8 Electromagnetic Waves

Lesson at a Glance

• Electromagnetic Waves

Electromagnetic waves are self-sustaining oscillations of electric and magnetic fields in free space or vacuum. The electric and magnetic fields E_x and B_y are perpendicular to each other and to the direction of propagation. The E_x and B_y can be written

as $$E_x = E_0 \sin(kz - \omega t)$$

and $$B_y = B_0 \sin(kz - \omega t)$$

Here K is related to the wavelength k of the wave of the usual equation

$$k = \frac{2\pi}{\lambda}$$

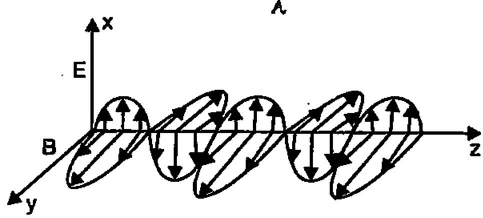

Fig. 8.1

• Spectrum of Electromagnetic Waves

The distribution of energy wavelength-wise is called spectrum. The spectrum of electromagnetic waves can be given wavelength-wise or frequency-wise as $\upsilon = \frac{c}{\lambda} = \frac{3 \times 10^8}{\lambda}$.

The spectrum of electromagnetic waves wavelength-wise is given in Fig. 8.2.

γ-ray	x-ray	Ultraviolet wave	Visible VIBGYOR	Infrared	Micro wave	Radio TV

← 10^{-10} m 10^{-8} m 4×10^{-7} m 7×10^{-7} m 1 m 100 m →

Fig. 8.2

• Displacement Current

According to Maxwell *the current produced due to change in electric field is called displacement current.*

- The displacement current is given by

$$I_D = \varepsilon_0 \frac{\partial Q}{\partial t}$$

where ∂Q is the change in magnetic flux in time ∂t.

- Maxwell used four equations to explain the behaviour of electromagnetic waves. These equations are called Maxwell's equations. Following are the Maxwell's equations.

(*i*) $\oint_B dl = \mu_o \left[\varepsilon_0 \frac{\partial Q}{\partial t} + I_c\right]$ This is modified Ampere's circuital law

(*ii*) $\oint_E dl = -\frac{\partial Q}{\partial t}$ (Faraday's law)

(*iii*) $\oint_E ds = \frac{1}{\varepsilon_0}(q)$ (Gauss theorem)

(*iv*) $\oint_B .ds = 0$ (Gauss theorem)

• Velocity of Electromagnetic Waves

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

This is the expression for the velocity of electromagnetic wave in free space.

- The velocity of electromagnetic wave in medium having relative permeability μ_r and relative permittivity ε_r.
  The Velocity of electromagnetic wave in medium

or $$C_m = \frac{1}{\sqrt{\mu_0 \mu_r . \varepsilon_0 \varepsilon_r}}$$

- The refractive index of the medium,

or $$n = \frac{C}{C_m} \quad \frac{1/\sqrt{\mu_0 \varepsilon_0}}{1/\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

or $$n = \sqrt{\mu_r \varepsilon_r}$$

TEXTBOOK QUESTIONS SOLVED

8.1. *Figure shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.*

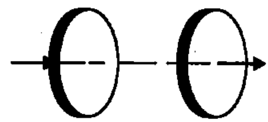

Fig. 8.3

(a) Calculate the capacitance and the rate of charge of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

Sol. (*a*) Given,
$$R = 12 \text{ cm} = 0.12 \text{ m}$$
$$d = 5.0 \text{ mm} = 5 \times 10^{-3}$$
$$I = 0.15 \text{ A}$$
$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$\therefore$ Area, $A = \pi R^2 = 3.14 \times (0.12)^2 \text{ m}^2$

Capacitance of parallel plate capacitor is given by

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

$$= \frac{8.85 \times 10^{-12} \times (3.14) \times (0.12)^2}{5 \times 10^{-3}}$$

$$= 80.1 \times 10^{-12} = 80.1 \text{ pF}$$

Now, $q = CV$

or, $\dfrac{dq}{dt} = C \times \dfrac{dV}{dt}$

or, $I = C \times \dfrac{dV}{dt}$ $\quad \left[\because I = \dfrac{dq}{dt}\right]$

or, $\dfrac{dV}{dt} = \dfrac{I}{C} = \dfrac{0.15}{80.1 \times 10^{-12}}$

$$= 1.87 \times 10^9 \text{ Vs}^{-1}$$

(*b*) Displacement current is equal to the conduction current *i.e.,* 0.15 A.

