5 Magnetism and Matter

Lesson at a Glance

• Magnetic Dipole Moment

It is product of the pole strength of either pole and length of the magnet. It is a vector quantity its direction is from south to north pole.

$$\vec{M} = 2m\vec{l}$$

Where m is pole strength and it is the length of the magnet. Its unit Am^2 or JT^{-1}.

• Magnetic Force Between Two Poles

The magnetic force between two poles of strength m_1 and m_2 separated by distance r is given as

$$F = \frac{\mu_o}{4\pi}\frac{m_1 m_2}{r^2}$$

In vector form, $$\vec{F} = \frac{\mu_o}{4\pi}\frac{m_1 m_2 \vec{r}}{|\vec{r}|^3}$$

• Magnetic Field Intensity

The magnetic field intensity is the force experienced by a magnetic pole of strength unity.

If a pole of strength m_0 experience a force $F = \frac{\mu_o}{4\pi}\frac{m m_0}{r^2}$ due the pole of strength m at a distance r, then the magnetic field intensity

$$B = \frac{F}{m_0} = \frac{\mu_o}{4\pi}\frac{m m_0}{r^2}\Big/ m_0 \quad \text{or} \quad B = \frac{\mu_0 m}{4\pi r^2}$$

In vector form, $$\vec{B} = \frac{\mu_0 m \vec{r}}{4\pi|\vec{r}|^3}.$$

• Magnetic Field Due to a Dipole

(*i*) **At axial point.** Let there be magnetic dipole of dipole movement $\vec{P} = m(\vec{Il})$ and a point P at the distance of r from the mild point of the dipole where magnetic field intensity is to be determined.

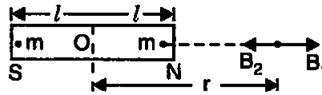


Fig. 5.1

The magnetic field intensity due to north pole at *P*,

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(r-l)^2}(\vec{OP})$$

and magnetic field due to south pole

at *P*, $$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(r+l)^2}(\vec{OP})$$

$\because \quad r >> l \quad \therefore \quad l^2$ can be neglected

$$\therefore \qquad \vec{B} = \frac{\mu_0}{4\pi}\frac{2M(r)}{(r^4)}(\vec{OP}) \quad \text{or} \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$$

(*ii*) **At equatorial point.** The magnetic field at *P* due to a dipole of dipole moment $\vec{M} = (2l)\vec{m}$ at point *P* at the distance of l_1 from its mid point at equatorial position is to be determined the magnetic field due to north pole,

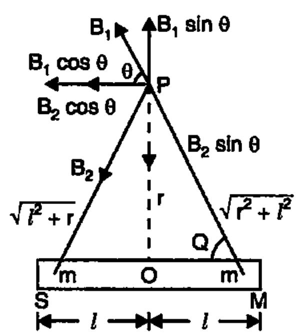


Fig. 5.2

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{\left(\sqrt{r^2+l^2}\right)^2}$$

and the magnetic field due to south pole at *P*,

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{\sqrt{r^2+l^2}}$$

If $r >> l$ then l^2 can be neglected, then

$$|\vec{B}| = \frac{\mu_0|\vec{M}|}{4\pi r^3}$$

• Dipole in Uniform Magnetic Field

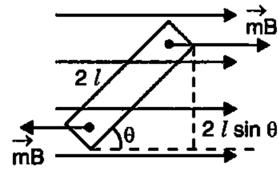


Fig. 5.3

When a magnetic dipole of dipole moment $\vec{M} = 2m\vec{l}$ is placed in uniform magnetic field of strength $\vec{B}$, its north pole experiences a force mB in the direction of field and south pole experiences the same force in opposite direction of field. As a result no net force acts on the dipole, but due to this pair of force a torque acts on the dipole. The torque is given by,

$$\tau = \text{force} \times \perp \text{distance}$$
$$= mB\,2l \sin\theta$$
$$= |\vec{M}|\,B \sin\theta$$

or

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of the torque is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$.

- The angle between freely suspended magnet and horizontal axis of earth is called angle of dip. It is 0° at the equator of earth and 90° at the poles.
- The angle between magnetic meridian and geographical meridian is called angle of declination.
- The magnetic field in which a magnetic material is placed for its magnitisation is called magnetising field. In a magnetising field the ratio of magnetising field $\vec{B_0}$ to the permeability of free space is called *intensity of magnetising field.*

$$\vec{H} = \frac{\vec{B_0}}{\mu_0} \quad \text{or} \quad \vec{B_0} = \mu_0 \vec{H}$$

  Its unit is Am.
- When a magnetic material is magnetised by placing it in a magnetising field, the induced dipole moment per unit volume in the material is called *intensity of magnetisation.*

$$\vec{I} = \frac{\vec{M}}{V}$$

- The ratio of magnitude of intensity of magnetisation to that of magnetising field is called *magnetic susceptibility.*

$$\chi = \frac{I}{H}$$

It is a scalar quantity having no unit.

