4 Magnetic Effect of Current

Lesson at a Glance

- The S.I. unit of magnetic field intensity or magnetic induction flux density is tesla or Wbm^{-2}.

• Biot-Savart law

The law states that the magnetic field (dB) due to a small current element of length 'dl' carrying current I is directly proportional to the strength of current, perpendicular length of the conductor and inversely proportional to the square of the distance from the current element to the point where magnetic field is determined as shown in figure.

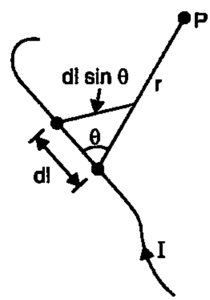

Fig. 4.1

$$dB \propto I$$
$$\propto dl \sin\theta$$
$$\propto \frac{1}{r^2}$$

or
$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

or
$$dB = \frac{\mu_0}{4\pi}\frac{Idl \sin\theta}{r^2}$$

where θ is the angle between line joining the point and current element and μ_0 is magnetic permeability of free space.

• Magnetic field due to circular loop at its centre

Let there be a circular loop of radius R carrying current I. The magnetic field at the centre of the loop due to small element of length 'dl',

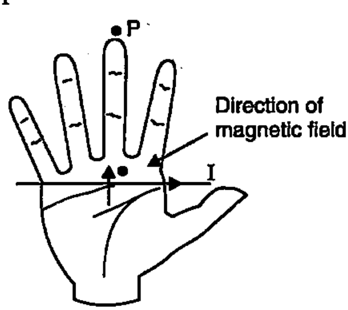

Fig. 4.2

$$dB = \frac{\mu_0 Idl \sin 90^\circ}{4\pi R^2}$$

(applying Biot-Savart law)

• Magnetic field due to circular loop at axial point

Let there be a circular loop of radius *R*, carrying current *I* and an axial point *P* at the distance of *x* from its centre where magnetic field is to be determined. (Fig. 4.3). The magnetic field due to a small element of length '*dl*' at point *P*,

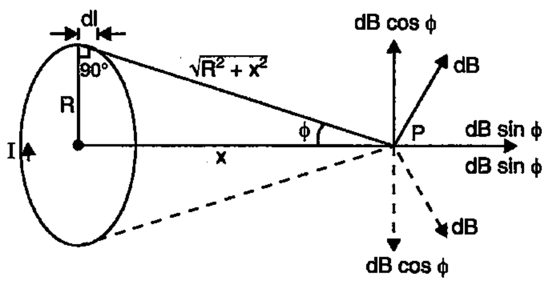

Fig. 4.3

$$dB = \frac{\mu_0 I dl \sin 90^\circ}{4\pi (R^2 + x^2)} = \frac{\mu_0 I dl}{4\pi(R^2 + x^2)}$$

• Ampere's circuital law

According to Ampere's circuital law, the line integral of the magnetic field $\vec{B}$ around any closed circuit is equal to μ_0 (permeability) times the total current *I* passing through the closed circuit or path

$$\therefore \qquad \oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

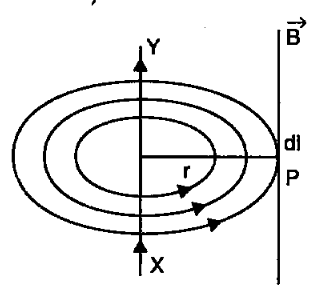

Fig. 4.4

• Magnetic field due to an infinitely long straight current carrying conductor

Let there be an infinitely long straight conductor carrying current *I* and a point *P* at the distance (shortest) *R* from the conductor (Fig. 4.4) where magnetic field is to be determined. Applying the Ampere's circuital law,

$$\oint B dl = \mu_0 (I)$$

$$B(2\pi R) = \mu_0 I \quad \text{or } B = \frac{\mu_0 I}{2\pi R}$$

• Magnetic field due to a hollow cylindrical conductor

Let there be a hollow cylindrical conductor of radius R carrying current I and a point P at the distance of x from the axis of the cylinder from magnetic field where magnetic field is to be determined as shown in Fig. 4.5.

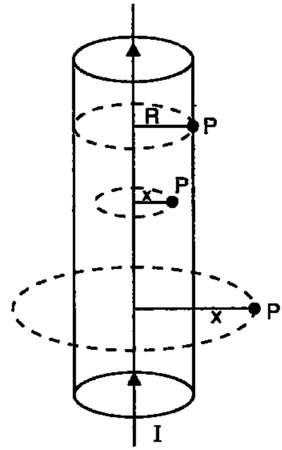

Fig. 4.5

Applying Ampere's circuital law for

(i) $x < R$, current threading the *wp* = 0

$$\therefore \quad \oint Bdl = \mu_0(0)$$

$$B(2\pi x) = 0 \quad \text{or} \quad B = 0$$

(ii) For $x = R$,

$$\oint Bdl = \mu_0(I)$$

$$B(2\pi R) = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi R}$$

(iii) For $x > R$

$$\oint Bdl = \mu_0(I)$$

$$B(2\pi x) = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi x}$$

• Magnetic field due to a solid cylindrical conductor

Let there be a solid cylindrical conductor of radius R carrying current I and a point P at the distance of x from its axis where magnetic field is to be determined. (Fig. 4.6)

