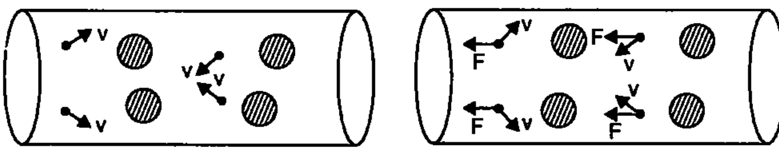
# 3 Current Electricity

## Lesson at a Glance

### • Motion of Free Electrons in a Conductor

In a conductor free electrons move randomly in all possible directions with all possible velocities. If an electric field is applied across a conductor the free electrons experience a force in the opposite direction of electric field and move in the direction of force.




(a) Motion of free electrons in the absence of electric field

(b) Motion of free electrons in presence of electric field

**Fig. 3.1**

• The average velocity of free electrons in presence of electric field in a conductor is called **drift velocity.**

$$\therefore \quad V_d = \frac{eV}{ml} \cdot \tau$$

### • Electric Current

If potential difference $V$ is applied across a conductor of area of cross-section $A$ and length $l$ having $n$ of free electrons per unit volume, then the rate of flow of charge, called electric current, is given by

$$I = \frac{q}{t}$$

• Electric current per unit area of cross-section is called **current density** ($J$)

Thus, $$J = \frac{I}{A}$$

• Resistivity is the property of the conductor, due to which they offer, resistance to the current flowing through the conductor.

$\rho = \dfrac{m}{ne^2C}$ is called resistivity of the conductor.

• When **temperature increases** the relaxation time decreases and the resistivity $\rho = \frac{m}{ne^2 e}$ increases. Correspondingly the **resistance increases.**

• The increase in resistance is directly proportional to the increase in temperature and the original resistance.

$$R = R_0 (1 + \alpha \Delta\theta)$$

where $\alpha$ in the temperature coefficient of resistance.

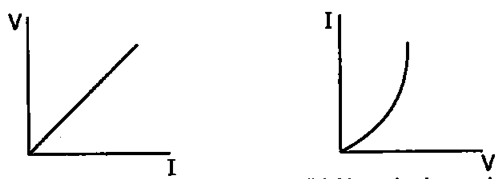
• The ratio of potential difference and current flowing through a conductor is constant provided all physical conditions are uncharged.

$$\frac{V}{I} = \text{Constant} - (R) \quad \text{or} \quad \frac{V}{I} = R$$

This statement is called Ohm's law.

• If $V - I$ graph for conductor is a straight line then it is called an **Ohmic conductor.**

• If the $V - I$ graph is not a straight line, the conductor or device is called a **non-ohmic conductor.**



(a) Ohmic conductor (b) Non-ohmic conductor

**Fig. 3.2**

• Cell is a device which converts chemical energy into electrical energy. It consists of two electrodes of different materials and an electrolyte.

## • Combination of Resistors

(i) **Series combination.** When resistors are connected one by one and equal current is passed through all, the combination is called series combination as shown in the figure. Let $R_1$, $R_2$ and $R_3$ resistors are connected in series. The potential drop across $R_1$, $R_2$ and $R_3$ respectively is given by

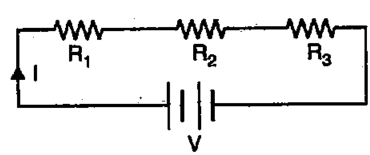


**Fig. 3.3**

$$V_1 = IR_1, \quad V_2 = IR_2 \quad \text{and} \quad V_3 = IR_3$$

but $$V = V_1 + V_2 + V_3$$

If $R$ be the equivalent resistance of the combination, then $V = IR$

or $$IR = IR_1 + IR_2 + IR_3$$

or $$R = R_1 + R_2 + R_3$$

(*ii*) **Parallel combination.** When resistors are connected across two same points and kept at the same potential difference, the combination is called parallel combination. Let resistors $R_1$, $R_2$ and $R_3$ are connected in parallel as shown in the figure. The current in resistors $R_1$, $R_2$ and $R_3$ respectively can be given as