- When a magnetic material is placed in a magnetic field, the ratio of magnitude of intensity of magnetising field is called *magnetic permiability.*

$$\mu = \frac{B}{H}$$

- The ratio of permeability of a medium to that of free space is called *relative permeability*

$$\mu_r = \frac{\mu}{\mu_0}$$

- When a material is placed in a magnetising field of intensity $\vec{H}$, induced field is developed of intensity $\vec{I}$ and net field becomes,

$$\vec{B} = \vec{H} + \vec{I}$$

or

$$\frac{B}{H} = 1 + \frac{I}{H} \quad \text{or} \quad \mu_r = 1 + \chi$$

• Magnetic Materials

On the basis of atomic theory, the materials are classified into three categories.

(*i*) **Diamagnetic Materials.** The materials which when placed in magnetic field get feebly magnetised in the opposite direction of field are called diamagnetic materials.

(*ii*) **Paramagnetic materials:** These are the materials which when placed in magnetic field get feebly magnetised in the direction of field.

• Ferromagnetic materials

These are the material which when placed in magnetic field get strongly magnetised in the direction of field. They have tendency to move from weak magnetic field to strong magnetic field.

TEXTBOOK QUESTIONS SOLVED

5.1. *Answer the following questions regarding earth's magnetism:*

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18°. Would you expect a greater or smaller dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

Sol. (*a*) The three independent quantities conventionally used to specify the earth's magnetic field are—Magnetic declination, angle of dip and horizontal component of earth's magnetic field.

(*b*) Since Britain is closer to the magnetic north pole, we can expect a greater dip angle in Britain. It is about 70°.

(*c*) Field lines of $\vec{B}$ due to the earth's magnetism would seem to come out of the ground.

(*d*) A compass is free to move in a horizontal plane, while the earth's field is exactly vertical at the magnetic poles. So the compass can point in any direction there.

(*e*) Magnetic field B at an equatorial point of the earth's magnetic dipole is given by

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}}$$

Now $m = 8 \times 10^{22}$ JT^{-1}, $r = 6.4 \times 10^6$ m

$$B = 10^{-7} \times \frac{8 \times 10^{22}}{(6.4 \times 10^6)^3} \text{ T}$$
$$= 0.3 \times 10^{-4} \text{ T} = 0.3 \text{ G}$$

Which is of the same order of magnitude as that of the observed field on the earth.

(*f*) The earth's field is only approximately a dipole field. Local *N-S* poles may arise due to, for instance, magnetised mineral deposits.

5.2. *Answer the following questions:*

(a) The earth's magnetic field varies from point to point in space. Does it also change with time ? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past ?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence? Explain.

*[**Note:** Question 5.2 is meant mainly to arouse your curiosity. Answers to some questions above are tentative or unknown. Brief answers wherever possible are given at the end. For details, you should consult a good text on geomagnetism.]*

Sol. (*a*) Yes, it does change with time. Time scale for appreciable change is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

(*b*) Because *molten iron* (which is the phase of the *iron at the high temperatures* of the *core*) is not ferromagnetic due to temperature beyond Curie temp.

(c) Radioactivity may be one of the possible sources for the charged current in the outer conducting regions of the earth's core which are thought to be responsible for earth's magnetism.

(d) Earth's magnetic field gets weakly 'recorded' in certain rocks during solidification. Analysis of this rock magnetism offers clues to geomagnetic history.

(e) At large distance, the field gets modified due to the field of ions in motion (in the earth's ionosphere). The field of these ions, in turn, is sensitive to extraterrestrial disturbances such as the solar wind.

(f) From the relation $R = \frac{mv}{eB}$, $\left[\because \frac{mv^2}{R} = qvB\right]$ we find that an extremely minute field bends charged particles in a circle of very large radius. Over a small distance, the deflection due to the circular orbit of such large R may not be noticeable, but over the gigantic interstellar distances, the deflection can significantly affect the passage of charged particles, *e.g.*, cosmic rays.

