2 Electrostatic Potential and Capacitance

Lesson at a Glance

• Electric Potential Energy

When an electric charge is brought near to another electric charge, the work is to be done against electrostatic force. This amount of work done is stored in the form of electrostatic energy in the system.

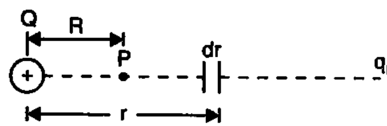

Fig. 2.1

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq_0}{R}$$

• Electric Potential

It is defined as the amount of work done in bringing a unit positive charge from infinity to the point of consideration in electric field.

Thus, electric potential,

$$V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq_0}{R} / q_0$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} \qquad \text{[where } Q \text{ is the source charge]}$$

• Potential Difference

Difference of potential across two points or the amount of work done in bringing a unit positive charge from one point to another is called the potential difference.

$$V_{AB} = \frac{1}{4\pi\varepsilon_0} Q \left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$

• Electric Potential Due to a Dipole

(i) At axial position

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{r^2}$$

***(ii)* At equitorial position**

$$V = 0$$

• Equipotential Surface

The surface at which potential at all points is same called equipotential surface. The work done across any two points on equipotential surface is always zero. The electric field is always normal to the equipotential surface.

• Capacitance

$$C = \frac{q}{V}$$

• Capacitance of a Spherical Body

Let q charge is given to a body of radius R to raise its potential by V volt.

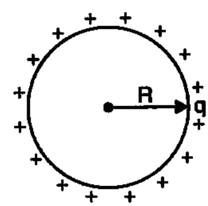

Fig. 2.2

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

and capacitance,

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}}$$

or $$C = 4\pi\varepsilon_0 R$$

For a medium of dielectric constant ε_r

$$C = 4\pi\varepsilon_0\varepsilon_r R.$$

• Capacitor

When a metallic plate is placed near to a charged plate the electric field in the surrounding of these plates becomes zero and the surrounding medium remains unaffected. This combination can be given now more charge and thus is called a capacitor. A dielectric medium can be put between the plate of the capacitor to decrease the intensity of electric field.

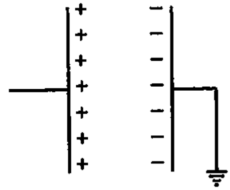
Fig. 2.3

In presence of dielectric, the capacitor can store more charge.

• Capacitance of a Parallel Plate Capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

• Combination of Capacitors

(i) Series Combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

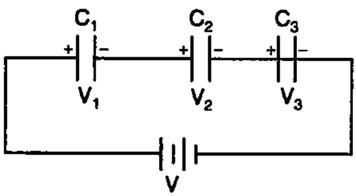

Fig. 2.4 Series Combination

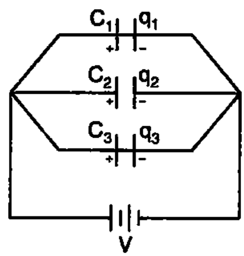

Fig. 2.5 Parallel Combination

(ii) Parallel Combination

$$C = C_1 + C_2 + C_3$$

• Capacitance of a Parallel Plate Capacitor Having Dielectric Partially Filled between Its Plate

$$C = \frac{\varepsilon_0 A k}{d}$$

• Energy Stored in a Capacitor

$$U = \frac{1}{2}\frac{q^2}{C}$$

• Common Potential

When two capacitors, charged to the different potential, are connected the redistribution of charge takes place and the capacitors acquired same potential called common potential.

Common potential $$V_C = \frac{\text{Net charge stored}}{\text{Net capacitance}}$$

$$= \frac{q_1 + q_2}{C_1 + C_2} \quad \text{or} \quad V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

• Energy Loss

$$\Delta U = \frac{1}{2}\frac{C_1C_2(V_1 - V_2)^2}{(C_1 + C_2)}$$

$\because$ ΔU is always positive $\therefore$ There is always loss of energy due to heat dissipation in connecting wire.

TEXTBOOK QUESTIONS SOLVED

2.1. *Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.*

Sol. The other possibility of point where total potential due to q_1 and q_2 are zero may be that point lies out side the segment joining q_1 and q_2

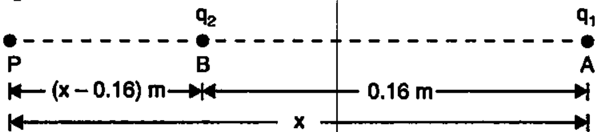

Fig. 2.6

Let required point P lies x m from q_1 then $V_1 + V_2 = 0$

$$\frac{Kq_1}{x} + \frac{Kq_2}{(x-0.16)} = 0$$

$$K\left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{(x-0.16)}\right] = 0$$

or
$$\frac{5}{x} - \frac{3}{x-0.16} = 0$$

$$\frac{5x - 0.8 - 3x}{x(x-0.16)} = 0$$

$$2x - 0.8 = 0$$

$$x = \frac{0.8}{2}$$

$$x = 0.4 \text{ m} = 40 \text{ cm}$$

required point is 40 cm away from q_1 and (40 – 16) = 24 cm from q_2.

2.2. *A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.*

Sol. Given,
$$q = 5 \times 10^{-6} \text{ C}$$
$$r = 10 \text{ cm} = 0.1\text{m}$$

Potential at the centre of the hexagon is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{q}{r}+\frac{q}{r}+\frac{q}{r}+\frac{q}{r}+\frac{q}{r}\right]$$

$$= 6\times\left(\frac{1}{4\pi\varepsilon_0}\right)q/r$$

or, $$V = \frac{6\times9\times10^9\times5\times10^{-6}}{0.1}$$

$$= 2.7 \times 10^6 \text{ V}.$$

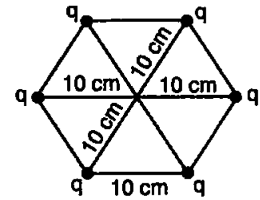

Fig. 2.7

2.3. *Two charges 2μC and –2μC are placed at points A and B, 6 cm apart.*

(a) Identify an equipotential surface of the system.

(b) What is the direction of electric field at every point on this surface?

Sol. Given, $q_1 = 2\mu C = 2 \times 10^{-6}$ C

$q_2 = -2\mu C = -2 \times 10^{-6}$ C

$r = 0.06$ m

(*a*) Potential will be zero due to both charges at equipotential surface

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x}+\frac{q_2}{(0.06-x)}\right] = 0$$

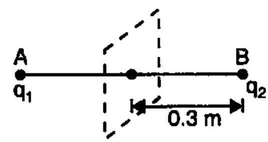

Fig. 2.8

or, $$\frac{q_1}{x} = -\frac{q_2}{(0.06-x)}$$

or, $$\frac{2\times10^{-6}}{x} = -\frac{(-2\times10^{-6})}{[(0.06)-x]}$$

or, $$x = 0.06 - x$$

$$x = \frac{0.06}{2} = 0.03 \text{ m}$$

i.e., the plane normal to *AB* and passing through its mid-point has zero potential everywhere.

(*b*) The direction of electric field is normal to the plane in the *AB* direction.

2.4. *A spherical conductor of radius 12 cm has a charge of 1.6 × 10⁻⁷ C distributed uniformly on its surface. What is the electric fields*

(a) inside the sphere,

(b) just outside the sphere,

(c) at a point 18 cm from the centre of the sphere?

Sol. Given, $q = 1.6 \times 10^{-7}$ C, $r = 0.12$ m

(*a*) Electric field is zero inside the sphere, because charge reside on outer surface of conductor.

