects.
Electric charge interacts another electric charge.

• Unit of Electric Charge

The SI unit of electric charge is coulomb (C).

• Coulomb's Law

It states that the electrostatic force between two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between the charges along the line joining them.

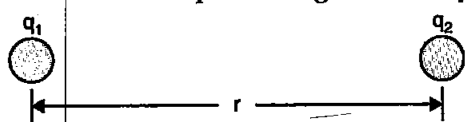

Fig. 1.1

$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{1 . q_1 q_2}{4\pi\varepsilon\, r^2}$$

• Principle of Superposition

Let point charges $q_1, q_2, q_3 \cdots q_N$ are placed at the position $\vec{r}_1, \vec{r}_2, \vec{r}_3 \cdots \vec{r}_N$ respectively. According to principle of superposition the net force on charge is the vector sum of all the forces due to other charges.

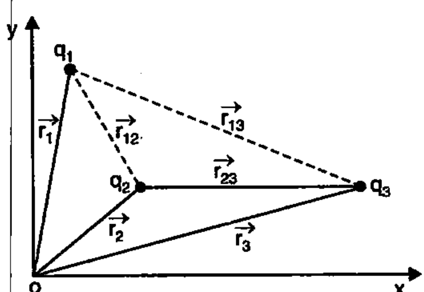

Fig. 1.2

• Electric Field

It is the region in which an electric charge experiences a force. The intensity of electric field is the force experienced by unit positive charge. If a test charge q_0 experiences a force F in electric field, then the intensity of electric field,

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\lim_{q_0 \to 0}$$

• Electric Field Intensity Due to a Point Charge

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

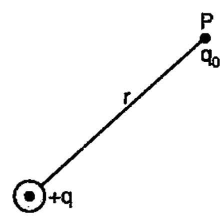

Fig. 1.3

• Electric Dipole

It is a system of two equal and opposite charges separated by an infinitely small separation.

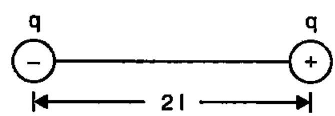

Fig. 1.4

- The dipole moment of an electric dipole is the product of the either charge and length of the dipole. It is vector quantity. Its direction is from –ve charge to +ve charge. Therefore dipole moment,

$$\vec{P} = (\vec{q})\,(2l) \quad \text{or} \quad \vec{P} = 2\vec{q}\,l$$

• Potential Energy of Electric Dipole

- Self potential energy of the dipole is the amount of work done during the formation of dipole. It is given as

$$U_1 = -\frac{1}{4\pi\varepsilon_0}\frac{(q)(q)(2l)}{(2l)^2}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l)^2}(q)\,(2l) = -E.P$$

or $\quad U_1 = -PE.$

• Electric Flux

Electric flux through an elementary area dS is the scalar product of area and electric field intensity.

$$d\phi = \vec{E}.\vec{dS} = E\,dS\cos\theta$$

or $$\phi = \int \vec{E}.\vec{dS} = \int E\,dS\cos\theta$$

where θ is the angle between electric field and normal drawn to the surface.

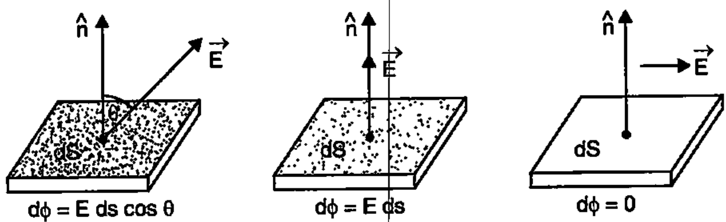

Fig. 1.5

- Electric flux is scalar quantity and its unit is $NC^{-1}m^2$ or Vm.
- Normal to the surface is always drawn outward.

• Gauss's Theorem

It states that the total electric flux linked with a closed surface is $\left(\frac{1}{\varepsilon_0}\right)$ times the total charge enclosed within the surface.

or $$\oint E\,dS\cos\theta = \frac{1}{\varepsilon_0}(q)$$

• Electric field intensity due to a point charge

To find the electric field intensity at P at a distance r from the source charge q a Gaussian surface of radius r can be drawn. Applying Gauss theorem,

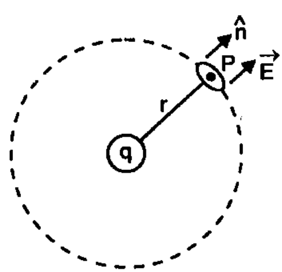

Fig. 1.6

$$\oint E\,dS\cos\theta = \frac{1}{\varepsilon_0}(q)$$

∵ $$\theta = 0^\circ$$

∴ $$E(4\pi r^2) = \frac{1}{\varepsilon_0}(q)$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- **Electric field intensity due to a linear charge**

$$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{q}{rl}$$

- **Electric field intensity due to a surface charge distribution**

$$E = \frac{\sigma}{2\varepsilon_0}$$

- **Electric field intensity due to a uniformly charged shell**

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

- **Electric field intensity due to a sphere of charge**

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

TEXTBOOK QUESTIONS SOLVED

1.1. *What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?*

Sol. Here, $q_1 = 2 \times 10^{-7}$ C, $q_2 = 3 \times 10^{-7}$ C

$r = 30$ cm $= 0.3$ m

$\therefore$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2}$$

$= 6 \times 10^{-3}$ N (Repulsive)

1.2. *The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge –0.8 μc in air is 0.2 N.*

(a) What is the distance between the two spheres?