5.3. *A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?*

Sol. Given, $\theta = 30°$, $B = 0.25$ T, $\tau = 4.5 \times 10^{-2}$ J

Using formula, $\tau = MB\sin\theta$

$\therefore$ $M = 2ml$ [m = pole strength]

$$\therefore \quad M = \frac{\tau}{B\sin\theta} = \frac{4.5 \times 10^{-2}}{0.25 \times \sin 30°}$$

$$m = 0.36 \text{ JT}^{-1}$$

5.4. *A short bar magnet of magnetic moment $M = 0.32$ JT^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?*

Sol. (a) If magnetic moment is parallel to $\vec{B}$, far stable equilibrium. Then potential energy, $\boxed{U = -MB\cos\theta}$ $[\theta = 0°]$

$$= -0.32 \times 0.15 \text{ J}$$
$$= -4.8 \times 10^{-2} \text{ J}$$

(*b*) If magnetic moment is antiparallel to $\vec{B}$, for unstable equilibrium then

$$\theta = 180° \quad \text{so } \cos\theta = -1$$
$$U' = +MB = 0.32 \times 0.15$$
$$= 4.8 \times 10^{-2} \text{ J.}$$

5.5. *A closely wound solenoid of 800 turns and area of cross-section 2.5×10^{-4} m^2 carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?*

Sol. Given, $N = 800$, $A = 2.5 \times 10^{-4}$ m^2, $I = 3.0$ A

Magnetic dipole moment, $\boxed{M = NIA}$

Putting values, $M = 800 \times 3.0 \times 2.5 \times 10^{-4}$

$$= 0.6 \text{ JT}^{-1}$$

It is along the axis of the solenoid. The direction is determined by the sense of flow of the current. Solenoid acts like a bar magnet.

5.6. *If the solenoid in Question 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?*

Sol. Given, $N = 800$, $I = 3$A, $A = 2.5 \times 10^{-4}$ m^2

$B = 0.25$ T, $\theta = 30°$

Magnetic moment, $\boxed{M = NIA}$

$$= 800 \times 3.0 \times 2.5 \times 10^{-4}$$
$$= 0.6 \text{ JT}^{-1}$$

Torque, $\boxed{\tau = MB\sin\theta} = 0.6 \times 0.25 \times \sin 30°$

$$= 0.150 \times 0.5 = 7.5 \times 10^{-2} \text{ J.}$$

5.7. *A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.*

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

Sol. $M = 1.5$ JT^{-1}, $B = 0.22$ T (*a*) (*i*) Work required to turn the magnet normal to the field direction

$$W_1 = -MB[\cos 90° - \cos 0°]$$
$$W_1 = +MB$$
$$W_1 = MB = 1.5 \times 0.22 = 0.33 \text{ J}$$

(*ii*) Work required to turn the magnet opposite to the field direction

$$W_2 = -MB[\cos 180° - \cos 0°]$$
$$W_2 = 2MB$$
$$W = -MB[\cos \theta_2 - \cos \theta_1]$$
$$W_2 = 2MB = 2 \times 0.33 = 0.66 \text{ J}$$

(*b*) (*i*) $\tau = MB \sin 90° = MB = 0.33$ J.

It works in the direction that tends to align the magnetic moment vector along B.

(*ii*)
$$\tau = MB \sin \theta$$
$$\theta = 180°$$
$$\tau = 1.5 \times 0.22 \times \sin 180°$$
$$= 1.5 \times 0.22 \times 0 = 0.$$

5.8. *A closely wound solenoid of 2000 turns and area of cross-section 1.6 × 10^{-4} m^2, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.*

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5 × 10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

Sol. (*a*) Magnetic moment

$$\boxed{M = NIA} = 2000 \times 4 \times 1.6 \times 10^{-4}$$
$$= 1.28 \text{ Am}^2$$

The direction of $\vec{M}$ is along the axis of the solenoid in the direction related to the sense of current via the right handed screw rule.

(*b*) The magnetic field is given to be uniform. So, the force on the solenoid is zero.

$$\text{Torque, } \boxed{\tau = MB \sin \theta}$$
$$= 1.28 \times 7.5 \times 10^{-2} \sin 30°$$
$$= 1.28 \times 7.5 \times 10^{-2} \times \frac{1}{2} \text{ J} = 0.048 \text{ J}.$$

The direction of the torque is such that the solenoid tends to align the axis of the solenoid (magnetic moment vector) along $\vec{B}$.

