13 Nuclei

Lesson at a Glance

• Discovery of Neutrons

Neutrons were discovered by Chadwick in 1932. When beryllium nuclei are bombarded by α-particles, highly penetrating radiations are emitted, which consists of neutral particles, each having mass nearly that of a proton. These particles were called neutrons.

$$^{9}_{4}Be + ^{4}_{2}He \longrightarrow ^{1}_{0}n + ^{12}_{6}C$$

• Composition of Nucleus

Neutrons and protons are the constituents of a nucleus. Almost the whole mass of the atom is in its nucleus.

Atomic number: The number of protons in the nucleus is called the atomic number. It is denoted by Z.

Mass number: The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by A.

Number of protons in an atom = Z

Number of electrons in an atom = Z

Number of neutrons in an atom $\Rightarrow N = A - Z$

where A is number of nucleons (protons + neutrons).

Nuclear mass: It was observed in Rutherford's α-particle scattering experiment that mass of an atom is concentrated within a very small positively charged region at the centre called nucleus. The total mass of neucleons in the nucleons is called as nuclear mass.

Nuclear mass = mass of protons + mass of neutrons.

• Isotopes, Isobars, Isotones and Isomers

Isotopes: Atoms having different mass number (A) but having same atomic number (Z), *e.g.*, $^{235}_{92}U$, $^{238}_{92}U$.

Isobars: The atoms having the same mass number but different atomic number are called isobars. For example: $^{54}_{24}Cr$, $^{54}_{26}Fe$.

Isotones: Nuclei containing same number of neutrons are called isotones. *e.g.*, $^{37}_{17}Cl$ and $^{39}_{17}K$.

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Isomers: These are the nuclei with same atomic number and same mass number but in different energy states.

• Electron Volt

It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ MeV} = 10^6 \text{ eV} = 1.602 \times 10^{-13} \text{ J}.$$

• Atomic Mass Unit

It is $\frac{1}{12}$th of the actual mass of a carbon atom of isotope $^{12}_{6}C$. It is denoted by 'amu' or just by '*u*'.

$$1\ u = 1.66 \times 10^{-27} \text{ kg}$$

1 a.m.u. represents the average mass of a nucleon.

$$1\ u = 931.25 \text{ MeV}.$$

• Mass-Energy Relation

Einstein proved that it is necessary to treat mass as another form of energy. He gave the mass-energy equivalence relation.

$$E = mc^2$$

where m is the mass and c is the velocity of light in vacuum.

$$c \approx 3 \times 10^8 \text{ m/s}.$$

• Radioactive Radiations

The radiations emitted by a radioactive element are found to be of three kinds.

(*i*) **Alpha rays:** *They consist of alpha particles. Alpha particle is nothing but a helium nucleus* (^4_2He) *having 2 protons and 2 neutrons. It has a positive charge equal to the charge of 2 protons.*

(*ii*) **Beta rays:** *They are negative electrons (or positrons) whose energy as well as ionising power are much less than those of* α*-rays.*

(*iii*) **Gamma rays:** *They are electromagnetic waves of very short wavelengths. They originate in the nucleus, have a very low ionising power but a very high penetrating power.*

• Laws of Radioactive Decay

(*i*) The rate of decay of nuclei at any instant is proportional to the number of undecayed radioactive nuclei present at that instant.

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$$\frac{dN}{dt} = -\lambda N$$

where λ is called **decay constant** or disintegration constant.

Also, $$N = N_0 e^{-\lambda t}$$

where N_0 is the number of radioactive nuclei present originally.

• Half-Life and Average Life

The half-life (T) of a radioactive element is the time in which the number of its nuclei is reduced to one-half its initial value.

It can be given as

$$T = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

The average life T_{av} of a radioactive element is the reciprocal of the disintegration constant.

$$T_{av} = \frac{1}{\lambda}$$

• Units of Radioactivity

The following units have been used for 'measuring' the radioactivity of a radioactive element.

(*i*) **The curie:** The curie is the activity of a radioactive element that is disintegrating at the rate of 3.7×10^{10} disintegration per second. *i.e.,*

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ disintegration per second.}$$

(*ii*) **The rutherford:** The rutherford is the activity of a radioactive element that is disintegrating at the rate of 1 million disintegrations per second. *i.e.,*

$$1 \text{ rutherford} = 10^6 \text{ disintegrations per second.}$$

(*iii*) **The becquerel:** This is the unit of radioactivity in the *SI* units.

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ disintegration per second.}$$

• Nuclear Fission

When a heavy nucleus splits into two or more medium intermediate mass nuclei the phenomenon is called nuclear fission. For example when uranium - 235 is bombarded by a thermal neutron, it splits Krypton and Barium with 2 or 3 neutrons and large amount of energy.

$$= {}^{235}_{92}\text{U} + {}^{1}_{0}n \longrightarrow {}^{90}_{36}\text{Kr} + {}^{144}_{56}\text{Ba} + 2\,{}^{1}_{0}n + 179.4 \text{ MeV}$$

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• Nuclear Reactors

It is a device which converts nuclear energy into electrical energy (useful energy.)