(*c*) Yes, Kirchhoff's first rule is valid at each plate of the capacitor provided. We take the current to be the sum of the conduction and displacement currents.

8.2. *A parallel plate capacitor (Fig.) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.*

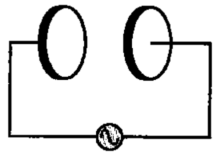

Fig. 8.4

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

Sol. (*a*) Here,

$$R = 6.0 \text{ cm}$$
$$C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$$
$$\omega = 300 \text{ rad s}^{-1}$$
$$E_{rms} = 230 \text{ V}$$

$$\boxed{I_{rms} = \frac{E_{rms}}{X_C}} = \frac{E_{rms}}{\frac{1}{\omega C}} = E_{rms} \times \omega C$$

$$\therefore \quad I_{rms} = 230 \times 300 \times 100 \times 10^{-12}$$
$$= 6.9 \times 10^{-6} \text{ A} = 6.9 \ \mu\text{A}$$

(*b*) Yes, $I = I_D$, whether I is steady *d.c.* or *a.c.*

This is shown below:

$$I_D = \varepsilon_0 \frac{d(\phi_E)}{dt} = \varepsilon_0 \frac{d}{dt}(EA) \qquad (\because \phi_E = EA)$$

or,
$$I_D = \varepsilon_0 A \frac{dE}{dt}$$

$$= \varepsilon_0 A \frac{d}{dt}\left(\frac{Q}{\varepsilon_0 A}\right) \qquad \left(\because E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}\right)$$

or,
$$I_D = \varepsilon_0 A \times \frac{1}{\varepsilon_0 A} \frac{dQ}{dt} = \frac{dQ}{dt} = I$$

(*c*) We know that

$$\boxed{B = \frac{\mu_0}{2\pi} \frac{r}{R^2} I_D}$$

This formula goes through even if I_D (and therefore B) oscillates in time. The formula shows that they oscillate in phase. Since $I_D = I$, we have

$$B = \frac{\mu_0 \, rI}{2\pi R^2}$$

If $I = I_0$, the maximum value of current, then amplitude of B = maximum value of B

$$= \frac{\mu_0 r I_0}{2\pi R^2} = \frac{\mu_0 r \sqrt{2}\, I_{rms}}{2\pi R^2} \left(\because I_0 = \sqrt{2}\, I_{rms}\right)$$

$$= \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2 \times 3.14 \times (0.06)^2} \text{ T}$$

$$= 1.63 \times 10^{-11} \text{ T}.$$

8.3. *What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Œ and radio waves of wavelength 500 m?*

Sol. The speed for X-ray, red light and radio waves in vacuum is the same and is $C = 3 \times 10^8 \text{ ms}^{-1}$.

8.4. *A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?*

Sol. $\vec{E}$ and $\vec{B}$ lie in x-y plane and are mutually perpendicular,

since $c = \nu\lambda$

Thus $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{30 \times 10^6} = 10 \text{ m}.$

8.5. *A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?*

Sol. $\lambda_1 = \frac{c}{\nu_1} = \frac{3 \times 10^8}{7.5 \times 10^6} = 40 \text{ m}$

$\lambda_2 = \frac{c}{\nu_2} = \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}$

Hence, wavelength band is 40 m – 25 m.

8.6. *A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?*

Sol. The frequency of electromagnetic wave is the same as that of oscillating charged particle about its equilibrium position; which is 10^9 Hz.

8.7. *The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?*

Sol. Given $B_0 = 510 \text{ nT} = 510 \times 10^{-9} \text{ T}$

$C = 3 \times 10^8 \text{ m s}^{-1}$

For electromagnetic waves,

$$\boxed{C = \frac{E_0}{B_0}}$$

or,

$$E_0 = C B_0 = 3 \times 10^8 \times 510 \times 10^{-9} = 153 \text{ NC}^{-1}$$

8.8. *Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120$ N/C and that its frequency is $\nu = 50.0$ MHz.*
(a) Determine, B_0, ω, k, and λ. (b) Find expressions for **E** *and* **B**.

