12 Atoms

Lesson at a Glance

• Rutherford's α-Particle Scattering Experiment

In 1911, Rutherford suggested an experiment for α-particle scattering. For the experiment, *H*-Guiger and *E*. Marsden considered $^{214}_{83}Bi$ as a source of α-particles. A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on a 2.1×10^{-7} m thick gold foil.

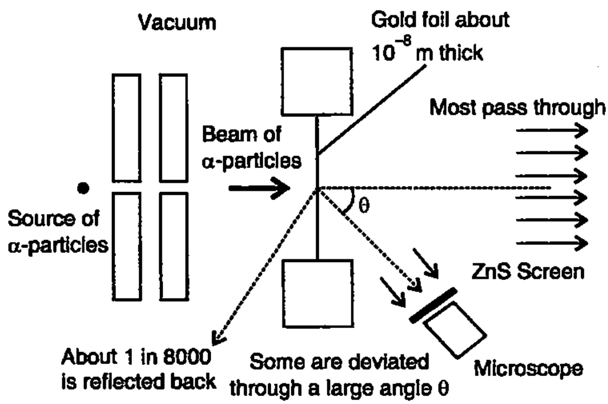

Fig. 12.1

• Distance of Closest Approach

When a α-particle of mass *m* and velocity *v* moves directly towards a nucleus of atomic number '*z*', its distance of closest approach is given by

$$r_0 = \frac{2k\,Z_e^2}{E} = \frac{4k\,Z_e^2}{mv^2}$$

where, $$E = \frac{1}{2}mv^2$$

and $$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{ C}^2$$

• Impact Parameter

Impact parameter is defined as the perpendicular distance of the initial velocity vector of the alpha particle from the central line of the nucleus, when the particle is far away from the nuclus of the atom.

$$\text{Impact parameter,} \quad b = \frac{ze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0 E} = \frac{ze^2 \cot\frac{\theta}{2}}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

• Velocity of Electron in its Orbit

Let an electron is revolving in an atom of atomic number z in its orbit of radius R. The required centrepetal force for this electron is provided by electrostatic force between the electron and nucleus.

Thus, $$\frac{mv^2}{R} = \frac{k(e)(Ze)}{R^2}$$

or $$(mvR)v = kZe^2$$

or $$\frac{nh}{2\pi}.v = kZe^2$$

or $$v = \frac{2\pi kZe}{nh} \qquad \text{...}(i)$$

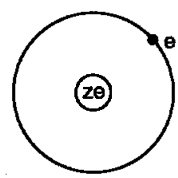

Fig. 12.2

• Radius of the Electron's Orbit

$\because$ $$\frac{mv^2}{R} = \frac{k(Ze)(e)}{R^2}$$

$\therefore$ $$R = \frac{kZe^2}{mv^2}$$

Putting the value of v from Eq. (i)

$$R = \frac{kZe^2}{m\left(\frac{2\pi kZe^2}{nh}\right)^2}$$

or $$R = \frac{n^2h^2}{4\pi^2 kZe^2 m} \qquad \text{...}(ii)$$

• Energy of the Electron in its Orbit

(*i*) **Kinetic energy**

$$\because \qquad \frac{mv^2}{R} = \frac{k(Ze)(-e)}{R^2}$$

$$\therefore \qquad \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{kZe^2}{R} \qquad ...(iii)$$

(*ii*) **Potential energy**

Potential energy of two charges (e) and (Ze) separated by R is given by

$$U = \frac{k(Ze)(-e)}{R} = -\frac{kZe^2}{R} \qquad ...(iv)$$

• Spectral Lines in Hydrogen Atom

- When electron jumps from any higher energy level to lower orbit, the distribution of energy wave length wise is called *spectrum*

