7 Alternating Current

Lesson at a Glance

- The current which changes in magnitude and direction both with time is called alternating current and the corresponding emf is called alternating emf.
- When a coil is rotated in uniform magnetic field with angular speed ω, the induced emf

$$\varepsilon = NBA\omega \sin \omega t$$

or $$\varepsilon = \varepsilon_0 \sin \omega t \quad ...(i)$$

$\Rightarrow$ $$\frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t$$

or $$I = I_0 \sin \omega t \quad ...(ii)$$

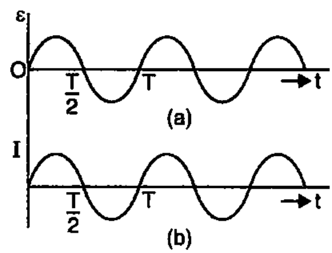


Fig. 7.1

Eqs (i) and (ii) are the equation of alternating emf and alternating current respectively.

These equations can be represented by graph as shown in Fig. 7.1.

- The average value of alternating current is zero.
- The average value of a.c. for half cycle = $\frac{2I_0}{\pi}$.

• Root Mean Square Value of AC

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

or $$I_0 = I_{rms}\sqrt{2}$$

Similarly $$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}}$$

or $$\varepsilon_0 = \varepsilon_{rms}\sqrt{2}$$

• Alternating Current Circuits

An alternating current circuit may have three elements separately or simultaneously. These three elements are

(i) Resistance (R) (ii) Inductance (L) (iii) Capacitance (C)

• Circuit Containing Resistance Only

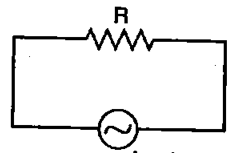
Let the voltage $\varepsilon = \varepsilon_0 \sin \omega t$ ...(i)

is applied across a circuit containing resistance only. The current in the circuit will be

$$\frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t$$

or $I = I_0 \sin \omega t$ ...(ii)

where $\frac{\varepsilon}{I}$ or $\frac{\varepsilon_0}{I_0}$ is the resistance of the circuit.



$\varepsilon = \varepsilon_0 \sin \omega t$

Fig. 7.2

• Circuit containing Inductor only

Let a voltage $\varepsilon = \varepsilon_0 \sin \omega t$ ...(i)

is applied across the circuit containing inductance L only. The current in the circuit will be

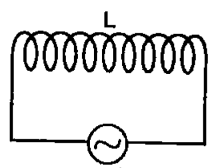
$$\int dI = \int \frac{\varepsilon_0}{L} \sin \omega t \quad \left(\because \ \varepsilon = L\frac{dI}{dt}\right)$$

or $$I = \frac{-\varepsilon_0}{\omega L} \cos \omega t$$

or $$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad ...(ii)$$

where $$I_0 = \frac{\varepsilon_0}{\omega L}$$

or $\frac{\varepsilon_0}{I_0} = \omega L = (X_L)$ is the resistance offered by inductor known as inductive reactance.



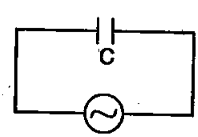
$\varepsilon = \varepsilon_0 \sin \omega t$

Fig. 7.3

• Circuit Containing Capacitor Only

Let a voltage $\varepsilon = \varepsilon_0 \sin \omega t$ ...(i)

is applied across a circuit containg capacitance C only. The current in the circuit will be



$\varepsilon = \varepsilon_0 \sin \omega t$

Fig. 7.4

$$I = \frac{dq}{dt} = \frac{d}{dt}(C\varepsilon)$$

$$= \frac{d}{dt}(C\varepsilon_0 \sin \omega t)$$

or $$I = \omega C\varepsilon_0 \cos \omega t$$

or $$I = I_0 \sin(\omega t + \pi/2) \quad ...(ii)$$

where $I_0 = \omega C \varepsilon_0$ or $E_0 = \frac{I_0}{\omega C} = I_0 X_C$

or $\frac{\varepsilon_0}{I_0} = \frac{1}{\omega C} - (X_C)$ is the resistance offered by capacitor known as capacitive reactance.