(*b*) Electric field just outside the sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 9\times10^9\times\frac{1.6\times10^{-7}}{(0.12)^2} = 10^5\ NC^{-1}$$

(*c*) At a point 0.18 m from the centre of the sphere

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 9\times10^9\times\frac{1.6\times10^{-7}}{(0.18)^2}$$

$$= 4.4 \times 10^4\ NC^{-1}$$

2.5. *A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12}F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?*

Sol. Given, $C = 8\ pF = 8 \times 10^{-12}$ F

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{or,} \quad \frac{\varepsilon_0 A}{d} = 8 \times 10^{-12}$$

If the distance is reduced to half *i.e.*, $d/2$ and space between the plates is filled by a substance of $k = 6$.

$$C = kC_0$$

Then, $$C = \frac{k\varepsilon_0 . A}{d/2} = \frac{2k(\varepsilon_0 A)}{d}$$

$$= 2 \times 6 \times 8 \times 10^{-12} = 96 \times 10^{-12}\ \text{F}$$

or, $C = 96$ pF.

2.6. *Three capacitors each of capacitance 9 pF are connected in series:*

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Sol. Given, three capacitors

$$C_1 = C_2 = C_3 = 9\ \text{pF}$$
$$= 9 \times 10^{-12}\ \text{F}$$

(*a*) Since the capacitors are connected in series.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{9\times10^{-12}} + \frac{1}{9\times10^{-12}} + \frac{1}{9\times10^{-12}}$$

$$= \frac{1}{10^{-12}}\left[\frac{1}{9}+\frac{1}{9}+\frac{1}{9}\right]$$

or, $$\frac{1}{C} = \frac{1}{10^{-12}}\times\frac{3}{9} = \frac{1}{3\times10^{-12}}$$

or, $$C = 3\times10^{-12} = 3 \text{ pF}$$

(*b*) Given, $$V = 120 \text{ volt}$$

$$\boxed{V = V_1 + V_2 + V_3}$$

as capacitances are equal so

$$V_1 = V_2 = V_3 = V' \text{ (let)}$$

$$V = 3V'$$

$$V' = \frac{V}{3} = \frac{120}{3} \qquad (\because C_1 = C_2 = C_3 = C)$$

$$= 40 \text{ volt.}$$

2.7. *Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.*

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Sol. Given, $$C_1 = 2 \text{ pF} = 2\times10^{-12} \text{ F}$$

$$C_2 = 3 \text{ pF} = 3\times10^{-12} \text{ F}$$

$$C_3 = 4 \text{ pF} = 4\times10^{-12} \text{ F}$$

(*a*) Since the capacitors are connected in parallel then

$$\boxed{C = C_1 + C_2 + C_3}$$

$$= (2+3+4)\times10^{-12}$$

$$= 9\times10^{-12} = 9 \text{ pF}$$

(*b*) Given, $$V = 100 \text{ volt}$$

$$\boxed{q_1 = C_1V} = 2\times10^{-12}\times100$$

$$= 2\times10^{-10} \text{ C}$$

$$q_2 = C_2V = 3\times10^{-12}\times100$$

$$= 3\times10^{-10} \text{ C}$$

$$q_3 = C_3V = 4\times10^{-12}\times100$$

$$= 4\times10^{-10} \text{ C}$$

2.8. *In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?*

Sol. Given,

$$d = 3 \times 10^{-3}\ \text{m}$$
$$A = 6 \times 10^{-3}\ \text{m}^2$$
$$C = \frac{\varepsilon_0 \cdot A}{d} = \frac{9\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$
$$= 18 \times 10^{-12}\ \text{F} = 18\text{pF}$$

Here,

$$V = 100\ V$$
$$q = CV = 18 \times 10^{-12} \times 100$$
$$= 1.8 \times 10^{-9}\ \text{C}$$

2.9. *Explain what would happen if in the capacitor given in Question 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,*

(a) while the voltage supply remained connected.

(b) after the supply was disconnected.

Sol. (*a*) Given,

$$k = 6$$
$$t = d = 3\ \text{mm} = 3 \times 10^{-3}\ \text{m}$$
$$C = kC_0$$
$$C = 6 \times 18 \times 10^{-12}\ \text{F}$$
$$= 108 \times 10^{-12}\ \text{F}$$
$$= 108\ \text{F}$$

As,

$$Q = CV = 108 \times 10^{-12} \times 100$$
$$= 108 \times 10^{-10}$$

or,

$$Q = 1.08 \times 10^{-8}\ \text{C}$$

while voltage supply remained connected,

$$C = 108\ \text{pF and } Q = 1.08 \times 10^{-8}\ \text{C}$$

(*b*) After the supply was disconnected, the charge remains same

i.e.,

$$q = 1.8 \times 10^{-9}\ \text{C}$$

As,

$$Q = CV$$
$$V = \frac{Q}{C} = \frac{1.8\times10^{-9}}{108\times10^{-12}} = 16.66\ \text{V}$$

The charge remains constant *i.e.,* $Q = 1.08 \times 10^{-8}$ C after the supply was disconnected, the voltage will come down to 16.6 V.

2.10. *A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?*

Sol. Given,

$$V = 50\ V$$
$$C = 12\ \text{pF} = 12 \times 10^{-12}\ \text{F}$$

Electrostatic energy stored in the capacitor is given as

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times 12\times 10^{-12}\times (50)^2$$

or, $U = 1.5 \times 10^{-8}$ J.

2.11. *A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?*

Sol. Given, $V = 200$ V

$C = 600 \text{ pF} = 600 \times 10^{-12}$ F

$$\boxed{Q = CV}$$

$= 600 \times 10^{-12} \times 200$

or, $Q = 12 \times 10^{-8}$ C

Electrostatic energy stored in capacitor

$$\boxed{U_1 = \frac{1}{2}CV^2} = \frac{1}{2}\times 600\times 10^{-12}\times (200)^2$$

$= 12 \times 10^{-6}$ J

Now, the supply is disconnected and the capacitor is connected to another similar uncharged capacitor. Therefore, the *charge is divided equally between the two capacitors.*

Hence, $$Q_1 = Q_2 = \frac{12\times 10^{-8}}{2} = 6 \times 10^{-8}\text{ C}$$

and $$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{6\times 10^{-8}}{600\times 10^{-12}} = 100\text{ V}$$

Total capacitance $C = C_1 + C_2$

$= 600 \times 10^{-12}\text{ F} + 600 \times 10^{-12}$ F

$= 1200 \times 10^{-12}$ F

Now, the electrostatic energy stored is given as

$$U_2 = \frac{1}{2}CV^2 = \frac{1}{2}\times 1200\times 10^{-12}\times (100)^2$$

or, $U_2 = 6 \times 10^{-6}$ J

Electrostatic energy lost in the process

$= U_1 - U_2$

$= 12 \times 10^{-6} - 6 \times 10^{-6} = 6 \times 10^{-6}$ J.

2.12. *A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P(0, 0, 3 cm) to a point Q(0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).*

Sol. Given charge q = 8 mC = 8×10^{-3} C is located at origin and the small charge ($q_0 = -2 \times 10^{-9}$ C) is taken from point P (0, 0, 3 cm) to a point Q (0, 4, cm, 0) through point R (0, 6 cm, 9 cm) which is shown in the figure.