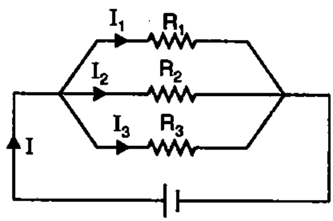


**Fig. 3.4**

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_3}$$

and $$I_3 = \frac{V}{R_3}$$

But $$I = I_1 + I_2 + I_3$$

If $R$ be the equivalent resistance, then

$$I = \frac{V}{R}$$

or $$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

## • Kirchhoff's Rules of Electricity

(*i*) **Junction rule or point rule.** It states that algebraic sum of all currents meeting at a point is zero.

$$\Sigma I = 0$$

(*ii*) **Loop rule.** It states that the algebraic sum of the emfs in a closed circuit is equal to the algebraic sum of the product of current and respective resistances.

$$\Sigma \varepsilon = \Sigma IR$$

## • Wheatstone Bridge

Wheatstone bridge is a network of four resistors say $P$, $Q$, $R$ and $S$ in the form of an quadrilateral having a galvanometer in one diagonal and the battery in another diagonal as shown in the figure.

$$\frac{P}{Q} = \frac{R}{S}$$

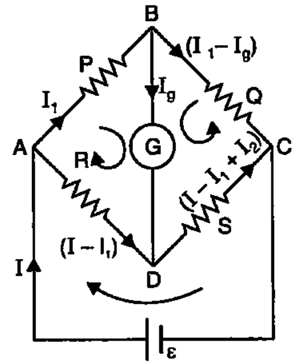



Fig. 3.5

## • Meter Bridge

It is a device used to measure the resistance and resistivity of the given wire.

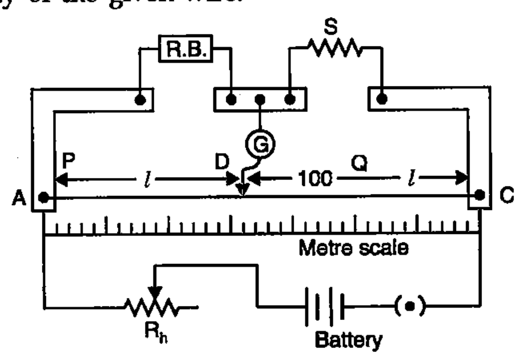



Fig. 3.6

$$S = R\frac{(100 - l)}{l}$$

## • Potentiometer

It is a device used to compare the emfs of primary cells and to determine the internal resistance of the cell.

$$E_s = IR_s$$

$$K = \frac{E_s}{L_s} = \text{Potential gradient}$$

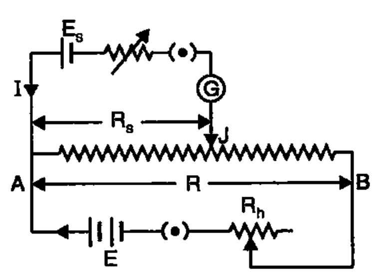



Fig. 3.7

## • Heating Effect of Electric Current

When potential difference is applied across a conductor of resistance $R$, the free electrons in a conductor moves from lower potential to higher potential and strike with the atoms of the conductor inelastically. In these inelastic collisions the energy is released in the form of heat. This heat released is equal to the amount of work done

$$\therefore \quad H = \text{Work done}$$

$$= V_q$$

$$H = VIt$$

or $$H = I^2Rt$$

or $$H = \frac{V^2}{R}t$$

- The electric power is the rate of doing work,

$$P = \frac{W}{t} = \frac{VIt}{t}$$

or $$P = VI$$

or $$P = I^2R$$

or $$P = \frac{V^2}{R}$$

The S.I. unit of electric power is watt.

- Kilowatt hour is the electrical energy consumed by a device of power one kilowatt in one hour.

$$1 \text{ KWH} = 3.6 \times 10^6 \text{ J}$$

- If an electric bulb of power $P$ rated at voltage $V$ is operated at voltage $V'$, then power consumed by the bulb,

$$P' = \left(\frac{V'}{V}\right)P$$

- If bulbs of power $P_1$ and $P_2$ are connected in parallel, the equivalent power of the combination will,

$$P = P_1 + P_2$$

- If bulbs of power $P_1$ and $P_2$ are connected in series, then equivalent power of the combination $P$ is given by

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

- Power transfer to the load by the cell will be

$$P = I^2R = \frac{\varepsilon^2 R}{(R+r)^2}$$

*i.e.*, $P$ will be minimum when $R = 0$ or $\infty$

and will be maximum when $\frac{dP}{dR} = 0$

## TEXTBOOK QUESTIONS SOLVED

**3.1.** *The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?*

**Sol.** When the external resistance in the circuit is zero *i.e.*, $R = 0$ then the maximum current is drawn from a battery.