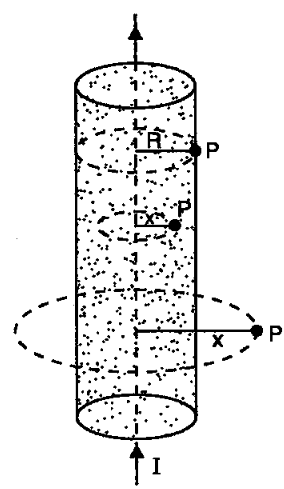

Fig. 4.6

Applying Ampere's circuital law

(i) for $x < R$, current threading the loop

$$= \frac{I}{(\pi R^2)}(\pi x^2) = \frac{Ix^2}{R^2}$$

$$\therefore \quad \oint Bdl = \mu_0 \left(\frac{Ix^2}{R^2}\right)$$

$$\text{or} \quad B\,(2\pi x) = \frac{\mu_0 I x^2}{R^2}$$

$$\text{or} \quad B = \frac{\mu_0 I(x)}{2\pi R^2}$$

(*ii*) For $x = R$

$$\oint B dl = \mu_0 (I)$$

or $$B(2\pi R) = \mu_0 (I) \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi R}$$

(*iii*) For $x > R$

$$\oint B dl = \mu_0 (I)$$

or $$B(2\pi x) = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi x}$$

- **Magnetic field due to a solenoid (straight)**

$$B = n\mu_0 I$$

- **Magnetic field due to a torodial solenoid**

Let there be a torodial solenoid of radius R, number of turns N and carrying current I. There be a point on its axis within the core where magnetic field is to be determined as shown in Fig. 4.7. Applying Ampere's circuital law,

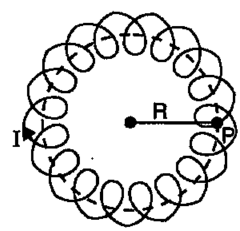

Fig. 4.7

$$\oint B dl = N\mu_0 (I)$$

$$B(2\pi R) = N\mu_0 (I)$$

or $$B = \frac{N\mu_0 I}{2\pi R}.$$

- There is no magnetic field outside the solenoid and inside the toroid.

- **Force on a current carrying conductor in a magnetic field**

Through experiments Ampere established that when a current element $\vec{dl}$ is placed in a magnetic field $\vec{B}$, it experiences a force

$$\vec{dF} = I\vec{dl} \times \vec{B}$$

and the magnitude force

$$dF = Idl\, B \sin\theta$$

or $$F = IBl \sin\theta$$

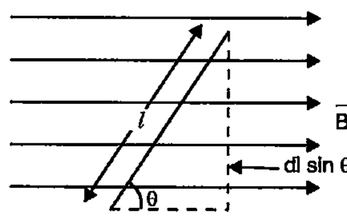

Fig. 4.8

- The direction of force is determined with the help of Fleming's left hand Rule. (Fig. 4.9)

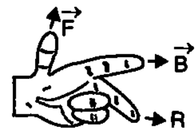

Fig. 4.9

- **Torque on a current loop in magnetic field**

$$\tau = NIBA \sin \alpha.$$

- **Moving Coil Galvanometer**
  - Current sensitivity of the galvanometer is the deflection per unit current *i.e.*,

$$\frac{\theta}{I} = \frac{NAB}{C}$$

  - Voltage sensitivity is the deflection per unit volt *i.e.*,

$$\frac{\theta}{V} = \frac{\theta}{IR}$$

or $$\text{Voltage sensitivity} = \frac{\text{current sensitivity}}{\text{Resistance}}$$

- **Force on a moving charge particle in magnetic field**

$\because$ $$F = IBl \sin \theta$$

where, $$I = \frac{q}{t} \quad \text{and} \quad l = v.t$$

$\therefore$ $$F = \frac{q}{t} Bv \,.\, t \sin \theta \quad \text{or} \quad F = qBv \sin \theta$$

QUESTIONS FROM TEXTBOOK SOLVED

4.1. *A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil?*

Sol. Given, $I = 0.40$ A, $r = 8.0$ cm $= 8 \times 10^{-2}$ m

$n = 100$

$$\boxed{B = \frac{\mu_0 n I}{2r}} = \frac{4\pi \times 10^{-7} \times 100 \times 0.4}{2 \times 8.0 \times 10^{-2}} \text{T}$$

$$= 3.1 \times 10^{-4} \text{ T.}$$

4.2. *A long straight wire carries a current of 35 A. What is the magnitude of the field* **B** *at a point 20 cm from the wire?*

Sol. Given, $I = 35\text{ A}, r = 20\text{ cm} = 0.2\text{ m}$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 35}{2\pi \times 0.20}\text{T}$$

$$= 3.5 \times 10^{-5}\text{ T.}$$

4.3. *A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of* **B** *at a point 2.5 m east of the wire.*

Sol. Given, $I = 50\text{ A}, r = 2.5\text{ m}$

$B = ?$

As, $$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 50}{2\pi \times 2.5}$$

$$= 4 \times 10^{-6}\text{ T}$$

Applying right hand thumb rule to find the direction of M.F. in east direction of wire it comes out upward direction.