- **Circuit Containing Inductor and Resistor**

$$X_L = \sqrt{R^2 + X_L^2}$$

- **Circuit Containing Capacitor and Resistor**

$$Z_C = \sqrt{R^2 + X_C^2}$$

- **Circuit Containing Inductor, Capacitor and Resistor**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- **Circuit Containing Inductor and Capacitor**

$$Z_{LC} = X_L - X_C$$

- **Energy Stored in Inductor**

The amount of work done to oppose the change in magnetic flux linked with the inductor stores in the form of magnetic energy in the inductor.

$$\therefore \quad U = \int \varepsilon . dq$$

$$= \int \frac{LdF}{dt} . dq$$

$$= \int LIdI$$

or

$$\boxed{U = \frac{1}{2} LI^2}$$

- **Average Power in AC Circuit**

$$P_{av} = \varepsilon_{rm} I_{rms} . \left(\frac{R}{Z}\right)$$

- **A.C. Generator**

It is a device which converts mechanical energy into electrical energy.

• Transformer

It is a device which is used to change the current or voltage in the circuit.

• Eddy Currents

When a metallic block is placed in a changing magnetic field, the flux linked with the coil changes and induced currents are produced in the block. The direction of these currents is circular in nature, hence, these are called eddy current. The currents are also called *Focault current.*

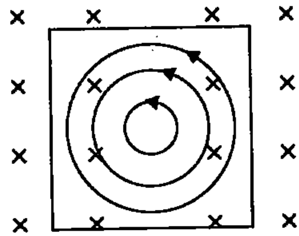


Fig. 7.5

TEXTBOOK QUESTIONS SOLVED

7.1. *A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.*

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Sol. Given, $R = 100\ \Omega$, $V = 220\ V_{eff}$, $f = 50$ Hz

∴ $\omega = 2\pi f = 2 \times 3.14 \times 50 = 314$

(*a*) Using the relation

$$I_{eff} = \frac{V_{eff}}{R}$$

Putting values, $I_{eff} = \frac{220}{100} = 2.2$ A

(*b*) Power consumed = current × voltage = $I_{eff} \times V_{eff}$
= 2.2 × 200 = 484 watt.

7.2. *(a) The peak voltage of an ac supply is 300 V. What is the rms voltage?*

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Sol. Given, $E_0 = 300$ V, $I_{rms} = 10$ A

(*a*) Using relation

$$E_{rms} = \frac{E_0}{\sqrt{2}}$$

$$= \frac{300}{\sqrt{2}} = \frac{300}{1.414} = 212.13\ \text{V}$$

(b) $I_{rms} = \frac{I_0}{\sqrt{2}}$

or, $I_0 = \sqrt{2}\ I_{rms} = \sqrt{2} \times 10 = 14.1$ A.

7.3. *A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.*

Sol. Given, $L = 44$ mH $= 44 \times 10^{-3}$ H

$f = 50$ Hz, $E_{rms} = 220$ V

$$X_L = L.\omega = L \cdot 2\pi f$$

$$= 44 \times 10^{-3} \times 2 \times 3.14 \times 50$$

Now, $$I_{rms} = \frac{E_{rms}}{X_L} = \frac{220}{44 \times 10^{-3} \times 2 \times 3.14 \times 50}$$

$= 15.9$ A.

7.4. *A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.*

Sol. Given, $C = 60\ \mu F = 60 \times 10^{-6}$ F

$E_{rms} = 110$ V

$f = 60$ Hz

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_{rms}}{\frac{1}{C.\omega}} = E_{rms} \cdot C \cdot 2\pi f$$

Putting the values,

$$I_{rms} = 110 \times 60 \times 10^{-6} \times 2 \times 3.14 \times 60 = 2.49 \text{ A.}$$

7.5. *In Questions 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.*

Sol. Net power absorbed by the circuit over a complete cycle is zero. Since power is not absorbed by pure inductor or capacitor and it is only resistance which absorbs the power. Power for pure inductor or capacitor circuit $= P_{av} = V_{eff}\, I_{eff} \cos\left(\pm\frac{\pi}{2}\right)$