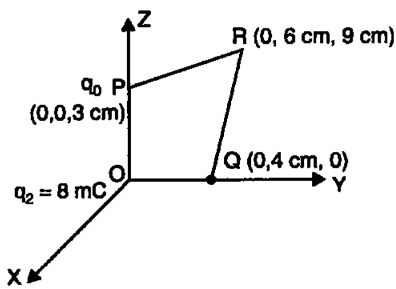

Fig. 2.9

Initial separation between q_0 and q is r_P = 3 cm = 0.03 m

Final separation between q_0 and q is r_Q = 4 cm = 0.4 m

Work done in taking the charge q_0 from point P to Q does not dependent on the path followed and depends only upon r_P and r_Q *i.e.*, initial and final positions.

$$W = \frac{1}{4\pi\varepsilon_0} q q_0 \left(\frac{1}{r_Q} - \frac{1}{r_P}\right)$$

or, $$W = 9 \times 10^9 \times 8 \times 10^{-3} \times (-2 \times 10^{-9}) \times \left(\frac{1}{0.04} - \frac{1}{0.03}\right)$$

$$= 1.2 \text{ J.}$$

2.13. *A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to these charges array at the centre of the cube.*

Sol. Diagonal DF of cube

$$DF = \sqrt{b^2 + b^2 + b^2}$$

$$DF = b\sqrt{3}$$

Thus, $DO = \frac{DF}{2} = \frac{\sqrt{3}}{2} b$

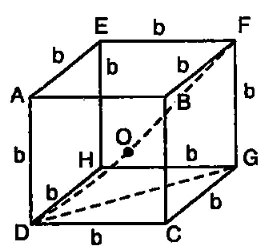

Fig. 2.10

Due to one charge q the potential at the centre O is given by

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}} = \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{\frac{\sqrt{3}b}{2}}\right)$$

Due to eight charges the total potential at the centre O is given as

$$V = 8\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{\sqrt{3}}{2}b}\right) = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

Remark: Due to two opposite corners D and F electric field intensity at the centre 'O' are equal in magnitude and opposite in direction. Therefore, they cancel out each other. Similarly all other intensities cancel out each other and the total electric field at centre is zero.

2.14. *Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:*

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this mid-point in a plane normal to the line and passing through the mid-point.

Sol. (*a*) Potential at the mid-point of the line joining the two charges is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$

$$= 9\times10^9\left[\frac{15\times10^{-6}}{0.15}+\frac{2.5\times10^{-6}}{0.15}\right]\text{V}$$

$$= 9\times10^9\times10^{-6}\left[10+\frac{50}{3}\right] = 9\times10^3\times\frac{80}{3}$$

or, $V = 2.4 \times 10^5$ V

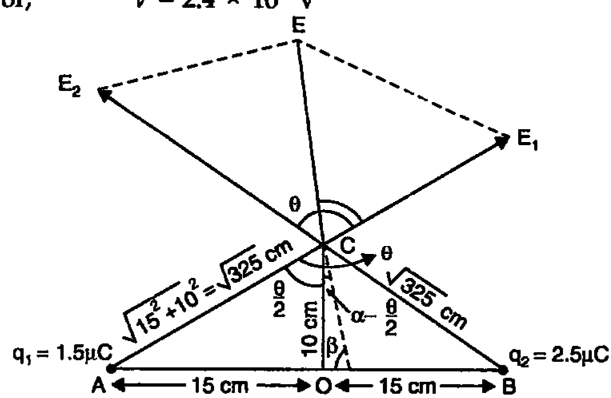

Fig. 2.11

Electric field at the mid-point O due to charge at A

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} = 9\times10^9\times\frac{1.5\times10^{-6}}{(0.15)^2}$$

$= 6 \times 10^5$ Vm^{-1} along OB

Electric field at the mid-point O due to charge at B

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} = 9\times10^9\times\frac{2.5\times10^{-6}}{(0.15)^2}$$

$= 10 \times 10^5$ Vm^{-1} along OA

Thus, the total electric field at the mid-point O is

$E = 10 \times 10^5 - 6 \times 10^5$

$= 4 \times 10^5$ Vm^{-1} (along BA)

(*b*) Potential at the point C due to the two charges is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$

$$= 9\times10^9\left[\frac{1.5\times10^{-6}}{\sqrt{325}\times10^{-2}}+\frac{2.5\times10^{-6}}{\sqrt{325}\times10^{-2}}\right]V$$

$$= \frac{9\times10^9\times10^{-6}}{10^{-2}}\cdot\frac{4.0}{\sqrt{325}}\text{V}$$

$$= \frac{9\times4}{18.02}\times10^5 V = 2 \times 10^5 \text{ V}$$

Electric field at C due to charge at A

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}$$

$$= 9 \times 10^9 \times \frac{1.5\times10^{-6}}{\left(\sqrt{325}\times10^{-2}\right)^2}\text{Vm}^{-1}$$

$$= \frac{9\times1.5}{325}\times10^7\text{ Vm}^{-1} = 4.15 \times 10^5 \text{ Vm}^{-1}$$

Electric field at C due to charge at B

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}$$

$$= 9\times10^9\times\frac{2.5\times10^{-6}}{325\times10^{-4}} = 6.92 \times 10^5 \text{ Vm}^{-1}$$

If the angle between E_1 and E_2 be θ, then

$$\tan\frac{\theta}{2} = \frac{0.15}{0.10} = 1.5$$

$$\theta/2 = 56.3^\circ \text{ or, } \theta = 112.6^\circ$$

Thus, magnitude of resultant field at C is

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

$$= \sqrt{(4.15\times10^5)^2 + (6.92\times10^5)^2 + 2\times4.15\times10^5\times6.92\times10^5\cos 112.6^\circ}$$

$$= 10^5\sqrt{17.2+47.8-2\times4.15\times6.92\cos 67.4^\circ}$$

$(\because \cos(180-\theta) = -\cos\theta)$

$$= 10^5\sqrt{43} = 6.56\times10^5\ \text{Vm}^{-1}$$

Let the field E makes angle α with the field E_1.

Now, $$\alpha = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta}$$

$$= \frac{6.92\times10^5\times0.9232}{4.15\times10^5 - 6.92\times10^5\times0.384}$$

$$= \frac{6.39}{4.15-2.66} = \frac{6.39}{1.49} = 4.2876$$

∴ $\alpha = \tan^{-1}(4.2876) \approx 76.9^\circ$

If field E makes angle β with the direction BA, then

$$\beta = 90^\circ - \left(\alpha - \frac{\theta}{2}\right) = 90^\circ + \frac{\theta}{2} - \alpha$$

$$= 90^\circ + 56.3^\circ - 76.9^\circ = 69.4^\circ$$

Therefore, angle of 69.4° is made by the electric field with the line joining the two charges. q_2 to q_1.

2.15. *A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.*

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Sol. (*a*) As a '+ q' charge place at the centre of the shell, it will create a '-q' charge on the inner surface of the shell.

The charge on the outer surface will increase by + q due to the $-q$ charge on the inner surface by induction. Therefore, there will be total $(Q + q)$ charge on the outer surface of the shell and $-q$ charge on the inner surface of the shell.

Now surface area of inner surface $= 4\pi r_1^2$

and surface area of outer surface $= 4\pi r_2^2$

Thus, charge density on the outer surface $= \dfrac{Q+q}{4\pi r_2^2}$

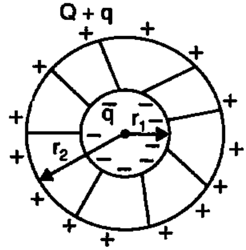

Fig. 2.12

and charge density on the inner surface $= \dfrac{-q}{4\pi r_1^2}$.