(b) What is the force on the second sphere due to the first?

Sol. (*a*) Here, $F = 0.2$ N

$q_1 = 0.4\ \mu C = 0.4 \times 10^{-6}$ C

$q_2 = 0.8\ \mu C = 0.8 \times 10^{-6}$ C

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Thus, $$r^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{F}$$

$$r^2 = \frac{9\times10^9 \times 0.4\times10^{-6}\times0.8\times10^{-6}}{0.2}$$

$$r^2 = 36 \times 4 \times 10^{-4} = 144 \times 10^{-4}$$

$$r = 12 \times 10^{-2}\text{ m} = 12\text{ cm.}$$

(*b*) Force on the second sphere due to the first is same, *i.e.*, 0.2 N and force is attractive as charges are unlike.

1.3. *Check that the ratio* $\frac{ke^2}{Gm_e m_p}$ *is dimensionless. Look up a table of physical constants and determine the value of this ratio. What does the ratio signify?*

$$F = K\frac{q_1 q_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

Sol. $G = 6.67 \times 10^{-11}\text{ N m}^2\text{ kg}^{-2}$

$m_e = 9.1 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$e = 1.6 \times 10^{-19}$ C

and $$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}$$

Now,

$$\frac{ke^2}{Gm_e m_p} = \frac{9\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\times1.6\times10^{-19}\text{C}\times1.6\times10^{-19}\text{C}}{6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}\times9.1\times10^{-31}\text{kg}\times1.67\times10^{-27}\text{kg}}$$

$= 2 \times 27 \times 10^{39}$ which is dimensionless.

It also establishes that the electrostatic force is about 10^{39} times stronger than the gravitational force.

1.4. *(a) Explain the meaning of the statement electric charge of a body is quantised.*

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Sol. (*a*) **Quantisation of Electric Charges:** Electric charge is due to transfer of electron. The electric charge is always an integral multiple of *e* which is termed as quantisation of charge.

i.e., $q = \pm ne$

Here $+e$ is taken as charge on a proton while $-e$ is taken as charge on an electron. The charge on a proton and an electron are numerically equal *i.e.,* 1.6×10^{-19} C but opposite in sign. "Quantisation is a property due to which charge exists in discrete packets in multiple of $\pm 1.6 \times 10^{-19}$ rather than in continuous amounts."

(*b*) Based on many practical phenomena, we may ignore quantisation of electric charge and consider the charge to be continuous. Large scale electric charge may be considered as integral multiple of the basic unit '*e*'. The "graininess" of charge can be ignored and it can be imagined that this large scale charge can be charged continuously and its quantisation is insignificant and can be ignored.

1.5. *When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.*

Sol. Total charge of an isolated system of objects is always conserved. As a consequence of conservation of charge, when two charged conductors of same size and same material carrying charges Q_1 and Q_2 respectively are brought in contact and separated, the charge on each conductor will be $\frac{Q_1+Q_2}{2}$. This condition, however, does not hold true if the conductors are of different sizes or of different material. In that case the charges on the conductors will be Q_1' and Q_2' respectively,

where $Q_1 + Q_2 = Q_1' + Q_2'$.

Example. When a glass rod is rubbed with silk cloth, glass rod becomes positively charged while silk cloth becomes negatively charged. The amount of positive charge on the glass rod is found to be exactly the same as negative charge on silk cloth. Thus, the system of glass rod and silk cloth, which was neutral before rubbing, still possesses no net charge after rubbing.

1.6. *Four point charges $q_A = 2$ μC, $q_B = -5$ μC, $q_C = 2$ μC, and $q_D = -5$ μC are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?*

Sol. Suppose a square ABCD with each side of 10 cm and centre O. At the centre, the charge of 1 μC is placed.

$q_D = -5\ \mu C$
$q_C = 2\ \mu C$
$q_A = 2\ \mu C$
$q_B = -5\ \mu C$

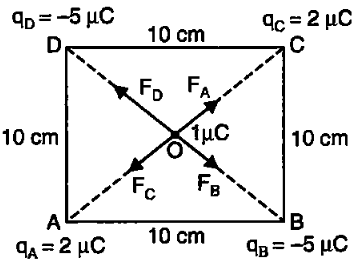

Fig. 1.7

As $q_A = q_C$, the charge of 1 μC experiences equal and opposite forces F_A and F_C due to charges q_A and q_C.

At the same time, the charge 1 μC experiences equal and opposite forces. F_B and F_D due to equal charges q_B and q_D at B and D. Thus, the net force on charge of 1 μC due to the given charges is zero.

1.7. *(a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?*

(b) Explain why two field lines never cross each other at any point?

Sol. (*a*) Lines of forces are the path of a small +ve change in electric field and path of test charge cannot be in breaking. It is always point to point or continuous. The direction of electric field at a point is displayed by the tangent at that point on a line of force. Generally the direction of electric field changes from point to point. Therefore, the lines of force are generally, curved lines. Further, they are continuous curves and cannot have sudden breaks. Even if it is so, the absence of electric field at the break points will be indicated by it.

(*b*) The path of small +ve charge at a point of intersection of field lines must be in two directions which is never possible. So electric field lines can never intersect. At the point of intersection, we can draw two tangents to the lines of force. That is why two field lines never cross each other at any point.

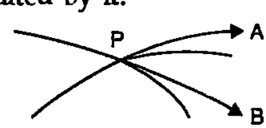

Fig. 1.8

This would mean two directions of electric field intensity at the point of intersection, which is not possible.