7.6. *Obtain the resonant frequency ω_r of a series LCR circuit with L = 2.0H, C = 32 μF and R = 10 Ω. What is the Q-value of this circuit?*

Sol. Here, $L = 2.0$ H, $C = 32\ \mu F = 32 \times 10^{-6}$ F

$R = 10$ Ohm

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}} = \frac{10^3}{8} = 125 \text{ rad/s}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$Q = \frac{1000}{40} = 25.$$

7.7. *A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?*

Sol. Given, $C = 30\ \mu F = 30 \times 10^{-6}$ F

$L = 27$ mH $= 27 \times 10^{-3}$ H

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}$$

$$= \frac{10^4}{9} = 1.1 \times 10^3 \text{ s}^{-1}.$$

7.8. *Suppose the initial charge on the capacitor in Question 7.6 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?*

Sol. Given, $Q = 6$ mC $= 6 \times 10^{-3}$ C

Since, $E = \frac{1}{2}\frac{Q^2}{C}$ (energy stored)

or, $$E = \frac{1}{2} \times \frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}} = \frac{36}{60} = 0.6 \text{ J}.$$

As there is no loss of energy, the total energy remains the same.

7.9. *A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?*

Sol. When the frequency of the *a.c.* supply is equal to the natural frequency, then

$z = R$ and $\phi = 0°$

∴ $z = 20\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{200}{20} = 10 \text{ A}$$

Average power transferred

$$P = E_v I_v \cos 0^\circ$$
$$= 200 \times 10 \times 1 = 2000 \text{ watt.}$$

7.10. *A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?*
[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radiowave.]

Sol.
$$f_1 = 800 \text{ kHz} = 800 \times 10^3 \text{ Hz},$$
$$f_2 = 1200 \text{ kHz} = 1200 \times 10^3 \text{ Hz}$$
$$L = 200\ \mu\text{H} = 200 \times 10^{-6} \text{ H}$$

We know that the resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad f^2 = \frac{1}{4\pi^2 LC}$$

or,
$$C = \frac{1}{4\pi^2 L f^2}$$

Now,
$$C_1 = \frac{1}{4\pi^2 L f_1^2} = \frac{49}{4(22)^2 \times 200 \times 10^{-6} (800 \times 10^3)^2}$$
$$C_1 = 197.73 \text{ pF.}$$

Similarly,
$$C_2 = \frac{1}{4\pi^2 L f_2^2}$$
$$= \frac{49}{4(22)^2 \times 200 \times 10^{-6} (1200 \times 10^3)^2} F$$

or,
$$C_2 = 87.88 \text{ pF.}$$

The range of the variable condenser is from 87.88 pF to 197.73 pF.

7.11. *Figure shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80 μF, R = 40 Ω.*

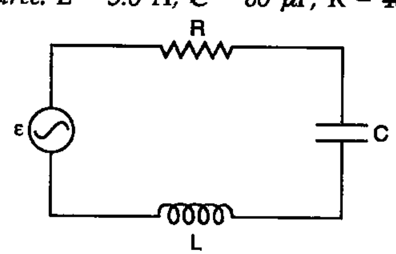


Fig. 7.6

(a) *Determine the source frequency which drives the circuit in resonance.*

(b) *Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.*

(c) *Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.*

Sol. Here,
$$L = 5.0 \text{ H}, \quad R = 40\ \Omega$$
$$C = 80\ \mu\text{F} = 80 \times 10^{-6} \text{ F}$$
$$E_v = 230 \text{ volt}$$
$$E_0 = \sqrt{0}\ E_v = \sqrt{2} \times 230 \text{ V}$$

(a) Resonance angular frequency,
$$\omega_r = \frac{1}{\sqrt{LC}}$$
$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{1}{2 \times 10^{-2}} = 50 \text{ rad/sec.}$$

(b) Impedance
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance,
$$\omega L = \frac{1}{\omega C}$$
$$Z = \sqrt{R^2} = R = 40\ \Omega$$