(*b*) As charge in shell reside on outer surface so, the net charge on the inner surface of the cavity is zero as per the Gaussian theorem. Although the net charge is zero yet the electric field may not be zero if the cavity is not spherical. The surface may not have equal number of positive and negative charges. We assume a loop for this reason, some portion of which is inside the cavity and rest of its part is inside the conductor. Now, consider that there is some electric field inside the cavity. Since inside the conductor total electric field is zero and net work done by the field in bringing a test charge over this loop will not be zero. But this is not possible for an electrostatic field. Therefore, we must conclude that there *is no electric field inside the cavity irrespective of its shape.*

2.16. *(a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by*

$$(\mathbf{E}_2 - \mathbf{E}_1).\hat{n} = \frac{\sigma}{\varepsilon_0}$$

where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence, show that just outside a conductor, the electric field is $\sigma\,\hat{n}/\varepsilon_0$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. **[Hint:** *For (a), use Gauss's law. (b) For, use the fact that work done by electrostatic field on a closed loop is zero.]*

Sol. (*a*) Near a plane sheet of charge, electric field is given as

$$E = \frac{\sigma}{2\varepsilon_0}$$

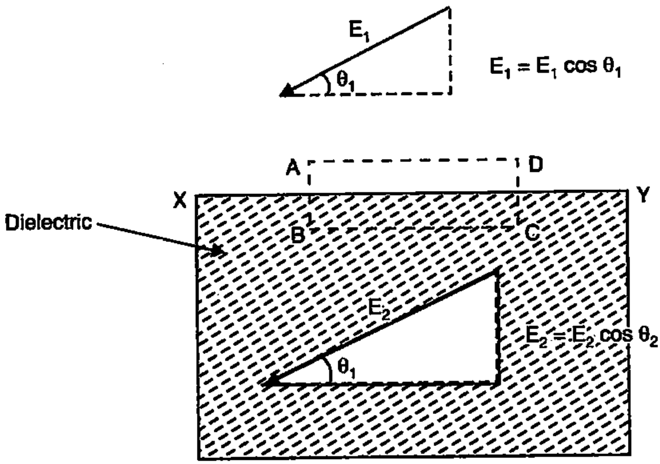

Fig. 2.13

Electric field on side 2 if $\hat{n}$ is a unit vector normal to the sheet from side 1 to side 2.

$$E_2 = \frac{\sigma}{2\varepsilon_0} \text{ (In the outward direction}$$

normal to the side 2)

Now, electric field on side 1 is given as

$$E_1 = \frac{\sigma}{2\varepsilon_0} \text{ (In the outward direction}$$

normal to the side 1)

As E_1 and E_2 are in opposite directions so will have opposite sign

$$\therefore \quad (E_2 - E_1)\,\hat{n} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}\hat{n}$$

There must be discontinuity at the sheet of charge since E_1 and E_2 act in opposite directions.

Now, electric field inside the conductor vanishes.

Hence, $E_1 = 0$

Therefore, electric field outside the conductor

$$E = E_2 = \frac{\sigma}{\varepsilon_0}\hat{n}$$

(*b*) Consider that E_1 and E_2 be the electric field on the two sides of the charged surface and *xy* be the charged surface of dielectric as shown in the figure.

Let a rectangular loop *ABCD* with length *l* and negligible small breadth. Line integral along the closed path *ABCD* will be

$$\oint E.dl = E_1 l - E_2 l = 0$$

or,
$$E_1 l \cos\theta_1 - E_2 l \cos\theta_2 = 0$$
$$(E_1 \cos\theta_1 - E_2 \cos\theta_2) l = 0$$
$$(E'_1 - E'_2) = 0$$

Where E'_1 and E'_2 are the tangential components of E_1 and E_2 respectively. Hence

$$E'_1 = E'_2$$

Therefore, the tangential component of the electrostatic field is continuous across the surface.

2.17. *A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?*

Sol. Two co-axial cylindrical shells *A* and *B* of radii *a* and *b* are possessed by a cylindrical capacitor. Assume *l* be length of the cylindrical shell. Due to the introduction of +*q* charge on the

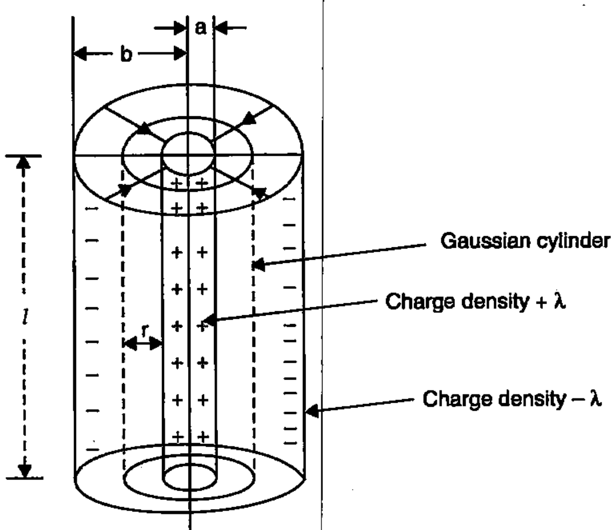

Fig. 2.14

inner cylindrical shell A, equal but opposite charge $-q$ is induced on the inner surface of the outer cylindrical shell B. The induced charge $+q$ on its outer surface will flow to earth if the shell B is earthed.

The capacitance of the cylindrical capacitor is given as follows if V is potential difference between the cylindrical shells A and B.

$$C = \frac{q}{V}$$

By applying the Gaussian theorem, we first need to find electric field E in the space between two shells to find out potential difference between the cylindrical shells A and B. Let a cylinder of radius r (such that $b > r > a$) and length l as the Gauss surface. Charge enclosed by the Gaussian surface is λl and if λ is charge per unit length on the shell A.

The electric flux will cross through only curved surface of the cylinder (Gauss surface). As the area of curved surface of cylinder is $2\pi rl$, we have by Gaussian theorem

$$\oint E.ds = \frac{q}{\varepsilon_0}$$

$$E.2\pi rl = \frac{\lambda l}{\varepsilon_0} \quad \text{or,} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The field lines are radial and normal to the axis of charged cylinder.

2.18. *In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:*

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Œ separation?

Sol. Given,

$$q_1 = -1.6 \times 10^{-19} \text{ C (electron)}$$

$$q_2 = 1.6 \times 10^{-19} \text{ C (proton)}$$

$$r = 0.53 \text{ Å} = 0.53 \times 10^{-10} \text{ m}$$

(*a*) Potential energy $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$

$$= \frac{(9\times10^9)(-16\times10^{-19})(16\times10^{-19})}{(0.53\times10^{-10})} \text{ J}$$

$$= -43.47 \times 10^{-19} \text{ J}$$

or, $$= -\frac{43.47\times10^{-19}}{1.6\times10^{-19}} eV = -27.17 \text{ eV}$$

Potential energy is zero at infinite separation. Hence, the potential energy of the system is (–27.17 – 0) or 27.17 *eV* if zero of potential energy is taken at infinite separation.

(*b*) Electron's $$\text{K.E.} = \frac{1}{2}\text{P.E.} = +\frac{27.2}{2}$$

$$= 13.6 \text{ eV}$$

Total energy of electron = – 27.2 + 13.6 = – 13.6 eV

Amount of work required to free the electron = Increase in energy of electron = 0 – (–13.6) = 13.6 eV

(*c*) At 1.06 Å (1.06 × 10^{-10} m) separation, the potential energy of system

$$W = 9\times10^9\left[\frac{(1.6\times10^{-19})\times(1.6\times10^{-19})}{1.06\times10^{-10}}\right]$$

$$= 21.74 \times 10^{-19} \text{ J} = -\frac{21.74\times10^{-19}}{1.6\times10^{-19}} = -13.585 \text{ eV}$$

If it is taken as zero of potential energy, then potential energy of the system

$$= -27.17 - (-13.585) = -13.585 \text{ eV}$$

2.19. *If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of a H_2^+, the two protons are separated by roughly 1.5Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.*

Sol. Suppose that the distance between two protons p_1 and p_2 be r_{12} and electron (e^-) is placed at r_{13} and r_{23} distance from protons p_1 and p_2 respectively.