• Nuclear Fusion

When two or more light mass nuclei combine to form a single nucleus the phenomenon is called nuclear fusion. For example when four hydrogen nuclei combine to form a nucleus of helium large amount of energy is released with few positrons.

$$4\,{}_1^1\text{H} \rightarrow {}_2^4\text{He} + 2\,{}_1^0e + 17.6 \text{ MeV}.$$

TEXTBOOK QUESTIONS SOLVED

Data useful in solving the exercises:

$e = 1.6 \times 10^{-19}$ C	$N = 6.023 \times 10^{23}$ per mole
$1/(4\pi\varepsilon_0) = 9 \times 10^9$ N m²/C²	$k = 1.381 \times 10^{-23}$ J K⁻¹
1 MeV = 1.6×10^{-13} J	1 u = 931.5 MeV/c^2
1 year = 3.154×10^7 s	
$m_H = 1.007825$ u	$m_n = 1.008665$ u
$m({}_2^4\text{He}) = 4.002603$ u	$m_e = 0.000548$ u

13.1. (a) *Two stable isotopes of lithium ${}_3^6Li$ and ${}_3^7Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.*

(b) *Boron has two stable isotopes, ${}_5^{10}B$ and ${}_5^{11}B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.81u. Find the abundances of ${}_5^{10}B$ and ${}_5^{11}B$.*

Sol. (a) Atomic weight = weighted average of the isotopes

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{(7.5 + 92.5)}$$

$$= \frac{45.1134 + 648.98}{100} = 6.941 \text{ u}$$

(b) Let relative abundance of ${}_5^{10}$B be x%

$\therefore$ Relative abundance of ${}_5^{11}$B $= (100 - x)$%

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Proceeding as above,

$$10.811 = \frac{10.01294x + 11.00931 \times (100 - x)}{100}$$

$$x = 19.9\% \quad \text{and} \quad (100 - x) = 80.1\%.$$

13.2. *The three stable isotopes of neon: $^{20}_{10}Ne$, $^{21}_{10}Ne$ and $^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.*

Sol. The average atomic mass of neon is

$$m\,(Ne) = [90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99] \times 10^{-2}$$

$$= 20.18 \text{ u.}$$

13.3. *Obtain the binding energy (in MeV) of a nitrogen nucleus $\left(^{14}_{7}N\right)$, given $m\left(^{14}_{7}N\right) = 14.00307\,u$.*

Sol. $^{14}_{7}N$ nucleus is made up of 7 protons and 7 neutrons. Mass of nucleons forming nucleus

$= 7\,m_p + 7\,m_n$

= Mass of 7 protons + Mass of 7 neutrons

$= 7 \times 1.00783 + 7 \times 1.00867$ u

$= 7.05431 + 7.06069$ u

$= 14.11550$ u

Mass of nucleus, $m_N = 14.00307$ u

Mass defect = 14.11550 − 14.00307 = 0.11243 a.m.u.

Energy equivalent to mass defect = 0.11243 × 931

= 104.67 MeV

∴ Binding energy = 104.67 MeV.

13.4. *Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:*

$m\,(^{56}_{26}Fe) = 55.934939\,u$; $m\,(^{209}_{83}Bi) = 208.980388\,u$

Which nucleus has greater binding energy per nucleon? Take 1 u = 931.5 MeV.

Sol. (*i*) $^{56}_{26}F$ nucleus contains 26 protons and (56 − 26) = 30 neutrons

Mass of 26 protons = 26 × 1.007825 = 26.20345 u

Mass of 30 neutrons = 30 × 1.008665 = 30.25995 u

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Total mass of 56 nucleons = 56.46340 u

Mass of $^{56}_{26}Fe$ nucleus = 55.934939 u

$\therefore$ Mass defect, Δm = 56.46340 − 55.934939 = 0.528461 u

Total binding energy = 0.528461 × 931.5 MeV = 492.26 MeV

Average binding energy per nucleon = $\frac{492.26}{56}$ = 8.790 MeV.

(*ii*) $^{209}_{83}Bi$ nucleus contains 83 protons and (209 − 83) = 126 neutrons.

Mass of 83 protons = 83 × 1.007825 = 83.649475 u

Mass of 126 neutrons = 126 × 1.008665 = 127.091790 u

Total mass of nucleons = 210.741260 u

Mass of $^{209}_{83}Bi$ nucleus = 208.980388 u

Mass defect, Δm = 210.741260 − 208.980388 = 1.760872

Total B.E. = 1.760872 × 931.5 MeV

= 1640.26 MeV.

Average binding energy per nucleon

$= \frac{1640.26}{209}$ = 7.848 MeV

Hence, $^{56}_{26}Fe$ has greater B.E. per nucleon than $^{209}_{83}Bi$.

13.5. *A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of* $^{63}_{29}Cu$ *atoms (of mass 62.92960 u).*

Sol. Mass of atom = 62.92960 u

Mass of 29 electrons = 29 × 0.000548 u = 0.015892 u

Mass of nucleus = (62.92960 − 0.015892) u

= 62.913708 u

Mass of 29 protons = 29 × 1.007825 u = 29.226925 u

Mass of (63 − 29) *i.e.,* 34 neutrons

= 34 × 1.008665 u = 34.29461 u

Total mass of protons and neutrons

= (29.226925 + 34.29461) u

= 63.521535 u

Binding energy = (63.521535 − 62.913708) × 931.5 MeV

= 0.607827 × 931.5 MeV

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Required energy

$$= \frac{6.023 \times 10^{23}}{63} \times 3 \times 0.607827 \times 931.5 \text{ MeV}$$

$$= 1.6 \times 10^{25} \text{ MeV} = 2.6 \times 10^{12} \text{ J.}$$

13.6. *Write nuclear reaction equation for*

(*i*) *α-decay of* ${}^{226}_{88}Ra$ (*ii*) *α-decay of* ${}^{242}_{94}Pu$

(*iii*) *β⁻-decay of* ${}^{32}_{15}P$ (*iv*) *β⁻-decay of* ${}^{210}_{83}Bi$

(*v*) *β⁺-decay of* ${}^{11}_{6}C$ (*vi*) *β⁺-decay of* ${}^{97}_{43}Tc$

(*vii*) *Electron capture of* ${}^{120}_{54}Xe$.