4.4. *A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?*

Sol. $$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 90}{2\pi \times 1.5} T$$

$$= \frac{180}{1.5} \times 10^{-7}\text{T} = 1.2 \times 10^{-5}\text{T}$$

It is an example of magnetic field due to current in a wire of infinite length.

Applying the right-hand thumb rule, we find that the magnetic field at the observation point is directed towards south.

4.5. *What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T?*

Sol. Given, $I = 8\text{ A}, B = 0.15\text{ T}, \theta = 30°$

Force acting on wire $F = BIl \sin\theta$

Force per unit length $= \frac{F}{l} = BI \cdot \sin\theta$

$= 0.15 \times 8 \times \sin 30°$

$= 0.6\text{ N/m}$

4.6. *A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?*

Sol. Here, $l = 3.0 \text{ cm} = 3 \times 10^{-2}$ m

$I = 10$ A, $B = 0.27$ T, $\theta = 90°$, F =?

By the formula

$$F = BIl \sin\theta$$

$$= 0.27 \times 10 \times (3 \times 10^{-2}) \times \sin 90°$$

$$= 0.27 \times 10 \times 3 \times 10^{-2} \times 1$$

or, $F = 8.1 \times 10^{-2}$ N

4.7. *Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.*

Sol. Given, $I_1 = 8.0$ A, $I_2 = 5$ A

$r = 4.0 \text{ cm} = 4 \times 10^{-2}$ m

$l = 10 \text{ cm} = 0.1$ m

$F = ?$

Force on length l,

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{r}$$

$$= \frac{(4\pi \times 10^{-7}) \times 8 \times 5 \times 0.1}{2\pi \times 0.04} = 2 \times 10^{-5}\text{N}$$

4.8. *A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of* **B** *inside the solenoid near its centre.*

Sol. Here, $l = 80 \text{ cm} = 0.80$ m, $N = 5 \times 400 = 2000$

$I = 8.0$ A, D = 1.8 cm, n = no. of turns per unit length

Magnitude of magnetic field inside a solenoid near its centre

$$n = \frac{\text{Total turn}}{\text{length}}$$

$$n = \frac{2000}{0.80}$$

$$B = \mu_0 nI = \frac{4\pi \times 10^{-7} \times 2000 \times 8.0}{0.80}$$

$$= 8\pi \times 10^{-3} \text{ T} = 2.5 \times 10^{-2} \text{ T.}$$

4.9. *A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?*

Sol. Here, $l = 10 \text{ cm} = 0.10 \text{ m}, N = 20, I = 12 \text{ A}$

$\theta = 30°, B = 0.80 \text{ T}, \tau = ?$

Area, $A = l \times l = 0.1 \times 0.1 = 0.01 \text{ m}^2$

$$\tau = NBIA \sin\theta$$

$$= 20 \times 0.80 \times 12 \times (0.1)^2 \times \sin 90°$$

$$= 0.96 \text{ Nm}$$

4.10. *Two moving coil meters, M_1 and M_2 have the following particulars:*

$R_1 = 10\ \Omega;\ N_1 = 30,$

$A_1 = 3.6 \times 10^{-3}\ m^2,\ B_1 = 0.25\ T$

$R_2 = 14\ \Omega;\ N_2 = 42,$

$A_2 = 1.8 \times 10^{-3}\ m^2,\ B_2 = 0.50\ T$

(The spring constants k are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

Sol. (*a*) Current sensitivity of first meter

$$I_s = \frac{\theta}{I} = \frac{BAN}{k}$$

$$A = \frac{\theta}{I} = \frac{B_1 A_1 N_1}{k} = \frac{0.25 \times 3.6 \times 10^{-3} \times 30}{k}$$

$$= \frac{27 \times 10^{-3}}{k} \quad ...(i)$$

Current sensitivity of second meter

$$B = \frac{\theta}{I} = \frac{B_2 A_2 N_2}{k} = \frac{0.50 \times 1.8 \times 10^{-3} \times 42}{k}$$

$$= \frac{37.8 \times 10^{-3}}{k} \quad ...(ii)$$

Ratio of current sensitivity

$$\left(\frac{B}{A}\right) = \frac{37.8 \times 10^{-3}}{k} \Big/ \frac{27 \times 10^{-3}}{k} = 1.4$$

(*b*) Voltage sensitivity of first meter

$$= \frac{\theta}{V} = \frac{\theta}{I \cdot R} = \frac{27 \times 10^{-3}}{k \times 10} = \frac{2.7 \times 10^{-3}}{k}$$

Voltage sensitivity of second meter

$$= \frac{\theta}{R.I} = \frac{37.8 \times 10^{-3}}{k \times 10} = \frac{2.7 \times 10^{-3}}{k}$$

Hence, the ratio of voltage sensitivity = 1.

