11 Dual Nature of Radiation and Matter

Lesson at a Glance

- Many discoveries and observations of Roentgen, J.J. Thomson, R.A. Millikan, etc., established that electrons are fundamental, universal constituent of matter. Electrons are negatively charged particles having charge to mass ratio

$$e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}.$$

• Electron Emission

The phenomenon of emission of electrons from the surface of a metal is called electron emission.

(*i*) *Thermionic Emission.* The emission of electrons from the surface of metal with the help of thermal energy is called thermionic emission.

(*ii*) *Photoelectric Emission.* Electrons emitted from a metal surface with the help of suitable electromagnetic radiations.

• Effect of Electric and Magnetic Field on the Motion of an Electron

(*a*) *Electric field.* The force F_E experienced by an electron of charge e in the electric field of intensity E is given by

$$F_E = eE$$

(*b*) *Magnetic field.* The force experienced by an electron 'e' in a magnetic field of strength B weber/m^2 is given by

$$F_B = Bev$$

• Photoelectric Effect

Photoelectric effect is the phenomenon of emission of electrons from the surface of metals, when radiations of suitable frequency fall on them.

• Particle Nature of Light

(*i*) In interaction of radiation with matter, radiation as if it is made up of particles called photons.

(*ii*) Each photon has energy E (= $h\nu$) and momentum

$$p\left(=\frac{h\nu}{c}=\frac{h}{\lambda}\right)$$

• Wave Nature of Matter

de-Broglie hypothesis. According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle or a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving particle is called *matter wave* or *de-Broglie wave* whose wavelength called de-Broglie wavelength, is given by

$$\lambda = \frac{h}{mv}$$

where m and v are the mass and velocity of the particle and h is a Planck's constant.

• Photoelectric Cells

Photoelectric cell is a device that converts light energy into electrical energy. It is of three types:

(*i*) Photoemissive cell

(*ii*) Photovoltaic cell

(*iii*) Photoconductive cell

• Compton Scattering

It is the phenomenon of increase in the wavelength of X-ray photons which occurs when these radiations are scattered on striking an electron. The difference in the wavelength of scattered and incident photons is called compton shift, which is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

where ϕ is the angle of scattering of the X-ray photon and m_0 is the rest mass of the electron.

• Millikan's Oil Drop Method

R.A. Millikan obtained a value for the charge of the electron by observing the effect of an electric field on the motion of a charged oil drop. He first used Stokes' law to determine the radius r of the drop by using the relation

$$\frac{4\pi}{3}r^3(\rho - \sigma)g = 6\pi\eta r v$$

or,
$$r = \frac{9}{2}\left[\frac{\eta v}{(\rho - \sigma)g}\right]$$

Mass of electron. The mass of the electron was calculated using the value of e/m obtained by Thomson and of 'e' obtained by Millikan.

$$m = \frac{e}{\left(e/m\right)}$$

• Thomson's Method for e/m Measurement of Cathode Rays

J.J. Thomson determined e/m of the cathode rays by subjecting them to crossed electric and magnetic field, *i.e.*, fields that are at right angles to each other.

Force due to electric field, F_E = Force due to magnetic field F_B

or,
$$eE = Bev \Rightarrow v = \frac{E}{B}$$

Also,
$$\frac{e}{m} = \frac{E}{B^2R} = \frac{V/d}{B^2R} = \frac{Vx}{B^2 lLd}$$

Where,

R = radius of circular arc in the presence of magnetic field B

x = shift of the electron beam on the screen

V = potential difference between the two electrodes (*i.e.*, P and Q)

d = distance between the two electrodes

l = length of the field

L = distance between the centre of the field and the screen.

TEXTBOOK QUESTIONS SOLVED

11.1. *Find the*

(*a*) *maximum frequency, and*

(*b*) *minimum wavelength of X-rays produced by 30 kV electrons.*

Sol. Here, $V = 30 \text{ kV} = 30 \times 10^3 \text{V} = 3 \times 10^4 \text{V}$

(*a*) $h\nu_{max} = eV$

$$\nu_{max} = \frac{eV}{h} = \frac{1.6\times10^{-19}\times3\times10^{4}}{6.63\times10^{-34}}\text{ Hz}$$

$$= 7.24 \times 10^{18}\text{ Hz}$$

(b) $$\lambda_{max} = \frac{c}{\nu_{max}} = \frac{3\times10^{8}}{7.24\times10^{18}}\text{ m}$$

$\Rightarrow$ $$\lambda_{max} = 0.414 \times 10^{-10}\text{ m} = 0.414\text{ Å}$$

$$= 0.0414\text{ nm.}$$

11.2. *The work function of caesium metal is 2.14 eV. When light of frequency 6 × 10^14 Hz is incident on the metal surface, photoemission of electrons occurs. What is the*

(a) maximum kinetic energy of the emitted electrons,

(b) stopping potential, and

(c) maximum speed of the emitted photoelectrons?