Given, $\varepsilon = 12$ V, $r = 0.4$

$$\boxed{I_{max} = \frac{\varepsilon}{r}} = \frac{12}{0.4} = 30 \text{ A}$$

**3.2.** *A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?*

**Sol.** Since, $\boxed{I = \frac{\varepsilon}{R+r}}$ or $R = \frac{\varepsilon}{I} - r$

Putting given values $\varepsilon = 10$ V, $r = 3$ Ω and

$$I = 0.5 \text{ A}$$

$$R = \frac{10}{0.5} - 3 = 17 \ \Omega$$

Terminal voltage, $V = IR = 0.5 \times 17 = 8.5$ V

**3.3.** *(a) Three resistors 1 Ω, 2 Ω, and 3 Ω are combined in series. What is the total resistance of the combination?*

*(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.*

**Sol.** (*a*) Resistance of series combination

$$\boxed{R = R_1 + R_2 + R_3}$$

$$= 1 + 2 + 3 = 6 \ \Omega$$

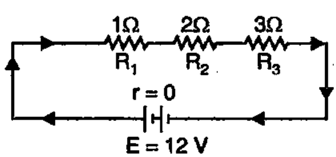



**Fig. 3.8**

(*b*) Current through the circuit,

$$I = \frac{E}{R+r} = \frac{12}{6+0} A = 2\text{A}$$

Potential drop across $R_1 = 2 \times 1$ V = 2 V

Potential drop across $R_2 = 2 \times 2$ V = 4 V

Potential drop across $R_3 = 2 \times 3$ V = 6 V

**3.4.** *(a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?*

*(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.*

**Sol.** (*a*) Total resistance of parallel combination,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$= \frac{10+5+4}{20} = \frac{19}{20}\,\Omega$$

or $$R = \frac{20}{19}\,\Omega$$

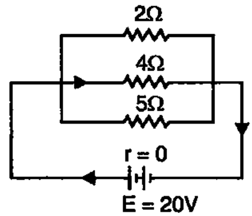



Fig. 3.9

(*b*) Given, voltage across the parallel

Combination $V = 20$ volt

Let the current through resistances 2Ω, 4Ω and 5Ω are $I_1$, $I_2$ and $I_3$ respectively.

Now, $$I_1 = \frac{V}{R_1} = \frac{20}{2} = 10\text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{20}{4} = 5\text{ A}$$

$$I_3 = \frac{V}{R_3} = \frac{20}{5} = 4\text{ A}$$

Total current, $I = I_1 + I_2 + I_3 = 10 + 5 + 4 = 19$ A.

**3.5.** *At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature co-efficient of the material of the resistor is $1.70 \times 10^{-4}$ °C$^{-1}$.*

**Sol.** Given, $R_t = 117, \quad R_{27} = 100$

Since, $$\boxed{R_t = R_{27}\,[1 + \alpha\,(t - 27)]}$$

Putting values, $$117 = 100\,[1 + 1.70 \times 10^{-4}\,(t - 27)]$$

$$t = 1000 + 27 = 1027\text{ °C}$$

**3.6.** *A negligibly small current is passed through a wire of length 15 m and uniform cross-section $6.0 \times 10^{-7}$ m$^2$, and its resistance is measured to* *be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?*

**Sol.** Given, $R = 5$ W

$l = 15$ m

$A = 6 \times 10^{-7}$ m$^2$ (Area of cross-section of wire)

Resistivity $\boxed{\rho = \frac{RA}{l}} = \frac{5 \times 6.0 \times 10^{-7}}{15}$

$= 2.0 \times 10^{-7}$ Ωm.