Sol. (*i*) ${}^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$ (*ii*) ${}^{242}_{94}\text{Pu} \longrightarrow {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He}$

(*iii*) ${}^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^{-1} + \bar{\nu}$ (*iv*) ${}^{210}_{83}\text{Bi} \longrightarrow {}^{210}_{84}\text{Po} + e^{-1} + \bar{\nu}$

(*v*) ${}^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^{+} + \nu$ (*vi*) ${}^{97}_{43}\text{Tc} \longrightarrow {}^{97}_{42}\text{Mo} + e^{+} + \nu$

(*vii*) ${}^{120}_{54}\text{Xc} + e^{-} \longrightarrow {}^{120}_{53}\text{I} + \nu$

13.7. *A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to (a) 3.125%, (b) 1% of its original value?*

Sol. (*a*) The fraction of the original sample left $= \frac{3.125}{100}$

$$= \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

Hence, there are 5 half lives of T years spent. Thus, the time taken is 5T years.

(*b*) The fraction of the original sample left $= \frac{1}{100} = \left(\frac{1}{2}\right)^n$

or, $2^n = 100 \Leftrightarrow n \log 2 = \log 100$

Hence, $n = \frac{\log 100}{\log 2} = \frac{2}{0.301} = 6.64$

Hence, there are 6.64 half lives of *T* years spent. Thus, the time taken is 6.64 T years.

13.8. *The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive* ${}^{14}_{6}C$ *present with the stable carbon isotope* ${}^{12}_{6}C$*. When the organism is dead, its*

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interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus Valley Civilisation.

Sol. Here, normal activity, $R_0 = 15$ decays/min

Present activity $R = 9$ decays/min, $T = 5730$ years,

Age t =?

As activity is proportional to the number of radioactive atoms, therefore,

$$\frac{N}{N_0} = \frac{R}{R_0} = \frac{9}{15}$$

But $$\frac{N}{N_0} = e^{-\lambda t} \quad \therefore \quad e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$e^{+\lambda t} = \frac{5}{3}$$

$$\lambda t \log_e e = .\log_e \frac{5}{3} = 2.3026 \log 1.6667$$

$$\lambda t = 2.3026 \times 0.2218 = 0.5109$$

$$t = \frac{0.5109}{\lambda}$$

But $$\lambda = \frac{0.693}{T} = \frac{0.693}{5730} \text{Yr}^{-1}$$

$\therefore$ $$t = \frac{0.5109}{0.693/5730} = \frac{0.5109 \times 5730}{0.693}$$

$$t = 4224.3 \text{ years.}$$

13.9. *Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.*

Sol. Strength of radioactive source

$= 8.0 \text{ mCi} = 8.0 \times 10^{-3} \text{ Ci}$

$= 8.0 \times 10^{-3} \times 3.7 \times 10^{10}$ disintegrations s^{-1}

$= 29.6 \times 10^{7}$ disintegrations s^{-1}

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Since the strength of the source decreases with time,

$$\therefore \quad \frac{dN}{dt} = -29.6 \times 10^7$$

But $\quad \frac{dN}{dt} = -\lambda N$

$$\therefore \quad -\lambda N = -29.6 \times 10^7$$

or, $\quad \lambda N = 29.6 \times 10^7 \quad$ or, $\quad N = \frac{29.6 \times 10^7}{\lambda}$

$$\text{or,} \quad N = \frac{29.6 \times 10^7 \times T}{0.693} \quad \left(\because \lambda = \frac{0.693}{T}\right)$$

$$= \frac{29.6 \times 10^7 \times 5.3 \times 365 \times 24 \times 60 \times 60}{0.693}$$

$$= 7.139 \times 10^{16}$$

Number of atoms in 60 g of cobalt = 6.023×10^{23}

$$\text{Mass of 1 atom of cobalt} = \frac{60}{6.023 \times 10^{23}} \text{ g}$$

$$\text{Mass of } 7.139 \times 10^{16} \text{ atoms} = \frac{60}{6.023 \times 10^{23}} \times 7.139 \times 10^{16} \text{ g}$$

$$= 7.11\ \mu\text{g}.$$

13.10. *The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?*

Sol. Since, $\quad \lambda = \frac{0.693}{T}$

$$\Rightarrow \quad \lambda = \frac{0.693}{28 \times 365 \times 24 \times 60 \times 60} = 7.85 \times 10^{-10}\ s^{-1}$$

90 g of *Sr* contains 6.023×10^{23} atoms

$$\text{15 mg of Sr contains, } N_0 = \frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90} \text{ atoms}$$

$$N_0 = 1.0038 \times 10^{20} \text{ atoms}$$

$$\text{Disintegration rate, } \frac{dN}{dt} = -\lambda N_0$$

$$= -7.85 \times 10^{-10} \times 1.0038 \times 10^{20}$$

$$= 7.88 \times 10^{10} \text{ dps or } Bq$$

$$= \frac{7.88 \times 10^{10}}{3.7 \times 10^{10}} \text{ Ci} = 2.13 \text{ Ci}.$$

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13.11. *Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.*