Sol. (a) $$K.E._{max} = h\nu - \phi_0 = \frac{6.63\times10^{-34}\times6\times10^{14}}{1.6\times10^{-19}} - 2.14$$

$$E_{max} = 0.346\text{ eV} = 0.35\text{ eV}$$

(b) $$eV_0 = K.E._{max}$$

$\Rightarrow$ $$V_0 = \frac{K.E._{max}}{e} = \frac{0.35\text{ eV}}{e} = 0.35\text{ V}$$

(c) $$\frac{1}{2}mv^2_{max} = 0.346\text{ eV} = 0.346 \times 1.6 \times 10^{-19}\text{ J}$$

$\Rightarrow$ $$v_{max} = \sqrt{\frac{0.346\times1.6\times10^{-19}\times2}{9.1\times10^{-31}}}\text{ ms}^{-1}$$

$$= 3.488 \times 10^{5}\text{ ms}^{-1} = 348.8\text{ km s}^{-1}$$

or, $$v_{max} = 349\text{ km s}^{-1}.$$

11.3. *The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?*

Sol. $$K.E._{max} = eV_0$$

$$= 1.5 \times 1.6 \times 10^{-19}\text{ J}$$

$$= 2.4 \times 10^{-19}\text{ J.}$$

11.4. *Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.*

(*a*) *Find the energy and momentum of each photon in the light beam.*

(*b*) *How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and*

(*c*) *How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?*

Sol. Given,

$$\text{Wavelength, } \lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$$

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{632.8 \times 10^{-9}} \text{ Hz}$$

$$= 4.74 \times 10^{14} \text{ Hz}$$

(*a*)

$$E = h\nu$$

$$= 6.63 \times 10^{-34} \times 4.74 \times 10^{14} \text{ J}$$

$$= 3.14 \times 10^{-19} \text{ J.}$$

$$p \text{ (momentum)} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}}$$

$$= 1.05 \times 10^{-27} \text{ kg ms}^{-1}$$

(*b*) Power emitted, $P = 9.42$ mW $= 9.42 \times 10^{-3}$ W

$P = nE$

$$n = \frac{P}{E} = \frac{9.42 \times 10^{-3} \text{ W}}{3.14 \times 10^{-19} \text{ J}} = 3 \times 10^{16} \text{ photons/sec.}$$

(*c*) Velocity of hydrogen atom

$$= \frac{\text{Momentum '}p\text{' of } H_2 \text{ atom } (mv)}{\text{Mass of } H_2 \text{ atom} (m)}$$

$$\Rightarrow \quad v = \frac{1.05 \times 10^{-27}}{1.673 \times 10^{-27}} \text{ ms}^{-1} = 0.63 \text{ ms}^{-1}.$$

11.5. *The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m². How many photons (nearly) per square metre are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.*

Sol. Energy flux of sunlight

= Total energy per square metre per second

$= 1.388 \times 10^3 \text{ Wm}^{-2}$

Energy of each photon,

$$E = \frac{hc}{\lambda}$$

$$\therefore \quad P = nE = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}}\text{J} = 3.62 \times 10^{-19}\,\text{J}$$

Number of photons incident on earth's surface per square metre per second

$$= \frac{\text{Total energy per square metre per second}}{\text{Energy of one photon}}$$

$$= \frac{1.388\times10^{3}}{3.62\times10^{-19}} = 3.8 \times 10^{21}.$$

11.6. *In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} Vs. Calculate the value of Planck's constant.*

Sol. The slope of the cut-off voltage versus frequency of incident light

$$\frac{\Delta V}{\Delta \nu} = 4.12 \times 10^{-15}\,\text{Vs} = 4.12 \times 10^{-15}\,\frac{\text{J.s.}}{\text{C}}$$

So, By multiplying it with the charge of an electron, which is the fundamental charge ($e = 1.6 \times 10^{-19}$ C) we get,

$$\because \quad E = h\nu$$

$$h = \frac{E}{\nu} = \text{J.s.}$$

$$h = 4.12 \times 10^{-15} \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad h = 6.592 \times 10^{-34}\,\text{Js.}$$

11.7. *A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?*

Sol. Given,

$$P\text{ (power)} = 100\text{ W}$$

$$\lambda = 589 \times 10^{-9}\,\text{m}$$

(*a*) Energy of each photon

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{589\times10^{-9}}\text{ J}$$

$\Rightarrow$ $E = 3.38 \times 10^{-19}$ J

(*b*) Number of photons delivered to sphere per second

$$n = \frac{\text{Energy radiated per second}}{\text{Energy of each photon}} \qquad P = nE$$

or, $$n = \frac{100}{3.38\times10^{-19}} = 3 \times 10^{20} \text{ photons/s.}$$

11.8. *The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.*

Sol. Given,

$$\nu_0 = 3.3 \times 10^{14} \text{ Hz}$$
$$\nu = 8.2 \times 10^{14} \text{ Hz}$$
$$h = 6.63 \times 10^{-34} \text{ Js}$$

Using Einstein's photoelectric equation,

$$\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0 = eV_0$$

$\Rightarrow$ $$V_0 = \frac{h(\nu-\nu_0)}{e}$$

$\Rightarrow$ $$V_0 = \frac{6.62\times10^{-34}}{1.6\times10^{-19}}(8.2 \times 10^{14} - 3.3 \times 10^{14})$$

or, $V_0 = 2.03$ V.

11.9. *The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?*

Sol. $\phi_0 = 4.2 \text{ eV} = 4.2 \times 1.6 \times 10^{-19} \text{ J} = 6.72 \times 10^{-19}$ J

$$E = \frac{hc}{\lambda}$$

$\Rightarrow$ $$E = \frac{6.62\times10^{-34}\times3\times10^{8}}{330\times10^{-9}} = 6.018 \times 10^{-19} \text{ J}$$

As energy of incident photon $E < \phi_0$, hence no photoelectric emission will take place.