4.11. *In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8 × 10^6 m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. (e = 1.6 × 10^{-19} C, m_e = 9.1 × 10^{-31} kg)*

Sol. Here, $B = 6.5 \times 10^{-4}$ T, $v = 4.8 \times 10^6$ *m/s*, $e = 1.6 \times 10^{-19}$ C;

$\theta = 90°$; $m = 9.1 \times 10^{-31}$ kg; r =?

(*i*) Force on the moving electron due to magnetic field will be,

$$F = evB \sin \theta.$$

The direction of this force is perpendicular to $\vec{v}$ as well as $\vec{B}$ therefore, this force will only change the direction of motion of the electron without affecting its velocity *i.e.*, this force will provide the centripetal force to the moving electron and hence, the electron will move on the circular path. If r is the radius of circular path traced by electron, then

$$evB \sin 90° = mv^2/r \text{ or } r = \frac{mv}{Be} = \frac{(9.1 \times 10^{-31}) \times (4.8 \times 10^6)}{(6.5 \times 10^{-4}) \times (1.6 \times 10^{-19})}$$

$$= 4.2 \times 10^{-2} \text{ m} = 4.2 \text{ cm}$$

4.12. *In Question 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.*

Sol. Frequency is given by

$$v = \frac{Bq}{2\pi m} = \frac{6.5 \times 10^{-4} \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$v = \frac{10.4 \times 10^{-23}}{51.148 \times 10^{-31}} = 0.18198 \times 10^8$$

$$v = 18 \times 10^6 \text{ Hz} = 18 \text{ MHz}$$

4.13. *(a) A circular coil of 30 turns and radius 8.0 cm, carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal to the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.*

(b) Would your answer change if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered).

Sol. Given, $N = 30, I = 6.0 \text{ A}, B = 1.0 \text{ T}, \alpha = 60°$

$r = 8.0 \text{ cm} = 8 \times 10^{-2} \text{ m}.$

Area of the coil,

$$A = \pi r^2$$

$$= \frac{22}{7} \times \left(8 \times 10^{-2}\right)^2$$

$$A = 2.01 \times 10^{-2} \text{ m}^2$$

(*a*) Now, $\tau = NBIA \sin \alpha$

$$= 30 \times 6.0 \times 1.0 \times (2 \times 10^{-2}) \times \sin 60°$$

$$\tau = 30 \times 6 \times 1 \times 2 \times \frac{\sqrt{3}}{2} \times 10^{-2} = 3.12 \text{ Nm}.$$

(*b*) If the area of the loop is the same, the torque will remain unchanged as the torque on the planar loop does not depend upon the shape.

4.14. *Two concentric coils X and Y of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of current in X is anti clockwise and in Y, clockwise, for an observer looking at the coil facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.*

Consider X, X′ axis in East, West directions respectively and Y, Y′ in North, South direction. Plane of coil is in Y–Z axis plane M.F. due to coil X is in East, and due to coil Y is in west direction.

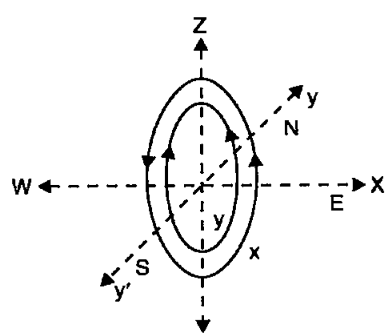

Fig. 4.10

Sol. Given, for coil X

$$r_x = 16 \text{ cm} = 0.16 \text{ m}$$
$$N_x = 20,\ I_x = 16 \text{ A}$$

Magnetic field at the centre of coil X is

$$B_x = \frac{\mu_0 I_x N_x}{2r_x}$$

$$= \frac{4\pi \times 10^{-7} \times 16 \times 20}{2 \times 0.16}$$
$$= 4\pi \times 10^{-4} \text{ T}$$

The current in the coil X is anticlockwise, the field B_x is directed towards east.

Given, for coil Y

$$r_y = 10 \text{ cm} = 0.10 \text{ m},\ N_y = 25,\ I_y = 18 \text{ A}$$

Magnetic field at the centre of coil Y is

$$B_y = \frac{\mu_0 I_y N_y}{2r_y}$$

$$= \frac{4\pi \times 10^{-7} \times 18 \times 25}{2 \times 0.10} = 9\pi \times 10^{-4} \text{ T}$$

The direction of magnetic field induction B_y is towards west.

$$\text{Net magnetic field} = B_y - B_x$$
$$= 9\pi \times 10^{-4} - 4\pi \times 10^{-4}$$
$$= 5\pi \times 10^{-4}$$
$$= 1.6 \times 10^{-3} \text{ T (Towards west).}$$

4.15. *A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross section about 10^{-3} m^2. The maximum current carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.*

Sol. Given, $B = 100 \text{ G} = 10^{-2} \text{ T}$

$$I = 15 \text{ A},\ n = 1000 \text{ m}^{-1}$$

Magnetic field inside a solenoid is

$$B = \mu_0 nI$$

$$nI = \frac{B}{\mu_0} = \frac{10^{-2}}{4\pi \times 10^{-7}} = \frac{10^5}{4\pi} = 7955$$

We may have $I = 10\ A$ and $n = 800$

The solenoid may have length 50 cm and cross section $5 \times 10^{-3}\ m^2$ (five times given values) so as to avoid *edge effects* etc.