1.8. *Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum.*

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

Sol.

$$q_A = 3\ \mu C = 3 \times 10^{-6}\ C$$
$$q_B = -3\ \mu C = -3 \times 10^{-6}\ C$$

and

$$d = 20\ cm$$

(*a*)

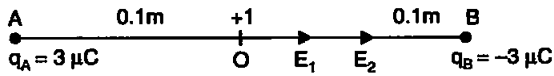

Fig. 1.9

Let us assume that a unit positive test charge is placed at *O*. q_A will repel this test charge while q_B will attract. Hence, $\vec{E}_1$ and $\vec{E}_2$ both are directed towards $\overrightarrow{OB}$.

$\therefore$
$$E = \vec{E}_1 + \vec{E}_2$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}[q_A + q_B]$$

$$= \frac{9 \times 10^9}{(0.1)^2}[3 \times 10^{-6} + 3 \times 10^{-6}]$$

$$= 5.4 \times 10^6\ NC^{-1} \text{ along } OB.$$

(*b*) As a negative test charge of $q_0 = -1.5 \times 10^{-6}$ C is placed at *O*. q_A will attract it while q_B will repel. Therefore, the net force

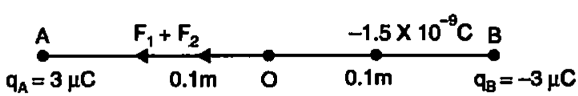

Fig. 1.10

$$F = F_1 + F_2$$

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$$

$$F = |F_1| + |F_2|$$

$$= \frac{Kq_A q_0}{r^2} + \frac{Kq_B q_0}{r^2} = \frac{Kq_0}{r^2}[q_A + q_0]$$

$$= \frac{9\times10^{9}\times1.5\times10^{-19}\,[3\times10^{-6}+3\times10^{-6}]}{(0.1)^2}$$

$$= \frac{9\times1.50\times6\times10^{-6+9-9}}{0.1\times0.1}$$

$$= \frac{9\times10^{9}\times3\times10^{-6}\times1.5\times10^{-9}}{(0.1)^2} + \frac{9\times10^{9}\times3\times10^{-6}\times1.5\times10^{-9}}{(0.1)^2}$$

$$= \frac{9\times10^{9}\times3\times10^{-6}\times1.5\times10^{-9}\times2}{(0.1)^2}$$

$= 8.1 \times 10^{-3}$ N.

1.9. *A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, –15 cm) and B: (0, 0, + 15 cm), respectively. What are the total charge and electric dipole moment of the system?*

Sol. Total charge, $q = q_A + q_B$

$= 2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

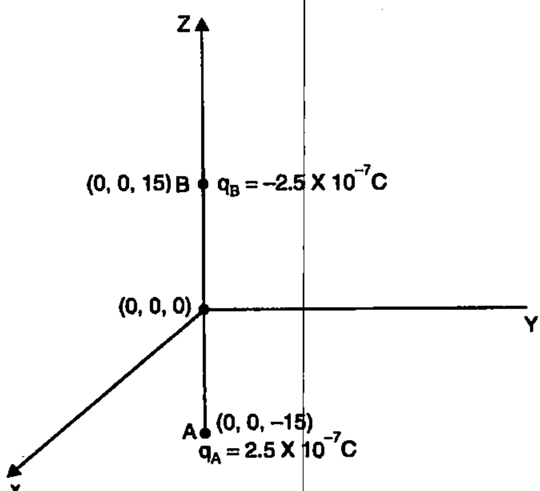

Fig. 1.11

a = AB = 15 – (– 15) = 30 cm = 0.3 m

Electric dipole moment

$p = q \cdot a$

$= 2.5 \times 10^{-7}$ (0.3 m)

$= 7.5 \times 10^{-8}$ cm (along –Z-axis).

1.10. *An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.*

Sol. Given,

$$P = 4 \times 10^{-9} \text{ cm}; \quad \theta = 30^\circ$$
$$E = 5 \times 10^4 \text{ NC}^{-1}$$

Torque,

$$\boxed{\tau = p \times \vec{E}} = p.E \sin\theta$$
$$= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30^\circ$$

or

$$\tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \frac{1}{2}$$

or

$$\tau = 10^{-4} \text{ Nm}$$

1.11. *A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C.*

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Sol. (*a*) Given,

$$q = -3.2 \times 10^{-7} \text{ C}$$
$$e = -1.6 \times 10^{-19} \text{ C}$$

∴ number of electrons transferred

$$n = \frac{q}{e} = \frac{-3.2 \times 10^{-7}}{-1.6 \times 10^{-19}} = 2 \times 10^{12}$$

Electrons are transferred from wool to polythene during rubbing as polythene has negative charge.

(*b*) From wool to polythene, certainly there is a transfer of mass.

Mass of an electron $= 9.1 \times 10^{-31}$ kg

Thus, amount of mass transferred

$$= 2 \times 10^{12} \times 9.1 \times 10^{-31} \text{ kg}$$
$$= 18.2 \times 10^{-19} \text{ kg.}$$

1.12. *(a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C ? The radii of A and B are negligible compared to the distance of separation.*

(b) What is the force of repulsion if each sphere is charged double the above amount, and th. distance between them is halved?