11.10. *Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?*

Sol. Here,

$$\nu = 7.21 \times 10^{14} \text{ Hz}$$
$$v_{max} = 6.0 \times 10^5 \text{ ms}^{-1}$$
$$m = 9 \times 10^{-31} \text{ kg}$$

Applying Einstein's photoelectric equation,

$$\text{K.E.}_{max} = \frac{1}{2} m v^2_{max} = h(\nu - \nu_0)$$

$$\Rightarrow \quad \frac{1}{2} m v^2_{max} = h(\nu - \nu_0) \quad \Rightarrow \quad \nu_0 = \nu - \frac{m v^2_{max}}{2h}$$

$$\Rightarrow \quad \nu_0 = 7.21 \times 10^{14} - \frac{(9.1 \times 10^{-31}) \times (6 \times 10^5)^2}{2 \times (6.63 \times 10^{-34})}$$
$$= 4.74 \times 10^{14} \text{ Hz.}$$

11.11. *Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.*

Sol. Given,

Wavelength, $\lambda = 488 \text{ nm} = 488 \times 10^{-9}$ m

Stopping potential, $V_0 = 0.38$V

As,

$$eV_0 = h\nu - \phi_0$$

$$\Rightarrow \quad eV_0 = h\frac{c}{\lambda} - \phi_0 \quad \Rightarrow \quad \phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{488 \times 10^{-9} \times 1.6 \times 10^{-19}} - \frac{1.6 \times 10^{-19} \times 0.38}{1.6 \times 10^{-19}} \right)$$

$$= (2.55 - 0.38) \text{ eV} = 2.17 \text{ eV.}$$

11.12. *Calculate the*

(a) momentum, and

(b) de-Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Sol. Energy of electron accelerated through potential difference of 56 V = 56 eV = 56 × 1.6 × 10^{-19} J

(*a*) As, $E = \dfrac{p^2}{2m}$ $\quad [p = mv,\ E = \frac{1}{2}mv^2]$

∴ $p^2 = 2mE \Rightarrow p = \sqrt{2mE}$

or, $p = \sqrt{2 \times 9 \times 10^{-31} \times 56 \times 1.6 \times 10^{-19}}$

$p = 4.02 \times 10^{-24}$ kg ms^{-1}

(*b*) As, $p = \dfrac{h}{\lambda}$

∴ $\lambda = \dfrac{h}{p} = \dfrac{6.62 \times 10^{-34}}{4.02 \times 10^{-24}}$

$= 1.64 \times 10^{-10}$ m $= 0.164 \times 10^{-9}$ m

or, $\lambda = 0.164$ nm.

11.13. *What is the*

(*a*) *momentum,* (*b*) *speed, and*

(*c*) *de-Broglie wavelength of an electron with kinetic energy of 120 eV.*

Sol. (*a*) $p = \sqrt{2mE}$

$\Rightarrow$ $p = \sqrt{2 \times (9 \times 10^{-31}) \times (120 \times 1.6 \times 10^{-19})}$

$= 5.88 \times 10^{-24}$ kg ms^{-1}

(*b*) $p = mv = \dfrac{p}{m} = \dfrac{5.88 \times 10^{-24}}{9.1 \times 10^{-31}}$

or, $v = 6.46 \times 10^6$ m/s

(*c*) $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{5.88 \times 10^{-24}} = 1.13 \times 10^{-10}$ m

$= 1.13$ Å.

11.14. *The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which*

(*a*) *an electron, and*

(*b*) *a neutron, would have the same de-Broglie wavelength.*

Sol. $\lambda = \dfrac{h}{\sqrt{2mE}} \Rightarrow E = \dfrac{h^2}{2\lambda^2 m}$

(*a*) For electron, $E = \dfrac{(6.63\times10^{-34})^2}{2\times(589\times10^{-9})^2\times9\times10^{-31}}$

$= 7.03 \times 10^{-25}$ J

(*b*) For neutron, $E = \dfrac{(6.63\times10^{-34})^2}{2\times(589\times10^{-9})^2\times1.66\times10^{-27}}$

$= 3.81\times10^{-28}$ J.

11.15. *What is the de-Broglie wavelength of*

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s.

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s and

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Sol. (*a*)

$m = 0.040$ kg

$v = 1\ \text{kms}^{-1} = 10^3\ \text{ms}^{-1}$

$p = mv$

$= 0.040 \times 10^3 = 40\ \text{kg ms}^{-1}$

$\therefore \quad \lambda = \dfrac{h}{p} = \dfrac{6.62\times10^{-34}}{40} = 1.7 \times 10^{-35}$ m

(*b*)

$m = 0.060$ kg

$v = 1.0\ \text{ms}^{-1}$

$p = mv = 0.060\ \text{kg ms}^{-1}$

$\lambda = \dfrac{h}{p} = \dfrac{6.62\times10^{-34}}{0.060} = 1.1 \times 10^{-32}$ m

(*c*)

$m = 1.0 \times 10^{-9}$ kg

$v = 2.2\ \text{ms}^{-1}$

$p = mv = 2.2 \times 10^{-9}$

$\lambda = \dfrac{h}{p} = \dfrac{6.62\times10^{-34}}{2.2\times10^{-9}} = 3 \times 10^{-25}$ m.

11.16. *An electron and a photon each have a wavelength of 1.00 nm. Find*

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

(Take $h = 6.63 \times 10^{-34}$ Js).