4.16. *For a circular coil of radius R and N turns carrying current I, the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by,*

$$B = \frac{\mu_0 I R^2 N}{2(x^2 + R^2)^{3/2}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius R, and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R, and is given by,

$$B = 0.72 \frac{\mu_0 NI}{R}, \text{ approximately.}$$

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as Helmholtz coils.]

Sol. (*a*) Given, that $B = \frac{\mu_0 I R^2 N}{2(x^2 + R^2)^{3/2}}$ (axial line)

Putting $x = 0$ (centre of coil)

$$B = \frac{\mu_0 I R^2 N}{2R^3}$$

or

$$B = \frac{\mu_0 I N}{2R}$$

which is same as the standard result.

(*b*) In figure, O is a point which is mid-way between the two coils X and Y.

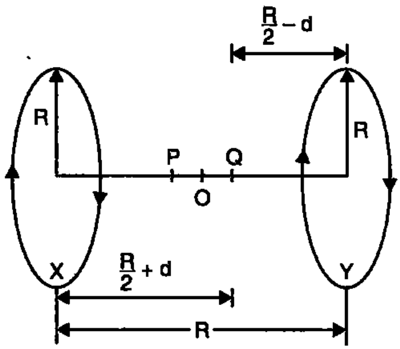

Fig. 4.11

Let B_x be the magnetic field at Q due to coil X.

Then,
$$B_x = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}+d\right)^2 + R^2\right]^{3/2}}$$

If B_y is the magnetic field at Q due to coil Y, then

$$B_y = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}-d\right)^2 + R^2\right]^{3/2}}$$

The currents in both the coils X and Y are flowing in the same direction. So, the resultant field is given by

$$B = B_x + B_y$$

$$B = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}+d\right)^2 + R^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}-d\right)^2 + R^2\right]^{3/2}}$$

$$B = \frac{\mu_0 NIR^2}{2}\left[\frac{1}{\left[\left(\frac{R}{2}+d\right)^2 + R^2\right]^{3/2}} + \frac{1}{\left[\left(\frac{R}{2}-d\right)^2 + R^2\right]^{3/2}}\right]$$

$$B = \frac{\mu_0 NIR^2}{2}\left[\frac{1}{\left[\frac{R^2}{4}+d^2+Rd+R^2\right]^{3/2}}+\frac{1}{\left[\frac{R^2}{4}+d^2-Rd+R^2\right]^{3/2}}\right]$$

$$B = \frac{\mu_0 NIR^2}{2}\left[\frac{1}{\left[\frac{5R^2}{4}+Rd\right]^{3/2}}+\frac{1}{\left[\frac{5R^2}{4}-Rd\right]^{3/2}}\right] \qquad \because \; d^2 << R^2$$

$$B = \frac{\mu_0 NIR^2}{2\left(\frac{5}{4}R^2\right)^{3/2}}\left[\frac{1}{\left[1+\frac{4}{5}\frac{d}{R}\right]^{3/2}}+\frac{1}{\left[1-\frac{4}{5}\frac{d}{R}\right]^{3/2}}\right]$$

$$= \frac{\mu_0 NIR^2}{2\left(\frac{5}{4}\right)^{3/2} R^3}\left[\left[1-\frac{3}{2}\times\frac{4}{5}\times\frac{d}{R}\right]+\left[1+\frac{3}{2}\cdot\frac{4}{5}\cdot\frac{d}{R}\right]\right]$$

$$= \frac{\mu_0 NI \cdot \cancel{2}}{\cancel{2}\left(\frac{5}{4}\right)^{3/2} R} = \frac{\mu_0 NI}{R}\left(\frac{5}{4}\right)^{3/2} = 0.72\,\frac{\mu_0 NI}{R} \text{ (approx.)}$$

4.17. *A toroid has a core (non-ferromagnetic material) of inner radius 25 cm and outer radius 26 cm around which 3500 turns of wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid (b) inside the core of the toroid (c) in the empty space surrounded by the toroid?*

Sol. Given, $r_1 = 0.25$ m, $r_2 = 0.26$ m N = 3500

$I = 11$ A

(*a*) The magnetic field is zero outside the toroid.