Sol. (*a*)

$$q_1 = 6.5 \times 10^{-7} \text{ C}$$
$$q_2 = 6.5 \times 10^{-7} \text{ C}$$
$$r = 50 \text{ cm} = 0.50 \text{ m}$$
$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$
$$F = ?$$

Coulomb's law

$$F = k\frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 6.5 \times 10^{-7} \times 6.5 \times 10^{-7}}{(0.50)^2} \text{ N}$$

$$F = 1.5 \times 10^{-2} \text{ N}$$

(*b*) Now, if each sphere is charged double, and the distance between them is halved then the force of repulsion is:

$$F = k.\frac{2q_1\, 2q_2}{(r/2)^2}$$

$$F = 16\,k.\frac{q_1 q_2}{r^2} = 16 \times 1.5 \times 10^{-2} = 24 \times 10^{-2}$$

$$F = 0.24 \text{ N.}$$

1.13. *Suppose the spheres A and B in question 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?*

Sol. Charge on each of the sphere A and B

$$= q = 6.5 \times 10^{-7} \text{ C}$$

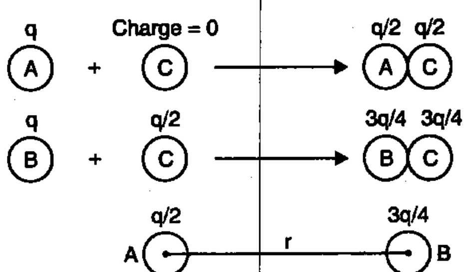

Fig. 1.12

When a similar but uncharged sphere C is placed in contact with sphere A, each sphere shares a charge $q/2$, equally.
Now, if the sphere C is placed in contact with sphere B, the charge is equally redistributed,
so that

Charge on sphere B or $C = \frac{1}{2}(q + q/2) = 3q/4$

Thus, the force of repulsion between A and B is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\frac{3q}{4} \cdot q/2}{(r/2)^2}$$

$$= \frac{3}{8} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2} = \frac{3}{8} \times 1.5 \times 10^{-2} \text{ N}$$

$$= 0.5625 \times 10^{-2} \text{ N} = 5.7 \times 10^{-3} \text{ N}.$$

1.14. *Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?*

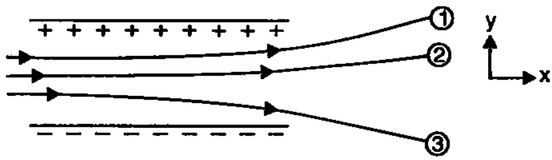

Fig. 1.13

Sol. Particles (1) and (2) are negatively charged and particle (3) is positively charged, since the charged particles are deflected towards oppositely charged plates.

Further, as the displacement $y \propto (e/m)$ therefore, particle (3) having maximum value of y has the highest charge to mass ratio.

1.15. *Consider a uniform electric field E = 3 × 10³ $\hat{i}$ N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?*

Sol. Given $\vec{E} = 3 \times 10^3 \hat{i}$ NC^{-1}

(*a*) ΔS (Area of the square)

$= 10 \times 10 = 100$ cm^2

$= 10^{-2}$ m^{-2}

The area of a surface can be represented as a vector along normal to the surface. Since normal to the square is along x-axis, we have

$$\Delta \vec{S} = 10^{-2} \hat{i} \text{ m}^{-2}$$

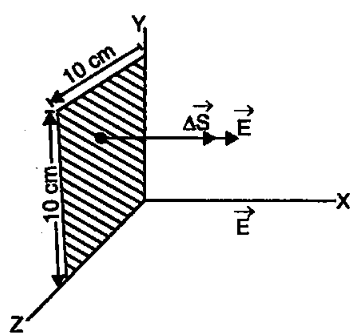

Fig. 1.14

Electric flux through the square

$$\boxed{\phi = \vec{E} \cdot \overrightarrow{\Delta S}}$$

$$= \left(3 \times 10^3\, \hat{i}\right) . \left(10^{-2}\, \hat{i}\right) = 30 \text{ Nm}^2 \text{ C}^{-1}$$

(*b*) Given, the angle between area vector and the electric field is 60°. Therefore,

$$\phi = \vec{E} . \Delta \vec{S} = E . dS \cos 60°$$

$$= 3 \times 10^3 \times 10^{-2} \times \frac{1}{2} = 15 \text{ Nm}^2 \text{ C}^{-1}$$

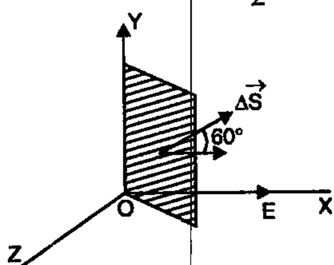

Fig. 1.15

1.16. *What is the net flux of the uniform electric field of question 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?*

Sol. Given, $\vec{E} = 3 \times 10^3\, \hat{i}$ NC^{-1}

The area of each face out of the six faces of the cube = 20 × 20 = 400 cm^2 = 4 × 10^{-2} m^2.

The area vector of four faces of cube *ABGH, OBGF, OCDF* and *ACDH* is along +Y axis, –Z axis, –Y axis and +Z axis. The direction of E (+X axis). $\overrightarrow{\Delta S}$ is perpendicular to each other so flux through there surfaces are zero.