Sol. As, $R \approx A^{1/3}$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{197}{107}\right)^{1/3} = (1.84)^{1/3}$$

$$\Rightarrow \quad \log_{10}\left(\frac{R_1}{R_2}\right) = \log_{10}(1.84)^{1/3}$$

$$\Rightarrow \quad \log_{10}\left(\frac{R_1}{R_2}\right) = \frac{1}{3}\log_{10}(1.84)$$

$$= \frac{1}{3} \times 0.2648 = 0.08827$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \text{antilog}\,(0.08827) = 1.23.$$

13.12. *Find the Q-value and the kinetic energy of the emitted α-particle in α-decay of (a) $^{226}_{88}Ra$ and (b) $^{220}_{86}Rn$.*

Given $m\left(^{226}_{88}Ra\right) = 226.02540\ u$; $m\left(^{222}_{86}Rn\right) = 222.01750\ u$;

$m\left(^{220}_{86}Rn\right) = 220.01137\ u$; $m\left(^{216}_{84}Po\right) = 216.00189\ u$.

Sol. (*a*) The difference in mass between the original nucleus and the decay products

$= 226.02540\ \text{u} - (222.01750\ \text{u} + 4.00260\ \text{u})$

$= +\ 0.0053\ \text{u}$

Energy equivalent $= 0.0053 \times 931.5$ MeV

$= 4.93695$ MeV $= 4.94$ MeV

The decay products would emerge with total kinetic energy 4.94 MeV. Momentum is conserved. If the parent nucleus is at rest, the daughter and the α-particle have momenta of equal magnitude p but opposite direction. Kinetic energy, $K = \frac{p^2}{2m}$. Since p is the same for the two particles therefore the kinetic energy divides inversely as their masses. The α-particle gets $\frac{222}{222+4}$ of the total *i.e.*, $\frac{222}{226} \times 4.94$ MeV or 4.85 MeV.

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(*b*) The difference in mass between the original nucleus and the decay products

$$= 220.01137 \text{ u} - (216.00189 \text{ u} + 4.00260 \text{ u})$$
$$= 0.00688 \text{ u}$$

Energy equivalent = 0.00688 × 931.5 MeV = 6.41 MeV

$$E_\alpha = \frac{216}{216+4} \times 6.41 \text{ MeV} = 6.29 \text{ MeV}.$$

13.13. *The radionuclide ^{11}C decays according to $^{11}_{6}C \rightarrow ^{11}_{5}B + e^+ + \nu$; $T_{1/2} = 20.3$ min.*

The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:

$$m\left(^{11}_{6}C\right) = 11.011434 \text{ u and } m\left(^{11}_{5}B\right) = 11.009305 \text{ u}.$$

Calculate Q and compare it with the maximum energy of the positron emitted.

Sol. Mass defect

$$= [m\,(^{11}_{6}C) - 6\,m_e] - [m\,(^{11}_{5}B) - 5\,m_e + m_e]$$
$$= m\,(^{11}_{6}C) - m\,(^{11}_{5}B) - 2\,m_e$$
$$= 11.011434 \text{ u} - 11.009305 \text{ u} - 2 \times 0.000548 \text{ u}$$
$$= 0.001033 \text{ u}$$
$$Q = 0.001033 \times 931.5 \text{ MeV} = 0.962 \text{ MeV}$$
$$Q = E_d + E_e + E_\nu$$

The daughter nucleus is too heavy compared to e^+ and ν. So, it carries negligible energy ($E_d \approx 0$). If the kinetic energy (E_ν) carried by the neutrino is minimum (*i.e.*, zero), the positron carries maximum energy, and this is practically all energy Q. Hence, maximum $E_e \approx Q$.

13.14. *The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted from the following data:*

$$m\,(^{23}_{10}Ne) = 22.994466 \text{ u}$$
$$m\,(^{23}_{11}Na) = 22.989770 \text{ u}$$

Sol. The β-decay of $^{23}_{10}Ne$ may be represented as $^{23}_{10}Ne \longrightarrow ^{23}_{11}Na - ^{0}_{-1}e + \bar{\nu} + Q$

Ignoring the rest mass of antineutrino ($\bar{\nu}$) and electron

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Mass defect, $\Delta m = m\,(^{23}_{10}Ne) - m\,(^{23}_{11}Na)$

$= 22.994466 - 22.989770 = 0.004696$ u

$\therefore$ $Q = 0.004696 \times 931$ MeV $= 4.372$ MeV.

As $^{23}_{11}$Na is very massive, this energy of 4.3792 MeV, is shared by e^- and $\bar{\nu}$ pair. The maximum K.E. of e^- = 4.372 MeV, when energy carried by $\bar{\nu}$ is zero.

13.15. *The Q-value of a nuclear reaction*
$A + b \longrightarrow C + d$ is defined by $Q = [m_A + m_b - m_c - m_d]\,c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) $^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$ *(ii)* $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

Atomic masses are given to be

$m\,(^2_1H) = 2.014102$ u; $m\,(_1H^3) = 3.016049$ u

$m\,(^{12}_6C) = 12.000000$ u; $m\,(_{10}Ne^{20}) = 19.992439$ u.