Given, r_{12} = 1.5Å = 1.5 × 10^{-10} m

$r_{13} = r_{23}$ = 1Å = 10^{-10} m

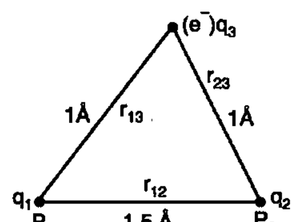

Fig. 2.15

$$q_1 = q_2 = 1.6 \times 10^{-19}\ C \text{ (protons)}$$
$$q_3 = -1.6 \times 10^{-19}\ C \text{ (electrons)}$$

Now, the potential energy of the system

$$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= 9\times10^9\left[\frac{1.6\times10^{-19}\times1.6\times10^{-19}}{1.5\times10^{-10}} + \frac{1.6\times10^{-19}\times(-1.6\times10^{-19})}{10^{-10}} + \frac{1.6\times10^{-19}\times(-1.6\times10^{-19})}{10^{-10}}\right]$$

$$W = \frac{1.6\times10^{-19}\times1.6\times10^{-19}\times9\times10^9}{10^{-10}}\left[\frac{1}{1.5} - \frac{1}{1} - \frac{1}{1}\right]$$

$$W = 1.6 \times 1.6 \times 9 \times 10^{-19-19+9+10}\left[\frac{2}{3} - 2\right]$$

$$W = -2.56 \times 9 \times 10^{-19} \times \frac{4}{3} = -2.56 \times 12 \times 10^{-19}\ J$$

$$W = \frac{-2.56\times12\times10^{-19}}{1.6\times10^{-19}}\ eV = -1.6 \times 12\ eV = -19.2\ eV$$

2.20. *Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.*

Sol. Suppose that two connected conducting spheres of radii a and b possess charges q_1 and q_2 respectively. On the surface of the two spheres, the potential will be

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b}$$

Till the potentials of two conductors become equal the flow of charges continue.

$$V_1 = V_2$$

or, $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b}$$

or, $$\frac{q_1}{q_2} = \frac{a}{b}$$

Now, the electric field on the two spheres is given as

$$\boxed{E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{a^2}}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{b^2}$$

or, $$\frac{E_1}{E_2} = \frac{q_1}{q_2}\cdot\frac{b^2}{a^2} = \frac{a}{b}\cdot\frac{b^2}{a^2} = b/a$$

Therefore, $b : a$ is the ratio of the electric field of the first sphere to that of the second sphere.

The surface charge densities of the two spheres are given as

$$\sigma_1 = \frac{q_1}{4\pi a^2}$$ (As the charges are distributed uniformly over the surfaces of conducting spheres)

$$\sigma_2 = \frac{q_2}{4\pi b^2}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2}\cdot\frac{b^2}{a^2} = \frac{a}{b}\cdot\frac{b^2}{a^2} = b/a$$

Therefore, the surface charge densities are *inversely related with the radii* of the sphere. The surface charge density on the sharp and pointed ends of a conductor is higher than on its flatter portion since a flat portion may be taken as a spherical surface of large radius and a pointed portion as that of small radius.

2.21. *Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively.*

(a) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?

(b) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Sol. (*a*) When the point $p(0, 0, z)$ is closer to charge $+q$ as shown in figure below. Electrostatic potential at the point $p\,(0, 0, z)$ is given as follows: (Fig. 2.16)

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{z-a} + \frac{-q}{z-(-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{z+a-z+a}{z^2-a^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q\cdot 2a}{z^2-a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{p}{z^2-a^2}$$

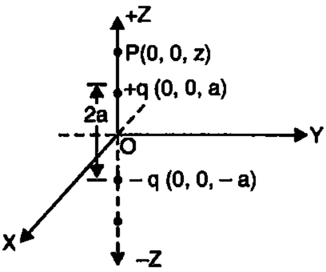

Fig. 2.16

where $p = q.2a$ (dipole moment)

Electrostatic potential at the point $p(0, 0, z)$ is given as follows when the point is closer to charge $-q$ as figure 2.16.

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{(z+a)} + \frac{-q}{(z-a)}\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{z-a-(z+a)}{z^2-a^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{z-a-z-a}{z^2-a^2}\right] = -\frac{1}{4\pi\varepsilon_0}.\frac{2qa}{z^2-a^2}$$

$$V = -\frac{1}{4\pi\varepsilon_0}.\frac{p}{z^2-a^2}$$

So potential at point P (0, 0, ± z) is

$$V = \pm\frac{1}{4\pi\varepsilon_0}.\frac{p}{(z^2-a^2)}$$

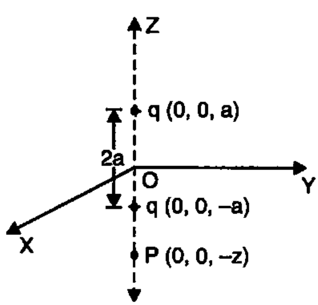

Fig. 2.17

Electric potential at point $(x, y, 0)$ is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(x-0)^2+(y-0)^2+(0-a)^2}} + \frac{-q}{\sqrt{(x-a)^2+(y-0)^2+(0+a)^2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+a^2}} - \frac{q}{\sqrt{x^2+y^2+a^2}}\right] = 0$$

(*b*) At a distance $OP = r$ from the origin electric potential

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{PA} - \frac{q}{PB}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{PA} - \frac{1}{PB}\right]$$

Now $$\frac{1}{PA} = \frac{1}{\sqrt{x^2+y^2+(z-a)^2}}$$

$$= [x^2 + y^2 + z^2 - 2az + a^2]^{-1/2}$$

$$= [r^2 - 2az + a^2]^{-1/2} \qquad [\because \ r^2 = x^2 + y^2 + z^2]$$

$$= [r^2 - 2az]^{-1/2} \qquad [\because \ r^2 >>> a^2] \quad (\because a \le r)$$

$$\frac{1}{PA} = \frac{1}{r}\left[1 - 2\frac{az}{r^2}\right]^{-1/2} = \frac{1}{r}\left[1 + \frac{az}{r^2}\right]$$

(Using Binomial theorem)

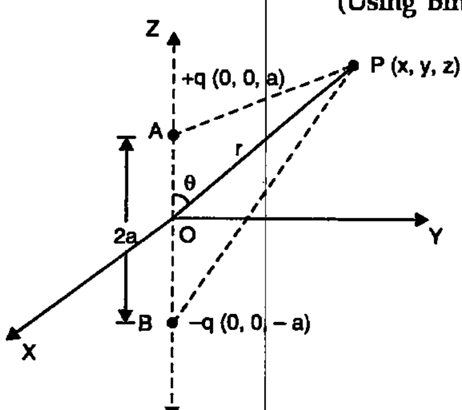

Fig. 2.18

Similarly, $$\frac{1}{PB} = \frac{1}{\sqrt{x^2+y^2+(z+a)^2}}$$

$$= \frac{1}{\sqrt{x^2+y^2+z^2+a^2+2az}}$$

$$= \frac{1}{r}\left[1-\frac{az}{r^2}\right]$$

$[\because\ x^2 + y^2 + z^2 = r^2 \text{ and } a^2 <<< r^2]$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1+\frac{az}{r^2}\right)-\frac{1}{r}\left(1-\frac{az}{r^2}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}.\frac{2az}{r^3}$$

$$V = \frac{Pr\cos\theta}{4\pi\varepsilon_0 r^2} \qquad [\because p = q.2a,\ z = r\cos\theta]$$

θ = angle between r and $+z$ axis

$\therefore$ The dependence of potential V on r is $\frac{1}{r^2}$ type, like that of a point charge for $a << r$.