Amplitude of current at resonating frequency
$$I_0 = \frac{E_0}{z} = \frac{\sqrt{2} \times 230}{40} = 8.13 \text{ amp.}$$
$$I_v = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} = 5.75 \text{ amp.}$$

(c) Potential drop across L
$$V_{L\ rms} = I_v\, \omega_r\, L = 5.75 \times 50 \times 5.0 = 1437.5 \text{ V}$$

Potential drop across R
$$V_{R\ rms} = I_v \times R = 5.75 \times 40 = 230 \text{ volt}$$

Potential drop across C
$$V_{C\ rms} = I_v \left(\frac{1}{\omega_r C}\right) = 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}}$$

$$= \frac{5.75}{4} \times 10^3 = 1437.5 \text{ V}$$

Potential drop across LC circuit

$$V_{LC\,rms} = V_{L\,rms} - V_{C\,rms} = 0$$

7.12. *An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.*

(a) What is the total energy stored initially? Is it conserved during LC oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored

(i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Sol. (*a*) Total initial energy

$$\boxed{E = \frac{Q_0^2}{2C}} = \frac{10^{-2} \times 10^{-2}}{2 \times 50 \times 10^{-6}} \text{ J} = 1 \text{ J}$$

This energy shall remain conserved in the absence of resistance.

(*b*) Angular frequency, $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{(20 \times 10^{-3} \times 50 \times 10^{-6})^{1/2}} \text{Hz} = 10^3 \text{ rad s}^{-1}.$$

$$\upsilon = \frac{10^3}{2\pi} \text{Hz} = 159 \text{ Hz}.$$

(*c*) $Q = Q_0 \cos \omega t$

or $Q = Q_0 \cos \frac{2\pi}{T} t$,

where $T = \frac{1}{\upsilon} = \frac{1}{159} \text{s} = 6.3 \text{ ms}$

Energy stored is completely electrical at $t = 0, T/2, T, 3T/2, \ldots$

Electrical energy is zero *i.e.,* energy stored is completely magnetic at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$

(*d*) At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots$

$$\left[\because Q = Q_0 \cos\frac{\omega T}{8} = Q_0 \cos\frac{\pi}{4} = \frac{Q_0}{\sqrt{2}}\right]$$

$\therefore$ Electrical energy $= \frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C}$, which is half of the total energy.

(*e*) R damps out the LC oscillations eventually. The whole of the initial energy 1.0 J is eventually dissipated as heat.

7.13. *A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V 50 Hz ac supply.*

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Sol. Here, $L = 0.50$ H, $R = 100\ \Omega$

$E_v = 240$ V, $f = 50$ Hz

$\omega = 2\pi f = 100\ \pi$

$E_0 = \sqrt{2}\, E_v = \sqrt{2} \times 240$ V

(*a*) $$I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\sqrt{2} \times 240}{\sqrt{10^4 + (100\pi \times 0.5)^2}} = 1.82 \text{ A}$$

(*b*) In LR circuit,

If $E = E_0 \cos \omega t$, $I = I_0 \cos(\omega t - \phi)$

At $t = 0$, $E = E_0$ *i.e.,* voltage is maximum.

At $t = \frac{\phi}{\omega}$, $I = I_0 \cos(\phi - \phi) = I_0 \times 1$, current is maximum

$\therefore$ Time lag between voltage maximum and current maximum $= \frac{\phi}{\omega}$

As $$\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.50}{100}$$

$$= \frac{22}{7 \times 2} = 1.571$$

$$\phi = \tan^{-1}(1.571) = 57.5^\circ = \frac{57.5\pi}{180} \text{ radian}$$

$$\therefore \boxed{\text{Time lag} = \frac{\phi}{\omega}} = \frac{57.5\pi}{180 \times 2\pi f} = \frac{57.5}{180 \times 2 \times 50}$$

$$= 3.19 \times 10^{-3} \text{ s.}$$

7.14. *Obtain the answers to (a) and (b) in Exercise 13, if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence explain statement that at very high frequency, an inductor in circuit nearly amounts to an open circuit. How does an inductor behave in a d.c. circuit after the steady state?*