Sol. (i) $^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$

$Q = \Delta m \times 931.5$ MeV

$= [m\,(^1_1H) + m\,(^3_1H) - 2m\,(^2_1H)] \times 931$ MeV

$= [1.007825 + 3.016049 - 2 \times 2.014102] \times 931$ MeV

$= -4.03$ MeV

$\therefore$ The reaction is endothermic

(ii) $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

$Q = \Delta m \times 931$ MeV

$= [2\,m\,(^{12}_6C) - m\,(^{20}_{10}Ne) - m\,(^4_2He)] \times 931$ MeV

$= [24.000000 - 19.992439 - 4.002603] \times 931$ MeV

$= +4.61$ MeV

$\therefore$ The reaction is exothermic.

13.16. *Suppose, we think of fission of a $^{56}_{26}$Fe nucleus into two equal fragments $^{28}_{13}$Al. Is the fission energetically possible? Argue by working out Q of the process. Given*

$m\,(^{56}_{26}Fe) = 55.93494$ u, $m\,(^{28}_{13}Al) = 27.98191$ u.

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Sol. $Q = [m\,(^{56}_{26}\text{Fe}) - 2\,m\,(^{28}_{13}\text{Al})] \times 931.5$ MeV

$= [55.93494 - 2 \times 27.98191] \times 931.5$ MeV

$Q = -0.02886 \times 931.5$ MeV $= -26.88$ MeV, which is negative.

The fission is not possible energetically.

13.17. *The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?*

Sol. Energy released per fission of

$$^{239}_{94}\text{Pu} = 180 \text{ MeV}$$

Quantity of fissionable material = 1 kg

In 239 gm Pu, number of fissionable atom or nuclei

$$= 6.023 \times 10^{23}$$

In 1 g of Pu, number of fissionable atom or nuclei

$$= \frac{6.023 \times 10^{23}}{239}$$

In 1000 gm of Pu, number of fissionable atom or nuclei

$$= \frac{6.023 \times 10^{23}}{239} \times 1000$$

$$= 25.2 \times 10^{23}$$

Energy released in fission of single Pu nucleus

$$= 180 \text{ MeV}$$

Energy released in fission of 25.2×10^{23} Pu nucleus or in fission of 1 kg pure Pu

$$= 180 \times 25.2 \times 10^{23}$$

$$= 4536 \times 10^{23} \text{ MeV}$$

$$= 4.5 \times 10^{26} \text{ MeV.}$$

13.18. *A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially? Assume that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.*

Sol. Power of reactor = 1000 MW = 10^3 MW = 10^9 W = 10^9 Js^{-1}

Energy generated by reactor in 5 years

$$= 5 \times 365 \times 24 \times 60 \times 60 \times 10^9 \text{ J}$$

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Energy generated per fission = 200 MeV

$= 200 \times 1.6 \times 10^{-13}$ J

Number of fission taking place or number of U^{235} nuclei required

$$= \frac{5 \times 365 \times 24 \times 60 \times 60 \times 10^9}{200 \times 1.6 \times 10^{-13}}$$

$= 8.2125 \times 10^{26} \times 6 = 49.275 \times 10^{26}$

Mass of 6.023×10^{23} nuclei of U

$= 235$ gm $= 235 \times 10^{-3}$ kg

Mass of 8.2125×10^{26} nuclei of U

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 6 \times 8.2125 \times 10^{26} = 1932 \text{ kg}$$

$\frac{1}{2}$ of fuel = 1932 kg

Total fuel = 3864 kg.

13.19. *How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as:*

$$^2_1H + ^2_1H \rightarrow ^3_2He + n + 3.2\,MeV.$$

Sol. When two nuclei of deuterium fuse together, energy released = 3.2 MeV

Number of deuterium atoms in 2 kg

$$= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$$

When 6.023×10^{26} nuclei of deuterium fuse together, energy released

$$= \frac{3.2}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.2}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-13} \text{ J}$$

$= 1.54 \times 10^{14}$ J or Ws

Power of electric lamp= 100 W

If the lamp glows for time t, then the electrical energy consumed by the lamp is 100 t.

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$\therefore \quad 100\,t = 1.54 \times 10^{14} \quad \text{or} \quad t = 1.54 \times 10^{12}$ s

$$= \frac{1.54 \times 10^{12}}{3.154 \times 10^{7}} \text{ years} = 4.88 \times 10^{4} \text{ years.}$$

13.20. *Calculate the height of the potential barrier for a head on collision of two deuterons. Assume that they can be taken as hard spheres of radius 2.0 fm.*

Sol. (**Hint:** *The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other.*) Suppose the two particles are fired at each other with the same kinetic energy K so that they are brought to rest by their mutual Coulomb repulsion when they are just touching each other. We can take this value of K as a representative measure of the height of Coulomb barrier.

P.E. = 2K.E.

$$\frac{e^2}{4\pi\varepsilon_0(2R)} = 2K_e$$

$$2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{(2R)}$$

$$K_e = \frac{e^2}{16\pi\varepsilon_0 R} = \frac{(1.6\times10^{-19})^2}{16\times3.14\times8.85\times10^{-12}\times2\times10^{-15}}\text{ J}$$

$$= 2.8788 \times 10^{-14} \text{ J}$$

$$= \frac{2.8788\times10^{-14}}{1.6\times10^{-19}\times10^{3}}\text{ keV} = 179.9 \text{ keV.}$$

13.21. *From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e., independent of A).*

Sol. It is found that a nucleus of mass number A has a radius

$$R = R_0\, A^{1/3}$$

where, $\quad R_0 = 1.2 \times 10^{-15}$ m.

This implies that the volume of the nucleus, which is proportional to R^3 is proportional to A.