(*c*) Due to the dipole, electrostatic potential at point (5, 0, 0) is given as

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(5-0)^2+(0-0)^2+(0-a)^2}} - \frac{q}{\sqrt{(5-0)^2+(0-0)^2+(0+a)^2}}\right] = 0$$

Due to dipole electrostatic potential at point (–7, 0, 0) is given as

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{(-7-0)^2+(0-0)^2+(0-a)^2}} - \frac{q}{\sqrt{(-7-0)^2+(0-0)^2+(0+a)^2}}\right] = 0$$

In moving small test charge q from the point (5, 0, 0) to (–7, 0, 0) the work done

$$W = q(V_1 - V_2) = q \times 0 = 0$$

As both points lies on X axis so, workdone by any charge along X or Y axis or on equitorial line is zero as potential on the equitorial line does not change.

Since the work done by the electrostatic field between two points is not dependent on the path connecting the two points. Therefore the answer will not change if the test charge between the same point is not along X-axis.

2.22. *Figure shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).*

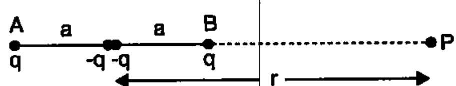

Fig. 2.19

Sol.

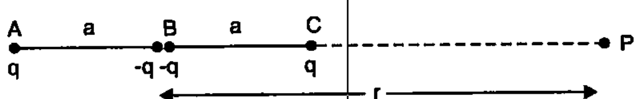

Fig. 2.20

Due to charge $+q$ at A, potential at $P = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r+a}$

Due to charge $-2q$ at B, potential at $P = \dfrac{1}{4\pi\varepsilon_0}\dfrac{-2q}{r}$

Due to charge $+q$ at C, potential at $P = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r-a}$

Total electrostatic potential at P

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r^2 - a^2) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - \cancel{ar} - 2r^2 + 2a^2 + r^2 + \cancel{ar}}{r\left(r^2 - a^2\right)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{2a^2}{r\left(r^2 - a^2\right)}$$

as $\frac{r}{a} >> 1$ *i.e.,* $r^2 >> a^2$, $r^2 - a^2 \rightarrow r^2$

Thus $\qquad V = \frac{2a^2 q}{4\pi\varepsilon_0}.\frac{1}{r^3}$

i.e., $\qquad V \propto \frac{1}{r^3}$ for a quadrupole

but $\qquad V \propto \frac{1}{r^2}$ for a dipole

and $\qquad V \propto \frac{1}{r}$ for a monopole

2.23. *An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.*

Sol. Let possible arrangement requires N capacitors of each 1 μF is n capacitors in series and m series arrangement in parallel

Total capacitors $\qquad N = m \times n$

As arrangement works on 1000 V.

P.D. across each capacitor in series arrangement is 400 V given

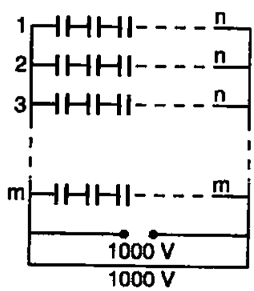

Fig. 2.21

So $\qquad \frac{1000}{n} = 400$

$n = 2.5$

As number of capacitor cannot be in fraction $\therefore$ $n = 3$ equivalent capacitors in each row of series.

$$\frac{1}{C_s} = \left[\frac{1}{n} + \frac{1}{n} + \ldots n \text{ lines}\right] = \frac{1}{C_s} = m$$

$$C_s = \frac{1}{n}\,\mu\text{F}$$

as the $\frac{1}{n}$ capacitors are in m rows so resultant capacitance of all capacitors equal to

$$\frac{1}{n}+\frac{1}{n}+\frac{1}{n}...+m \text{ lines} = 2$$

$$\frac{m}{n} = 2$$

$$\frac{m}{3} = 2$$

$$m = 6 \text{ rows}$$

$$n = 6 \times 3 = 18$$

2.24. *What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electorlytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductor.]*

Sol. Given, the separation between plates (d) = 0.5 cm = 5×10^{-3} m the capacitance (c) = 2F

Now, $$C = \varepsilon_0 \frac{A}{d}$$

$$A = \frac{Cd}{\varepsilon_0} \qquad (\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1})$$

$$= \frac{2\times5\times10^{-3}}{8.85\times10^{-12}}$$

or, $$A = 1.13 \times 10^{9}\ m^2$$

$$= 1.13 \times 10^{3}\ km^2 = 1130\ km^2$$

The area of plates should be in kilometres in order to get the capacitance in Farads. Therefore, the ordinary capacitors are in the range of μF.

2.25. *Obtain the equivalent capacitance of the network shown in figure alongside. For a 300 V supply, determine the charge and voltage across each capacitor.*

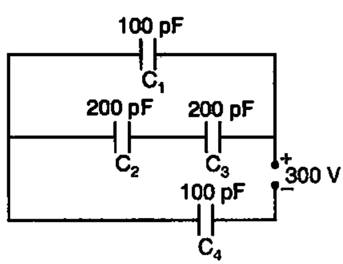

Fig. 2.22

Sol. A similar network is drawn below as given in the problem.

Here, $C_1 = C_4 = 100pF$

$C_2 = C_3 = 200pF$

Assume that the series combination of C_2 and C_3 is C_{23}.

$$\frac{1}{C_{23}} = \frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{200}+\frac{1}{200} = \frac{1}{100}$$

or, $$C_{23} = 100 \text{ pF}$$

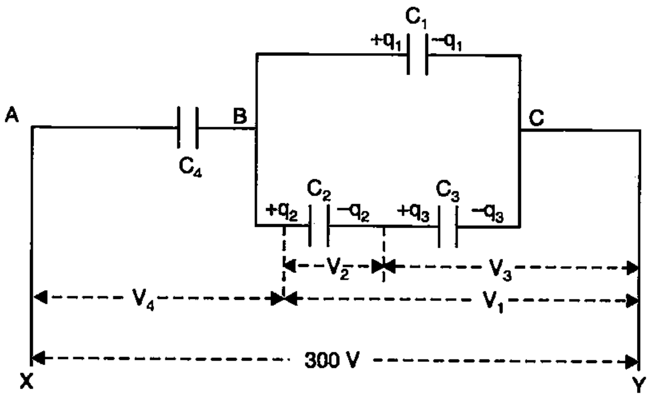

Fig. 2.23

Suppose that the parallel combination of C_1 and C_{23} is C' which is given as

$$C' = C_1 + C_{23} = 100 + 100 = 200 \text{ pF}$$

Let the series combination of C_4 and C' is C which is given as

$$\frac{1}{C} = \frac{1}{C_4}+\frac{1}{C'} = \frac{1}{100}+\frac{1}{200} = \frac{3}{200}$$

or, $$C = \frac{200}{3} \text{pF}$$

Total charge on all capacitors.