**3.7.** *A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature co-efficient of resistivity of silver.*

**Sol.** Given, $R_1 = 2.1$ Ω, $t_1 = 27.5$ °C, $R_2 = 2.7$ Ω

$t_2 = 100$ °C

Since, $\boxed{R_2 = R_1 [1 + \alpha (t_2 - t_1)]}$

or, $\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)} = \frac{2.7 - 2.1}{2.1 (100 - 27.5)}$

$= \frac{0.6}{2.1 \times 72.5} = \frac{0.6}{152 \cdot 25} = 0.0039$ °C$^{-1}$

**3.8.** *A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0°C? Temperature co-efficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}$ °C$^{-1}$.*

**Sol.** Here, $R_1 = \frac{230}{3.2} = 71.87$ Ω

$R_2 = \frac{230}{2.8} = 82.14$ Ω

$\alpha = 1.7 \times 10^{-4}$ °C$^{-1}$

$t_1 = 27°$

Since, $\boxed{R_2 = R_1 [1 + \alpha (t_2 - t_1)]}$

Thus, $t_2 = \frac{R_2 - R_1}{R_1 \cdot \alpha} + t_1$

or, $t_2 = \frac{82.14 - 71.87}{71.87 \times 1.7 \times 10^{-4}} + 27$

$= 840.56 + 27 = 867.56$ °C $= 867$ °C.

**3.9.** *Determine the current in each branch of the network shown in figure.*

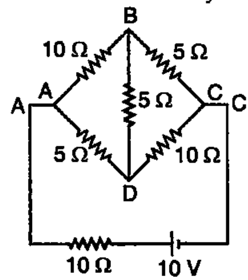



**Fig. 3.10**

**Sol.** Applying Kirchhoff's second law to the mesh *ABDA*,

$$-10I_1 - 5I_g + (I - I_1)5 = 0$$

or $$3I_1 - I + I_g = 0 \quad ...(i)$$

Again, applying Kirchhoff's second law to the mesh *BDCB*,

$$-5I_g - 10(I - I_1 + I_g) + 5(I_1 - I_g) = 0$$

or $$3I_1 - 2I - 4I_g = 0 \quad ...(ii)$$

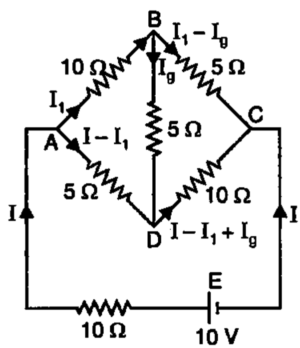



**Fig. 3.11**

Applying Kirchhoff's second law to the mesh *ABCEA*,

$$-10I_1 - 5(I_1 - I_g) - 10I + 10 = 0$$

or $$3I_1 + 2I - I_g = 2 \quad ...(iii)$$

Adding (*i*) and (*iii*), we get

$$6I_1 + I = 2 \quad ...(iv)$$

Multiplying (*i*) by 4 and adding in (*ii*), we get

$$15I_1 - 6I = 0 \quad ...(v)$$

Solving equations (*iv*) and (*v*), we get

$$I_1 = \frac{4}{17} A = 0.235 \text{ A}$$

So, current in branch *AB* is 0.235 A.

Putting the value of $I_1$ in equation (*v*) and simplifying, we get

Total current, $$I = \frac{10}{17} = 0.588 \text{ A}$$

Putting the values of *I* and $I_1$ in equation (*iii*) and simplifying, we get

$$I_g = \frac{2}{17} A = -0.118 \text{ A}$$

The negative sign indicates that the direction of current is opposite to that shown in Fig. above.

So, current in branch *BD* is "– 0.118 A".

Current in branch *BC* is $(I_1 - I_g)$ *i.e.*, $\frac{4}{17} - \left(-\frac{2}{17}\right)$

*i.e.*, $\frac{6}{17}$ or 0.353 A.