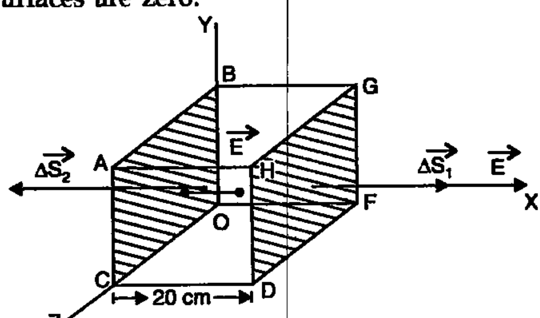

Fig. 1.16

Hence, the net electric flux through the cube

$$\phi = \vec{E}.\Delta\vec{S}_1 + \vec{E}.\Delta\vec{S}_2$$

Now, $|\Delta\vec{S}_1| = |\Delta\vec{S}_2| = 4 \times 10^{-2}$ m^2 and the angle between $\vec{E}$ and $\Delta\vec{S}_1$ is 0°, whereas the angle between $\vec{E}$ and $\Delta\vec{S}_2$ is 180°.

Thus,
$$\phi = E\,\Delta S_1 \cos 0° + E\,\Delta S_2 \cos 180°$$
$$= E \times 4 \times 10^{-2} \times 1 + E \times 4 \times 10^{-2} \times (-1)$$
$$= 0$$

Now, it is established that if some electric flux enters the cube the same amount of flux leaves through the other face, so that the net flux is zero.

1.17. *Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0 × 10^3 Nm^2/C. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?*

Sol. Given, $\phi = 8.0 \times 10^3$ Nm^2/C

(*a*) As $\boxed{\phi = \dfrac{q}{\varepsilon_0}}$

Hence, $q = \phi \,.\, \varepsilon_0$

or $q = 8.0 \times 10^3 \times 9 \times 10^{-12} = 0.07 \times 10^{-6}$

i.e., $q = 0.07$ μC

(*b*) No, a conclusion cannot be made that there was no charge in the box. Perhaps, the net charge inside the box is zero.

1.18. *A point charge + 10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. What is the magnitude of the electric flux through the square?*

(Hint: Think of the square as one face of a cube with edge 10 cm).

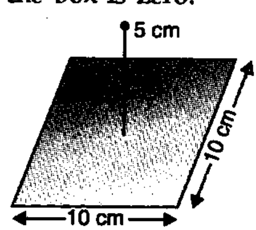

Fig. 1.17

Sol. Let us assume that the charge $q = \pm 10$ μC $= 10^{-5}$ C is placed at a distance of 5 cm from the square *ABCD* of each side 10 cm. The square *ABCD* can be considered as one of the six faces of a cubic Gaussian surface of each side 10 cm.

Now, the total electric flux through the faces of the cube as per Gaussian theorem

$$\phi = \frac{q}{\varepsilon_0}$$

Therefore, the total electric flux through the square *ABCD* will be

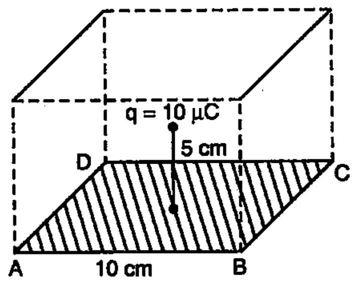

Fig. 1.18

$$\phi_E = \frac{1}{6} \times \phi = \frac{1}{6} \times \frac{q}{\varepsilon_0}$$

$$= \frac{1}{6} \times \frac{10^{-5}}{8.854 \times 10^{-12}} = 1.88 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}$$

1.19. *A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?*

Sol. Given, $q = 2.0 \ \mu C = 2.0 \times 10^{-6}$ C

The total flux through the surface of the cube (using Gaussian theorem) is given by

$$\phi = \frac{q}{\varepsilon_0}$$

$$= \frac{2.0 \times 10^{-6}}{8.854 \times 10^{-12}} = 2.26 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}$$

1.20. *A point charge causes an electric flux of -1.0×10^3 Nm²/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?*

Sol. Given, $\phi = -1.0 \times 10^3$ Nm²/C

$r_1 = 0.1$ m, $r_2 = 0.2$ m

(*a*) Doubling the radius of Gaussian surface will not affect the electric flux since the charge enclosed is the same in the two cases.

Thus, the flux will remain be the same *i.e.*, -1.0×10^3 Nm²/C

(*b*) $$\phi = \frac{q}{\varepsilon_0}$$

$\therefore \quad q = \phi \cdot \varepsilon_0$

or, $\quad q = -1.0 \times 10^3 \times 8.8 \times 10^{-12}$

$= -8.8 \times 10^{-9}$ C

$q = -8.8$ NC

1.21. *A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?*

Sol. Given, $r = 10$ cm $= 0.1$ m

$E = 1.5 \times 10^3$ N/C at $d = 0.2$ m

As, $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

then, $q = E \cdot 4\pi\,\varepsilon_0 \cdot r^2$

or, $$q = 1.5 \times 10^3 \times 4\pi \times \left(\frac{1}{4\pi \times 9 \times 10^9}\right) \times (0.2)^2$$

or, $$q = \frac{6}{9} \times 10^{-8} = \frac{60}{9} \times 10^{-9}$$

$= 6.67 \times 10^{-9}$ C

Here, q is negative since electric field is directed inward.

Thus, $q = 6.67 \times 10^{-9}$ C

$= -\,6.67$ nC.