5.9. *A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?*

Sol. Here,

$$N = 16, \quad r = 10 \text{ cm} = 0.1 \text{ m}$$
$$I = 0.75 \text{ A}, \quad B = 5.0 \times 10^{-2} \text{ T}$$
$$\upsilon = 2.0 \text{ s}^{-1}$$
$$\boxed{M = NIA} = NI\pi r^2$$
$$= 16 \times 0.75 \times \frac{22}{7} (0.1)^2 = 0.377 \text{ JT}^{-1}$$

Using formula

$$\boxed{\upsilon = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}}$$

$$\therefore \quad \upsilon^2 = \frac{MB}{4\pi^2 I} \quad \text{or} \quad I = \frac{MB}{4\pi^2\upsilon^2}$$

Putting values,

$$I = \frac{0.377 \times 5.0 \times 10^{-2}}{4 \times \left(\frac{22}{7}\right)^2 \times 2^2} = 1.2 \times 10^{-4} \text{ kg m}^2$$

5.10. *A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.*

Sol. Here, $\delta = 22°, \quad B_H = 0.35$ G

Since, $B_H = B_E \cos\delta$

$$\therefore \quad B_E = \frac{B_H}{\cos\delta} = \frac{0.35}{\cos 22°} = \frac{0.35}{0.9272}$$

or, $B_E = 0.38$ G.

5.11. *At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.*

Sol. Given, declination, $\theta = 12°$ west

dip, $\delta = 60°$

$B_H = 0.16$ gauss $= 0.16 \times 10^{-4}$ tesla

Since,

$$B_H = B_E \cos \delta$$

$$\therefore \quad B_E = \frac{B_H}{\cos \delta} = \frac{0.16 \times 10^{-4}}{\cos 60°}$$

or,

$$B_E = \frac{0.16 \times 10^{-4}}{1/2} = 0.32 \times 10^{-4} \text{ T.}$$

The earth's field lies in a vertical plane 12° west of geographic meridian at an angle of 60° above the horizontal.

5.12. *A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet.*

Sol. Given $M = 0.48 \text{ JT}^{-1}$, $r = 10$ cm $= 0.1$ m

(*a*) Magnetic field at its axis

$$B_a = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$$

Putting values,

$$B_a = \frac{4\pi \times 10^{-7} \times 2 \times 0.48}{4\pi \times (0.1)^3}$$

or, $B_a = 960 \times 10^{-7}$ T $= 960 \times 10^{-3}$ G

or, $B_a = 0.96$ G along *N-S* line.

(*b*) Magnetic field along the equatorial line

$$B_e = \frac{\mu_0}{4\pi} \frac{M}{r^3}$$

or,

$$B_e = \frac{4\pi \times 10^{-7} \times 0.48}{4\pi \times (0.1)^3}$$

or, $B_e = 480 \times 10^{-7}$ T $= 480 \times 10^{-3}$ G

or, $B_e = 0.48$ G along *N-S* line.

5.13. *A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null point (i.e., 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field).*

Sol. Bar magnet is placed so that its axis is aligned along the magnetic *N-S* direction. As the null points are found on the axis of the magnet, it shows that south pole of magnet faces Geographical North (as shown in fig.).

In this case $\vec{M}$ is antiparallel to the earth's field. Since angle of dip is zero, the horizontal component of the earth's magnetic field equals the field itself. In this case

$$\frac{\mu_0 2M}{4\pi r^3} = 0.36 \text{ G}$$

$$B_{ax} = -0.36 \text{ G}$$

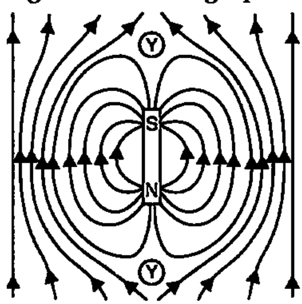


Fig. 5.4

From the figure, total magnetic field at the normal bisector of the magnet at the same distance ($r = 14$ cm) from the centre of magnet is given by

$$= B_{eq} + B_H$$

$$= \frac{\mu_0 M}{4\pi r^3} + 0.36 = \frac{0.36}{2} + 0.36 = 0.54$$

i.e., total magnetic field is 0.54 G in the direction of earth's field.

5.14. *If the bar magnet in exercise 13 is turned around by 180°, where will the new null points be located?*

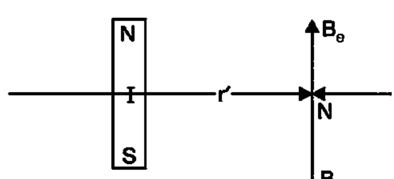


Fig. 5.5

Sol. In this case, the neutral point will be on the equatorial line. If r' is the distance of the neutral point from the centre of the magnet, then

$$\frac{\mu_0}{4\pi}\frac{M}{r^B} = 0.36 \times 10^{-4} = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

or, $$(r')^3 = \frac{r^3}{2}$$

or, $$r' = \frac{r}{.2^{1/3}} = 14\ (2)^{-1/3} = 11.1\ \text{cm}$$

It is on the normal bisector.