Sol. (*a*) (*i*)

$$\boxed{\frac{E_0}{B_0} = C}$$

or

$$B_0 = \frac{E_0}{C} = \frac{120}{3 \times 10^8} \text{ T}$$

$$= 40 \times 10^{-8} \text{ T} = 400 \times 10^{-9} \text{ T} = 400 \text{ nT}$$

(*ii*)

$$\boxed{\omega = 2\pi \nu} = 2\pi \times 50 \times 10^6 = 3.14 \times 10^8 \text{ rad s}^{-1}$$

(*iii*)

$$\boxed{k = \frac{2\pi}{\lambda}} = \frac{2\pi \nu}{\nu\lambda} = \frac{2\pi \nu}{C}$$

$$= \frac{\omega}{C} = \frac{\pi \times 10^8}{3 \times 10^8} \text{ rad m}^{-1}$$

$$= \frac{\pi}{3} \text{ rad m}^{-1} = 1.05 \text{ rad m}^{-1}$$

(*iv*)

$$\boxed{C = \nu\lambda;} \ \lambda = \frac{C}{\nu} = \frac{3 \times 10^8}{50 \times 10^6} \text{ m} = \frac{300}{50} = 6 \text{ m.}$$

(*b*) Let the electromagnetic wave travel along +*x*-axis, and $\vec{E}$ and $\vec{B}$ are along *y*-axis and *z*-axis respectively. Then,

$$\boxed{\vec{E}_y = E_0 \sin (kx - \omega t) \hat{j}}$$

$$= 120 \sin (1.05x - 3.14 \times 10^8 \, t) \hat{j} \text{ NC}^{-1}$$

$$\vec{B}_z = B_0 \sin (kx - \omega t) \hat{k}$$

$$= 400 \sin (1.05x - 3.14 \times 10^8 \, t) \hat{k} \text{ nT}$$

8.9. *The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula E = hν (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation.*

Sol. Energy of photon

$$E = h\nu \quad \text{or} \quad E = h\frac{C}{\lambda}$$

$$h = 6.62 \times 10^{-34} \text{ js}, \quad C = 3 \times 10^{8} \text{ m s}^{-1}$$

If λ is in metre, E in J, then we divide by 1.6×10^{-19} to convert E into eV.

$$\therefore \qquad E = \frac{hc}{\lambda \times 1.6 \times 10^{-19}} \text{ eV}$$

(1) **γ-rays.** 1 ranges from 10^{-10} m to less that 10^{-14} m

$$\therefore \qquad \text{Energy} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{10^{-10} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 12.4 \times 10^{3}\ eV \approx 10^{4} \text{ eV}$$

Thus for $\lambda = 10^{-10}$ m, energy = 10^{4} eV.

For $\lambda = 10^{-14}$ m, energy = 10^{8} eV.

Energy of γ-rays between 10^{4} to 10^{8} eV.

(2) **X-rays.** 1 ranges from 10^{-8} m to 10^{-13} m.

For $\lambda = 10^{-8}$

$$\therefore \qquad \text{Energy} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{10^{-8} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 124 \approx 10^{2} \text{ eV}$$

For $\lambda = 10^{-13}$ m, energy = 10^{7} eV.

(3) **Ultraviolet radiations.** λ ranges from 4×10^{-7} m to 6×10^{-10} m.

For $\lambda = 4 \times 10^{-7}$

$$\therefore \qquad \text{Energy} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 3.1 \text{ eV} \approx 10^{0} \text{ eV}$$

Energy of ultraviolet radiations vary between 10^{0} to 10^{3} eV.

(4) **Visible radiations.** λ ranges from 4×10^{-7} m to 7×10^{-7} m.

For $\lambda = 4 \times 10^{-7}$,

energy $= 10^0$ eV (as proved above)

For $\lambda = 7 \times 10^{-7}$

$$\therefore \quad \text{Energy} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{7 \times 10^{-7} \times 1.6 \times 10^{-19}} \text{eV}$$

$$= 1.77 \text{ eV} \approx 10^0 \text{ eV}$$

(5) **Infrared radiations.** λ ranges from 7×10^{-7} m to 7×10^{-4} m.