1.22. *A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m². (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?*

Sol. Given, $$r = \frac{2.4}{2} = 1.2 \text{ m}$$

$\sigma = 80 \times 10^{-6}$ C/m²

(*a*) Charge on sphere

$$q = \sigma \cdot A = \sigma \cdot 4\pi r^2$$

or, $q = 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$

$q = 1.45 \times 10^{-3}$ C

(*b*) The total electric flux leaving the surface of the sphere

$$\phi = \frac{q}{\varepsilon_0} = \frac{1.45 \times 10^{-3}}{9 \times 10^{-12}} = 1.6 \times 10^8 \text{ Nm}^2/\text{C}$$

1.23. *An infinite line charge produces a field of 9×10^4 N/C at distance of 2 cm. Calculate the linear charge density.*

Sol. Given, $E = 9 \times 10^4$ N/C

$r = 2 \times 10^{-2}$ m

As, $$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

$\therefore$ $$\lambda = E \cdot 2\pi r \cdot \varepsilon_0$$

$$\lambda = \frac{E \cdot 2\pi r}{4\pi K}$$

or, $$\lambda = \frac{9 \times 10^4 \times 2\pi \times 2 \times 10^{-2}}{4\pi \times 9 \times 10^9} = \lambda = \frac{E \, . \, 2\pi r}{4\pi \, k}$$

$$= 10^{-7} = 10 \times 10^{-6}$$

or, $\lambda = 10$ μC/m.

1.24. *Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0 × 10^{-22} C/m². What is E: (a) in the outer region of the first plate, (b) in the outer region of second plate, and (c) between the plates?*

Sol. Given, $\sigma = 17.0 \times 10^{-22}$ C/m²

(*a*) To the left of the plates, electric fields are equal and opposite as plates are close to each other electric field is zero as surface charge density in outer side is zero.

(*b*) To the right of the plates, electric fields are equal and opposite as plates are close to each other electric field is zero.

(*c*) Electric fields between the plates are in same direction as total E.F. on both sides of plate due to σ surface charge density $= \dfrac{\sigma}{\varepsilon_0}$

So EF. of inner side of plate $= \dfrac{\sigma}{2\varepsilon_0}$

and for both plate $$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \sigma \times 4\pi \times 9 \times 10^9$$

or, $$E = 17.0 \times 10^{-22} \times 4 \times 3.14 \times 9 \times 10^9$$

or, $$E = 1921.7 \times 10^{-13} = 1.92 \times 10^{-10} \text{ N/C.}$$

1.25. *An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55 × 10^4 NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. (g = 9.81 ms^{-2}: e = 1.60 × 10^{-19} C).*

Sol. Given, $E = 2.25 \times 10^4 \text{ NC}^{-1}$, $n = 12$

$\rho = 1.26 \text{ gm cm}^{-3}$ or $1.26 \times 10^3 \text{ kg m}^{-3}$

Since, the droplet is stationary weight of the droplet = force due to the electric field

$$\therefore \quad \frac{4}{3}\pi r^3 \rho g = Ene \qquad \boxed{mg = Eq}$$

or, $$r^3 = \frac{3Ene}{4\pi \rho g}$$

or $$r^3 = \frac{3 \times 2.55 \times 10^4 \times 12 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81} = 0.9 \times 10^{-18}$$

or $$r = (0.9 \times 10^{-18})^{1/3}$$

$$r = 9.81 \times 10^{-7} \text{ m.}$$

1.26. *Which among the curves shown in the figure cannot possibly represent electrostatic field lines?*

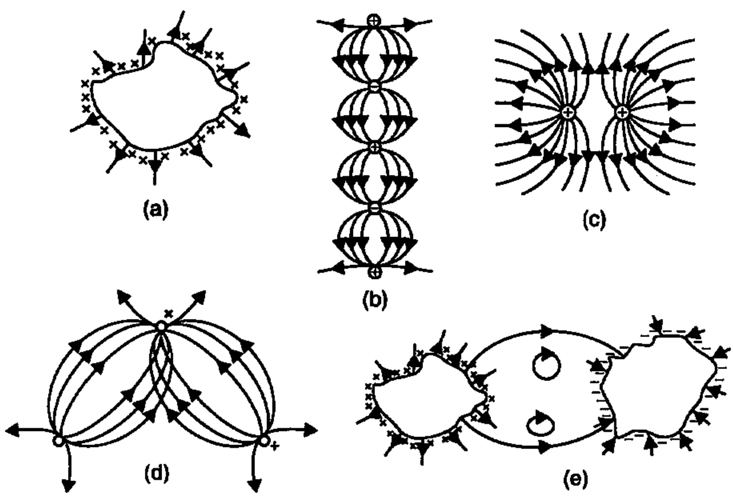

Fig. 1.19

Sol. (*a*) Figure (*a*) cannot represent electrostatic field lines since electrostatic field lines start or end only at 90° to the surface of the conductor.

(*b*) Figure (*b*) too cannot represent electrostatic field lines as electrostatic field lines do not start from a negative charge.

(*c*) Electrostatic field lines are represented by figure (*c*).

(*d*) Figure (*d*) cannot represent electrostatic field lines since no two such lines of force can intersect each other.

(*e*) As electrostatic field lines cannot form closed loop, therefore figure (*d*) also does not represent electrostatic field lines.