Sol. $\lambda_e = \lambda_p = 1.00 \text{ nm} = 1 \times 10^{-9}$ m

(*a*) $$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{1.00\times10^{-9}} \text{ kg ms}^{-1}$$

$$= 6.63 \times 10^{-25} \text{ kg ms}^{-1}$$

(*b*) $$E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1\times10^{-9}} \text{J}$$

$$= 1.989 \times 10^{-16} \text{J} = \frac{1.989\times10^{-16}}{1.6\times10^{-19}} \text{eV}$$

$$= 1.243 \text{ keV}$$

(*c*) $$\frac{h}{\sqrt{2mE_k}} = \lambda$$

$\Rightarrow$ $$\sqrt{2mE_k} = \frac{h}{\lambda} \quad \text{or,} \quad E_k = \frac{h^2}{2m\lambda^2}$$

$\Rightarrow$ $$E_k = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-9})^2}$$

$$= \frac{43.96\times10^{-68}}{18.2\times10^{-49}} \text{J} = 2.4 \times 10^{-19} \text{J}$$

or, $$E_k = \frac{2.4\times10^{-19}}{1.6\times10^{-19}} \text{eV} = 1.5 \text{ eV.}$$

11.17. (*a*) *For what kinetic energy of a neutron will the associated de-Broglie wavelength be* 1.40×10^{-10} *m?*

(*b*) *Also find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of* $\frac{3}{2}kT$ *at 300 K.*

Sol. (*a*) Here, $\lambda = 1.40 \times 10^{-10}$ m

$m = 1.675 \times 10^{-27}$ kg, $h = 6.63 \times 10^{-34}$ Js

As, $$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda}$$

or, $$v = \frac{6.63\times10^{-34}}{1.675\times10^{-27}\times1.40\times10^{-10}}$$

$$= 28.28 \times 10^{2} \text{ m/s}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.675 \times 10^{-27} \times (28.28 \times 10^2)^2$$
$$= 6.634 \times 10^{-21} \text{ J.}$$

(b) $$E = \frac{3}{2}kT = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 300 = 6.21 \times 10^{-21} \text{ J}$$

(Here, k is Boltzmann constant. The value of k is 1.38×10^{-38} JK^{-1})

As, $$\lambda = \frac{h}{\sqrt{2mE}}$$

$\Rightarrow$ $$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.675 \times 10^{-27}}}$$

or, $$\lambda = 1.45 \times 10^{-10} \text{ m.}$$

11.18. *Show that the wavelength of electromagnetic radiation equal to the de-Broglie wavelength of its quantum (photon).*

Sol. de-Broglie wavelength of a photon,

$$\lambda = \frac{h}{p}$$

Momentum of a photon, $p = \dfrac{h\nu}{c}$

Hence, $$\lambda = \frac{h}{\frac{h\nu}{c}} = \frac{c}{\nu}$$

It is same as wavelength of electromagnetic radiation.

11.19. *What is the de-Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u).*

Sol. Temperature, $T = 300$ K

Mass of nitrogen molecule, $m = 2 \times 14.0076$ u
$= 2 \times 14.0076 \times 1.6606 \times 10^{-27}$ kg
$= 46.52 \times 10^{-27}$ kg

de-Broglie wavelength, $\lambda = \dfrac{h}{mv} = \dfrac{h}{p}$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2m(3/2\, k_B T)}} = \frac{h}{\sqrt{3mk_BT}} \quad \left(\because\ E_k = \frac{3}{2}k_B T\right)$$

(Where k_B is Boltzmann constant)

$$\Rightarrow \qquad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 46.52 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= \frac{6.63}{240.37} \times 10^{-9}\text{ m} = 0.0276\text{ nm} = 0.276\text{ Å}.$$

11.20. (*a*) *Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The specific charge of the electron, i.e., its e/m is given to be* 1.76×10^{11} *C kg*$^{-1}$.

(*b*) *Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?*

Sol. (*a*) Here, $V = 500$ V

$$\frac{e}{m} = 1.76 \times 10^{11}\text{ C kg}^{-1}$$

The work done on the electron by potential difference between the cathode and the anode appears as its kinetic energy. Thus,

$$\frac{1}{2} mv^2 = eV$$

$$\Rightarrow \qquad v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 1.76 \times 10^{11} \times 500}$$

or, $v = 1.327 \times 10^7$ ms^{-1}

(*b*) Here, $V = 10$ MV $= 10 \times 10^6 = 10^7$ V

$$\frac{e}{m} = 1.76 \times 10^{11}\text{ C kg}^{-1}$$

Now, $$v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 1.76 \times 10^{11} \times 10^7}$$

or, $v = 1.876 \times 10^9$ ms^{-1}

As this speed is greater than the speed of light, so this speed is not possible, because as v approaches c, mass of electron becomes infinite $\left[\because m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$ so the formula has to be modified.

11.21. *(a)* *A monoenergetic electron beam with electron speed of 5.20×10^6 ms^{-1} is subject to a magnetic field of 1.30×10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76×10^{11}C kg^{-1}.*

(b) *Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?*

Sol. *(a)* Velocity, $v = 5.20 \times 10^6$ m/s

Magnetic field, $B = 1.30 \times 10^{-4}$ T

$e/m = 1.76 \times 10^{11}$ C kg^{-1}

Centripetal force is provided by the force exerted by magnetic field on electron,

∴ [illegible]

or, $$r = \frac{mv}{Be} = \frac{v}{B(e/m)}$$

or, $$r = \frac{5.20\times10^6}{1.30\times10^{-4}\times1.76\times10^{11}} \text{ m} = 0.227 \text{ m} = 22.7 \text{ cm.}$$

(b) $$\frac{1}{2}mv^2 = 20 \text{ MeV} = 20 \times 1.6 \times 10^{-13} \text{ J}$$

$$v = \sqrt{\frac{2\times20\times1.6\times10^{-13}}{9.1\times10^{-31}}} \text{ m/s} = 2.65 \times 10^9 \text{ m/s}$$

The velocity is greater than velocity of light. It appears something is wrong with data. However, the electron is clearly moving at relativisitic speed. So, the non-relativistic formula $r = \frac{m_0 v}{eB}$ is not valid. We should use relativistic formula:

$$r = \frac{mv}{eB}$$

[illegible].