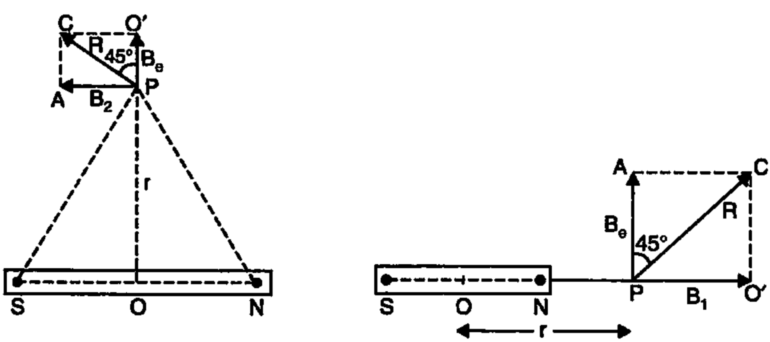
5.15. *A short bar magnet of magnetic moment 5.25×10^{-2} JT^{-1} is placed with its axis perpendicular to earth's field direction. At what distance from the centre of the magnet, is the resultant field inclined at 45° with earth's field on (i) its normal bisector, (ii) its axis? Magnitude of earth's field at the place 0.42 G. Ignore the length of the magnet in comparison to the distances involved.*

Sol. Here, $M = 5.25 \times 10^{-2}\ JT^{-1}$

$r = ?$

Earth's field $\vec{B}_e = 0.42\ G = 0.42 \times 10^{-4}\ T$

(*i*) At a point *P* distant *r* on normal bisector, fig. (*a*), field due to the magnet is



(*a*) (*b*)

Fig. 5.6

Magnetic field B_2 due to magnet at equitorial line

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{M}{r^3} \text{ along } PA \parallel NS$$

The resultant field $\vec{R}$ will be inclined at 45° to the earth's field along *PQ'*, only when

$$|\vec{B}_2| = |\vec{B}_e|$$

$$\frac{\mu_0}{4\pi}\frac{M}{r^3} = 0.42 \times 10^{-4}$$

$$\frac{10^{-7} \times 5.25 \times 10^{-2}}{r^3} = 0.42 \times 10^{-4}$$

which gives, $r = 0.05 \text{ m} = 5 \text{ cm}.$

(*ii*) When the point P lies on axis of the magnet such that $OP = r$, field due to magnet [Fig. (*b*),] is

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2M}{r^3}, \text{ along } PO,$$

Earth's field $\vec{B}_e$ is along $\overrightarrow{PA}$.

The resultant field $\vec{R}$ will be inclined at 45° to earth's field only when

$$\left|\vec{B}_1\right| = \left|\vec{B}_e\right|$$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{2M}{r^3} = 0.42 \times 10^{-4}$$

which gives $r = 6.3 \times 10^{-2} \text{ m} = 6.3 \text{ cm}.$

5.16. *Answer the following questions:*

(*a*) *Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?*

(*b*) *Why is diamagnetism, in contrast, almost independent of temperature?*

(*c*) *If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?*

(*d*) *Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?*

(*e*) *Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point). Why?*

(*f*) *Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of a ferromagnet?*

Sol. (*a*) The tendency to disrupt the alignment of dipoles (with the magnetising field) arising from random thermal motion is reduced at lower temperatures.

(*b*) The induced dipole moment in a diamagnetic sample is always opposite to the magnetising field, no matter what the internal motion of the atoms is.

(*c*) Slightly less, since bismuth is diamagnetic.

(*d*) No, as is evident from the magnetisation curve. From the slope of magnetisation curve, it is clear that μ is greater for lower fields.

(*e*) Proof of this important fact (of much practical use) is based on boundary conditions of magnetic field ($\vec{B}$ and $\vec{H}$) at the interface of two media. (When one of the media has $\mu >> 1$, the field lines meet this medium nearly normally).

(*f*) Yes. Apart from minor differences in strength of the individual atomic dipoles of two different materials, a paramagnetic sample with saturated magnetisation will have the same order of magnetisation. But of course, saturation requires impractically high magnetising fields.

5.17. *Answer the following questions:*

(*a*) *Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.*

(*b*) *The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?*

(*c*) *'A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory ?' Explain the meaning of this statement.*

(*d*) *What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer?*

(*e*) *A certain region of space is to be shielded from magnetic fields. Suggest a method.*

Sol. (*a*) The atomic dipoles are grouped together in domains in a ferromagnetic substance. All the dipoles of a domain are aligned in the same direction and have net magnetic moment. In an unmagnetised substance these domains are randomly distributed so that the resultant magnetisation is zero.