$$q = CV$$

$$= \frac{200}{\cancel{3}} \times 10^{-12} \times \overset{100}{\cancel{300}}$$

$$q = 2 \times 10^{-8} \text{ C}$$

$$V_4 = \frac{q_4}{C_4} = \frac{2\times 10^{-8}}{100 \times 10^{-12}}$$

$$V_4 = 2 \times 100 = 200 \text{ V.}$$

$$V_1 = [300 - 200] = 100 \text{ V}$$

$$q_1 = C_1 V_1$$

$$q_1 = 100 \times 10^{-12} \times 100 = \mathbf{10^{-8}} \textbf{ C}$$

$$V_2 = V_3$$
$$V_2 + V_3 = 100 \text{ V}$$
$$2V_2 = 100$$
$$V_2 = 50\text{V}$$
$$V_2 = V_S = 50\text{V}$$
$$q_2 = C_2V_2 = 200 \times 10^{-12} \times 50$$
$$q_2 = 10000 \times 10^{-12} = 10^{-8} \text{ C}$$
$$q_3 = C_3V_3 = 200 \times 10^{12} \times 50 = 10^{-8} \text{ C}$$
$$V_1 = 100 \text{ V} \qquad q_1 = 10^{-8} \text{ C}$$
$$V_2 = 500 \text{ V} \qquad q_2 = 10^{-8} \text{ C}$$
$$V_3 = 50 \text{ V} \qquad q_3 = 10^{-8} \text{ C}$$
$$V_4 = 200 \text{ V} \qquad q_4 = 2 \times 10^{-8} \text{ C.}$$

2.26. *The plates of a parallel plate capacitor have an area of 90 cm² each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.*

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Sol. Given,
$$d = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$$
$$V = 400 \text{ V}$$
$$A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2$$

(*a*) Stored electrostatic energy in the capacitor

$$\boxed{U = \frac{1}{2}CV^2} = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

$$= \frac{1}{2}\frac{(8.85\times10^{-12})90\times10^{-4}(400)^2}{2.5\times10^{-3}}$$

$$= \frac{8.85\times90\times16}{2\times2.5}\times10^{-9} = 2.55 \times 10^{-6} \text{ J}$$

(*b*) Volume of the medium between the plates

$$= A \times d$$
$$= 90 \times 10^{-4} \times 2.5 \times 10^{-3} = 225 \times 10^{-7} \text{ m}^3$$

Per unit volume, energy stored $= \dfrac{2.55\times10^{-6}}{255\times10^{-7}}$

$$U = 0.113 \text{ Jm}^{-3}$$

Relation between E and U

$$U = \frac{U}{A.d} = \frac{\frac{1}{2}CV^2}{A.d} = \frac{1}{2}\frac{V^2}{A.d}\frac{\varepsilon_0 A}{d}$$

$$U = \frac{1}{2}\varepsilon_0\frac{V^2}{d^2}$$

$\because$ $$E = \frac{V}{d}$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

2.27. *A 4 μF capacitor is charged by a 200V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?*

Sol. When 4 μF capacitor is charged by 200V then charge on it is given as

$$Q = CV = 4 \times 10^{-6} \times 200 = 8 \times 10^{-4}\ \text{C}$$

Now it is connected to another uncharged capacitor of capacitance 2 μF (2 × 10^{-6}F).

Thus, $(4 + 2) = 6\ \mu\text{F}$

Until both the capacitor acquire a common potential charge on the first capacitor is shared between them.

After the combination, the common potential = Q/C

$$V' = \frac{8\times10^{-4}C}{6\times10^{-6}F} = 1.33 \times 10^2\ \text{V}$$

$$V' = 133\ \text{V}$$

Before the combination the electrostatic potential energy of the first capacitor

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}(4\times10^{-6})(200)^2$$

$$U_1 = 8 \times 10^{-2}\ \text{J}$$

After the combination electrostatic potential energy of the system

$$U_2 = \frac{1}{2}C'V'^2 = \frac{1}{2}(6\times10^{-6})(133)^2$$

$$U_2 = 5.30 \times 10^{-2}\ \text{J}$$

Now, lost electrostatic energy by the first capacitor in the form of heat and electromagnetic radiation

$$U = U_1 - U_2 = (8 \times 10^{-2} - 5.3 \times 10^{-2})$$
$$= 2.7 \times 10^{-2}\ \text{J}$$

2.28. *Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\left(\frac{1}{2}\right)QE$, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.*

Sol. Consider surface charge density of the capacitor σ and A as the plate area.

Now,
$$Q = \sigma A$$
$$E = \sigma/\varepsilon_0 \quad \text{or} \quad \varepsilon_0 = \frac{\sigma}{E} \qquad ...(i)$$

If the separation of the capacitor plates is increased by a small distance Δx against the force F. Then, work done by the external agency $= F \cdot \Delta x$

Let u be the energy stored per unit volume or the energy density of capacitor, then *increase in the potential energy* of the capacitor

$$= u \times \text{increase in volume}$$
$$= u.\ A.\ \Delta x$$
$$F.\ \Delta x = uA.\ \Delta x$$
$$F = uA = \left(\frac{1}{2}\varepsilon_0 E^2\right)A$$
$$F = \frac{1}{2}\frac{\sigma}{E}E^2 A \qquad \text{[From } (i)\text{]}$$
$$= \frac{1}{2}\sigma AE = \frac{1}{2}QE$$

Therefore, the origin of the factor $\frac{1}{2}$ lies in the fact that field is zero just outside the conductor and it is E inside. Hence, the average value $E/2$ contributes to the force.

2.29. *A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports. Show that the capacitance of a spherical capacitor is given by* $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ *where r_1 and r_2 are the radii of outer and inner spheres respectively.*

Sol. As a charge $-Q$ is introduced on the inner sphere of radius r_2, it is distributed uniformly on its outer surface. A charge $+Q$ is induced on the outer surface of spherical shell of radius r_1 and Q is induced on its inner surface. The positive charge of the outer surface of shell flows to earth as it is earthed.

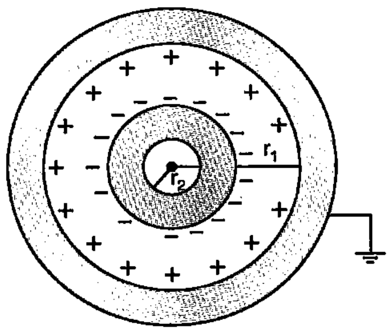

Fig. 2.24

Electric field inside sphere of radius r_2 is zero due to electrostatic shielding.

$$E = 0 \text{ for } r < r_2$$
$$E = 0 \text{ for } r > r_1$$

Electric field exists in between and is directed radially outward.

Electrostatic potential of inner sphere of radius r_2

$$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q}{r_2} - \frac{Q}{r_1}\right\} = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{r_2} - \frac{1}{r_1}\right\}$$

Potential of outer spherical shell = 0

Potential difference = $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

If C is the capacitance of spherical capacitor

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]}$$

$$\boxed{C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}}$$

2.30. *A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.*

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the sphere?