Sol. Here, $L = 0.50$ H, $R = 100\ \Omega$

$$V_{rms} = 240 \text{ V}, \quad f = 10 \text{ kHz} = 10^4 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 10^4 \text{ rad s}^{-1}$$

Peak voltage, $V_0 = \sqrt{2}\ V_{rms} = \sqrt{2} \times 240 = 339.36$ V

Maximum current, $I_0 = \dfrac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$$= \frac{339.36}{\sqrt{(100)^2 + (2\pi \times 10^4 \times 0.5)^2}} \text{ A}$$

$$= \frac{339.36}{31416} \text{ A} \qquad \text{(Neglecting } R\text{)}$$

$$= 0.01212 \text{ A} = 1.12 \times 10^{-2} \text{ A}$$

This current is much smaller than for the low frequency case (1.82 A in above question) showing that the inductive reactance is very large at high frequencies and L nearly amounts to an open circuit. In *d.c.* circuit (after steady state) $\omega = 0$.

$$\therefore \qquad Z_L = \omega L = 0$$

i.e., inductance L behaves like a pure inductor.

7.15. *A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.*

(a) What is the maximum current in the circuit?

(b) What is the time lag between current maximum and voltage maximum?

Sol. Here, $C = 100\ \mu F = 100 \times 10^{-6} F = 10^{-4}$ F, $R = 40\ \Omega$.

$$E_v = 110 \text{ volt}, \quad E_0 = \sqrt{2} \,.\, E_v = \sqrt{2} \times 110 \text{ V}$$

$v = 60$ Hz., $\omega = 2\pi v = 120\pi$ rad/s

$I_0 = ?$

In RC circuit, as $\boxed{Z = \sqrt{R^2 + X_c^2}} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$$\therefore \quad \boxed{I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}} = \frac{\sqrt{2} \times 110}{\sqrt{1600 + \frac{1}{(120\pi \times 10^{-4})^2}}}$$

$I_0 = 3.24$ amp.

In RC circuit, voltage lags behind the current by phase angle ϕ,

$$\text{where } \boxed{\tan\phi = \frac{1/\omega C}{R}} = \frac{1}{\omega CR} = \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6628$$

$$\phi = \tan^{-1}(0.6628) = 33.5^\circ = \frac{33.5\,\pi}{180} \text{ rad.}$$

$$\boxed{\text{Time lag} = \frac{\phi}{\omega}} = \frac{33.5\,\pi}{180 \times 120\,\pi} = 1.55 \times 10^{-3} \text{ sec.}$$

7.16. *Obtain the answers to (a) and (b) in Exercise 15, if the circuit is connected to 110 V, 12 kHz supply. Hence explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in d.c. circuit after the steady state.*

Sol. (*a*) For the high frequency,

$$\omega = 2\pi f = 2\pi \times 12 \times 10^3 \text{ rad s}^{-1}$$

$$\therefore \quad I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{2 E_v}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\text{or,} \quad I_0 = \frac{\sqrt{2} \times 110}{\sqrt{1600 + \frac{1}{4\pi^2 \times 144 \times 10^6 \times 10^{-8}}}} \text{ A}$$

$$= \frac{1.414 \times 110}{\sqrt{1600 + 0.0176}} \text{ A} = \frac{1.414}{40} \text{ A} = 3.9 \text{ A}$$

[It may be noted that the C term is negligible at higher frequencies.]

(b) $$\tan\phi = \frac{1}{2\pi \times 12 \times 10^3 \times 10^{-4} \times 40} = \frac{1}{96\pi}$$

ϕ is nearly zero at high frequency.

It is clear from here that at high frequency, C acts like a conductor. For a D.C. circuit, after steady state has been reached, $\omega = 0$ and C amounts to an open circuit.