Current in branch *AD* is $(I - I_1)$

*i.e.*, $\left(\frac{10}{17} - \frac{4}{17}\right) A$ *i.e.*, $\frac{6}{17} A$ or 0.353 A

Current in branch *DC* is $(I_1 - I_1 + I_g)$

*i.e.*, $\frac{6}{17} + \left(-\frac{2}{17}\right) A$ or $\frac{4}{17} A$ or 0.235 A

**3.10.** *(a) In a metre bridge, the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or Meter bridge made of thick copper strips?*

*(b) Determine the balance point of the bridge above if X and Y are interchanged?*

*(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?*

**Sol.** (a) Here, $l = 39.5$ cm, $R = X = ?$, $S = Y = 12.5\ \Omega$

$$S = \frac{100 - l}{l} \times R$$

$$\therefore \qquad 12.5 = \frac{100 - 39.5}{39.5} \times X$$

$$\text{or} \qquad X = \frac{12.5 \times 39.5}{60.5} = 8.16\ \Omega$$

Thick copper strips are used to minimise resistance of the connections which are not accounted in the formula.

(*b*) As $X$ and $Y$ are interchanged, therefore, $l_1$ and $l_2$ (*i.e.*, lengths) are also interchanged.

Hence, $l = 100 - 39.5 = 60.5$ cm

(*c*) The galvanometer will show no current.

**3.11.** *A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?*

**Sol.** Charging current,

$$I = \frac{V_a - E}{R + r}$$

$V_a \rightarrow$ supply voltage
$E \rightarrow$ E.m.f of battery
$R \rightarrow$ External resistance
$r \rightarrow$ internal resistance

$$\text{or,} \qquad I = \frac{120 - 8}{15.5 + 0.5} = \frac{112}{16} = 7A$$

Terminal voltage, $V = E + I\,r = 8 + 7 \times 0.5 = 11.5$ V

The series resistor limits the current from the external source. In its absence, the current may be dangerously high.

**3.12.** *In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?*

**Sol.** $\boxed{\dfrac{E_2}{E_1} = \dfrac{l_1}{l_2}}$ substituting values,

$$\frac{E_2}{1.25} = \frac{63}{35} \quad \text{or} \quad E_2 = 1.25 \times \frac{63}{35} \text{ volt} = 2.25 \text{ volt}$$

**3.13.** *The number density of free electrons in a copper conductor is $8.5 \times 10^{28}$ $m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is $2.0 \times 10^{-6}$ $m^2$ and it is carrying a current of 3.0 A.*

**Sol.** $n = 8.5 \times 10^{28}\ m^{-3}$, $I = 3.0$ A,

$A = 2.0 \times 10^{-6}\ m^2$, $l = 3.0$ m, $e = 1.6 \times 10^{-19}$ C

Drift velocity, $\boxed{v_d = \frac{I}{neA}}$

$$= \frac{3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-6}} \text{m s}^{-1}$$

$$= 1.103 \times 10^{-4}\ \text{m s}^{-1}$$

Time taken by electron to drift from one end to another,

$$t = \frac{l}{v_d} = \frac{3.0}{1.103 \times 10^{-4}} \text{s}$$

$$= 2.72 \times 10^4 \text{ s } (\approx 7.5 \text{ h}).$$

**3.14.** *The earth's surface has a negative surface charge density of $10^{-9}$ C $m^{-2}$. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe.)*

*(Radius of earth = $6.37 \times 10^6$ m).*

**Sol.** Given charge per unit area of surface of earth = $10^{-9}$ coulomb per sq. metre.

Current $I = 1800$ A

The radius of earth 6370 km = $6.37 \times 10^6$ m

Charge on entire surface of the earth

$$= 4\pi (6.37 \times 10^6)^2 \times 10^{-9} \text{ C}$$

As the rate of flow of charge is 1800 C per second, time required for the flow of entire charge

$$= \frac{4 \times 3.14 \times (6.37 \times 10^6)^2 \times 10^{-9}}{1800} = 283 \text{ seconds.}$$

**3.15.** *(a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?*

*(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?*

**Sol.** (*a*) Here, $E = 2.0$ V, $n = 6$, $r = 0.015\ \Omega$

and $R = 8.5\ \Omega$

Current, $I = \dfrac{nE}{R+nr} = \dfrac{6 \times 2.0}{8.5 + 6 \times 0.015} = 1.4\text{A}$

Terminal voltage $V = IR = 1.4 \times 8.5 = 11.9$ V

(*b*) Given, $E = 1.9$ V, $r = 380\ \Omega$

$$I_{max} = \frac{E}{r} = \frac{1.9}{380}$$

or $I_{max} = 0.005$ A

This amount of current cannot start a car because to start the motor, the current required is 100 A for few second.