11.22. *An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (~10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path* *can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine e/m from the data.*

Sol. Here, $V = 100$ V;

Magnetic field, $B = 2.83 \times 10^{-4}$ T

$r = 12.0$ cm $= 12.0 \times 10^{-2}$ m

When electrons are accelerated through V volt, the gain in K.E. of the electron is given by

$$\frac{1}{2}mv^2 = eV \quad \Rightarrow \quad v^2 = \frac{2eV}{m} \qquad ...(i)$$

Since the electron moves in circular orbit under magnetic field, therefore, force on the electron due to magnetic field provides the centripetal force to the electron.

$$\therefore \qquad evB = \frac{mv^2}{r}$$

or, $$eB = \frac{mv}{r} \quad \text{or,} \quad v^2 = \frac{e^2B^2r^2}{m^2} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$\frac{2eV}{m} = \frac{e^2B^2r^2}{m^2} \quad \Rightarrow \quad \frac{e}{m} = \frac{2V}{r^2B^2}$$

or, $$\frac{e}{m} = \frac{2 \times 100}{(12 \times 10^{-2})^2 \times (2.83 \times 10^{-4})^2}$$

$$= 1.73 \times 10^{11} \text{ C kg}^{-1}.$$

11.23. *(a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?*

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Sol. Here,

$\lambda = 0.45$ Å $= 0.45 \times 10^{-10}$ m

$h = 6.62 \times 10^{-34}$ Js

$c = 3 \times 10^8$ ms^{-1}

(*a*) The maximum energy of photon is given by

$$E = h\nu = h\frac{c}{\lambda}$$

or, $$E = \frac{6.62\times10^{-34}\times3\times10^{8}}{0.45\times10^{-10}} = 44 \times 10^{-16}\ \text{J}$$

or, $$E = \frac{44\times10^{-16}}{1.6\times10^{-19}}\ \text{eV} = 27.5 \times 10^{3}\ \text{eV} = 27.5\ \text{keV}.$$

(*b*) To produce electrons of 27.5 keV, accelerating potential of 27.5 kV or of the order of 30 kV is required.

11.24. *In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^9eV)*

Sol. Total energy of 2 γ-rays = 10.2 BeV = 10.2×10^9 eV

∴ Energy of each γ-ray

$$\Rightarrow \quad E = \frac{1}{2}(10.2 \times 10^9 \times 1.6 \times 10^{-19})\ \text{J} = 8.16 \times 10^{-10}\ \text{J}$$

Using the formula,

$$E = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{8.16\times10^{-10}} = 2.436 \times 10^{-16}\ \text{m}.$$

11.25. *Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about 'photons'. The second number tells you why our eye can never 'count photon' even in barely detectable light.*

(*a*) *The number of photons emitted per second by a medium wave transmitter of 10 kW power emitting radio waves of length 500 m.*

(*b*) *The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive (~10^{-10} Wm^{-2}). Take the area of the pupil to be about 0.4 cm^2, and the average frequency of white light to be about 6×10^4 Hz.*

Sol. (*a*) $\lambda = 500$ m

Energy of photon, $E = h\nu$

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{500}\ \text{J}$$

$$E = 3.98 \times 10^{-28}\ \text{J}$$

Number of photons emitted/s

$$= \frac{\text{Power of transmitter}}{\text{Energy of one photon}} \quad \because \boxed{P = ne}$$

$$= \frac{10^4 \text{ Js}^{-1}}{3.98 \times 10^{-28} \text{ J}} = 3 \times 10^{31} \text{ s}^{-1}$$

We see that the energy of a radiophoton is exceedingly small, and the number of photons emitted per second in a radio beam is enormously large. There is, therefore, negligible error involved in ignoring the existence of a minimum quantum of energy (photon) and treating the total energy of a radiowave as continuous.

(*b*) For $\nu = 6 \times 10^{14}$ Hz, $E = h\nu = 6.63 \times 10^{-34} \times 6 \times 10^{14}$ J $\simeq$ 4×10^{-19} J. Photon flux corresponding to minimum intensity

$$= \frac{10^{-10} \text{ Wm}^{-2}}{4 \times 10^{-19} \text{ J}} \quad \boxed{P = nE}$$

$$= 2.5 \times 10^8 \text{ m}^{-2} \text{ s}^{-1}$$

Number of photons entering the pupil per second = $2.5 \times 10^8 \times 0.4 \times 10^{-4}$ s^{-1} = 10^4 s^{-1}. Though this number is not as large as in (*a*) above, it is large enough for us never to 'sense' or 'count' individual photons by our eye.

11.26. *Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photocell made of molybdenum metal. If the stopping potential is –1.3 volt, estimate the work function of the metal. How would the photocell respond to a high intensity (~ 10^5 Wm^{-2}) red light of wavelength 6328 Å produced by He-Ne laser?*

Sol. Here, $V_0 = 1.3$ V

$\lambda = 2271$ Å $= 2271 \times 10^{-10}$ m

Now, $$\boxed{h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2 = \phi_0 + eV_0}$$

or, $$\phi_0 = \frac{hc}{\lambda} - eV_0$$

Taking, $h = 6.62 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$c = 3 \times 10^8$ ms^{-1}

We have, $\phi_0 = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{2271\times10^{-10}} - 1.6\times10^{-19}\times1.3$

$\Rightarrow$ $\phi_0 = 8.745\times10^{-19} - 2.08\times10^{-19} = 6.665\times10^{-19}$ J

or, $\phi_0 = \dfrac{6.665\times10^{-19}}{1.6\times10^{-19}} = 4.166$ eV.