1.27. *In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} cm in the negative z-direction?*

Sol. Force acting on an electric dipole in the positive z-direction which is placed in a non-uniform electric field.

$$F = p_x \frac{\partial E}{\partial x} + p_y \frac{\partial E}{\partial y} + p_z \frac{\partial E}{\partial z}$$

As, the electric field changes uniformly in the positive z-direction, only,

Thus,
$$\frac{\partial E}{\partial z} = +10^5 \text{ NC}^{-1} \text{ m}^{-1}$$
$$\frac{\partial E}{dy} = 0 \quad \text{and} \quad \frac{dE}{dx} = 0$$

As, the system has the total dipole moment equal to 10^{-7} cm in the *negative* z-direction, Thus,

$$p_x = 0, \quad p_y = 0, \quad p_z = -10^{-7} \text{ cm}$$

$\therefore$
$$F = 0 + 0 - 10^{-7} \times 10^5 = -10^{-2} \text{ N}$$

It is indicated by the negative sign that the force 10^{-2} N acts in the negative z-direction.

In an electric field $\vec{E}$, the torque on dipole moment $\vec{P}$ is given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$|\vec{\tau}| = pE \sin \theta$$

As $\vec{P}$ and $\vec{E}$ are acting in opposite direction,

$$\theta = 180°,$$

so,
$$|\vec{\tau}| = pE \sin 180° = 0.$$

1.28. *(a) A conductor A with a cavity as shown in figure (a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [Fig. (b)]. (c) A sensitive instrument* *is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.*

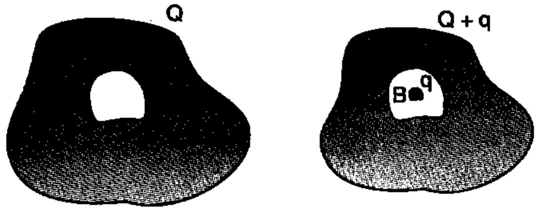

Fig. 1.20

Sol. (*a*) Let us take a Gaussian surface which is lying completely within the conductor and enclosing the cavity. According to the Gaussian theorem the charge enclosed by Gaussian surface must be zero as *electric field vanishes everywhere inside a conductor.* Thus, electric field vanishes inside the cavity. Therefore, charges which are supplied to the conductor reside on its outer surface.

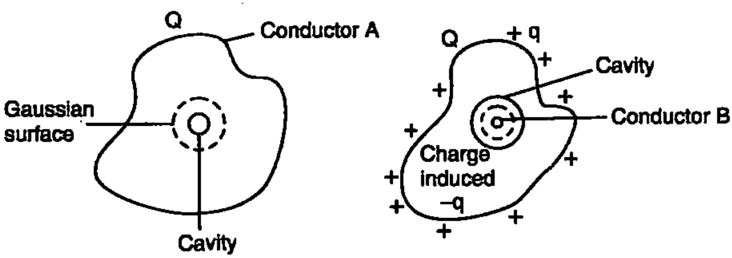

Fig. 1.21

(*b*) Let us take a Gaussian surface inside the conductor which is quite close to the cavity. According to the Gaussian theorem

$$\phi_E = \int E \cdot ds = \frac{\text{total charge}}{\varepsilon_0}$$

(as the electric field inside the conductor is zero)

The total charge enclosed by the Gaussian surface must be zero. This requires a charge of $-q$ units to be induced on the inner surface of the hollow conductor A. But an equal and opposite charge $+q$ units must appear on the outer surface of conductor A, so that the total charge on the outer surface of A is $Q + q$.

(c) Use a metallic surface to enclose the sensitive instrument fully safe. Because of the electrostatic shielding, the electric field inside the metal surface vanishes to zero and all charge reside on outer surface.

1.29. *A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is* $(\sigma/2\varepsilon_0)\,\hat{n}$, *where* $\hat{n}$ *is the unit vector in the outward normal direction, and* σ *is the surface charge density near the hole.*

Sol. Let us take a charged conductor with the hole filled up, as shown by shaded portion in the figure.

We find with the application of Gaussian theorem that field inside is zero and just outside is $\frac{\sigma}{\varepsilon_0}\hat{n}$.

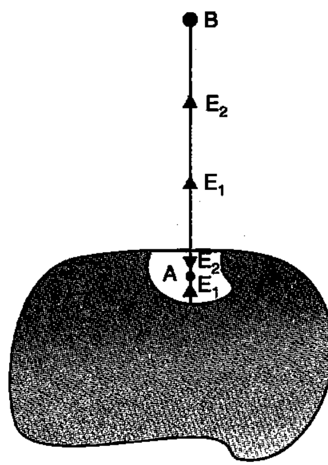

Fig. 1.22

This field can be viewed as the superposition of the field E_2 due to the filled up hole plus the field E_1 due to the rest of the charged conductor.

The two fields (E_1 and E_2) must be equal and opposite as the field vanishes inside the conductor.

Thus, $E_1 - E_2 = 0$

Now, the field outside the conductor is given by

$$E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$$

$$\therefore \qquad 2E_1 = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad E_1 = \frac{\sigma}{2\varepsilon_0}$$

Therefore, field in the hole (due to the rest of the conductor) is given as:

$$E_1 = \frac{\sigma}{2\varepsilon_0}\hat{n}$$ ($\hat{n} \rightarrow$ unit vector in the outward normal direction)

1.30. *Show that electric field due to line charge at any plane is same in magnitude and directed radially upward.*

Sol. A thin long straight line L of charge having uniform linear charge density λ is shown by figure. By symmetry, it follows that electric field due to line charge at distance r in any plane is same in magnitude and directed radially upward.