These domains align themselves in the direction of the field when the substance is placed in an external magnetic field. Some energy is spent in the process of alignment. These domains do not come back into their original random positions completely when the external field is removed.

Some magnetisation is retained by the substance. The energy spent in the process of magnetisation is not fully recovered. The balance of energy is lost as heat. This is the basic cause for irreversibility of the magnetisation curve of a ferromagnetic substance.

(*b*) Carbon steel piece, because heat lost per cycle is proportional to the area of hysteresis loop.

(*c*) Magnetisation of a ferromagnet is not a single-valued function of the magnetising field. Its value for a particular field depends both on the field and also on history of magnetisation (*i.e.*, how many cycles of magnetisation it has gone through etc.). In other words, the value of magnetisation is a record or 'memory' of its cycles of magnetisation. If information bits can be made to correspond to these cycles, the system displaying such a hysteresis loop can act as a device for storing information.

(*d*) Ceramics. (specially treated barium iron oxides, also called ferrites.)

(*e*) Surround the region by soft iron rings. Magnetic field lines will be drawn into the rings, and the enclosed space will be free of magnetic field. But this shielding is only approximate, unlike the perfect electric shielding of a cavity in a conductor placed in an external electric field.

5.18. *A long straight horizontal cable carries a current of 2.5 A in the direction 10° south of west to 10° north of east. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points (Ignore the thickness of the cable)?*

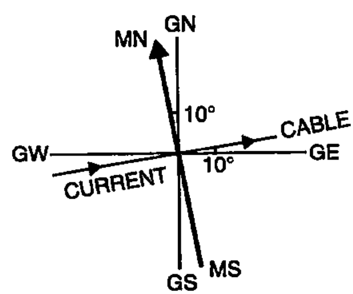


Fig. 5.7

Sol. Here,

$$I = 2.5 \text{ amp.}$$

$$R = 0.33\text{G} = 0.33 \times 10^{-4} \text{ T}$$

$$\delta = 0^\circ$$

Horizontal component of earth's field

$$H = R \cos \delta = 0.33 \times 10^{-4} \cos 0^\circ$$
$$= 0.33 \times 10^{-4} \text{ tesla}$$

Let the neutral points lie at a distance r from the cable.

Strength of magnetic field on this line due to current in the cable

$$= \frac{\mu_0 I}{2\pi r}$$

At neutral point, $$\frac{\mu_0 I}{2\pi r} = H$$

$$r = \frac{\mu_0 I}{2\pi H} = \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 0.33 \times 10^{-4}}$$
$$= 1.5 \times 10^{-2} \text{ m} = 1.5 \text{ cm}$$

Hence neutral points lie on a straight line parallel to the cable at a perpendicular distance of 1.5 cm above the plane of the paper.

5.19. *A telephonic cable at a place has four long straight horizontal wires carrying a current of 1.0 amp. in the same direction east to west. The earth's magnetic field at the place is 0.39 G and the angle of dip is 35°. The magnetic declination is almost zero. What are the resultant magnetic fields at points 4.0 cm below and above the cable?*

Sol. Given, earth's magnetic field,

$$B_e = 0.39 \text{ G}$$

and $$\delta = 35^\circ$$

∴ Horizontal component of earth's magnetic field,

$$B_H = B_e \cos \delta$$

or, $$B_H = 0.39 \cos 35^\circ = 0.3195 \text{ G}$$

Vertical component of earth's magnetic field,

$$B_V = B_e \sin \delta$$

or, $$B_V = 0.39 \sin 35^\circ = 0.224 \text{ G}$$

Magnetic field produced by telephone cable having 4 wires

$$B = \left(\frac{\mu_0 I . 2}{4\pi r}\right) \times 4$$

$$\boxed{B = \frac{\mu_0 F}{2\pi r}}$$

where, $I = 1.0$ A, $a = 4.0$ cm $= 4 \times 10^{-2}$ m

∴ $$B' = 10^{-7} \times \frac{2 \times 1.0 \times 4}{4 \times 10^{-2}} = 0.2 \times 10^{-4} \text{ T} = 0.2 \text{ G}$$

Resultant field below the cable. As per the right hand thumb rule, the direction of B' will be opposite to B_H at a point below the cable.