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi\ (R_0\, A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

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$$\text{Density of nucleus} = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$$

$$= \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Above derived equation shows that density of nucleus is constant, independent of A, for all nuclei and density of nuclear matter is approximately 2.3×10^7 kg m^{-3} which is very large as compared to ordinary matter, say water which is 10^3 kg m^{-3}.

13.22. *For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).*

$$e^+ + {}^A_Z X \rightarrow {}_{Z-1}^{\;\;A}Y + \nu$$

Show that if β^{++} emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Sol. Consider the two competing processes:

$${}^A_Z X \rightarrow {}_{Z-1}^{\;\;A}Y + e^+ + \nu_e + Q_1 \quad \text{(positron emission)}$$

$$e^- + {}^A_Z X \rightarrow {}_{Z-1}^{\;\;A}Y + \nu_e + Q_2 \quad \text{(electron capture)}$$

$$Q_1 = \left[m_N\left({}^A_Z X\right) - m_N\left({}_{Z-1}^{\;\;A}Y\right) - m_e\right]c^2$$

$$= \left[m\left({}^A_Z X\right) - Zm_e - m\left({}_{Z-1}^{\;\;A}Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[m\left({}^A_Z X\right) - m\left({}_{Z-1}^{\;\;A}Y\right) - 2m_e\right]c^2$$

$$Q_2 = \left[m_N\left({}^A_Z X\right) + m_e - m_N\left({}_{Z-1}^{\;\;A}Y\right)\right]c^2$$

$$= \left[m\left({}^A_Z X\right) - m\left({}_{Z-1}^{\;\;A}Y\right)\right]c^2$$

This means $Q_1 > 0$ implies $Q_2 > 0$ but $Q_2 > 0$ does not necessarily mean $Q_1 > 0$. Hence the result.

13.23. *In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}Mg$ (23.98504 u), ${}^{25}_{12}Mg$ (24.98584 u) and ${}^{26}_{12}Mg$ (25.98259 u). The*

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natural abundance of $^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Sol.

Isotope	Abundance Y	Atomic mass (Z)
$_{12}Mg^{24}$	78.99	23.98504
$_{12}Mg^{25}$	x	24.98584
$_{12}Mg^{26}$	$100 - (78.99 + x) = 21.1 - x$	25.98259
	$\Sigma Y = 100$	

Mean atomic mass = 24.312

$$\text{Average atomic mass} = \frac{\Sigma YZ}{\Sigma Y}$$

$$\Rightarrow 24.312 = \frac{78.99 \times 23.98504 + x \times 24.98584 + (21.01 - x)\,25.98254}{100}$$

or, $2431.2 = 1894.58 + 24.98584\,x + 545.89 - 25.98254\,x$

or, $2431.2 = 2440.47 - .99675\,x$

or, $.99675\,x = 2440.47 - 2431.2 = 9.27$

or, $x = \dfrac{9.27}{.99675} = 9.30$

$\therefore \quad 21.01 - x = 21.01 - 9.30 = 11.71$

Relative abundance of $_{12}Mg^{25}$ = 9.30%

Relative abundance of $_{12}Mg^{26}$ = 11.71%.

13.24. *The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:*

$$m\left(^{40}_{20}Ca\right) = 39.962591\ u$$

$$m\left(^{41}_{20}Ca\right) = 40.962278\ u$$

$$m\left(^{26}_{13}Al\right) = 25.986895\ u$$

$$m\left(^{27}_{13}Al\right) = 26.981541\ u.$$

Sol. The equation for the neutron separation in first case can be written as,

$$^{41}_{20}Ca + Q_1 \rightarrow {}^{40}_{20}Ca + {}^{1}_{0}n$$

$$\Delta m = m\,(^{40}_{20}Ca) + m\,(^{1}_{0}n) - m\,(^{41}_{20}Ca)$$

$$= 39.962591 + 1.008665 - 40.962278$$

$$= 0.008978\ u$$

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But, $1 \text{ u} \equiv 931.5 \text{ MeV}$

Hence, $0.008978 \text{ u} \equiv 0.008978 \times 931.5 = 8.363 \text{ MeV}$

The equation for the neutron separation in second case can be written as,

$$^{27}_{13}\text{Al} + Q_2 \rightarrow ^{26}_{13}\text{Al} + ^{1}_{0}n$$

$$\Delta m = m\,(^{26}_{13}\text{Al}) + m\,(^{1}_{0}n) - m\,(^{27}_{13}\text{Al})$$

But, $1 \text{ u} \equiv 931.5 \text{ MeV}$

Hence, $0.014019 \text{ u} \equiv 0.014019 \times 931.5 = 13.06 \text{ MeV}.$

13.25. *A source contains two phosphorous radio nuclides $^{33}_{15}P$ ($T_{1/2}$ = 14.3 d) and $^{32}_{15}P$ ($T_{1/2}$ = 25.3 d). Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?*

Sol. We know that $-\dfrac{dN}{dt} \propto N$.

So, clearly the initial ratio of the amounts of $^{33}_{15}\text{P}$ and $^{32}_{15}\text{P}$ is 1:9. We have to find the time after which the ratio is 9 : 1.

Initially, if the amount of $^{33}_{15}\text{P}$ is x, the amount of $^{32}_{15}\text{P}$ is $9x$. Finally, if the amount of $^{33}_{15}\text{P}$ is $9y$, the amount of $^{32}_{15}\text{P}$ is y.