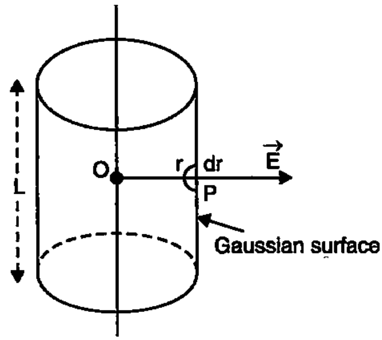

Fig. 1.23

1.31. *It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3) e, and the 'down' quark (denoted by d) of charge (–1/3) e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and a neutron.*

Sol. Charge on 'up' quark, $(u) = +\frac{2}{3}e$

Charge on 'down' quark, $(d) = -\frac{1}{3}e$

Charge on a proton = e

Charge on a neutron = 0

Let a proton contains x 'up' quarks and $(3 - x)$ 'down' quarks. Then total charge on a proton is

$$ux + d\,(3 - x) = e$$

or, $$+\frac{2}{3}ex - \frac{1}{3}e\,(3 - x) = e$$

or, $$+\frac{2}{3}x - 1 + \frac{x}{3} = 1$$

or, $$x = 2$$

and $$3 - x = 3 - 2 = 1$$

i.e., proton contains 2 'up' quarks and 1 'down' quark. Its quark composition should be *uud*.

Let a neutron contains y 'up' quarks and $(3 - y)$ 'down' quarks.

Then total charge on a neutron is

$$ny + d\,(3 - y) = 0$$

or, $$+\frac{2}{3}ey - \frac{1}{3}e(3-y) = 0$$

or, $$+\frac{2}{3}y - 1 + \frac{y}{3} = 1$$

or, $$y = 1$$

and $$3 - y = 3 - 1 = 2$$

i.e., neutrons contain 1 'up' quark and 2 'down' quarks. Its quark composition should be *udd*.

1.32. *(a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where* **E** *= 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.*

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Sol. (*a*) It can be proved by contradiction. Assume that the test charge placed at null point be in stable equilibrium. The test charge displaced slightly in any direction will experience a restoring force towards the null-point as the stable equilibrium requires restoring force in all directions. That is, all field lines near the null point should be directed inwards towards the null point. This indicates that there is a net inward flux of electric field through a closed surface around the null point. But, according to Gauss law, the flux of electric field through a surface enclosing no charge must be zero. This contradicts our assumption. Therefore, the test charge placed at null point must be necessarily in unstable equilibrium.

(*b*) On the mid-point of the line joining the two charges, the null point lies. The test charge will experience a restoring force if it is displaced slightly on either side of the null point along this line. While the net force takes it away from the null point if it is displaced normal to this line. That is no restoring force acts in the normal direction. But restoring force in all directions is demanded by stable equilibrium, therefore, test charge placed at null point will not be in stable equilibrium.

1.33. *A particle of mass m and charge −q enters the region between the two charged plates initially moving along x-axis with speed v_x. The length of plate is L and a uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m\,v_x^2)$.*

Sol. The particle is moving along x-axis in a uniformly charged electric field E between two oppositely charged metallic plates of length L. The motion of a charge particle in an electric field is analogous to the motion of a projectile in the gravitational field. The only difference is that here the constant electric field is upward direction and is limited to the region between the plates of length L.

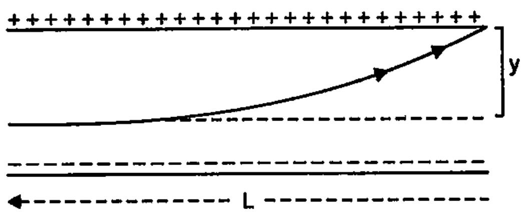

Fig. 1.24

Since x-component of the electric force is zero therefore, acceleration along x-axis is zero. So, the velocity v_x along x-axis is constant. If x is the horizontal distance covered in time t, then

$$x = v_x t \quad \text{or} \quad t = \frac{x}{v_x}$$

Force acting along y-axis, $F_y = qE$

Acceleration along y-axis, $a_y = \frac{qE}{m}$

where m is the mass of charged particle (electron)

If y is the vertical distance covered by the particle in time t, then

$$y = u_y + a_y t^2$$

$$y = \frac{1}{2} a_y t^2$$

[$\because$ Initial velocity is zero]

or,
$$y = \frac{1}{2} \frac{qE}{m} \left(\frac{x}{v_x}\right)^2$$

$\therefore$
$$y = \frac{qE}{2mv_x^2} x^2$$

So, within the electric field, the particle follows a parabolic path. Let y_1 be the vertical deflection suffered by the particle inside the electric field.

When $x = L$, then $y = y_1$

$$\therefore \quad y_1 = \frac{q E L^2}{2m v_x^2}$$

1.34. *Suppose that the particle in Question 1.33 is an electron projected with velocity $v_x = 2.0 \times 10^6$ m s^{-1}. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.)*

Sol. Acceleration, $a = \dfrac{qE}{m}$

$$= \frac{1.6 \times 10^{-19} \times 9.1 \times 10^2}{9.1 \times 10^{-31}} = 1.6 \times 10^{14} \text{ m/s}^2$$

Using formula $y = ut + \frac{1}{2}at^2$,

We get, $0.005 = 0 + \frac{1}{2} \times 1.6 \times 10^{14} \times t^2$

Simplifying for value of t, we get

$$t = 8 \times 10^{-9} \text{ s}$$

The electron covers vertical distance is shown as

$$y = v_x t$$
$$= 2.0 \times 10^6 \times 8 \times 10^{-9}$$
$$= 1.6 \times 10^{-2} \text{ m} = 1.6 \text{ cm}$$

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