Therefore at a point 4 cm *below* the cable, resultant horizontal component of earth's field

$$R_H = B_H - B' = 0.3195 - 0.2$$
$$= 0.1195 \text{ G}$$

Resultant vertical component of earth's field

$$R_v = B_v = 0.224 \text{ G (unchanged)}$$

∴ Resultant of earth's field

$$R = \sqrt{R_H^2 + R_V^2} = \sqrt{(0.1195)^2 + (0.224)^2}$$
$$= \sqrt{0.0143 + 0.0500} = \sqrt{0.0643} = 0.254 \text{ G}$$

Resultant field above the cable. As per right hand thumb rule, at a point above the cable, B' will be in the same direction as B_H. Hence, at a point 4 cm above the cable.

$$R_H = B_H + B' = 0.3195 + 0.2 = 0.5195 \text{ G}$$
$$R_V = B_V = 0.224 \text{ G}$$

∴ $$R = \sqrt{R_H^2 + R_V^2} = \sqrt{(0.5195)^2 + (0.224)^2}$$

or, $$R = 0.566 \text{ G.}$$

5.20. *A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45° with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.*

(a) Determine the horizontal component of the earth's magnetic field at the location.

(b) The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90° in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the places to be zero.

Sol. (*a*) $N = 30$, $I = 0.35$ A, $r = 12$ cm $= 0.12$ m

The magnetic field at the centre of the coil is $B = \frac{\mu_0 NI}{2r}$. It acts in a direction perpendicular to the plane of the coil. Its component parallel to the magnetic meridian is $\frac{\mu_0 NI}{2r} \cos 45°$.

The component perpendicular to the magnetic meridian is $\frac{\mu_0 NI}{2r}$ sin 45°.

As the needle points in the west-east direction,

$\therefore$ Horizontal component of earth's magnetic field is given by

$$B_H = \frac{\mu_0 NI}{2r} \cos 45°$$

$$= \frac{4\pi \times 10^{-7} \times 30 \times 0.35}{2 \times 0.12 \times \sqrt{2}} = 0.39 \times 10^{-4} \text{T}$$

or, $B_H = 0.39$ G

(*b*) In this case, the plane of the coil makes an angle of 45° with the magnetic meridian on the other side. The needle will rotate and will set in east to west direction.

5.21. *A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60° and one of the fields has a magnitude of 1.2 × 10⁻² tesla. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field ?*

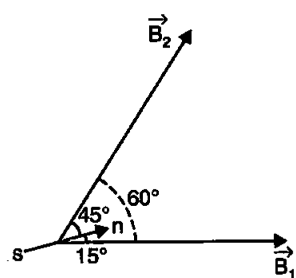


Fig. 5.8

Sol. Here, $\theta = 60°$;

$B_1 = 1.2 \times 10^{-2}$ tesla

$\theta_1 = 15°$;

$\theta_2 = 60° - 15° = 45°$.

In equilibrium, torques due to two fields must balance

i.e., $\tau_1 = \tau_2$

$M B_1 \sin \theta_1 = MB_2 \sin \theta_2$

$$B_2 = \frac{B_1 \sin \theta_1}{\sin \theta_2} = \frac{1.2 \times 10^{-2} \sin 15°}{\sin 45°}$$

$$B_2 = \frac{1.2 \times 10^{-2} \times 0.2588}{0.7071} = 4.4 \times 10^{-3} \text{ tesla}$$

5.22. *A monoenergetic (18 keV) electron beam initially in the horizontal direction is subjected to a horizontal magnetic field of 0.40 G normal to the initial direction. Estimate the up or down deflection of the beam* *over a distance of 30 cm. Given mass of electron 9.11×10^{-31} kg and charge on electron = 1.6×10^{-19} C.*

*[**Note:** Data in this exercise are so chosen that the answer will give you an idea of the effect of earth's magnetic field on the motion of electron beam from electron gun to the screen in a T.V. set.]*

Sol. Here,

energy $E = 18$ keV

$= 18 \times 1.6 \times 10^{-16}$ J

$B = 0.40$ G

$= 0.40 \times 10^{-4}$ T

$x = 30$ cm $= 0.3$ m

As $E = \frac{1}{2} m v^2$

$\therefore$ $v = \sqrt{2E/m}$

In a magnetic field, electron beam is deflected along a circular arc of radius r, such that