(*b*) Magnetic field inside the core of the toroid, $B = \mu_0 nI$

or $$B = \frac{\mu_0 NI}{l}$$

$$\left(\because n = \frac{N}{l} \rightarrow \text{Number of turns per unit length}\right)$$

$$l = 2\pi\left(\frac{r_1 + r_2}{2}\right)$$

$$= \pi\,(r_1 + r_2) = \pi\,(0.25 + 0.26) = \pi \times 0.51 \text{ m}$$

Putting the values

$$B = \frac{(4\pi \times 10^{-7}) \times 3500 \times 11}{\pi \times 0.51} = 3.02 \times 10^{-2} \text{ T}$$

(c) The magnetic field is zero in the empty space surrounded by the toroid.

4.18. *Answer the following questions:*

(a) *A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?*

(b) *A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?*

(c) *An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.*

Sol. (a) The force on a charged particle moving in a magnetic field is given by

$$F = qvB \sin \theta.$$

The charged particle will travel undeflected along a straight path with constant speed in a magnetic field if no force act on it *i.e.*, $F = 0$.

It is possible only when $\sin \theta = 0$

or $\theta = 0°, 180°$

i.e., initial velocity v is either parallel or anti-parallel to $\vec{B}$.

(b) Yes, because magnetic force can change the direction of $\vec{v}$, not its magnitude. The direction of force due to magnetic force on moving charge is perpendicular to v and BSO force component in the direction of v is zero. So charge zero is also zero.

(c) The electrostatic field is directed towards south. Since the electron is a negatively charged particle, therefore, the electrostatic field shall exert a force directed towards north. So, if the electron is to be prevented from deflection from straight path, by the magnetic force on the electron should be directed towards south. Now $\vec{F}_m = -e(\vec{v}\times\vec{B})$. $\vec{F}_m$ is towards south, $\vec{v}$ is due east. Applying Fleming's Left-Hand Rule, we find that magnetic field $\vec{B}$ should be in the vertically downward direction.

4.19. *An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30° with the initial velocity.*

Sol. Given, $V = 2$ kilo volt $= 2000$ volt

$B = 0.15$ T

(a) If magnetic field is transverse to initial velocity of electron. In this particular case, the velocity vector has no component in the direction of magnetic field.

$\therefore$Force on electron $= Bev \sin 90°$

$= Bev$

This force acts as the centripetal force:

$$\therefore \quad Bev = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{eB} \qquad \text{...(I)}$$

$$\text{But} \quad \frac{1}{2}mv^2 = eV \qquad \text{...(II)}$$

$$\text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

$$\therefore \quad r = \frac{m}{eB}\sqrt{\frac{2eV}{m}} \qquad \text{[From I, II]}$$

$$\text{or} \quad v = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

$$= \frac{1}{0.15}\sqrt{\frac{2\times 9.1\times 10^{-31}\times 2000}{1.6\times 10^{-19}}}\ \text{m}$$

$$= 10^{-3}\ \text{m} = 1.0\ \text{mm}.$$

The electron would move in a circular trajectory of radius 1.0 mm. The plane of the trajectory is normal to B.

(*b*) If v makes an angle 30° with the direction of magnetic field, the velocity can be resolved into $v_\perp$ and $v_\parallel$ *i.e.*, $v \cos 30°$ and $v \sin 30°$ respectively.

Due to $v_\perp$ the electron will move on a circular path. The resultant path will be a combination of straight line motion and circular motion which is called helical.

Thus, $$evB \sin \theta = \frac{m(v \sin \theta)^2}{r_n}$$

for circular motion of radius r_n

$$ev_\perp \times B = \frac{mv_\perp^2}{r_n}$$

$$v_\perp = v \sin \theta \quad \text{or} \quad r_n = \frac{mv \sin \theta}{eB}$$

$$r_n = \frac{9.1 \times 10^{-31} \times 2.65 \times 10^7 \times \sin 30}{1.6 \times 10^{-19} \times 0.15}$$

$$= 0.49 \times 10^{-3} \text{ m} = 0.49 \text{ mm} \simeq 0.5 \text{ mm}$$

The linear velocity $= v \cos \theta$

$$= 2.65 \times 10^7 \times \cos 30°$$

$$= 2.65 \times 10^7 \times \frac{\sqrt{3}}{2}$$

$$= 2.3 \times 10^7 \text{ ms}^{-1}$$

Thus, the electron moves in a helical path of radius 0.49 mm with a velocity component of 2.3×10^7 ms^{-1} in the direction of magnetic field.

4.20. *A magnetic field set up using Helmholtz coils is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single-species) charged particles, all accelerated through 15 kV, enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is 9.0×10^5 Vm^{-1}, make a simple guess as to what the beam contains. Why is the answer not unique?*

Sol. Given, $$\frac{1}{2}mv^2 = eV$$

$$\frac{e}{m} = \frac{v^2}{2V}$$

$$\frac{e}{m} = \frac{1.2\times10^6 \times 1.2\times10^6}{2\times15\times10^3}$$

$$\frac{e}{m} = \frac{0.24}{5}\times10^9$$

$$\frac{e}{m} = 0.048 \times 10^9$$

$$\frac{e}{m} = 4.8 \times 10^7 \text{ coulomb/kg}$$

This charge to mass ratio is equivalent to charge to mass ratio of proton so the charge particle may be deuterons. However, the answer is not unique. This is because He^{++} $\left(\frac{2e}{2m}\right)$ and Li^{++} $\left(\frac{3e}{3m}\right)$ have also the same value of $\frac{e}{m}$.