**3.16.** *Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{Al} = 2.63 \times 10^{-8}\ \Omega$ m, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega$ m, Relative density of Al = 2.7, of Cu = 8.9).*

**Sol.** $$R = \rho\frac{l}{A} = \rho\frac{l^2}{Al} = \frac{\rho l^2}{V} = \frac{\rho l^2 d}{m} \quad V = \frac{m}{d}$$

where $V$ = volume of wire = $Al$

$m$ = mass of wire

$d$ = density of wire material

mass = volume × density = $Ald$

For aluminium wire, $R_{Al} = \dfrac{\rho_{Al} l_{Al}^2 d_{Al}}{m_{Al}}$

For copper wire, $R_{cu} = \dfrac{\rho_{Cu} l_{Cu}^2 d_{Cu}}{m_{Cu}}$

Since, $R_{Al} = R_{Cu}$ and $l_{Al} = l_{Cu}$

$\therefore$ $$\frac{\rho_{Al} l_{Al}^2 d_{Al}}{m_{Al}} = \frac{\rho_{Cu} l_{Cu}^2 d_{Cu}}{m_{Cu}}$$

or $$\frac{m_{Cu}}{m_{Al}} = \frac{\rho_{Cu} d_{Cu}}{\rho_{Al} d_{Al}} = \frac{0.72 \times 10^{-8} \times 8.9}{2.63 \times 10^{-8} \times 2.7} = 2.2.$$

It indicates that aluminium wire is lighter than copper wire. Therefore, aluminium wires are preferred in overhead cables.

**3.17.** *What conclusion can you draw from the following observations on a resistor made of alloy manganin?*

| Current A | Voltage V | Current A | Voltage V |
|---|---|---|---|
| 0.2 | 3.94 | 3.0 | 59.2 |
| 0.4 | 7.87 | 4.0 | 78.8 |
| 0.6 | 11.8 | 5.0 | 98.6 |
| 0.8 | 15.7 | 6.0 | 118.5 |
| 1.0 | 19.7 | 7.0 | 138.2 |
| 2.0 | 39.4 | 8.0 | 158.0 |

**Sol.**

$$\frac{3.94}{0.2} = 19.7\ \Omega \qquad \frac{59.2}{3.0} = 19.7\ \Omega$$

$$\frac{7.87}{0.4} = 19.67\ \Omega \qquad \frac{78.8}{4.0} = 19.7\ \Omega$$

$$\frac{11.8}{0.6} = 19.66\ \Omega \qquad \frac{98.6}{5.0} = 19.72\ \Omega$$

$$\frac{15.7}{0.8} = 19.62\ \Omega \qquad \frac{118.5}{6.0} = 19.75\ \Omega$$

$$\frac{19.7}{1.0} = 19.7\ \Omega \qquad \frac{138.2}{7.0} = 19.74\ \Omega$$

$$\frac{39.4}{2.0} = 19.7\ \Omega \qquad \frac{158.0}{8.0} = 19.75\ \Omega$$

Since the ratio of voltage and current for different readings is same so Ohm's law is valid to high accuracy. The *resistivity of alloy is nearly independent of temperature.*

**3.18.** *Answer the following questions:*

*(a) A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor: current, current density, electric field, drift speed?*

*(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.*

*(c) A low voltage supply from which one needs high currents must have very low internal resistance. Why?*

*(d) A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?*

**Sol.** (*a*) Only current because it is given to be steady. The rest depends on the area of cross-section inversely.

(*b*) Ohm's law is not applicable for non-ohmic elements. For example; vacuum tubes, semi-conducting diode, liquid electrolyte etc. (see text).

(*c*) Maximum current drawn from a source $= \frac{E}{r}$.

(*d*) If accidently the circuit is shorted, the current drawn will exceed safety limit and will cause damage to circuit. Therefore, a high tension supply must have a large internal resistance.