7.17. *Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements L, C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 11 for this frequency.*

Sol. $V_{rms} = 230$ V, $L = 5.0$ H

$C = 80\ \mu F = 80 \times 10^{-6}$ F, $R = 40\ \Omega$

Using relation,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}}$$

$$= \frac{1}{\sqrt{400 \times 10^{-6}}} = 50 \text{ rad s}^{-1}$$

Since elements are in parallel, reactance X of L and C in parallel is given by

$$\frac{1}{X} = \frac{1}{\omega L} - \frac{1}{1/\omega C} = \frac{1}{\omega L} - \omega C$$

Impedance of R and X in parallel is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X^2}} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

or, $$\frac{1}{Z} = \frac{\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}}{R}$$

$$Z = \frac{R}{\left[1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2\right]}$$

which is less than resistance R. At resonant frequency,

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega C = \frac{1}{\omega L}$$

and $$\left(\frac{1}{\omega L} - WC\right) = 0$$

Then, impedance $Z = R$ and will be maximum. Hence, current will be minimum at resonant frequency in the parallel LCR circuit. From Ex. 11 $L = 5H.$; $C = 80 \times 10^{-6}$ F, $R = 40\ \Omega$

$$E_{rms} = 230 \text{ V}.$$

$$(I_R)_{rms} = \frac{V_{rms}}{R} = \frac{230}{40} = 5.75 \text{ A}.$$

$$(I_L)_{rms} = \frac{V_{rms}}{\omega L} = \frac{230}{50 \times 5} = 0.92 \text{ A}.$$

$$(I_C)_{rms} = \frac{V_{rms}}{1/\omega C} = 230 \times 50 \times 80 \times 10^{-6} = 0.92 \text{ A}.$$

Current through L and C will be in opposite phase, hence, I_{rms} in circuit will be only 5.75 A. $\left(= \frac{V_{rms}}{R}\right)$ as circuit impedance will be equal to R only.

7.18. *A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.*

(a) Obtain the current amplitude and rms value.

(b) Obtain the rms values of potential drop across each element.

(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

Sol. Here,
$$L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$$
$$C = 60\ \mu\text{F} = 60 \times 10^{-6} \text{ F}, \quad R = 0$$
$$E_v = 230 \text{ V}, \quad E_0 = \sqrt{2} \times E_v = \sqrt{2} \times 230 \text{ V}$$
$$f = 50 \text{ Hz}, \quad \omega = 2\pi f = 100\ \pi \text{ rad/s}$$

(a) $I_0 = ?\ I_v = ?$

$$I_0 = \frac{E_0}{\left(\omega L - \frac{1}{\omega L}\right)} = \frac{230\sqrt{2}}{\left(100\,\pi \times 80 \times 10^{-3} - \frac{1}{100\,\pi \times 60 \times 10^{-6}}\right)}$$

$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = \frac{230\sqrt{2}}{-27.91} = -11.63 \text{ amp.}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{1.414} = -8.23 \text{ amp.}$$

Negative sign appears as $\omega L < \frac{1}{\omega C}$.

∴ e.m.f. lags behind the current by 90°

(*b*) Across L, $V = I_v \times \omega L = 8.23 \times 100\,\pi \times 80 \times 10^{-3} = 206.74$ volt.

Across C, $V = I_v \times \frac{1}{\omega C} = 8.23 \times \frac{1}{100\,\pi \times 60 \times 10^{-6}}$

$= 436.84$ volt.

As voltages across L and C are 180° out of phase, therefore, they get subtracted.

That is why applied *r.m.s.* voltage = 436.84 − 206.74 = 230.1 volt.

(*c*) Average power transferred over a complete cycle by the source to inductor is always zero because of phase difference of $\pi/2$ between voltage and current through L.

(*d*) Average power transferred over a complete cycle by the source to the capacitor is also zero because of phase difference of $\pi/2$ between voltage and current through C.

(*e*) Total average power absorbed by the circuit is also, therefore zero.