4.21. *A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.*

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before?

(Ignore the mass of the wires.) $g = 9.8 \text{ m s}^{-2}$.

Sol. Given, $l = 0.45$ m, m = 60 g = 60×10^{-3} kg

$I = 5.0$ A

(*a*) Force needed to balance the weight of the rod,

$$F = mg = 0.06 \text{ kg} \times 9.8 = 0.588 \text{ N}$$

By the formula,

$$F = BIl$$

$$B = \frac{F}{Il} = \frac{0.588}{5.0\times0.45} = 0.26 \text{ T}$$

If the direction of current in horizontal conductor is from right to left then the direction of magnetic field is horizontal and normal to the conductor, the force due to magnetic field will be upwards by Fleming's left hand rule.

(*b*) 'BIl' and 'mg' will act vertically downwards if direction of current is reversed.

Total tension in the wires = $BIl + mg$

$$= 0.588 + 0.588 = 1.176 \text{ N.}$$

4.22. *The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?*

Sol. Given, $I_1 = 300\ A$, $I_2 = 300\ A$, $r = 1.5$ cm $= 1.5 \times 10^{-2}$ m

By using formula $F = \dfrac{\mu_0 I_1 I_2}{2\pi r}$

or force per unit length $= \dfrac{F}{l} = \dfrac{4\pi \times 10^{-7} \times 300 \times 300}{2\pi \times 0.015}$

$= 1.2 \text{ Nm}^{-1}$

The total force between the wires is

$$F = fl = 1.2 \times 0.70 \text{ N} = 0.84 \text{ N.}$$

The force is repulsive since the current will flow in opposite direction in the two wires connecting the battery to the starting motor.

4.23. *A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,*

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Sol. (*a*) Diameter of cylindrical region = 20 cm = 0.20 m

Clearly, $l = 0.20$ m. Also, $\theta = 90°$

$F = BIl \sin\theta = 1.5 \times 7 \times 0.20 \sin 90°$ N

$= 2.1$ N

Using Fleming's left-hand rule, we find that the force is directed vertically downwards.

(*b*) If l_1 is the length of the wire in the magnetic field, then,

$$F_1 = BIl_1 \sin 45°$$

But $\qquad l_1 \sin 45° = l$

$\therefore \qquad F_1 = BIl = 1.5 \times 7 \times 0.20 \text{ N} = 2.1 \text{ N}$

The force is directed vertically downwards by Fleming's left hand rule.

(c) When the wire is lowered by 6 cm, the length of the wire in the cylindrical magnetic field is $2x$.

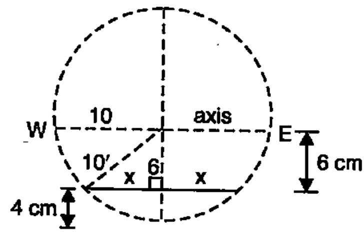

Fig. 4.12

Now, $\qquad x^2 = 10^2 - 6^2$

$$x = \sqrt{64} = 8 \text{ cm}$$

$\therefore \qquad 2x = 16$ cm.

$$F_2 = BIl_2$$

$$= 1.5 \times 7 \times 0.16 \text{ N} = 1.68 \text{ N}$$

The force is directed vertically downwards.

The result is true for any angle between current and direction of $\vec{B}$. This is because $l \sin \theta$ remains constant [illegible] 20 cm.

4.24. *A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current 12A. What is the torque on the loop in the different cases shown in the figure below. What is the force on each case? Which case corresponds to stable equilibrium?*

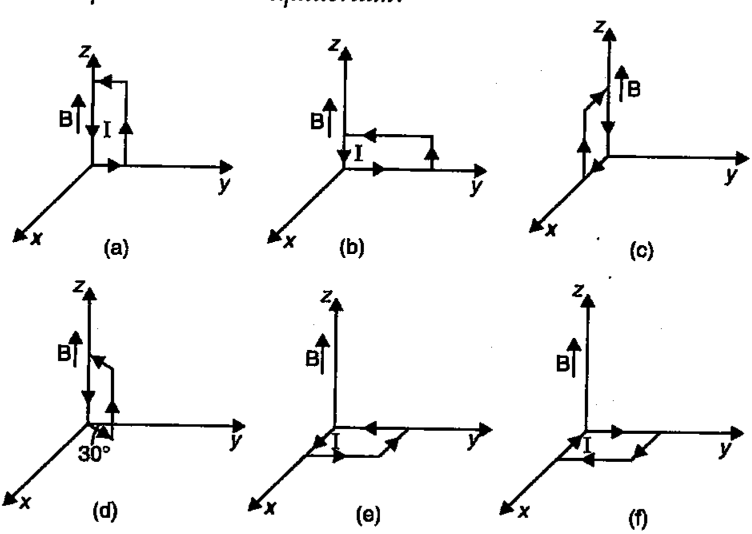

Fig. 4.13

Sol. (*a*) Torque on the loop,

$$\tau = BIA \cos \theta$$

where θ is the angle between the plane of loop and direction of magnetic field.