For $\lambda = 7 \times 10^{-7}$,

energy $= 10^0$ eV (as proved above)

For $\lambda = 7 \times 10^{-4}$,

the energy is $\frac{1}{1000}$ times, *i.e.*, of the order of 10^{-3} eV.

(6) **Micro waves.** λ ranges from 1 mm to 0.3 m.

For $\lambda = 1$ mm or 10^{-3},

$$\text{Energy is equal to } E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{10^{-3} \times 1.6 \times 10^{-19}} \text{eV}$$

$$= 1.24 \times 10^{-3} \text{ eV} \approx 10^{-3} \text{ eV.}$$

For $\lambda = 0.3$ m, energy $= 4.1 \times 10^{-6}$ eV $\approx 10^{-6}$ eV.

(7) **Radio waves.** λ ranges from 1 m to few km.

For $\lambda = 1$ m

$$\text{Energy is equal to } E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{10^0 \times 1.6 \times 10^{-19}} \text{eV}$$

$$= 1.24 \times 10^{-6} \text{ eV} \approx 10^{-6} \text{ eV.}$$

Energy for λ of the order of few km $\approx 10^{-6}$ ev.

Energy of a photon that a source produces indicates the spacing of relevant energy levels of the source.

8.10. *In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.*

(a) What is the wavelength of the wave?

(b) What is the amplitude of the oscillating magnetic field?

*(c) Show that the average energy density of the **E** field equals the average energy density of the **B** field. [$c = 3 \times 10^8$ m s^{-1}.]*

Sol. Here, $\nu = 2 \times 10^{10}$ Hz, $E_0 = 48$ Vm^{-1}.

(*a*)
$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{10}} \text{ m} = 1.5 \times 10^{-2} \text{ m}$$

(*b*) $$B_0 = \frac{E_0}{c} = \frac{48}{3\times 10^8}\,\text{T} = 1.6 \times 10^{-7}\,\text{T}$$

(*c*) Energy density in electric field,

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Energy density in magnetic field,

$$u_B = \frac{1}{2\mu_0}B^2$$

Using $E = cB$, $u_E = \frac{1}{2}\varepsilon_0 (cB)^2 = c^2\left(\frac{1}{2}\varepsilon_0 B^2\right)$

But $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\therefore$ $$u_E = \frac{1}{\mu_0 \varepsilon_0}\left(\frac{1}{2}\varepsilon_0 B^2\right) = \frac{1}{2\mu_0}B^2 = u_B$$

8.11. *Suppose that the electric field part of an electromagnetic wave in vacuum is* $\mathbf{E} = \{(3.1\ \text{N/C}) \cos [(1.8\ \text{rad/m})\, y + (5.4 \times 10^6\ \text{rad/s})t]\}\hat{i}$.

(a) What is the direction of propagation?

(b) What is the wavelength λ?

(c) What is the frequency ν?

(d) What is the amplitude of the magnetic field part of the wave?

(e) Write an expression for the magnetic field part of the wave.

Sol. (*a*) The wave is propagating along negative *y* direction *i.e.*, along $-\hat{j}$.

(*b*) Comparing the given equation with the equation

$$E = E_0 \cos (ky + \omega t)\hat{i}$$

$$E_0 = 3.1\ \text{NC}^{-1}, \quad k = 1.8\ \text{rad m}^{-1}$$

$$\omega = 5.4 \times 10^6\ \text{rad s}^{-1}$$

we know that

$$\omega = 2\pi\nu$$

or, $$5.4 \times 10^6 = 2\pi\nu = 2\pi \times \frac{c}{\lambda}$$

or, $$\lambda = \frac{2\pi c}{5.4\times 10^8} = \frac{2\pi\times 3\times 10^8}{5.4\times 10^8}$$

$$= 3.5\ \text{m}$$

(c) Again $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.5} = 0.86 \times 10^8$ Hz

$= 86 \times 10^6$ Hz = 86 MHz

(d) We know that

$$c = \frac{E_0}{B_0}$$

$\therefore$ $B_0 = \frac{E_0}{c} = \frac{3.1}{3 \times 10^8} = 1.03 \times 10^{-8}$ T

$= 0.0103 \times 10^{-6}$ T = 0.0103 μT

(e) Expression of magnetic field part of the wave

$B = B_0 \cos(ky + \omega t)$

$= 1.03 \times 10^{-8} \cos \{(1.8 \text{ rad/m})\, y + (5.4 \times 10^6 \text{ rad/s})\, t\}$