**3.19.** *Choose the correct alternative:*

*(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.*

*(b) Alloys usually have much (lower/higher) temperature co-efficients of resistance than pure metals.*

*(c) The resistivity of the alloy manganin is (nearly independent of/ increases) rapidly with increase of temperature.*

*(d) The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of ($10^{22}$/$10^3$).*

**Sol.** (*a*) Greater

(*b*) lower

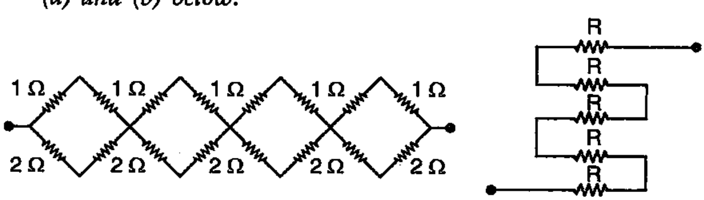
(*c*) nearly independent of

(*d*) $10^{22}$

**3.20.** *(a) Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?*

*(b) Given the resistance of 1 Ω, 2 Ω, 3 Ω, how will you combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5) Ω, (iii) 6 Ω, (iv) (6/11) Ω?*

*(c) Determine the equivalent resistance of networks shown in the figures (a) and (b) below.*




(a) (b)

**Fig. 3.12**

**Sol.** (*a*) (*i*) all in series, (*ii*) all in parallel; $n^2$ $\because$ $\frac{R_S}{R_b} = \frac{nR}{\frac{R}{n}} = n^2$

(*b*) (*i*) Join 1 Ω, 2 Ω in parallel and the combination in series with 3 Ω. (*ii*) parallel combination of 2 Ω and 3 Ω in series with 1 Ω. (*iii*) all in series (*iv*) all in parallel.

(*c*) **Equivalent resistance of network in figure (*a*).** The given network is a series combination of four identical units. Let us consider one such unit shown in Fig. It is equivalent to a parallel combination of 2 Ω and 4 Ω. Its equivalent resistance is

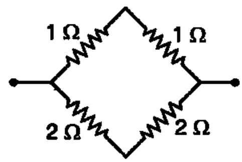


**Fig. 3.13**

$$R_p = \frac{2 \times 4}{2+4}\,\Omega \text{ i.e., } \frac{8}{6}\,\Omega \text{ i.e., } \frac{4}{3}\,\Omega$$

So, the given electrical network is a series combination of four resistors, each equal to $\frac{4}{3}\,\Omega$.

Thus, the combined resistance is $\frac{16}{3}\,\Omega$ or 5.33 Ω.

**Equivalent resistance of network in fig. (*b*).** Suppose a battery is connected between *A* and *B*. Same current will flow through all the resistors. So, all the resistors are connected in series.

$\therefore$ Equivalent resistance,

$$R_s = R + R + R + R + R = 5R$$

**3.21.** *Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the infinite network shown in figure. Each resistor has 1 Ω resistance.*

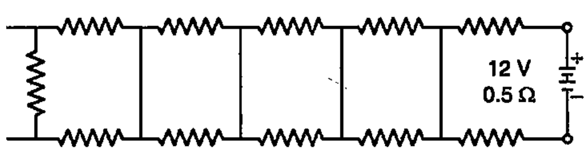


**Fig. 3.14**

**Sol.** Let *x* be the equivalent resistance of the infinite network. This network consists of infinite sets of three resistors of 1 Ω, 1 Ω and 1 Ω. Adding one more set across *AB* to the infinite network will not affect the equivalent resistance.

Resistance between $A$ and $B$

$$R_p = \frac{x}{x+1} \qquad \left(\because \frac{1}{R} = \frac{1}{x} + \frac{1}{1}\right)$$