Here, $\theta = 0°, B = 3000$ gauss

$= 3000 \times 10^{-4}$ T

$= 0.3$ T

$I = 12$ A, $A = 10 \times 10^5$ cm$^2 = 50 \times 10^{-4}$ m^2

$\tau = 0.3 \times 12 \times 50 \times 10^{-4} = 1.8 \times 10^{-2}$ Nm.

The direction of torque or force on arm 5 cm, lower arm $+x$ axis upper arm $-x$ axis by Fleming's left hand rule.

(*b*) Similar to (*a*) but torque act on side of 10 cm.

(*c*) $\tau = 1.8 \times 10^{-2}$ Nm along $-x$ direction of torque on lower arm of 5 cm towards $-y$ axis.

(*d*) This case is similar to (c). Direction of torque is 60°.

(*e*) zero. (∵ angle between plane of loop and direction of magnetic field is 90°)

(*f*) zero.

Force is zero in each case. Stable equilibrium is corresponded by case (*e*).

4.25. *A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the*

(a) total torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{-29} m^{-3}.)

Sol. Given, $N = 20, r = 10$ cm $= 10 \times 10^{-2}$ m

$B = 0.10$ T, $I = 5.0$ A

$\theta = 0°$ (angle between field and normal to the coil)

Area of the coil,

$$A = \pi r^2 = \pi \times (10 \times 10^{-2})^2 = \pi \times 10^{-2} \text{ m}^2$$

(*a*) Torque $\tau = NIBA \sin \theta$

$= 20 \times 5.0 \times 0.10 \times \pi \times 10^{-2} \sin 0°$

$= 20 \times 5.0 \times 0.10 \times \pi \times 10^{-2} \times 0 = 0$

(*b*) Net force on a planer current loop in a magnetic field is always zero, as net force due to couple of force is zero.

(*c*) If v_d is the drift velocity of electron

$$F = qv \times B$$
$$= ev_d \,.\, B \sin 90^\circ$$

Force on one electron = $Be\, v_d = Be \dfrac{I}{neA} = \dfrac{BI}{nA}$

Here, $n = 10^{29}\ \text{m}^{-3},\ A = 10^{-5}\ \text{m}^2$

$\therefore$ Force on one electron = $\dfrac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = 5 \times 10^{-25}$ N.

4.26. *A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to the axis: both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? $g = 9.8\ ms^{-2}$.*

Sol. Given, for solenoid, $l = 60$ cm $= 0.60$ m

$N = 3 \times 300 = 900$

For wire, $l_1 = 2.0$ cm $= 0.02$ m, m 2.5

g $= 2.5 \times 10^{-3}$ kg

$I_1 = 6.0$ A

Let current I be passed through the solenoid windings, the magnetic field produced inside the solenoid due to current is

$$B = \frac{\mu_0 NI}{l}$$

Force on wire $= I_1\, l_1\, B = I_1 l_1 \dfrac{\mu_0 NI}{l}$

The wire can be supported if the force on wire is equal to the weight of wire, *i.e.,*

$$I_1 l_1 = \frac{\mu_0 NI}{l} = mg$$

or $$I = \frac{mgl}{I_1 l_1 \mu_0 N} = \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{(6.0) \times (0.02) \times \left(4\pi \times 10^{-7}\right)} \times 900$$

$$= 108.27\ \text{A}$$

4.27. *A galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?*

Sol. Given, $G = 12\ \Omega,\quad I_g = 3\text{ mA} = 3 \times 10^{-3}\text{ A}$

$V = 18\ v,\ R = ?$

By using formula,

$$V = I_g\,(R + R_g)$$

or $$\frac{V}{I_g} = R + R_g$$

or $$R = \frac{V}{I_g} - R_g = \frac{18}{3\times10^{-3}} - 12$$

$$R = 6 \times 10^3 - 12 = 5988\ \Omega$$

4.28. *A galvanometer coil has a resistance of 15 Ω and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?*

Sol. Given, $G = 15\ \Omega,\ I_g = 4\text{ mA} = 4 \times 10^{-3}\text{ A}$

$I = 6\text{ A}$

Using formula, $$S = \frac{I_g \cdot G}{I - I_g}$$

Putting values, $$S = \frac{4\times10^{-3}\times15}{6-0.004}$$

$$= \frac{60\times10^{-3}}{5.996} = 10\times10^{-3}\ \Omega$$

or $$S = 10\text{ m}\Omega$$

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