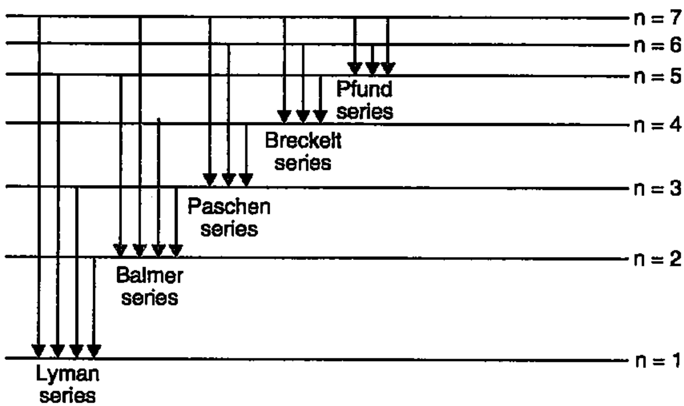

Fig. 12.3

TEXTBOOK QUESTIONS SOLVED

12.1. *Choose the correct alternative from the clues given at the end of the each statement:*

(*a*) *The size of the atom in Thomson's model is .... the atomic size in Rutherford's model.*
(much greater than/no different from/much less than)

(b) *In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/Rutherford's model)*

(c) *A classical atom based on .... is doomed to collapse. (Thomson's model/Rutherford's model)*

(d) *An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in .... (Thomson's model/Rutherford's model)*

(e) *The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models).*

Sol. (a) No different from

(b) Thomson's model, Rutherford's model

(c) Rutherford's model

(d) Thomson's model, Rutherford's model

(e) Both the models.

12.2. *Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?*

Sol. The nucleus of a hydrogen atom is a proton (mass 1.67 × 10^{-27} kg) which has only about one-fourth of the mass of an alpha particle (6.64 × 10^{-27} kg). Because the alpha particle is more massive, it won't bounce back in even a head-on collision with a proton. It is like a bowling ball colliding with a ping-pong ball at rest. Thus, there would be no large angle scattering in this case. In Rutherford's experiment, by contrast, there was large-angle scattering because a gold nucleus is more massive than an alpha-particle. The analogy there is a ping-pong ball hitting a bowling ball at rest.

12.3. *What is the shortest wavelength present in the Paschen series of spectral lines?*

Sol. The shortest wavelength of the spectral line (series limit) of Paschen series is given by

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$$

$$\Rightarrow \qquad \lambda_{min} = \frac{9}{R} = \frac{9}{1.097 \times 10^7}\ \text{m}$$

or, $$\lambda_{min} = \frac{9 \times 10^{-7} \times 10^{10}}{1.097} \text{Å} = 8204.2 \text{ Å}.$$

12.4. *A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?*

Sol. $$E_2 - E_1 = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} \text{ J}$$

$$\nu = \frac{E_2 - E_1}{h}$$

$\Rightarrow$ $$\nu = \frac{2.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

or, $$\nu = \frac{3.68 \times 10^{15}}{6.6} = 0.557 \times 10^{15} \text{ Hz}$$

$$= 5.6 \times 10^{14} \text{ Hz.}$$

12.5. *The ground state energy of hydrogen atom is – 13.6 eV. What are the kinetic and potential energies of the electron in this state?*

Sol. Here, Ground Energy, $E = -13.6$ eV

Kinetic Energy, $E_k = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r}$ $[E_k = 2E_p]$

and Potential Energy, $E_p = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$ $\left[\because E_p = \frac{-kq_1q_2}{r}\right]$

Total energy, $E = E_k + E_p$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{2r} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

$$E = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}\right)$$

or, $$-13.6 = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}\right)$$

$\therefore$ $$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r} = 27.2$$

$\therefore$ $$E_k = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{2r} = \frac{27.2}{2} \text{eV} = 13.6 \text{ eV}$$

$$E_p = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r} = -27.2 \text{ eV.}$$