Threshold wavelength is given by

$$\boxed{\lambda_0 = \frac{hc}{\phi_0}} = \frac{6.62\times10^{-34}\times3\times10^{8}}{6.665\times10^{-19}} = 2.98\times10^{-7}\text{m}$$

or, $\lambda_0 = 2980$ Å

Since wavelength 6328 Å is greater than λ_0, the photocell will not respond, when red light of wavelength 6328 Å produced by He-Ne laser is incident on the photocell.

11.27. *Monochromatic radiation of wavelength 640.2 nm (1 nm = 10^{-9} m) from a neon lamp irradiates a photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photocell. Predict the new stopping voltage.*

Sol. Here, for neon lamp,

$$\lambda = 640.2 \text{ nm} = 640.2\times10^{-9}\text{ m}$$
$$V = 0.54 \text{ V}$$

As, $$eV_1 = \frac{hc}{\lambda_1} - \phi_0$$

$$eV_2 = \frac{hc}{\lambda_2} - \phi_0$$

$$eV_2 - eV_1 = hc\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$V_2 - V_1 = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$V_2 = V_1 = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$V_2 = 0.54 + \frac{6.63\times10^{-24}\times3\times10^{8}}{1.6\times10^{-19}}\left[\frac{1}{427.2\times10^{-9}} - \frac{1}{640.2\times10^{-9}}\right]$$

$$= 0.54 + \frac{6.63\times3\times10^{-24+8+9}}{1.6}\left[\frac{1}{427.2} - \frac{1}{640.2}\right]$$

$$= 0.54 + \frac{6.63 \times 3}{1.6} \frac{[640.2 - 427.2]}{640.2 \times 427.2} \times 10^{-7}$$

$$= 0.54 + \frac{6.63 \times 3 \times 213 \times 10^{-7}}{1.6 \times 640.2 \times 427.2} = 0.54 + 0.97$$

$V_2 = 1.51$ V.

11.28. *A mercury lamp is convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photocell, the following lines from a mercury source were used:*

$$\lambda_1 = 3650 \text{ Å}, \lambda_2 = 4047 \text{ Å}, \lambda_3 = 4358 \text{ Å},$$
$$\lambda_4 = 5461 \text{ Å}, \lambda_5 = 6907 \text{ Å}$$

The stopping voltages, respectively were measured to be:

$$V_{01} = 1.28 \text{ V}, V_{02} = 0.95 \text{ V}, V_{03} = 0.74 \text{ V},$$
$$V_{04} = 0.16 \text{ V}, V_{05} = 0 \text{V}$$

(a) Determine the value of Planck's constant h.

(b) Estimate the threshold frequency and work function for the material.

Sol. (*a*) Using $\nu = \frac{c}{\lambda}$, we first determine frequency in each case and then plot a graph between stopping potential V_0 and frequency ν.

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3 \times 10^8}{3650 \times 10^{-10}} = 8.219 \times 10^{14} \text{ Hz}$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3 \times 10^8}{4047 \times 10^{-10}} = 7.412 \times 10^{14} \text{ Hz}$$

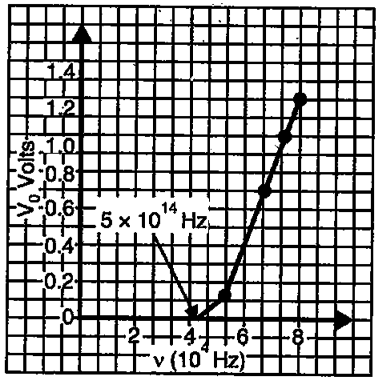
$$\nu_3 = \frac{c}{\lambda_3} = \frac{3 \times 10^8}{4358 \times 10^{-10}} = 6.884 \times 10^{14} \text{ Hz}$$

$$\nu_4 = \frac{c}{\lambda_4} = \frac{3 \times 10^8}{5461 \times 10^{-10}} = 5.493 \times 10^{14} \text{ Hz}$$

$$\nu_5 = \frac{c}{\lambda_5} = \frac{3 \times 10^8}{6907 \times 10^{-10}} = 4.343 \times 10^{14} \text{ Hz}$$

V_0 versus ν plot is shown in figure.

The first four points lie nearly on a straight line which intercepts the ν axis of threshold frequency $\nu_0 = 5.0 \times 10^{14}$ Hz.



The fifth point ν (= 4.3 × 10^{14} Hz) corresponds to $\nu < \nu_0$, so there is no photoelectric emission and not stopping voltage is required to stop the current. Slope of V_0 versus ν graph is

$$\frac{\Delta V}{\Delta \nu} = \frac{(1.28-0)V}{(8.2-5.0)\times 10^{14} s^{-1}} = 4.0 \times 10^{-15} \text{ Vs}$$

From Einstein's photoelectric equation,

$$\text{K.E.} = eV = h\nu - W_0$$

$\therefore$ $e\Delta V = h\Delta\nu$ [W_0 is a constant]

or, $$\frac{\Delta V}{\Delta \nu} = \frac{h}{e}$$

Hence, $$\frac{h}{e} = 4.0 \times 10^{-15} \text{ Vs}$$

Planck's constant, $h = e \times 4.0 \times 10^{-15}$ Js

or, $h = 1.6 \times 10^{-19} \times 4.0 \times 10^{-15}$ Js

$= 6.4 \times 10^{-34}$ Js

(*b*) Threshold frequency, $\nu_0 = 5.0 \times 10^{14}$ Hz

$\therefore$ Work function, $\phi_0 = h\nu_0 = 6.4 \times 10^{34} \times 5.0 \times 10^{14}$

or, $$\phi_0 = \frac{6.4 \times 5 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 2.00 \text{ eV}$$

11.29. *The work function for the following metals is given:*

Na: 2.75 eV; K: 2.30 eV; M_0: 4.17 eV; Ni: 5.15 eV.

Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He–Cd laser placed 1m *away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?*

Sol. (*i*) Work function of Na is

$$\phi_{Na} = 1.92 \text{ eV} = 1.92 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = 3300 \text{ Å} = 3300 \times 10^{-10} \text{ m}$$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10}} \text{ J}$$

$$E = \frac{6.6 \times 3 \times 10^{-34+8+10-2}}{33 \times 10} \text{ J}$$

$$= \frac{6 \times 10^{-18-1}}{1.6 \times 10^{-19}} \text{ eV} = \frac{60}{16} \text{ eV}$$

$$E = 3.75 \text{ eV}$$

It is observed that energy of incident radiation is less than Ni and Mo but larger than Na and K. So photoemission current take place from Na and K but not from Mo and Ni. Therefore, Mo and Ni will not give photoelectric emission. If the laser is brought closer the intensity of radiation increases without any change in frequency. This therefore, will not affect the result. However, photoelectric current from Na and K will increase.

11.30. *Light of intensity 10^{-5} W m^{-2} falls on a sodium photocell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?*

Sol. Number of atoms in 5 layers of sodium

$$= \frac{5 \times \text{area of each layer}}{\text{Effective area of atom}} = \frac{5 \times 2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

Assume that there is only one conduction electron per Na atom.

$\therefore$ Number of electrons in 5 layers $= 10^{17}$

Energy received by an electron per sec

$$= \frac{\text{Power of incident light}}{\text{Number of electrons}} = \frac{10^{-5} \times 2 \times 10^{-4}}{10^{17}} = 2 \times 10^{-26} \text{ W}$$

Time required for photoemission

$$= \frac{\text{Energy required per electron}}{\text{Energy absorbed per second per electron}}$$

$$= \frac{2\times1.6\times10^{-19}}{2\times10^{-26}} \text{ s} = 1.6 \times 10^{7} \text{ s}.$$

It is contrary to the observed fact that there is no time lag between the incidence of light and the emission of photoelectrons.

11.31. *Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy, an X-ray photon or the electron? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of interatomic spacing in the lattice)* *($m_e = 9.11 \times 10^{-31}$ kg).*

Sol. Energy of photon, $\boxed{E = h\nu = \frac{hc}{\lambda}}$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{10^{-10}} \text{ J}$$

$$= 19.89 \times 10^{-16} \text{ J}$$

or, $$E = \frac{19.89\times10^{-16}}{1.6\times10^{-19}} \text{ eV} = 12.43 \text{ keV}$$

For the case of electron,

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}} \Rightarrow \sqrt{2mE} = \frac{h}{\lambda}$$

or, $$2mE = \frac{h^2}{\lambda^2} \quad \text{or,} \quad E = \frac{h^2}{2m\lambda^2}$$

$\Rightarrow$ $$E = \frac{(6.63\times10^{-34})^2}{2\times9.11\times10^{-31}\times10^{-20}} \text{ J}$$

$$= \frac{(6.63\times10^{-34})^2}{2\times9.11\times10^{-31}\times10^{-20}\times1.6\times10^{-19}} \text{eV}$$

or, $$E = 150.8 \text{ eV}$$

For the same given wavelength, kinetic energy of a photon is much greater than that of electron.

11.32. (*a*) *Obtain the de-Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Question 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain.*

Given $m_n = 1.675 \times 10^{-27}$ *kg.*

(*b*) *Obtain the de-Broglie wavelength associated with thermal neutrons at room temperature (27°C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.*

Sol. (*a*) Here, K.E. of neutron, $E = 150$ eV $= 150 \times 1.6 \times 10^{-19}$ J,

Mass of neutron, $m = 1.675 \times 10^{-27}$ kg.

We know, K.E. of neutron, $E = \frac{1}{2}mv^2$ or $mv = \sqrt{2Em}$

$$\therefore \boxed{\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2Em}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 150 \times 1.6 \times 10^{-19} \times 1.675 \times 10^{-27}}} = 2.33 \times 10^{-12} \text{ m.}$$

The interatomic spacing ~ 1 Å (= 10^{-10} m) is about a hundred times greater than this wavelength. Therefore, a neutron beam of energy 150 eV is not suitable for diffraction experiment.

(*b*) Here, $T = 27 + 273 = 300$ K,

Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J mol^{-1} K^{-1}

We know, average K.E. of neutron at absolute temperature T is given by $E = \frac{3}{2}kT$. Where k is the Boltzmann's constant.

Now, $\boxed{\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}}$.

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} = 1.45 \times 10^{-10} \text{ m}$$

Since this wavelength is comparable to interatomic spacing (~ 1 Å) in a crystal, therefore, thermal neutrons are suitable probe for diffraction experiments: so a high energy neutron beam should be first thermalised before using it for diffraction.