Using, $$N = \frac{N_0}{2^{t/T}}$$

$$9y = \frac{x}{2^{t/25.3}}$$

$$y = \frac{9x}{2^{t/14.3}}$$

Dividing, $$9 = \frac{x}{2^{t/25.3}} \times \frac{2^{t/14.3}}{9x}$$

or, $$81 = 2^{\frac{t}{14.3} - \frac{t}{25.3}} \quad \text{or,} \quad 81 = 2^{\frac{11t}{361.79}}$$

or, $$\log_{10} 81 = \frac{11t}{361.79} \log_{10} 2 = \frac{11 \times 0.3010\, t}{361.79}$$

$$= 9.15 \times 10^{-3}\, t$$

$$9.15 \times 10^{-3}\, t = 1.91$$

or, $$t = \frac{1.91 \times 1000}{9.15} d = 208.7 \text{ d}.$$

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13.26. *Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:*

$$^{223}_{88}Ra \rightarrow \,^{209}_{82}Pb + \,^{14}_{6}C$$

$$^{223}_{88}Ra \rightarrow \,^{219}_{86}Rn + \,^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Sol. (*i*) For decay process $^{223}_{88}Ra \rightarrow \,^{209}_{82}Pb + \,^{14}_{6}C + Q$

Mass defect,

$$\Delta m = \text{mass of } Ra^{223} - (\text{mass of } Pb^{209} + \text{mass of } C^{14})$$

$$= 223.01850 - (208.98107 + 14.00324) = 0.03419 \text{ u}$$

$\therefore \quad Q = 0.03419 \times 931 \text{ MeV} = 31.83 \text{ MeV}$

(*ii*) For decay process $^{223}_{88}\text{Ra} \rightarrow \,^{219}_{86}\text{Rn} + \,^{4}_{2}\text{He} + Q$

Mass defect,

$$\Delta m = \text{mass of Ra}^{223} - (\text{mass of Rn}^{219} + \text{mass of He}^{4})$$

$$= 223.01850 - (219.00948 + 4.00260) = 0.00642 \text{ u}$$

$\therefore \quad Q = 0.00642 \times 931 \text{ MeV} = 5.98 \text{ MeV}$

As *Q*-values are positive in both the cases, therefore both the decays are energetically possible.

13.27. *Consider the fission of* $^{238}_{92}\text{U}$ *by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are* $^{140}_{58}\text{Ce}$ *and* $^{99}_{44}\text{Ru}$. *Calculate Q for this fission process. The relevant atomic and particle masses are*

$$m(^{238}_{92}\text{U}) = 238.05079 \text{ } u$$

$$m(^{140}_{58}\text{Ce}) = 139.90543 \text{ } u$$

$$m(^{99}_{44}\text{Ru}) = 98.90594 \text{ } u.$$

Sol. Fission reaction is

$$^{238}_{92}\text{U} + \,^{1}_{0}n \rightarrow \,^{140}_{58}\text{Ce} + \,^{99}_{44}\text{Ru} + Q$$

Q-value = (mass of U^{238} + mass of $^{1}_{0}n$ – mass of Ce^{140} – mass of Ru^{99}) × 931.5 MeV

= (238.05079 + 1.00867 – 139.90543 – 98.90594) × 931.5 MeV

= 231.1 MeV.

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13.28. *Consider the D–T reaction (deuterium-tritium fusion)*

$$^2_1H + ^3_1H \rightarrow ^4_2He + n$$

(*a*) *Calculate the energy released in MeV in this reaction from the data:*

$$m\left(^2_1H\right) = 2.014102\ u$$

$$m\left(^3_1H\right) = 3.016049\ u$$

(*b*) *Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the Coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?*

(**Hint:** *Kinetic energy required for one fusion event = average thermal kinetic energy available with the interacting particles = 2(3kT/2); k = Boltzman's constant, T = absolute temperature.)*

Sol. (*a*) For the process $^2_1H + ^3_1H \rightarrow ^4_2He + n + Q$

Q-value = [mass of ^2_1H + mass of ^3_1H – mass of ^4_2He – mass of n] × 931 MeV

= (2.014102 + 3.016049 – 4.002603 – 1.00867) × 931 MeV

= 0.018878 × 931 = 17.58 MeV

(*b*) Repulsive potential energy of two nuclei when they almost touch each other is

$$\frac{q^2}{4\pi\varepsilon_0 (2r)} = \frac{9\times 10^9\ (1.6\times 10^{-19})^2}{2\times 2\times 10^{-15}} \text{ joule}$$

$$= 5.76 \times 10^{-14}\ \text{J}$$

Classically, *K.E.* at least equal to this amount is required to overcome Coulomb repulsion. Using relation

$$KE = \frac{3}{2}kT$$

$$T = \frac{2\text{K.E.}}{3k} = \frac{2\times 5.76\times 10^{-14}}{3\times 1.38\times 10^{-23}} = 2.78\times 10^9\,\text{K}$$

In actual practice, the temperature required for triggering the reaction is somewhat less.