7.19. *Suppose the circuit in Question 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.*

Sol. Here, $R = 15\ \Omega$

∴ Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

or, $$Z = \sqrt{15^2 + \left(2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}\right)^2}$$

$$= \sqrt{225 + 779.5}\ \Omega = 31.7\ \Omega$$

$$\therefore\ I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{31.7} = 7.255 \text{ A}$$

Average power transferred to $L = E_v I_v \cos\frac{\pi}{2} = 0$

Average power transferred to $C = E_v I_v \cos\left(\frac{-\pi}{2}\right) = 0$

Average power transferred to $R = I^2_{rms} \times R = (7.255)^2 \times 15$ W

$= 789.5$ W.

7.20. *A series LCR circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply.*

(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the Q-factor of the given circuit?

Sol. (*a*) Here, $L = 0.12$ H, $R = 23\ \Omega$,

$C = 480$ nF $= 480 \times 10^{-9}$ F

$E_v = 230$ volt, $E_0 = \sqrt{2}\, E_v = \sqrt{2} \times 230$ volt.

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

I_0 would be maximum, when

$$\omega_r = \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}}$$

$= 4166.7$ rad s^{-1}

$$I_0 = \frac{E_0}{R} = \frac{\sqrt{2} \times 230}{23} = 14.14 \text{ amp.}$$

(*b*) Average power absorbed by the circuit is maximum, when $I = I_0$

$$P_{av} = \frac{1}{2} I_0^2 R = \frac{1}{2} (14.14)^2 \times 23 = 2299.3 \text{ watt}$$

(*c*) The two angular frequencies for which the power transferred to the circuit is half the power at the resonant frequency,

$$\omega = \omega_r \pm \Delta\omega$$

When $\boxed{\Delta\omega = \frac{R}{2L}} = \frac{23}{2 \times 0.12} = 95.83 \text{ rad s}^{-1}$

$\therefore$ angular frequencies at which power transferred is half

$= \omega_r \pm \Delta\omega$

$= 4166.7 \pm 95.83 = 4262.3$ and $4070.87 \text{ rad s}^{-1}$

current amplitude at these frequencies is

$$\frac{I_0}{\sqrt{2}} = \frac{14.14}{1.414} = 10 \text{ A.}$$

(*d*) $$\text{Q-factor} = \frac{\omega_r L}{R} = \frac{4166.7 \times 0.12}{23} = 21.74.$$

7.21. *Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 μF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.*

Sol. Given, $L = 3.0$ H, $C = 27\ \mu\text{F} = 27 \times 10^{-6}$ F

$R = 7.4\ \Omega$

Resonant frequency, $$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{3.0 \times 27 \times 10^{-6}}} = 111 \text{ rad s}^{-1}$$

Q-factor of the circuit, $$Q = \frac{\omega_r L}{R} = \frac{111 \times 3.0}{7.4} = 45$$

For improvement in sharpness of resonance by a factor of 2, Q should be doubled. To double Q with changing ω_r, R should be reduced to half, *i.e.*, to 3.7 Ω.

7.22. *Answer the following questions:*

(*a*) *In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?*

(*b*) *A capacitor is used in the primary circuit of an induction coil.*

(*c*) *An applied voltage signal consists of a superposition of a dc voltage and an a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.*

(*d*) *A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke* *causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an a.c. line.*

(e) *Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?*

Sol. (a) Yes, the applied instantaneous voltage is equal to the algebraic sum of the instantaneous voltages across the series elements because the voltage variations across each element will follow the variations of the supply voltage at all instants. But this is not true for rms voltage because voltage across different elements may not be in phase.

(b) When the circuit is broken, the large induced voltage is used up in charging the capacitor. Thus sparking etc. is avoided.

(c) For high frequency, the inductive reactance for *a.c.*, $X_L = \omega L = \infty$ and capacitance of reactance $X_C = \frac{1}{\omega C} = 0$. Hence, capacitor does not offer any resistance for a.c. Thus *a.c.* components of voltage appears across *L* only.

Consequently, X_L for *d.c.*, $X_L = \omega L = 0$ and

$$X_C = \frac{1}{\omega C} = \infty$$

Therefore, *d.c.* components of voltage appears across C only.