E is along $\hat{i}$ and c is along $-\hat{j}$, c is the direction of $\vec{E} \times \vec{B}$

$$-\hat{j} = \hat{i} \times ?$$

Clearly? is in the direction of $\hat{k}\left(\hat{k} \times \hat{i} = \hat{j}\right)$ and $\left(\hat{i} \times \hat{k} = -\hat{j}\right)$

Thus $\vec{B}$ is completely represented as

$\vec{B} = 1.03 \times 10^{-8} \cos \{(1.8 \text{ rad/m})\, y + (5.4 \times 10^6 \text{ rad/s})\, t\}\hat{k}$.

8.12. *About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation*

(a) at a distance of 1m from the bulb?

(b) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

Sol. Power converted into visible radiation,

$$P = \frac{5}{100} \times 100 \text{ W} = 5 \text{ W}$$

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$

(a) Intensity, $I = \frac{5}{4 \times 3.14 \times 1 \times 1} = 0.4 \text{ Wm}^{-2}$

(b) $I = \frac{5}{4 \times 3.14 \times 10 \times 10} \text{ Wm}^{-2} = 0.004 \text{ Wm}^{-2}$

8.13. *Use the formula $\lambda_m T = 0.29$ cm K to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?*

Sol. We know, every body at a given temperature T, emits radiations of all wavelengths in certain range. For a black body, the wavelength corresponding to maximum intensity of radiation at a given temperature T is given, according to Wein's law, by the radiation

$$\boxed{\lambda_m T = 0.29 \text{ cmK}} \quad \text{or} \quad T = \frac{0.29}{\lambda_m}$$

For $\lambda_m = 10^{-6}$ m $= 10^{-4}$ cm, $T = \dfrac{0.29}{10^{-4}} = 2900$ K.

Temperature for other wavelengths can be similarly found. These numbers tell us the temperature ranges required for obtaining radiations in different parts of the electro-magnetic spectrum. Thus to obtain visible radiation, say, $\lambda_m = 5 \times 10^{-5}$ cm, the source should have a temperature $T = \dfrac{0.29}{5 \times 10^{-5}} \simeq 6000$ K

It is to be noted that, a body at lower temperature will also produce this wavelength but not with maximum intensity.

8.14. *Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.*

(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).

(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift).

(c) 2.7 K [temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe].

(d) 5890 Å – 5896 Å [double lines of sodium].

(e) 14.4 keV [energy of a particular transition in ^{57}Fe nucleus associated with a famous high resolution spectroscopic method (Mössbauer spectroscopy)].

Sol. (*a*) Radio waves (short wavelength end)

(*b*) Radio waves (short wavelength end)

(*c*) Microwaves

(*d*) Visible region (yellow)

(*e*) X-rays (or soft γ-rays) region.

8.15. *Answer the following questions:*

(a) *Long distance radio broadcasts use short-wave bands. Why?*

(b) *It is necessary to use satellites for long distance TV transmission. Why?*

(c) *Optical and radiotelescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?*

(d) *The small ozone layer on top of the stratosphere is crucial for human survival. Why?*

(e) *If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?*

(f) *Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?*

Sol. (*a*) This is because ionosphere reflects waves in these bands.

(*b*) It is so because television signals are not properly reflected by the ionosphere. Therefore, for reflection of signals, satellites are needed.

(*c*) Atmosphere absorbs X-rays, while visible and radiowaves can penetrate it. That is why optical and radio telescopes can work on earth's surface but X-ray astronomical telescopes must be used on satellites orbiting the earth.

(*d*) The small ozone layer on the top of the stratosphere absorbs ultraviolet radiations, γ-rays etc., from the sun. It also absorbs cosmic radiations. So, these radiations, which can cause genetic damage to the living cells, are prevented from reaching the earth. Thus, the small ozone layer on top of the stratosphere is crucial for human survival.

(*e*) The temperature of the earth would be lower because the green house effect of the atmosphere would be absent.

(*f*) The clouds produced by a global nuclear war would perhaps cover substantial parts of the sky preventing solar light from reaching many parts of the globe. This would cause a 'winter'.

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