11.33. *An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?*

Sol. Accelerating voltage, $V = 50$ kV

$\therefore$ Energy of electrons, $E = eV = 50$ keV

or, $E = 50 \times 10^3 \times 1.6 \times 10^{-19}$ J

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \quad \lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\lambda = \frac{6.62 \times 10^{-34}}{1.21 \times 10^{-22}} = 5.47 \times 10^{-12} \text{ m.}$$

Resolving power of microscope $\propto \frac{1}{\lambda}$

$$\therefore \quad \frac{\text{R.P. of electron microscope}}{\text{R.P. of optical microscope}} = \frac{\lambda y}{\lambda} + \frac{5.9 \times 10^{-7}}{5.47 \times 10^{-17}} \cong 10^{+5}$$

λy = wavelength yellow light.

Resolving power of a microscope is inversely proportional to the wavelength of the radiation used. Since the wavelength of yellow light is 5990 Å or 5.99×10^{-7} m, so it follows that electron microscope will have resolving power 10^5 times that of optical microscope.

11.34. *The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)*

Sol. Applying formula, $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js}}{10^{-15} \text{ m}}$

$= 6.63 \times 10^{-19}$ kg ms^{-1}

Use the relativistic formula for energy:

$$E^2 = c^2p^2 + m_0^2c^4$$

or, $$E^2 = 9 \times (6.63)^2 \times 10^{-22} + (0.511 \times 1.6)^2 \times 10^{-26}$$

$$\simeq 9 \times (6.63)^2 \times 10^{-22}$$

the second term (rest mass energy) being negligible.

Therefore, $E = 1.989 \times 10^{-10}$ J

or, $$E = \frac{1.989 \times 10^{-10}}{1.6 \times 10^{-11}} \text{ BeV} = 1.24 \text{ BeV}.$$

Thus, electron energies from the accelerator must have been of the order of a few BeV.

11.35. *Find the typical de-Broglie wavelength associated with a He atom in helium gas at room temperature (27 °C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.*

Sol. Mass of atom,

$$m = \frac{\text{Atomic mass of He}}{\text{Avogadro's number}} = \frac{4}{6 \times 10^{23}} \text{g}$$

or, $$m = 6.67 \times 10^{-27} \text{ kg}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} \text{ m}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 6.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

or, $$\lambda = 7.3 \times 10^{-11} \text{ m}$$

Now, $$PV = RT = kNT$$

(The kinetic gas equation for one mole of a gas.)

or, $$\frac{V}{N} = \frac{KT}{P}$$

Mean separation,

$$r_0 = \left(\frac{\text{Molar volume}}{\text{Avogadro's number}}\right)^{1/3} = \left(\frac{V}{N}\right)^{1/3} = \left(\frac{kT}{P}\right)^{1/3}$$

$$\Rightarrow \quad r_0 = \left(\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5}\right)^{1/3} \text{m} \quad \text{or,} \quad r_0 = 3.4 \times 10^{-9} \text{ m}$$

The mean separation between two atoms is much larger than the de-Broglie wavelength.

11.36. *Compute the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.*

*[**Note:** Questions 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This indistinguishibility has many fundamental implications which you will explore in more advanced Physics courses.]*

Sol. Here, $T = 27° \text{ C} \Rightarrow T = 273 + 27 = 300 \text{ K}$

$m = 9.1 \times 10^{-31}$ kg

Using formula,

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$\Rightarrow \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 9.1 \times 10^{-31} \times \left(\frac{8.31 \times 10^{3}}{6 \times 10^{23}}\right) \times 300}}$$

or, $\lambda = 62.15 \times 10^{-10}$ m

Interelectronic separation, $r = 2 \times 10^{-10}$ m

Hence, $r < \lambda$

We find that wave-packets in metals strongly overlap with one another (This is not the case in gas atoms).

11.37. *Answer the following questions:*

(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+ 2/3)e; (–1/3e). Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination e/m? Why do we not simply talk of e and m separately?

(c) *Why should gases be insulators at ordinary pressures and start conducting at very low pressures?*

(d) *Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?*

(e) *The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations*

$$E = h\nu,\ p = \frac{h}{\lambda}$$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why?

Sol. (a) Quarks are thought to be confined within a proton or neutron by forces which grow stronger if one tries to pull them apart. Though bound fractional charges may exist in nature, independent charges are still integral multiples of e.

(b) Basic relations $eV = 1/2\ mv^2$ or $eE = ma$ and $eBv = mv^2/r$, for electric and magnetic fields, respectively, show that the dynamics of electrons is determined not by e, and m separately but by the combination e/m.

(c) At low pressures, ions have a chance to reach their respective electrodes and constitute a current. At ordinary pressures, ions have no chance to do so because of collisions with gas molecules and recombination.

(d) Work function only indicates the minimum energy required for the electron in the highest level of the conduction band to get out of the metal. Not all electrons in the metal belong to this level. They occupy a continuous band of levels. Consequently, for the same incident radiation, electrons knocked off from different levels come out with different energies.

(e) The absolute value of energy E (but not momentum p) of any particle is arbitrary to within an additive constant.

Hence, while λ is physically significant, absolute value of v of a matter wave of an electron has no direct physical meaning. The phase speed $v\lambda$ is likewise not physically significant. The group speed given by $v = \nu\lambda = \frac{\omega}{k}$

$$\frac{dw}{dk} = \frac{dw}{dk}\frac{dv}{d(1/\lambda)} = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m}$$

$$\because \quad \boxed{K = \frac{2\pi}{\lambda},\ \omega = 2\pi\nu} \text{ and } \lambda = \frac{h}{p}$$

is physically meaningful.