(d) For a steady state *d.c.*, *L* has no effect even if it is increased by an iron core. For *a.c.*, the lamp will shine dimly because of additional impedance of the choke. It will dim further when the iron core is inserted which increases the choke's impedance.

(e) A choke coil is needed in the use of fluorescent tubes to reduce *a.c.* without loss of power, if we use an ordinary resistor, a.c. will reduce, but if loss of power due to heating will be there.

Power dissipated $= E_v I_v \cos\phi$

In a resistor, $\phi = 0°$

$\therefore$ Power dissipated $= E_v I_v \cos 0° = E_v I_v = \text{max}$

In a choke coil, $\phi = 90°$

$\therefore$ Power dissipated $= E_v I_v \cos 90° = \text{zero}$.

7.23. *A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?*

Sol. $V_1 = 2300$ volt $\quad n_1 = 4000$

$V_2 = 230$ volt

$$\frac{V_2}{V_1} = \frac{n_2}{n_1}$$

or, $$n_2 = n_1 \frac{V_2}{V_1} = 4000 \times \frac{230}{2300} = 400 \text{ turns.}$$

7.24. *At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 m^3s^{-1}. If the turbine generator efficiency is 60%, estimate the electric power available from the plant (g = 9.8 ms^{-2}).*

Sol. Here, $h = 300$ m

Volume of the water flowing per second = 100 m^3

Mass of water flowing per second,

$$m = 100 \times 10^3 \text{ kg} = 10^5 \text{ kg}$$

$$g = 9.8 \text{ ms}^{-2}$$

Potential energy of water fall during one second

$$= mgh = 10^5 \times 9.8 \times 300$$

$$= 29.4 \times 10^7 \text{ J.}$$

$$\text{Input power} = 294 \times 10^6 \text{ Js}^{-1}$$

$$\text{Efficiency, } \eta = \frac{\text{output power}}{\text{input power}}$$

$$\therefore \quad \text{Output power} = \eta \times \text{input power}$$

$$= 0.6 \times 294 \times 10^6$$

$$= 176.4 \times 10^6 \text{ watt} = 176.4 \text{ MW.}$$

7.25. *A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.*

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterise the step-up transformer at the plant.

Sol. Power required ⇒ $P = 800$ kW $= 800 \times 10^3$ W

Total resistance of two wire lines

$$R = 2 \times 15 \times 0.5 = 15 \, \Omega.$$

Since supply is through 4000 – 230 V transformer

$\therefore \quad E_v = 4000$ volt

As $\quad P = E_v I_v$

$\therefore \quad 800 \times 10^3 = 4000\, I_v$

$$I_v = \frac{800 \times 10^3}{4000} = 200 \text{ amp}$$

(*a*) Line power loss in the form of heat

$= I_v^2 R$

$= (200)^2 \times 15 = 60 \times 10^4$ watt.

$= 600$ kW.

(*b*) If there is no power loss due to leakage,
then the essential plant supply = 800 + 600 = 1400 kW

(*c*) Voltage drop on the line = $I_v R$

$= 200 \times 15 = 3000$ volt

$\therefore$ Voltage from transmission = 3000 + 4000 = 7000 V

Since the power is generated at 440 volt, the step-up transformer needed at the plant is 440 V – 7000 V.

7.26. *Do the same exercise as above with the replacement of the earlier transformer by a 40,000-200 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?*

Sol. The rms current in the two-wire line

$$= \frac{800 \times 10^3 \text{ W}}{40000 \text{ V}} = 20 \text{ A}$$

(*a*) Line power loss = $I_v^2 R = (20)^2 \times 15 = 6000$ W = 6 kW

(*b*) Power supplied by the plant = 800 + 6 = 806 kW

(*c*) Voltage drop on the line = 20 × 15 = 300 V

Voltage output of the step-up transformer at the plant

= 40000 + 300 = 40300 V

$\therefore$ The step-up transformer at the plant is

440 V – 40300 V

Power loss is (exercise 25) $= \dfrac{600}{1400} \times 100 = 43\%$

Power loss in this exercise $= \dfrac{6}{806} \times 100 = 0.74\%$

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