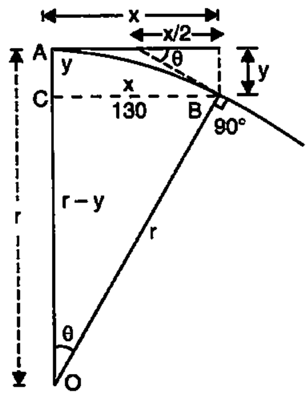


Fig. 5.9

$$Bev = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{Be}$$

$$r = \frac{m}{Be}\sqrt{\frac{2E}{m}} = \frac{1}{Be}\sqrt{2E\,m} = 11.3 \text{ m}$$

$$\sin\theta = \frac{x}{r}$$

$$\sin\theta = \frac{0.3}{11.3} = \frac{3}{113}$$

$$y = r - OC$$

$$= r - r\cos\theta = r(1 - \cos\theta) = r[1 - \sqrt{1 - \sin^2\theta}]$$

By x sin g binomial

$$y = r\left[1 - \left(1 - \frac{1}{2}\sin^2\theta\right)\right.$$

$$y = \frac{r}{2}.\sin^2\theta$$

$$y = \frac{11.3}{2} \times \frac{3 \times 3}{113 \times 113 \times 10} = \frac{9}{2260}$$

$$= 3.98 \times 10^{-3} \cong 4 \text{ mm.}$$

5.23. *A sample of paramagnetic salt contains 2.0 × 10^{24} atomic dipoles each of dipole moment 1.5 × 10^{-23} J T^{-1}. The sample is placed under a homogeneous magnetic field of 0.64 T, and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K? (Assume Curie's law)*

Sol. Magnetic dipole moment of each dipole = 1.5 × 10^{-23} J T^{-1}
Number of atomic dipoles = 2.0 × 10^{24}
∴ Possible magnetic dipole moment of the sample

$$M = 1.5 \times 10^{-23} \times 2.0 \times 10^{24} = 30 \text{ J T}^{-1}$$

At temperature of 4.2 K, the magnetic saturation is 15%.
∴ Dipole moment achieved at 4.2 K = 15% of M

$$M_1 = 30 \times \frac{15}{100} = 4.5 \text{ J T}^{-1}$$

According to Curie's law $\boxed{M \propto \frac{B}{T}}$

or $$\frac{M_1}{M_2} = \frac{B_1}{T_1} \times \frac{T_2}{B_2}$$

or, $$M_2 = M_1 \times \frac{T_1}{T_2} \times \frac{B_1}{B_2}$$

Here, $M_1 = 4.5 \text{ JT}^{-1}$, $T_1 = 4.2$ K, $T_2 = 2.8$ K
$B_1 = 0.84$ T and $B_2 = 0.98$ K

$$M_2 = \frac{4.5 \times 4.2 \times 0.98}{2.8 \times 0.84} = 7.875 \text{ JT}^{-1}$$

5.24. *A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field B in the core for a magnetising current of 1.2 A?*

Sol. Rowland ring is toroid with core of magnetic material

∴ $$B = \mu_0 n I$$

$$n = \frac{N}{l}$$

But $\mu = \mu_0 \mu_r$ and $l = 2\pi r$

∴ $$B = \frac{\mu_r \mu_0 \, NI}{2\pi r}$$

or, $$B = \frac{4\pi \times 10^{-7} \times 800 \times 3500 \times 1.2}{2\pi \times 0.15}$$

or, $B = 4.48$ tesla.

5.25. *The magnetic moment vectors μ_s and μ_l associated with the intrinsic spin angular momentum S and orbital angular momentum l, respectively, of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by:*

$$\mu_s = -(e/m)\,S,$$
$$\mu_l = -(e/2m)\,1$$

Which of these relations is in accordance with the result expected classically? Outline the derivation of the classical result.

Sol. The relation $\mu_1 = -\left(\frac{e}{2m}\right)l$ is in accordance with the result expected from classical physics. It can be derived as follows: Magnetic moment associated with the orbital motion of the electron is

$$\mu_l = \text{current} \times \text{area of the orbit}$$
$$= IA = \frac{-e}{T}\cdot \pi r^2$$

and angular momentum of the orbiting electron is given by

$$l = mvr = m\cdot\frac{2\pi r}{T}\cdot r = \frac{2\pi.mr^2}{T}$$

Here r is the radius of the circular orbit which the electron of mass m and charge $(-e)$ completes in time T.

$$\therefore \qquad \vec{\mu}_l = \frac{-e\pi r^2 l}{2\pi mr^2} = \frac{-e}{2m}\vec{l}$$

As charge of the electron is negative $(= -e)$ it is easily seen that μ_1 and l are antiparallel, both normal to the plane of the orbit.

Therefore, $\mu_1 = -\frac{e}{2ml}$ which is same result as predicted by quantum theory in contrast, $\mu_s/s = \frac{e}{m}$ is twice the classically expected value. This latter result (verified experimentally) is an outstanding consequence of modern quantum theory.

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