13.29. *Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Figure. You are given that*

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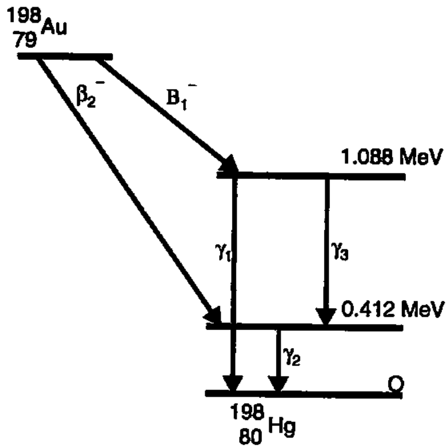
$m\,(^{198}Au) = 197.968233\ u$

$m\,(^{198}Hg) = 197.966760\ u.$



Sol. $$\nu = \frac{(E_2 - E_1)}{h}$$

$$\nu(\gamma_1) = \frac{(1.088 - 0) \times 1.6 \times 10^{-13}}{6.62 \times 10^{-34}}$$

$$\left[\because \begin{array}{l} 1\,\text{MeV} = 10^6 \times 1.6 \times 10^{-19}\,\text{J} \\ 1\,\text{MeV} = 1.6 \times 10^{-13}\,\text{J} \end{array}\right]$$

$$= 2.63 \times 10^{20}\ \text{s}^{-1}$$

$$\nu(\gamma_2) = \frac{(0.412 - 0) \times 1.6 \times 10^{-13}}{6.62 \times 10^{-34}} = 9.96 \times 10^{20}\,\text{s}^{-1}$$

$$\nu(\gamma_3) = \frac{(1.088 - 0.412) \times 1.6 \times 10^{-13}}{6.62 \times 10^{-34}} = 1.63 \times 10^{20}\,\text{s}^{-1}$$

The emission of β_1^- decay may be represented as:

$$^{198}_{79}\text{Au} \to {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + E(\beta_1^-) + E(\gamma_1)$$

Where, $E(\gamma_1) = 1.088$ MeV

Now, $E(\beta_1^-) = \left[m\left(^{198}_{79}\text{Au}\right) - m\left(^{198}_{80}\text{Hg}\right) - m_e\right] \times 931.5 - E(\gamma_1)$

where $m\left(^{198}_{79}\text{Au}\right)$ and $m\left(^{198}_{80}\text{Hg}\right)$ are masses of the $^{198}_{79}\text{Au}$ and $^{198}_{80}\text{Hg}$ nuclei.

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$$\therefore \quad E(\beta_1^-) = \Big[\{M(^{198}_{79}\text{Au}) - 79\, m_e\}$$

$$- \{M(^{198}_{80}\text{Hg}) - 80\, m_e\} - m_e\Big] \times 931.5 - E(\gamma_1)$$

$$= \big[M(^{198}_{79}\text{Au}) - M(^{198}_{80}\text{Hg})\big] \times 931.5 - 1.088$$

$$= (197.968233 - 197.966760) \times 931.5 - 1.088$$

$$= 1.372 - 1.088 = 0.284 \text{ MeV}$$

The emission of β_2^- decay may be represented as:

$$^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + E(\beta_2^-) + E(\gamma_2)$$

As in case of β_1^- decay, it can be deduced that

$$\therefore \quad E(\beta_2^-) = \big[M(^{198}_{79}\text{Au}) - M(^{198}_{80}\text{Hg})\big] \times 931.5 - E(\gamma_2)$$

$$= 1.372 - 0.412 = 0.960 \text{ MeV.}$$

13.30. *Calculate and compare the energy released by (a) fusion of 1.0 kg of hydrogen deep within the sun, and (b) the fission of 1.0 kg of U^{235} in a fission reactor.*

Sol. In sun, four hydrogen nuclei fuse to form a helium nucleus with the release of 26 MeV energy.

$\therefore$ Energy released by fusion of 1 kg of hydrogen

$$= \frac{6 \times 10^{23} \times 26}{4} \times 10^3 \text{ MeV}$$

$$E_1 = 39 \times 10^{26} \text{ MeV}$$

As energy released in fission of one atom of $_{92}U^{235}$

$$= 200 \text{ MeV}$$

$\therefore$ Energy released in fission of 1 kg of $_{92}U^{235}$

$$= \frac{6 \times 10^{23} \times 1000}{235} \times 200 \text{ MeV}$$

$$E_2 = 5.1 \times 10^{26} \text{ MeV}$$

$$\therefore \frac{E_1}{E_2} = \frac{39 \times 10^{26}}{5.1 \times 10^{26}} = 7.65$$

i.e., energy released in fusion is 7.65 times the energy released in fission.

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13.31. *Suppose India has a target of producting by 2020 A.D., 2×10^5 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that on an average, the efficiency of utilization (i.e., conversion to electrical energy) of thermal energy produced in a reactor is 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of U^{235} to be about 200 MeV.*

Sol. Target of producing electric power = 100,000 MW. Required electric power from nuclear plants

$$= 100000 \times \frac{10}{100} = 10{,}000 \text{ MW}$$

Therefore, required electric energy from nuclear plants per year

$$= (10{,}000 \times 10^6 \text{ W}) \times 365 \times 24 \times 60 \times 60$$
$$= 3.1536 \times 10^{17} \text{ J}$$

Electrical energy recovered from the fission of one U^{235} nucleus

$$= 200 \times \frac{25}{100} = 50 \text{ MeV}$$
$$= 50 \times 1.6 \times 10^{-13} = 8 \times 10^{-12} \text{ J}$$

∴ Number of fissions of U^{235} nucleus required.

$$= \frac{3.1536 \times 10^7}{8 \times 10^{-12}} = 3.942 \times 10^{28}$$

Number of moles of U^{235} required per year

$$= \frac{3.942 \times 10^{28}}{6.023 \times 10^{23}} = 6.5449 \times 10^4$$

Therefore, mass of U^{235} required per year

$$= 6.5449 \times 10^4 \times 235$$
$$= 1538.054 \text{ g} = 1.538054 \text{ kg}.$$

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