Resistance between $P$ and $Q$

$$R_s = 1 + \frac{x}{x+1} + 1 = 2 + \frac{x}{x+1}$$

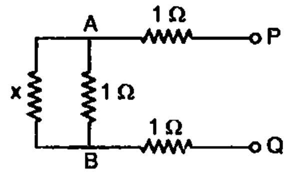



Fig. 3.15

This must be equal to the initial resistance $x$

$\therefore$ $$x = 2 + \frac{x}{x+1}$$

or $$x + x^2 = 2 + 2x + x$$

$$x^2 - 2x - 2 = 0$$

$$x = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$$

But $$x \neq 1 - \sqrt{3}$$

$\therefore$ $$x = \left(1 + \sqrt{3}\right)\Omega = 2.732\ \Omega$$

$$\text{Current } I = \frac{\text{e.m.f}}{\text{total resistance}} = \frac{12}{2.732 + 0.5}$$

or $$I = 3.7\ \text{A}.$$

**3.22.** *Figure shows a potentiometer with a cell of 2.0 V and internal resistance 0.4 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents up to a few mA) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf ε and the balance point found similarly, turns out to be at 82.3 cm length of the wire.*

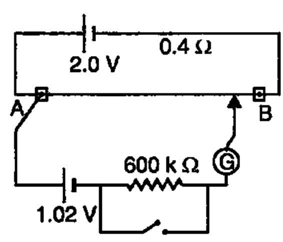



Fig. 3.16

*(a) What is the value of ε?*

*(b) What purpose does the high resistance of 600 kΩ have?*

*(c) Is the balance point affected by this high resistance?*

(*d*) *Is the balance point affected by the internal resistance of the driver cell?*

(*e*) *Would the method work in the above situation, if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?*

(*f*) *Would the circuit work well for determining extremely small emf, say of the order of a few mV (such as the typical emf of a thermocouple)? If not, how will you modify the circuit ?*

**Sol.** (*a*) Here, $E_1 = 1.02$ V; $l_1 = 67.3$ cm; $E_2 = E = ?$; $l_2 = 82.3$ cm

Since, $$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

$\therefore$ $$E = \frac{l_2}{l_1} \times E_1 = \frac{82.3}{67.3} \times 1.02 = 1.247 \text{ V}.$$

(*b*) The purpose of using high resistance of 600 kΩ is to allow very small current through the galvanometer when the movable contact is far from the balance point.

(*c*) No, the balance point is not affected by the presence of this resistance.

(*d*) No, the balance point is not affected by the internal resistance of the driver cell.

(*e*) No, it is necessary that the emf of the driver cell is more than the emf of the cells.

(*f*) For measurement of small emf, this circuit will not work well. The number of potentiometer wires is increased to 11 or 15 to get a potential gradient of 0.1 $Vm^{-1}$. The purpose discussed above will be served by a single 1 metre long wire with series resistance equal to 10 or 14 wires.

**3.23.** *Figure shows a potentiometer circuit for comparison of two resistors. The balance point with a standard resistor R = 10.0 Ω is found to be 58.3 cm, while that with the unknown resistor X is 68.5 cm. Determine the value of X. What might you do if you failed to find a balance point with the given cell of emf ε?*

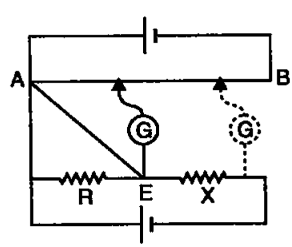



**Fig. 3.17**

**Sol.** Here, $$\frac{R}{X} = \frac{l_1}{l_2}$$

or, $$X = R\frac{l_1}{l_2} = \frac{10 \times 68.5}{58.3} = 11.75\ \Omega$$

The potential drop across $R$ and $X$ are greater than the potential drop across the potentiometer wire $AB$ if there is no balance point. The obvious thing to do is to reduce the current in the outside circuit (hence the potential drop across $R$ and $X$) suitably by putting a series resistor.

**3.24.** *Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.*

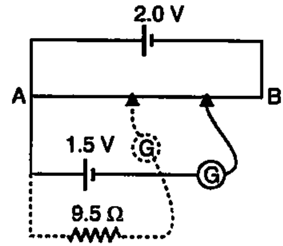


Fig. 3.18

**Sol.**

$$r = \frac{R(E-V)}{V} = \frac{R(l_1 - l_2)}{l_2}$$

$$= \frac{9.5(76.3 - 64.8)}{64.8} \text{ ohm} \qquad \left[\frac{E}{V} = \frac{l_1}{l_2}\right]$$

$$= \frac{9.5 \times 11.5}{64.8} \text{ ohm}$$

$$= 1.686 \text{ ohm} \sim 1.7\ \Omega$$

❑❑❑