12.6. *A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.*

Sol. Energy of an electron in n^{th} orbit of H atom

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

$$E_1 = -13.6 \text{ eV}$$

Energy is 4^{th} ($n = 4$) level

$$R_4 = \frac{-13.6}{4^2} = -0.85$$

$$\Delta E = E_4 - E_1$$

$$\Delta E = -0.85 - (-13.6) \text{ eV}$$

$$= -0.85 + 13.6$$

$$\Delta E = 12.75 \text{ eV}$$

$$h\nu = 12.75 \text{ eV}$$

$$h\nu = 12.75 \times 1.6 \times 10^{-19} \text{ J}$$

$$\nu = \frac{12.75 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$\nu = 3.078 \times 10^{15} \text{ Hz}$$

$$\lambda = \frac{c}{\lambda} = \frac{3 \times 10^8}{3.078 \times 10^{15}}$$

$$\lambda = 974.4 \text{ Å}.$$

12.7. *(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.*

Sol. (*a*) From $v = \frac{c}{n}\alpha$, where $\alpha = \frac{2\pi Ke^2}{ch} = 0.0073$

$$v_1 = \frac{3 \times 10^8}{1} \times 0.0073 = 2.19 \times 10^6 \text{ m/s}$$

$$v_2 = \frac{3 \times 10^8}{2} \times 0.0073 = 1.095 \times 10^6 \text{ m/s}$$

$$v_3 = \frac{3 \times 10^8}{3} \times 0.0073 = 7.3 \times 10^5 \text{ m/s}.$$

(*b*) Orbital period, $T = \frac{2\pi r}{v}$ As $r_1 = 0.53 \times 10^{-10}$ m

$$T_1 = \frac{2\pi \times 0.53 \times 10^{-10}}{2.19 \times 10^6} = 1.52 \times 10^{-16} \text{ s}$$

As $r_2 = 4\, r_1$ and $v_2 = \frac{1}{2} v_1$

$T_2 = 8\, T_1 = 8 \times 1.52 \times 10^{-16}$ s $= 1.216 \times 10^{-15}$ s

As $r_3 = 9\, r_1$ and $v_3 = \frac{1}{3} v_1$

$\therefore$ $T_3 = 27\, T_1 = 27 \times 1.52 \times 10^{-16}$ s $= 4.1 \times 10^{-15}$ s.

12.8. *The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n = 3 orbits?*

Sol. $r_0 = 5.3 \times 10^{-11}$ m, $r = r_0 \,.\, n^2$

(*i*) when $n = 2$, $r = 5.3 \times 10^{-11} \times (2)^2$

or, $r = 21.2 \times 10^{-11}$ m $= 2.12 \times 10^{-10}$ m

[$\because$ $r_n = 0.53 \times n^2$ Å]

(*ii*) when $n = 3$, $r = 5.3 \times 10^{-11}$ m $\times (3)^2$

$= 47.7 \times 10^{-11}$ m $= 4.77 \times 10^{-10}$ m.

12.9. *A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?*

Sol. In ground state, energy of gaseous hydrogen at room temperature = – 13.6 eV. When it is bombarded with 12.5 eV electro beam, the energy becomes –13.6 + 12.6 = – 1.1 eV.

$$\because \quad E_n = \frac{-13.6}{n^2} \quad \text{So} \quad n^2 = \frac{-13.6}{-1.1} = 12.3 \quad \Rightarrow \quad n = 3$$

The electron would jump from $n = 1$ to $n = 3$, where $E_3 = -\frac{13.6}{3^2} = -1.5$ eV. On de-excitation the electron may jump from $n = 3$ to $n = 2$ giving rise to Balmer series. It may also jump from $n = 3$ to $n = 1$, giving rise to Lyman series.

So, number of spectral line $= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$ spectral lines appear.

12.10. *In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)*

Sol. According to Bohr's theory

$$mvr = \frac{nh}{2\pi}$$

or,
$$n = \frac{2\pi\, mvr}{h}$$

or,
$$n = \frac{2 \times 3.14 \times 6.0 \times 10^{24} \times 3 \times 10^{4} \times 1.5 \times 10^{11}}{6.63 \times 10^{-34}}$$

$$= 2.56 \times 10^{74}.$$

12.11. *Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.*

(a) *Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?*

(b) *Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?*

(c) *Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?*

(d) *In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?*

Sol. (a) About the same.