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Sol. Given, $r_1 = 12 \text{ cm} = 12 \times 10^{-2}\ m$

$r_2 = 13 \text{ cm} = 13 \times 10^{-2}\ m$

$q = 2.5\ \mu C = 2.5 \times 10^{-6}$ C

$k = 32$

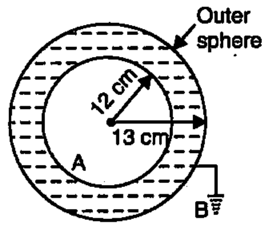

Fig. 2.25

(*a*) From formula, $C = kC_0$

$$C = k.4\pi\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1}$$

$$= \frac{32\times13\times10^{-2}\times12\times10^{-2}}{9\times10^{9}\left(13\times10^{-2}-12\times10^{-2}\right)} \quad \left[\because 4\pi\varepsilon_0 = \frac{1}{9\times10^{9}}\right]$$

$$= \frac{32\times13\times12}{9}\times10^{-11} = \frac{1644}{3}\times10^{-11}$$

$$= 5.54 \times 10^{-9} \text{ F}$$

(*b*) Potential of inner sphere,

$$V = \frac{q}{C} = \frac{2.5\times10^{-6}}{5.54\times10^{-9}} = 4.5 \times 10^{2} \text{ V}$$

(*c*) Capacitance of sphere

$$= 4\pi\varepsilon_0 = \frac{12\times10^{-2}}{9\times10^{9}} = 1.33 \times 10^{-11} \text{ F}$$

Total potential in case of concentric spheres is distributed over two spheres and the potential difference between the two spheres becomes smaller that is why the capacitance of an isolated shpere is much smaller than that of concentric spheres. Since the capacitance is inversely proportional to the potential difference ($C = Q/V$).

2.31. *Answer carefully:*

(a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2 / 4\pi\varepsilon_0 r^2$, where r is the distance between their centres?

(b) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

(c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(f) What meaning would you give to the capacitance of a single conductor?

(g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Sol. (*a*) Since the spheres are brought closer, the charge distribution will not remain uniform. Thus the magnitude of electrostatic force between them cannot be given by $Q_1Q_2/4\pi\varepsilon_0 r^2$ as they are not exactly point charges.

(*b*) No, because solid angle

$$\boxed{d\omega = \frac{ds\cos\theta}{r^2}} \text{ and } \neq \frac{ds\cos\theta}{r^3}$$

(*c*) Not definately. If the field line is a straight line then only the small test charge will move along the line of force. The direction of velocity is not given by line of force, it gives direction of acceleration.

(*d*) Whatsoever be the shape of the orbit work done is always zero because electron will be in the same energy state after it completes an orbit.

(*e*) No, since electric potential is a scalar quantity, it is continuous everywhere.

(*f*) The behaviour of a single conductor is like a capacitor with one plate at infinity. "The charge required to raise the potential of the conductor by a unit amount is termed as the capacitance of a single conductor".

(*g*) Water has a much greater dielectric constant than mica because of the high degree of association of water molecules each of which has a permanent dipole moment of about 0.6×10^{-29} cm.

2.32. *A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).*

Sol. Given,

$$q = 3.5\ \mu C = 3.5 \times 10^{-6}\ C$$
$$a = 1.4\ cm = 1.4 \times 10^{-2}\ m$$
$$b = 1.5\ cm = 1.5 \times 10^{-2}\ m$$
$$l = 15\ cm = 15 \times 10^{-2}\ m$$

$$C = \frac{2\pi\varepsilon_0 l}{2.303\log_{10}(b/a)}$$

$$= \frac{2\times\pi\times 8.854\times 10^{-12}\times 15\times 10^{-2}}{2.303\log_{10}\dfrac{1.5\times 10^{-2}}{1.4\times 10^{-2}}}$$

$$C = 1.21 \times 10^{-10}\ F$$

The potential of inner cylinder will be equal to the potential difference between inner and outer cylinder as outer cylinder is earthed.

Hence, potential of inner cylinder

$$V = \frac{q}{C} = \frac{3.5\times 10^{-6}}{1.21\times 10^{-10}} = 2.89 \times 10^{4}\ V.$$

2.33. *A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?*

Sol. Given

$$V = 1\ kV = 1000\ V$$
$$K = \varepsilon_r = 3$$

Dielectric strength = 10^7 V/m

Due to reasons of safety, electric field at the most should be 10% of dielectric strength.

E = 10% of 10^7 V/m = 10^6 V/m

$$A = ?$$
$$C = 50\ pF = 50 \times 10^{-12}\ F$$

As $$E = \frac{V}{d}$$

$\therefore$ $$d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}\text{ m}$$

Now, $$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$\therefore$ $$A = \frac{Cd}{\varepsilon_0 \varepsilon_r} = \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3}$$

or, $$A = 1.9 \times 10^{-3}\text{ m}^2$$

2.34. *Describe schematically the equipotential surfaces corresponding to*

(a) a constant electric field in the z-direction,

(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(c) a single positive charge at the origin, and

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Sol. The surfaces where the potential has a constant value are called equipotential surfaces.

(*a*) When an electric field acting in z-direction is constant, the potential in a direction perpendicular to z-axis remains constant. Therefore, equipotential surface is represented by the planes parallel to x-y plane.

(*b*) The answer is same as (*a*) since the potential in a direction perpendicular to the direction of field remains constant irrespective of the magnitude of the field.

(*c*) The equipotential surfaces are concentric sphere centered at the origin for a single positive charge at the origin. By a constant potential increases with increase in distance from the origin, the separation between the equipotentials differing.

(*d*) Near the grid, the equipotential surfaces are of periodically varying shape which gradually reach the shape of planes parallel to the grid at per distance.

2.35. *In a Van de Graaff type generator a spherical metal shell is to be a 15×10^5 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)*

Sol. Dielectric strength of gas surrounding the electrodes $E = 5 \times 10^7$ V/m.

Potential of sphere $V = 15 \times 10^5$ V

Suppose radius of the shell = R

For spherical shell $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$

As $V = \frac{kq}{R}$ ...(*i*)

$$E = \frac{kq}{R^2}$$

$$E = \frac{V}{R} \quad \text{[From } (i)\text{]}$$

$$R = \frac{V}{E} = 0.3 \text{ m} = 30 \text{ cm}$$

$$R = \frac{15 \times 10^5}{10\% \text{ of } 5 \times 10^7}$$

$$R = \frac{15 \times 10^5}{\frac{10}{100} \times 5 \times 10^5 \times 100} = \frac{3}{10} \text{ m}$$

$$R = 30 \text{ cm.}$$

2.36. *A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.*

Sol. Potential of inner sphere due to its one charge

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Potential of inner sphere due to its presence inside the shell

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

Thus total potential of inner sphere = $V_1 + V_2$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Potential of shell $V' = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

Potential difference between inner sphere and shell = $V - V'$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)-\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

Thus, we can conclude that potential difference is independent of the charge q_2 on the shell. Potential of sphere is positive since q_1 is positive. From inner sphere to shell, charge (if positive) will always flow no matter whatsoever charge q_2 on the shell is.

2.37. *Answer the following:*

(a) *The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 V m^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)*

(b) *A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning?*

(c) *The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?*

(d) *What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?*

*[**Hint:** The earth has an electric field of about 100 V m^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} Cm^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)*

Sol. (a) The surface of earth and the equipotential surface are parallel. Human body is a good conductor. The original equipotential surfaces of open air get modified as we step out into the open but keeping our head and ground at the same potential and we do not get any electric shock.

(*b*) Yes, the aluminium sheet gradually is charged up by the steady discharging current in the atmosphere and raises its voltage to an extent depending on the capacitance of the capacitor (formed by the sheet, slab and the ground).

(*c*) Thunderstorms and ground lightning all over the globe charge the atmosphere continually and discharged through regions of ordinary weather. The two opposing currents are, on an average, in equilibrium.

(*d*) Electrical energy of the atmosphere is dissipated as

(*i*) light energy involved in lightning.

(*ii*) heat and sound energy in accompanying thunder.

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