(b) Much less.

(c) It suggests that scattering is predominantly due to a single collision, because the chance of a single collision increases linearly with the number of target atoms, and hence linearly with the thickness of the foil.

(d) In Thomson's model, a single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. So it is wrong to ignore multiple scattering in Thomson's model. In Rutherford's model, most of the scattering comes through a single collision and multiple scattering effects can be ignored as a first approximation.

12.12. *The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.*

Sol. The radius of the first orbit of hydrogen atom in Bohr's model is given by

$$r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

$$r = \frac{\varepsilon_0 h^2}{\pi me^2} = \frac{4\pi\varepsilon_0}{e^2}\left(\frac{h^2}{4\pi^2 m}\right) \qquad \left[\text{here } k = \frac{1}{4\pi\varepsilon_0},\ Z = 1, n = 1\right]$$

If electrostatic force $\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2}$ is replaced by gravitational force $\frac{GMm}{r^2}$, we put GMm in place of $\frac{e^2}{4\pi\varepsilon_0}$ in above expression.

Hence radius of first orbit under gravitational force

$$r_G = \frac{1}{GMm}\cdot\frac{h^2}{4\pi^2 m} = \frac{h^2}{4\pi^2 GMm^2} \qquad \left[\begin{array}{l} M = \text{mass of proton} \\ m = \text{mass of electron}\end{array}\right]$$

or,

$$r_G = \frac{(6.26\times 10^{-34})^2}{4\times(3.14)^2\,(6.67\times 10^{-11})\times(1.672\times 10^{-27})\times(9.1\times 10^{-31})^2}$$

$$= \frac{6626\times 6626\times 10^{-74}}{4\times 3.14\times 3.14\times 667\times 16724\times 19\times 19\times 10^{-112}}$$

$= 1.21 \times 10^{29}$ m.

It is larger than the size of the universe.

12.13. *Obtain an expression for the frequency of radiations emitted when a hydrogen atom de-excites from level n to level (n − 1). For large n, show that the frequency equals the classical frequency of revolution of the electron in the orbit.*

Sol. The frequency ν of the emitted radiation when a hydrogen atom de-excites from level n to level $(n - 1)$ is

$$E = h\nu = E_2 - E_1$$

$$\nu = \frac{1}{2}\frac{mc^2\alpha^2}{h}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

where $\alpha = \dfrac{2\pi Ke^2}{ch}$ = fine structure constant

$$\nu = \frac{1}{2}\frac{mc^2\alpha^2}{h}\left[\frac{1}{(n-1)^2}-\frac{1}{n^2}\right] = \frac{mc^2\alpha^2}{2h}\left[\frac{n^2-(n-1)^2}{n^2\,(n-1)^2}\right]$$

$$= \frac{mc^2\alpha^2\,[(n+n-1)\,(n-n+1)]}{2hn^2\,(n-1)^2}$$

$$\nu = \frac{mc^2\alpha^2\,(2n-1)}{2h\,n^2\,(n-1)^2}.$$

For large n, $(2n-1) \approx 2n$, and $(n-1) \approx n$

$$\nu = \frac{mc^2\alpha^2.2n}{2h\,n^2.n^2} = \frac{mc^2\alpha^2}{hn^3}$$

Putting $\alpha = \dfrac{2\pi Ke^2}{ch}$, we get $\nu = \dfrac{mc^2}{hn^3}\cdot\dfrac{4\pi^2K^2e^4}{c^2h^2}$

$$\nu = \frac{4\pi^2mK^2e^4}{n^3h^3}$$

In Bohr's atom model, velocity of electron in nth orbit is $v = \dfrac{nh}{2\pi\,mr}$ and radius of nth orbit is $r = \dfrac{n^2h^2}{4\pi^2mKe^2}$

$(\because \quad Z = 1)$

$\therefore$ frequency of revolution of electron

$$\nu = \frac{v}{2\pi r} = \frac{nh}{2\pi\,mr}\left(\frac{4\pi^2mKe^2}{2\pi\cdot n^2h^2}\right)$$

$$\nu = \frac{Ke^2}{nh\cdot r} = \frac{Ke^2}{nh}\left(\frac{4\pi^2mKe^2}{n^2h^2}\right)$$

$$\nu = \frac{4\pi^2mK^2e^4}{n^3h^3}$$ which is the same as (*i*).

Hence for large values of n, classical frequency of revolution of electron in nth orbit is the same as the frequency of radiation emitted when hydrogen atom de-excites from level (n) to level ($n-1$).

12.14. *Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10} m).*

(a) *Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.*

(b) *You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.*

Sol. (a) From Coulomb's law for force between hydrogen nucleus and electron.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \cdot e}{r^2}$$

$$\Rightarrow \qquad r = \frac{1}{4\pi\varepsilon_0} \frac{e \cdot e}{F \cdot r}$$

But $F.r$ (force × distance) = work or energy = mc^2

$$\therefore \qquad r = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mc^2} = 2.8 \times 10^{-15} \text{ m.}$$

It is much smaller than typical atomic size.

(b) From Bohr's formula for first hydrogen orbit.

$$r = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10} \text{ m}$$

It is of the order of atomic size.

12.15. *The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV.*

(*a*) *What is the kinetic energy of the electron in this state?*

(*b*) *What is the potential energy of the electron in this state?*

(*c*) *Which of the answers above would change if the choice of the zero of potential energy is changed?*

Sol. In Bohr's model, $mvr = \frac{nh}{2\pi}$ and $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

which gives $E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$; $r = \frac{4\pi\varepsilon_0 h^2}{Ze^2 m}n^2$.

These relations have nothing to do with the choice of the zero of potential energy. Now, choosing the zero of potential energy at infinity, we have

$$E_p = \frac{-Ze^2}{4\pi\varepsilon_0 r} \text{ which gives } E_p = -2E_k$$

and $$E = E_k + E_p = -E_k$$

(*a*) The quoted value of $E = -3.4$ eV is based on the customary choice of zero of potential energy at infinity. Using $E = -E_k$, the kinetic energy of electron in this state is + 3.4 eV.

(*b*) Using $E_p = -2E_k$, potential energy of the electron is -2×3.4 eV = – 6.8 eV.

(*c*) If the zero of potential energy is chosen differently, kinetic energy does not change. Its value is + 3.4 eV. This is independent of the choice of the zero of potential energy. The potential energy, and the total energy of the state, however, would alter if a different zero of the potential energy is chosen.

12.16. *If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?*

Sol. Angular momenta associated with planetary motion are incomparably large relative to *h*. For example, angular momentum of the earth in its orbital motion is of the order of 10^{70} h. In terms of the Bohr's quantisation postulate, this corresponds to a very large value of n (of the order of 10^{70}). For such large values of n, the differences in the successive energies and angular momenta of the quantised levels of the Bohr model are so small compared to the energies and angular momenta respectively for the levels that one can practically consider the levels continuous.

12.17. *Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about 207 m_e orbits around a proton].*

Sol. The first Bohr's radius of H-atom is given by

$$r_1 = 4\pi\varepsilon_0 \frac{h^2}{4\pi^2 m_e e^2} = 5.29 \times 10^{-11} \text{ m}$$

If r_1' is the first Bohr's radius of muonic hydrogen atom, then

$$r_1' = 4\pi\varepsilon_0 \frac{h^2}{4\pi^2 (207\, m_e) e^2} = \frac{5.29 \times 10^{-11}}{207}$$

$$= 2.5 \times 10^{-12} \text{ m}$$

The ground state ($n = 1$) energy of H-atom is given by

$$E_1 = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 m_e e^2}{h^2} = 13.6 \text{ eV}$$

If E_1' is ground state energy of muonic hydrogen atom, then

$$E_1' = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 (207\, m_e) e^2}{h^2} = -13.6 \times 207$$

$$= -2815.2 \text{ eV.}$$

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